Online Appendix: Impact of Sourcing Flexibility on the Outsourcing of Services under Demand Uncertainty

Appendix A: Derivations

Derivation of (1)

$$\begin{aligned}
C_1(D_0, D) &= E\left[\int_0^{\tau_1(D)} e^{-rt}\left(v_I D_t + F_I\right)dt + \int_{\tau_1(D)}^{\infty} e^{-rt}\left(v_O D_t + F_O\right)dt + e^{-r\tau_1(D)} S_{IO}\right] \\
&= E\left[\int_0^{\infty} e^{-rt}\left(v_I D_t + F_I\right)dt\right] - E\left[\int_{\tau_1(D)}^{\infty} e^{-rt}\left[\left(F_I - F_O\right) - \left(v_O - v_I\right)D_t\right]dt - e^{-r\tau_1(D)} S_{IO}\right] \\
&= C_I\left(D_0\right) - E\left[e^{-r\tau_1(D)}\left(\int_{\tau_1(D)}^{\infty} e^{-r\left(t-\tau_1(D)\right)}\left[\left(F_I - F_O\right) - \left(v_O - v_I\right)D_t\right]dt - S_{IO}\right)\right] \\
&= C_I\left(D_0\right) - E\left[e^{-r\tau_1(D)}\right]\left(\int_0^{\infty} e^{-rt}\left[\left(F_I - F_O\right) - \left(v_O - v_I\right)E\left[D_{\tau_1(D)+t} \middle| D_{\tau_1(D)} = D\right]\right]dt - S_{IO}\right).
\end{aligned}$$

The remaining steps take advantage of three identities.

$$E[D_t] = D_0 e^{\mu t} \tag{A1}$$

$$\int_0^{\infty} e^{-rt}\left(ae^{\mu t} + b\right)dt = \frac{a}{r-\mu} + \frac{b}{r} \text{ for } r>\mu \tag{A2}$$

$$E\left[e^{-r\tau(D)}\right] = \left(\frac{D}{D_0}\right)^{\gamma} \text{ for } D<D_0 \tag{A3}$$

where

$$\gamma = \frac{\sqrt{\left(\mu - 0.5\sigma^2\right)^2 + 2\sigma^2 r} + \left(\mu - 0.5\sigma^2\right)}{\sigma^2}.$$

(A1) can be obtained by taking the expectation of a geometric Brownian motion process. (A2) can be obtained by standard integration rules. (A3) can be obtained from a Laplace transform involving the density of $\tau_1(D)$. Substituting identities (A1) – (A3) into $C_1(D_0, D)$ yields

$$\begin{aligned}
C_1(D_0, D) &= C_I\left(D_0\right) - \left(\frac{D}{D_0}\right)^{\gamma}\left[\left(\frac{F_I - F_O - rS_{IO}}{r}\right) - \left(\frac{v_O - v_I}{r-\mu}\right)D\right] \\
&= C_I\left(D_0\right) - V_1\left(D\right).
\end{aligned}$$

Derivation of (3)

Taking the derivative of $V_1(D)$, we have

$$V_1'(D) = \frac{D^{\gamma-1}}{D_0^{\gamma}}\left[\gamma\left(\frac{F_I - F_O - rS_{IO}}{r}\right) - \left(\gamma+1\right)\left(\frac{v_O - v_I}{r-\mu}\right)D\right].$$

We see that $V_1(D)$ has a unique positive stationary point $D^* = \dfrac{\gamma\left(r-\mu\right)\left(F_I - F_O - rS_{IO}\right)}{r\left(\gamma+1\right)\left(v_O - v_I\right)}$. Furthermore, $V_1'(D) \geq 0$ for all $D \in [0, D^*)$ and $V_1'(D) < 0$ for all $D \in (D^*, \infty)$. Thus $V_1(D)$ is quasi-concave over $D \in [0, \infty)$ and is maximized at D^*.

Derivation of (4)

Note that if $D_0 \le D^*$, then the current demand rate is at or below the threshold where it is optimal to switch to outsourcing. Thus the firm immediately switches to outsourcing and the value of outsourcing is

$$V_1^* = \left(\frac{F_I - F_O - rS_{IO}}{r}\right) - \left(\frac{v_O - v_I}{r-\mu}\right) D_0 .$$

In the event of $D_0 \ge D^*$, we substitute

$$D^* = \underset{D \ge 0}{\arg\max}\, V_1(D) = \frac{\gamma(r-\mu)(F_I - F_O - rS_{IO})}{r(\gamma+1)(v_O - v_I)}$$

Into $V_1(D)$ to get

$$\begin{aligned} V_1^* &= \left(\frac{D^*}{D_0}\right)^{\gamma} \left[\left(\frac{F_I - F_O - rS_{IO}}{r}\right) - \left(\frac{v_O - v_I}{r-\mu}\right) D^*\right] \\ &= \left(\frac{\gamma(r-\mu)}{D_0(v_O - v_I)}\right)^{\gamma} \left(\frac{F_I - F_O - rS_{IO}}{r(\gamma+1)}\right)^{\gamma+1} . \end{aligned}$$

Derivation of (5)

For geometric Brownian motion process $dD_t = D_t\mu dt + D_t\sigma dz$, random demand at time t is $D_t = D_0 e^{(\mu - 0.5\sigma^2)t + \sigma z(t)}$ where $z(t)$ is a standard Weiner process. Dixit (2001) gives the probability that an arithmetic Brownian motion process will hit a lower threshold, which can be used in conjunction with the following relationship to obtain (5).

$$P[\min\{D_t\} \le D^*] = P[\min\{X_t\} \le \ln(D^*/D_0)]$$

where $X_t = (\mu - 0.5\sigma^2)t + \sigma z(t)$ is arithmetic Brownian motion.

Derivation of (6)

Dixit (2001) gives the expected time that an arithmetic Brownian motion process will hit a lower threshold, which can be used via the transformation employed in the derivation of (5) to obtain (6).

Derivation of (13)

The derivation of (13) follows the derivation (9) that gives the optimal threshold for switching to a lower variable cost (and higher fixed cost), except that F_I, F_O, and S_{IO} are replaced with F_O, F_I, and S_{OI} respectively.

Derivation of (14)

Section 4.1 shows how to determine the value of the option to switch from a higher variable cost and a lower fixed cost (e.g., insourcing in Section 4.1, but outsourcing in Section 3.3) to a lower variable cost and a higher fixed cost (e.g., outsourcing in Section 4.1, but insourcing in Section 3.3). The value of the option to insource is obtained from equation (10) but the cost rates interchanged and initial demand rate D, i.e.,

$$V_2^*(D) = \begin{cases} \left(\dfrac{D(v_O - v_I)}{\beta(r-\mu)}\right)^{\beta} \left(\dfrac{r(\beta-1)}{F_I - F_O + rS_{OI}}\right)^{\beta-1} & \text{if } D \le D^{++} \\ \left(\dfrac{v_O - v_I}{r-\mu}\right) D - \left(\dfrac{F_I - F_O + rS_{OI}}{r}\right) & \text{if } D \ge D^{++} \end{cases}$$

where

$$\beta = \frac{\sqrt{(\mu - 0.5\sigma^2)^2 + 2\sigma^2 r} - (\mu - 0.5\sigma^2)}{\sigma^2} .$$

Derivation of (15)

In addition to $D \geq 0$, the optimization problem

$$D^{o} = \underset{D\geq 0, D\leq D_0, D<D^{++}}{\arg\max} V_3(D)$$

has two other constraints: $D \leq D_0$ and $D < D^{++}$. If $D > D_0$, then the firm immediately switches to outsourcing, and thus D_0 represents an upper limit on the threshold to switch to outsourcing (i.e., $D \leq D_0$). An outsource-to-insource threshold value satisfying $D \geq D^{++}$ is not viable because such a threshold implies that the moment the demand rate hits D, the firm switches from insourcing to outsourcing, then immediately back to insourcing, thus incurring two switching costs with no change in the operation. Thus, $D < D^{++}$ and expression (14) becomes

$$V_2^{*}(D) = \left(\frac{D(v_O - v_I)}{\beta(r-\mu)}\right)^{\beta}\left(\frac{r(\beta-1)}{F_I - F_O + rS_{OI}}\right)^{\beta-1}.$$

As an aside,

$$V_3(D) = = \left\{\left(\frac{D}{D_0}\right)^{\gamma}\left[\left(\frac{F_I - F_O - rS_{IO}}{r}\right) - \left(\frac{v_O - v_I}{r-\mu}\right)D\right]\right\} + \left\{\left(\frac{D}{D_0}\right)^{\gamma} V_2^{*}(D)\right\}$$

The first term in brackets is quasi-concave in D (see derivation of (3)) and, given $\beta > 1$ (which implies $\beta + \gamma > 2$), the second term in brackets is convex in D. Thus, $V_3(D)$ is not necessarily quasi-concave (which complicates the characterization of D^{o}).

Derivation of (7)

We make use of the following identity (Shackleton and Wojakowski 2002).

$$E\left[e^{-r\tau_2(D)}\right] = \left(\frac{D_0}{D}\right)^{\beta} \text{ for } D>D_0 \qquad \text{(A4)}$$

where

$$\beta = \frac{\sqrt{(\mu - 0.5\sigma^2)^2 + 2\sigma^2 r} - (\mu - 0.5\sigma^2)}{\sigma^2}.$$

Accordingly,

$$\begin{aligned}
C_2(D_0, D) &= E\left[\int_0^{\tau_2(D)} e^{-rt}(v_I D_t + F_I)\,dt + \int_{\tau_2(D)}^{\infty} e^{-rt}(v_O D_t + F_O)\,dt + e^{-r\tau_2(D)} S_{IO}\right] \\
&= E\left[\int_0^{\infty} e^{-rt}(v_I D_t + F_I)\,dt\right] - E\left[\int_{\tau_2(D)}^{\infty} e^{-rt}\left[(v_I - v_O)D_t - (F_O - F_I)\right]dt - e^{-r\tau_2(D)} S_{IO}\right] \\
&= C_I(D_0) - E\left[e^{-r\tau_2(D)}\left(\int_{\tau_2(D)}^{\infty} e^{-r(t-\tau_2(D))}\left[(v_I - v_O)D_t - (F_O - F_I)\right]dt - S_{IO}\right)\right] \\
&= C_I(D_0) - E\left[e^{-r\tau_2(D)}\right]\left(\int_0^{\infty} e^{-rt}\left[(v_I - v_O)E\left[D_{\tau_2(D)+t} \middle| D_{\tau_2(D)} = D\right] - (F_O - F_I)\right]dt - S_{IO}\right).
\end{aligned}$$

Substituting identities (A1) – (A4) into $C_2(D_0, D)$ yields

$$\begin{aligned}
C_2(D_0, D) &= C_I(D_0) - \left(\frac{D_0}{D}\right)^{\beta}\left[\left(\frac{v_I - v_O}{r-\mu}\right)D - \left(\frac{F_O - F_I + rS_{IO}}{r}\right)\right]. \\
&= C_I(D_0) - V_2(D).
\end{aligned}$$

Derivation of (9)
Taking the derivative of $V_2(D)$, we have

$$V_2'(D) = \frac{D_0^{\beta}}{D^{\beta+1}}\left[\beta\left(\frac{F_O - F_I + rS_{IO}}{r}\right) - (\beta-1)\left(\frac{v_I - v_O}{r-\mu}\right)D\right].$$

If $\beta \le 1$, then $V_2'(D) > 0$ for all $D \ge 0$, and the optimal threshold is $D^{**} = \infty$. If $\beta > 1$, then $V_2(D)$ has a unique positive stationary point $D^{**} = \frac{\beta(r-\mu)(F_O - F_I + rS_{IO})}{r(\beta-1)(v_I - v_O)}$. Furthermore, $V_2'(D) > 0$ for all $D \in (0, D^{**})$ and $V_2'(D) < 0$ for all $D \in (D^{**}, \infty)$. Thus $V_2(D)$ is quasi-concave over $D \in [0, \infty)$ and is maximized at D^{**}.

Derivation of (10)
Note that if $D_0 \ge D^{**}$, then the current demand rate is at or above the threshold where it is optimal to switch to outsourcing. Thus the firm immediately switches to outsourcing and the value of outsourcing is

$$V_2^* = \left(\frac{v_I - v_O}{r-\mu}\right)D_0 - \left(\frac{F_O - F_I + rS_{IO}}{r}\right).$$

In the event of $D_0 \le D^{**}$, we substitute

$$D^{**} = \underset{D\ge 0}{\arg\max}\, V_2(D) = \frac{\beta(r-\mu)(F_O - F_I + rS_{IO})}{r(\beta-1)(v_I - v_O)}$$

into $V_2(D)$ to get

$$\begin{aligned} V_2^* &= \left(\frac{D_0}{D^{**}}\right)^{\beta}\left[\left(\frac{v_I - v_O}{r-\mu}\right)D^{**} - \left(\frac{F_O - F_I + rS_{IO}}{r}\right)\right] \\ &= \left(\frac{D_0(v_I - v_O)}{\beta(r-\mu)}\right)^{\beta}\left(\frac{r(\beta-1)}{F_O - F_I + rS_{IO}}\right)^{\beta-1}. \end{aligned}$$

Derivation of (11)
Dixit (2001) gives the probability that an arithmetic Brownian motion process will hit an upper threshold, which can be used to obtain (11) (see Derivation of (5)).

Derivation of (12)
Dixit (2001) gives the expected time that an arithmetic Brownian motion process will hit an upper threshold, which can be used to obtain (12) (see Derivation of (6)).

Derivation of (16)
The derivation of (16) follows the derivation (3) that gives the optimal threshold for switching to a higher variable cost (and lower fixed cost), except that F_I, F_O, and S_{IO} are replaced with F_O, F_I, and S_{OI} respectively.

Derivation of (17)
Section 3.1 shows how to determine the value of the option to switch from a lower variable cost and a higher fixed cost (e.g., insourcing in Section 3.1, but outsourcing here) to a higher variable cost and a lower fixed cost (e.g., outsourcing in Section 3.1, but insourcing here). The value of the option to insource is obtained from equation (4) but the cost rates interchanged and initial demand rate D, i.e.,

$$V_1^*(D) = \begin{cases} \left(\frac{F_O - F_I - rS_{OI}}{r}\right) - \left(\frac{v_I - v_O}{r-\mu}\right)D & \text{if } D \le D^+ \\ \left(\frac{\gamma(r-\mu)}{D(v_I - v_O)}\right)^{\gamma}\left(\frac{F_O - F_I - rS_{OI}}{r(\gamma+1)}\right)^{\gamma+1} & \text{if } D \ge D^+ \end{cases}$$

where

$$\gamma = \frac{\sqrt{\left(\mu - 0.5\sigma^2\right)^2 + 2\sigma^2 r} + \left(\mu - 0.5\sigma^2\right)}{\sigma^2}.$$

Derivation of (18)

The optimization problem

$$D^{oo} = \underset{D \geq D_0, D > D^+}{\arg\max} V_4(D)$$

has two constraints: $D \geq D_0$ and $D > D^+$. If $D < D_0$, then the firm immediately switches to outsourcing, and thus D_0 represents a lower limit on the threshold to switch to outsourcing (i.e., $D \geq D_0$). An outsource-to-insource threshold value satisfying $D \leq D^+$ is not viable because such a threshold implies that the moment the demand rate hits D, the firm switches from insourcing to outsourcing, then immediately back to insourcing, thus incurring two switching costs with no change in the operation). Thus, $D > D^+$ and expression (17) becomes

$$V_1^*(D) = \left(\frac{\gamma(r-\mu)}{D(v_I - v_O)}\right)^{\gamma} \left(\frac{F_O - F_I - rS_{OI}}{r(\gamma+1)}\right)^{\gamma+1}.$$

As an aside,

$$V_4(D) = \left\{\left(\frac{D_0}{D}\right)^{\beta}\left[\left(\frac{v_I - v_O}{r-\mu}\right)D - \left(\frac{F_O - F_I + rS_{IO}}{r}\right)\right]\right\} + \left\{\left(\frac{D_0}{D}\right)^{\beta} V_1^*(D)\right\}$$

Given $\beta > 1$, the first term in brackets is quasi-concave in D (see derivation of (9)) and the second term in brackets is convex in D. Thus, $V_4(D)$ is not necessarily quasi-concave (which complicates the characterization of D^{oo}).

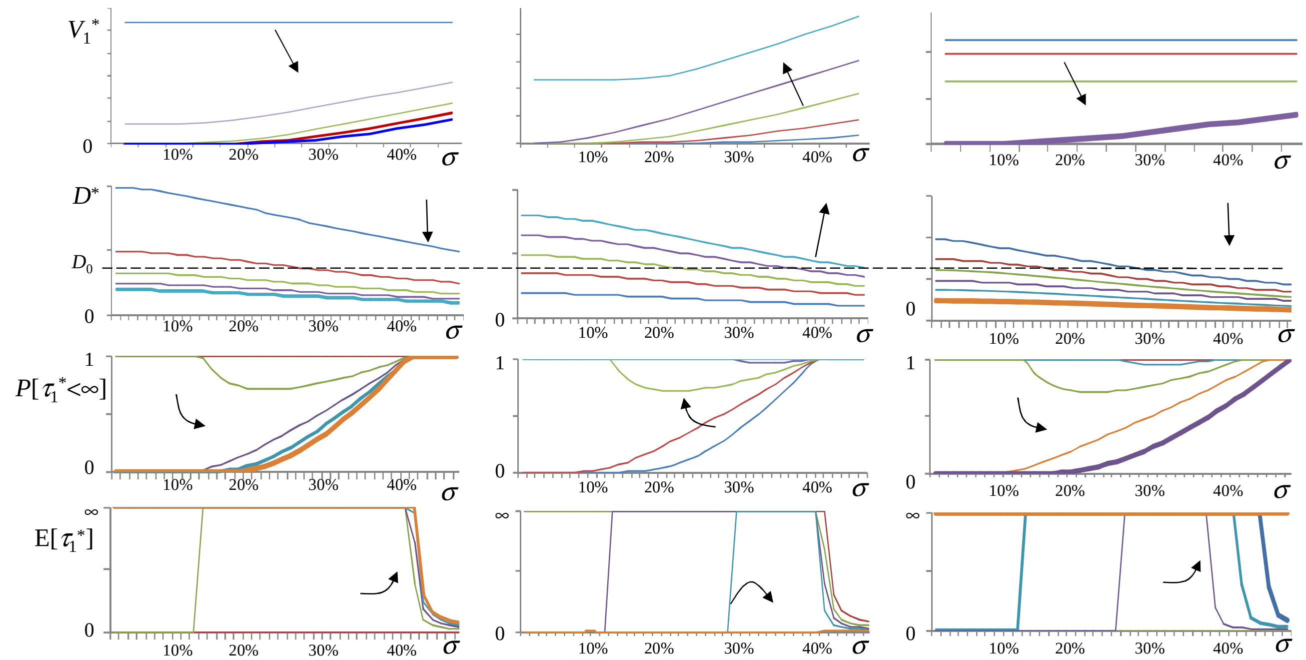
Appendix B: Base Case Numerical Simulation Results

This appendix illustrates the effect of changing demand volatility (i.e., demand coefficient of variation), variable cost differential, fixed cost differential, and demand growth rate on four measures: the value of the option to outsource (V_1^* in Regime 1, and V_2^* in Regime 2), the optimal switching demand threshold, the probability of a switch to outsourcing, and expected time to making the switch to outsourcing (see Figures B1 and B2).

The base models yield results similar to those of past studies, thus establishing their adequacy for the problem at hand, and they refine these results as well as produce new valuable insights. We start with the known results. First, the value of the option to outsource services increases in demand volatility. This result is consistent with findings in the real options theory literature (Dixit and Pindyk 1994). Moreover, for Regime 1 and Regime 2, respectively, the value of this option increases when: (1) the difference in variable costs decreases / increases, (2) the difference in fixed costs increases / decreases, and (3) the demand growth rate decreases / increases. Second, the hysteresis band is increasing in demand volatility. The demand threshold for triggering a switch to outsourcing of the process increases in Regime 1 (D^*) and decreases in Regime 2 (D^{**}) as a function of demand volatility, under all conditions. Thus, under Regime 1, where outsourcing has a higher variable cost but a lower fixed cost than in insourcing, outsourcing is preferable when the demand rate for the service process is low. And, under Regime 2, where outsourcing has a higher fixed cost but a lower variable cost, outsourcing is preferable when demand for the service process is high. In both regimes, the demand threshold decreases when: the difference in variable costs increases, the difference in fixed costs decreases, or the demand growth rate increases. These conditions make it less likely for a firm to make the switch to outsourcing the process in Regime 1, and more likely in Regime 2.

Beyond what is known from past studies, there are new insights from the base models. While a monotone behavior is observed for the demand threshold and the outsourcing option value, a more intricate behavior is observed for the likelihood of this option to be exercised. The probability of a switch to outsourcing and the expected time to making the switch follow a non-monotone behavior with the demand volatility in Regime 1 and a monotone behavior in Regime 2 (except when the demand growth rate is negative). Specifically, in Regime 1, for some problem parameters, the probability of a switch can be 100% for relatively small and for relatively large levels of demand volatility (i.e., the firm immediately switches to outsourcing), but for intermediate values of the demand volatility, the probability can exhibit both an increasing and decreasing behavior. By contrast, in Regime 2, for low levels of demand volatility, the probability of a switch can be 100% under most situational conditions, but the probability of a switch starts decreasing and continues to decrease as demand volatility increases. Thus, in Regime 2, higher demand volatility values imply that insourcing is a better alternative for the firm. In general, though, increasing values of the difference in variable costs, or increasing growth rates in demand, lowers the probability of a switch in Regime 1 and increases this probability in Regime 2. The opposite is seen for increasing values of the difference in the fixed costs. Nevertheless, what is consistent is that higher demand volatility values cause the probability to approach 100% in Regime 1 and to approach 0% in Regime 2.

In summary, in Regime 1, significant demand volatility increases the probability of outsourcing despite the fact it also decreases the demand threshold. Likewise, in Regime 2, significant demand volatility lowers the probability of outsourcing as it also increases the demand threshold. The expected time to making a switch to outsourcing, however, decreases with increasing demand volatility in Regime 1 and increases in Regime 2. Thus, under significantly high demand volatility, a firm in Regime 1 would prefer the outsourcing alternative earlier in time, and a firm in Regime 2 would postpone the switching decision. In fact, in Regime 2, the expected time to making a switch becomes unbounded when the probability of outsourcing is less than 100%.

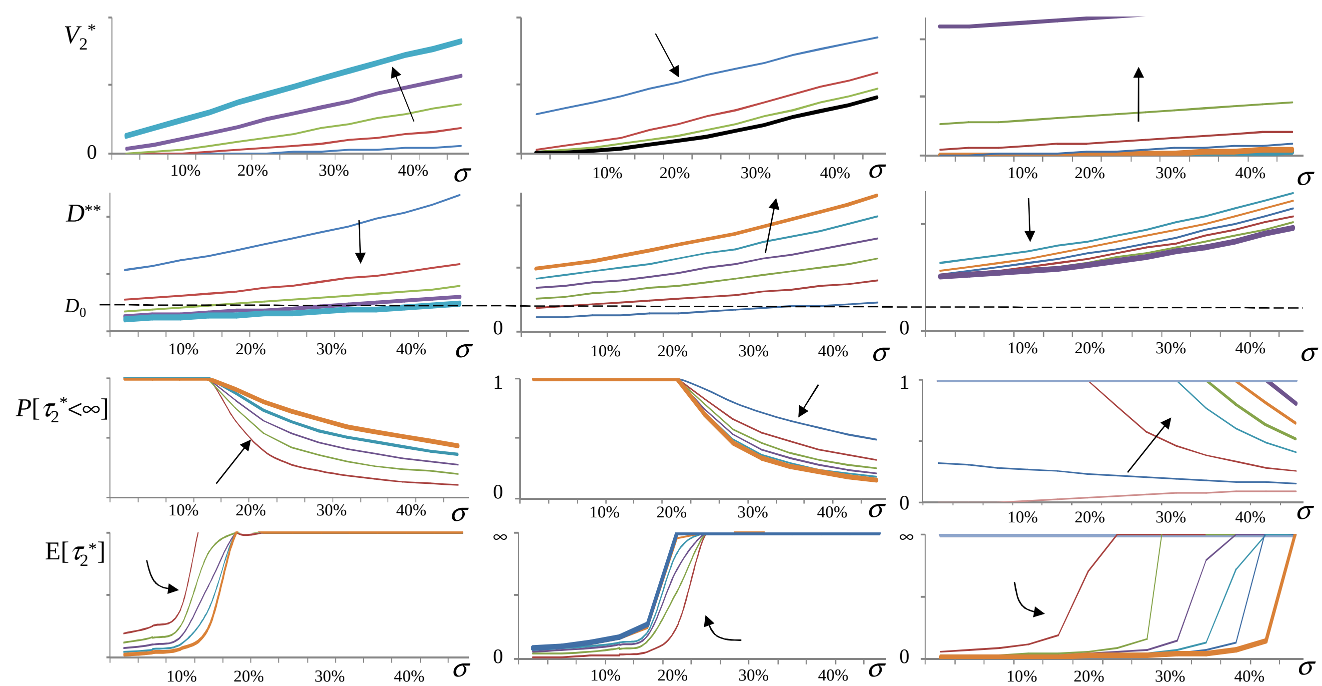


(a) Variable cost differential ($v_O - v_I$)
(v_O range is \$0.12 - \$0.2; μ=1%)

(b) Fixed cost differential ($F_I - F_O$)
(F_I range is \$230 - \$360)

(c) Demand growth rate (μ)
(μ range is 0% - 8%)

Figure B1: Analysis of Regime 1



(a) Variable cost differential ($v_I - v_O$)
(v_I range is \$0.12 - \$0.2)

(b) Fixed cost differential ($F_O - F_I$)
(F_O range is \$230 - \$360)

(c) Demand growth rate (μ)
(μ range is -2% - +8%; v_O=\$0.16)

Figure B2: Analysis of Regime 2

Appendix C: Backsourcing Case Numerical Simulation Results

This appendix shows the effect of changing volatility, variable cost differential, fixed cost differential, and demand growth rate on four measures: the incremental value of the option to backsource (given by $V_3(D^0) - V_1^*$ in Regime 1 and $V_4(D^{00}) - V_2^*$ in Regime 2), the optimal demand threshold for switching to outsourcing, the probability of a switch to outsourcing, and the expected time to making the switch to outsourcing (see Figures C1 and C2).

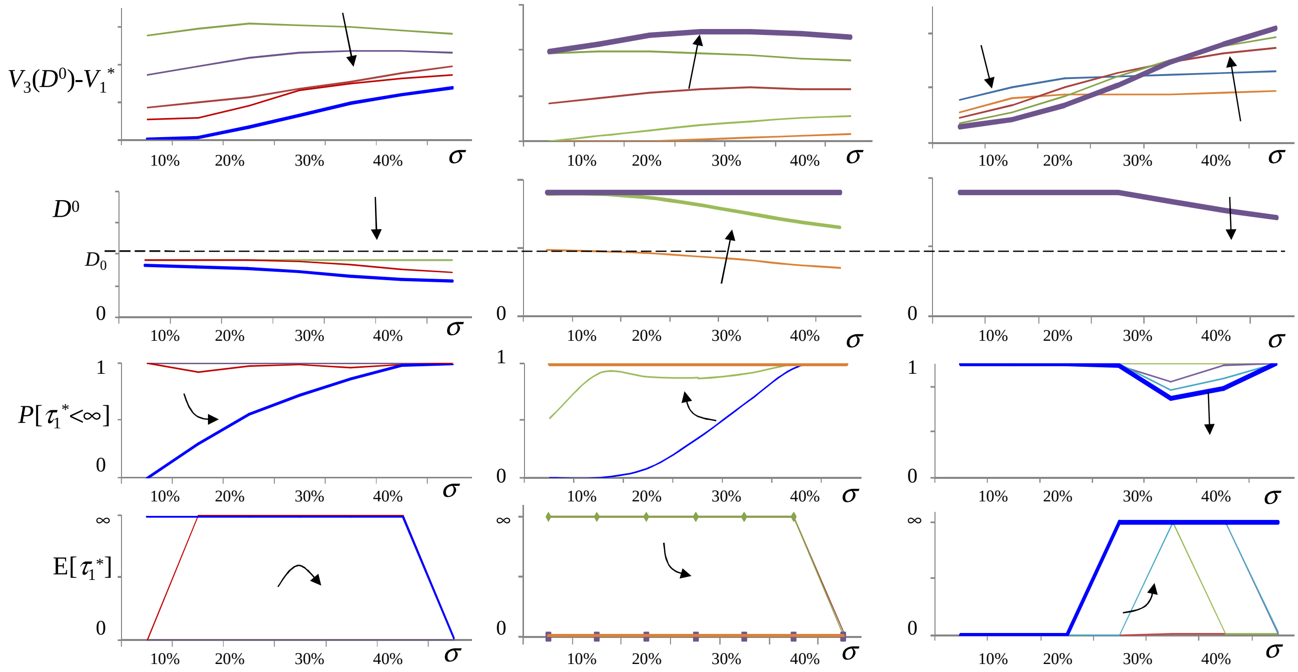
We start with the net added value of having backsourcing flexibility built into an outsourcing arrangement. Under Regime 1, this net added value generally exhibits an increasing pattern with a reversed bowl-shape for some problem parameters. It increases for relatively low values of the demand volatility, but decreases or just converges to a relatively stable value for higher demand volatilities. (The decrease occurs for lower values of the variable costs differential and demand growth rate, and for higher values of the fixed costs differential.) Under Regime 2, this net added value is significantly less when the demand volatility is low under all problem parameters, it increases with higher demand volatility, and it becomes significant only under relatively high values of the demand volatility. In both regimes, the net added value of backsourcing flexibility is significant only when the value of the "plain" option to outsource (with no backsourcing flexibility) is high; an exception is for higher demand growth rates. This is such because a low value of the plain option means a negligible probability that a switch to outsourcing will occur, in which case, there is a negligible value to having the flexibility to backsource.

The reversed-bowl shape in Regime 1 is the pattern that stands out and needs to be explained using properties of the model. As demand volatility increases, the stochastic demand process described by the geometric Brownian motion can hit the absorbent state (zero) with a higher probability. As the probability of experiencing zero demand increases, implying that the service process is no more viable, the option to backsource becomes less valuable, and its net value contribution starts decreasing with increasing volatility.

Importantly, the last result for Regime 1 refines the findings of earlier literature providing examples of how increasing market volatility creates a higher value for various forms of flexibility (Alvarez and Stenbecka 2007, Van Mieghem 1999 and 2003). Consistent with the findings in the real options literature, the common perception is that the value of flexibility increases with higher volatility. Yet, our result shows that the incremental value of additional flexibility, in the form of having the ability to backsource an outsourced process, departs from this common understanding.

Going beyond the net added value of backsourcing flexibility, what is more important is that this net added value has a reversed impact on the initial decision to switch to outsourcing. In Regime 2, while this net added value is strictly increasing in demand volatility, it does not necessarily make a switch to outsourcing more likely. When compared to the continuous increasing behavior of the demand threshold for the plain option (Figure 3), the presence of backsourcing flexibility generally lowers this threshold. However, the probability of a switch to outsourcing grows only marginally for intermediate and high values of the demand volatility and for all other problem parameters, and no beneficial impact can be observed for the expected time to making a switch to outsourcing. By contrast, in Regime 1, the net added value of backsourcing flexibility has a significant beneficial effect on the initial decision to switch to outsourcing. The demand threshold is consistently higher than for the plain option to outsource (Figure 2). And, while it continues to decrease monotonically under all problem parameters, there is a point where the decrease tapers off significantly with increasing values of the demand volatility. As a result, the probability of a switch to outsourcing increases under all problem parameters, and the expected time to making the switch goes to zero when the probability is close enough to 100%. Once again, though, the behavior of this probability is not monotone in the demand volatility,[5] and the expected time to making the switch to outsourcing becomes excessively large when the probability drops enough below 100%.

[5] For certain problem parameter values, the probability increases for low values of the demand volatility, then become wave-shaped or just U-shaped for intermediate values of demand volatility, and eventually becomes 100% for high demand volatilities. This is exactly the case seen in Figure 4 for: (1) decreasing values of the difference in variable costs, (2) increasing values of the difference in fixed costs, and (3) increasing values of the demand growth rate.

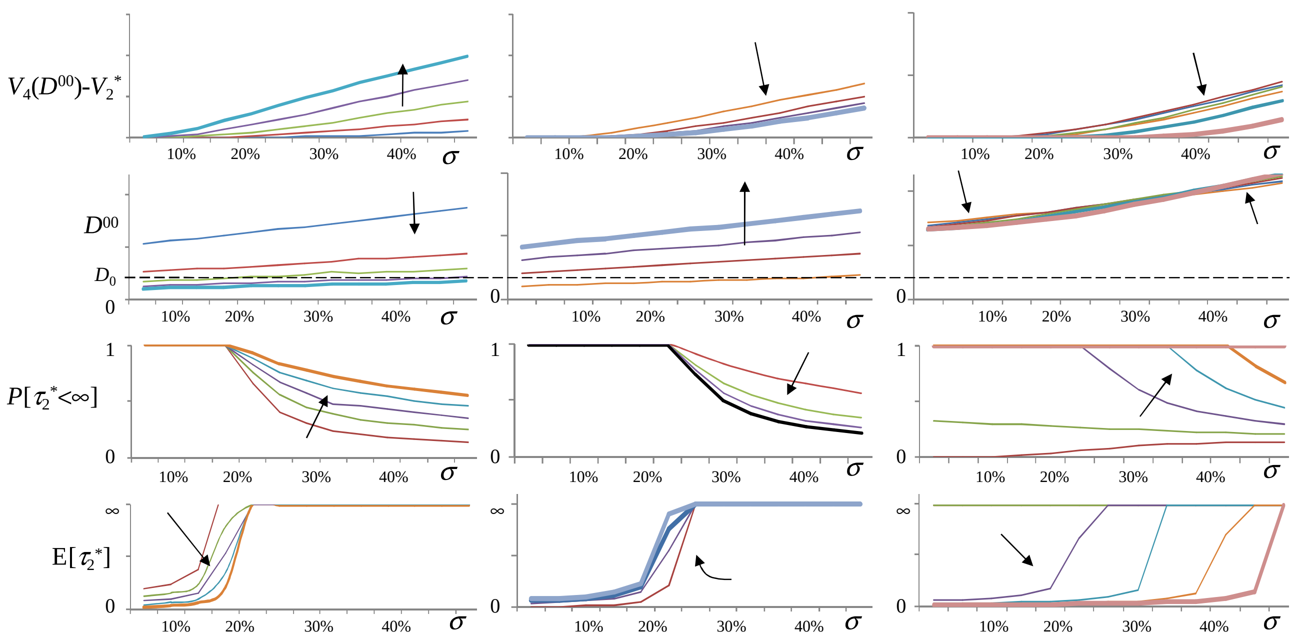


(a) Variable cost differential ($v_O - v_I$)
(v_O range is \$0.12 - \$0.2; μ=1%)

(b) Fixed cost differential ($F_I - F_O$)
(F_I range is \$230 - \$360)

(c) Demand growth rate (μ)
(μ range is 0% - 8%)

Figure C1: Analysis of Regime 1 with the Option to Backsource



(a) Variable cost differential ($v_I - v_O$)
(v_I range is \$0.12 - \$0.2)

(b) Fixed cost differential ($F_O - F_I$)
(F_O range is \$230 - \$360)

(c) Demand growth rate (μ)
(μ range is -2% - +8%; v_O=\$0.16)

Figure C2: Analysis of Regime 2 with the Option to Backsource

References (unique to Appendix)

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Shackleton, M., R. Wojakowski. 2002. The expected return and exercise time of Merton-style real options. *Journal of Business Finance & Accounting* **29**(3&4) 541-555.