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The Impact of Yield-Dependent Trading Costs on Pricing and Production Planning Under Supply Uncertainty

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This paper studies the role of the yield-dependent trading cost structure influencing the optimal choice of the selling price and production quantity for a firm that operates under supply uncertainty in the agricultural industry. The firm initially leases farm space, but its realized amount of fruit supply fluctuates because of weather conditions, diseases, etc. At the end of the growing season, the firm has three options: convert its crop supply to the final product, purchase additional supplies from other growers, and sell some (or all) of its crop supply in the open market without converting to the finished product. We consider the problem both from a risk-neutral and a risk-averse perspective with varying degrees of risk aversion. The paper offers three sets of contributions: (1) It shows that the use of a static cost exaggerates the initial investment in the farm space and the expected profit significantly, and the actual value gained from a secondary (emergency) option for an agricultural firm is lower under the yield-dependent cost structure. (2) It proves that although the risk-neutral firm does not benefit from fruit futures, a sufficiently risk-averse firm can benefit from the presence of a fruit futures market. The same risk-averse firm does not purchase fruit futures when it operates under static costs. Thus, fruit futures can only add value under yield-dependent trading costs. (3) Contrary to the results presented for the newsvendor problem under demand uncertainty, the firm does not always commit to a lower initial quantity (leased farm space) under risk aversion. Rather, the firm might lease a larger farm space under risk aversion.

Key words: supply uncertainty; risk aversion; yield-dependent trading costs; pricing; fruit futures
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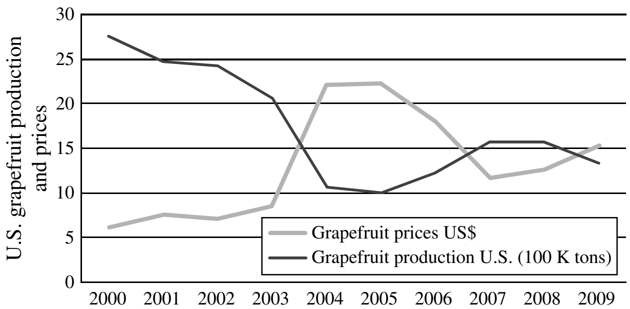
1. Introduction

Four devastating hurricanes in the state of Florida (Charley, Frances, Jeanne, Wilma) caused not only destruction, but led to fluctuating prices of citrus fruit and citrus juice. The grapefruit crop, for example, decreased by 48.47% from 2003 to 2004, increasing the cost of grapefruit by 156.88%. Figure 1 shows the annual grapefruit crop collected in the United States in the last decade and the average price of fresh grapefruit (primarily used for juice). This figure demonstrates the impact of crop yield on grapefruit fruit prices in the open market. Similarly, in December 2006, Martin (2006) reported that the average price of a gallon of orange juice had increased by 14%, going from $4.45 a year earlier to $5.09. The article states that the two largest orange juice producers in the United States, Tropicana and Minute Maid, warned consumers of more price increases because of what is expected to be a lackluster orange crop. As predicted, the orange supply in 2007 turned out to be the smallest in 17 years. In addition to the hurricanes, the state had to battle two highly contagious bacterial diseases, citrus canker and citrus greening. The U.S. Department of Agriculture forecasted Florida's 2009 orange crop to be 18% less than that of 2007, and in response, orange juice futures went up by 10.1% (Associated Press 2006). These events exemplify the impact of fruit supply fluctuations on the cost of citrus fruit and the selling price of an end product, such as the orange juice.

Because of the high fluctuations in the crop supply, agribusinesses typically perceive the price of fruit in the open market as random. The Ayvalık Chamber of Commerce in Turkey, the region that provides more than 70% of the country's production of olive oil, reports how influential crop yield fluctuations are in the purchasing cost of olives in the Turkish olive oil industry. Using the behavior of olive prices, Kazaz (2004) describes the relationship between crop yield and the purchasing cost of olives, and defines the unit purchasing cost as "yield dependent." Similar observations can be made in other agricultural industries, such as the wine industry. The labeling requirements with references to the growing region and the year of the vintage force this industry to operate under a yield-dependent purchasing cost, where the unit

Figure 1 **Fresh Grapefruit Production and Prices in the United States**



Source. National agricultural statistics services of the U.S. Department of Agriculture, annual reports 2000 through 2009.

cost is strongly related to the amount of the crop collected in the region in a specific year. In the case of Australian wine grapes, for example, the purchasing cost of grapes was averaging at $100 per ton in 2006 because of an oversupply going below the cost of growing the fruit. The cost of grapes increased five to thirty times the next year because of a long drought that cut the country's wine grape supply by more than 50%, and was ranging between $500 to $3,000 per ton in 2007 (Bradsher 2008).

The fluctuations in the citrus, olive, and grape supplies illustrate the necessity to incorporate a yield-dependent trading cost structure into pricing and production planning decisions. This paper responds to this need by examining a firm's combined decisions of price setting and production planning while operating under yield-dependent trading cost and revenue (generated from selling the fruit in the open market). By comparing traditional modeling approaches with static cost parameters, our study demonstrates that agribusinesses operate in an increased level of risk with lower expected profits because of the yield-dependent nature of the cost parameters.

Firms in the agricultural industry counter the uncertainty in cost and supply by employing three methods. The first method corresponds to leasing farm space to grow fruit in anticipation of reducing the future purchasing costs. When compared with the expected purchasing cost over supply uncertainty, the expected cost of growing the fruit is typically less expensive. Leasing farm space is a common practice among citrus juice producers in the United States, olive oil producers in Turkey, and many wine producers throughout the world. The lease is determined by the number of trees (or vines), and the producer incurs the cost of growing the fruit and maintaining the farm space (e.g., includes pruning, stem cutting, weed control, and insect and disease management). Although the producer faces the risk of supply uncertainty, an executive of one of the largest wine producers in the United States has described the benefit of leasing farm space as reducing the risk associated with a lower return on equity, and therefore, avoiding the potential decrease in the perceived value of the firm in financial markets.

The second method of battling supply uncertainty involves trading fruit in the open market after yield is realized. The firm can purchase additional fruit from other growers when the realized supply is insufficient, or alternatively, can sell fruit in the open market without converting to the final product. As described above, the unit purchasing cost and the unit spot-selling revenue of the fruit (e.g., citrus, olives, and grapes) change with the amount of realized supply.

It should also be emphasized that, unlike the practices observed in the manufacturing industry, agricultural businesses cannot use inventories strategically to battle supply uncertainty. For example, the olive oil producer needs to press its olives within 48 hours of collection to achieve the highest quality of its final product. As a result, the oil producer cannot hold inventories of olives for future production of olive oil. And olive oil stored for more than two years is undesirable as it begins to produce an acidic taste. Similar observations can be made for the production of fresh orange juice and wine. Freshly squeezed orange juice has a superior taste than the juice obtained from oranges stored in inventory. Therefore, high-quality orange juice is obtained by pressing the oranges immediately after collection. In wine production, grapes are also pressed immediately after their collection. Blackburn and Scudder (2008) provide other examples of deteriorating product quality in fresh produce (e.g., melons and sweet corn). As a result, as is the case with most perishable goods, the problem for the agricultural business can be modeled in a single-period context.

The third method for battling uncertainties is through pricing of the final product. As illustrated in the preceding examples, setting prices can be an important tool for managing supply uncertainty. In the case of Turkish olive oil, a single firm, the Ana Gıda Group, owns more than half of the country's extra virgin olive oil market share. The company's 56% market share in 2009 provides the manager with the price-setting capability. Considering these observations, we incorporate the price-setting behavior of the firm into the formulation.

In addition to the above three methods of battling the uncertainty in fruit costs and supply, the paper investigates the value that can be gained from purchasing fruit futures. Although there are futures for the finished product in the case of olive oil, orange juice, and some higher-quality wine, there is no futures market for olives, oranges, or grapes. Our analysis shows that a risk-neutral firm cannot increase its expected profit through the use of fruit futures;

however, a sufficiently risk-averse agricultural firm may benefit from fruit futures, as it relies less heavily on leased farm space and the open market for trading fruit. It is important to note that the same risk-averse firm does not purchase fruit futures when operating under static costs. Thus, fruit futures can add value to a risk-averse firm only under a yield-dependent cost structure.

Several distinguishing factors separate our work from earlier studies. Earlier research that examined production planning problems under supply uncertainty (as yield uncertainty) has considered prices to be exogenous. The body of literature that incorporates the price-setting behavior of a monopolistic firm into the decision-making process under supply uncertainty is limited. This paper advances these works (1) by highlighting the impact of the yield-dependent trading cost on pricing and production decisions under supply uncertainty, and (2) by analyzing both from a risk-neutral and a risk-averse perspective.

The paper is organized as follows. A review of the literature is provided in §2. The model is introduced in §3. Section 4 examines the impact of supply uncertainty and yield-dependent trading costs and provides numerical illustrations using data from the Turkish olive oil producers. Section 5 introduces risk aversion and shows when the firm may benefit from fruit futures. Section 6 summarizes the main conclusions. All proofs and derivations are presented in the appendices that can be found in the online supplement (available in the electronic companion).

2. Literature Review

The problem of production planning under supply uncertainty has received considerable attention. Price is commonly assumed to be exogenous. Yano and Lee (1995) provide an extensive review of production and inventory problems under yield uncertainty, which is the foundation for supply uncertainty. Grosfeld-Nir and Gerchak (2004) review the literature on multiple lot sizing decisions under random supply and demand in make-to-order systems. In addition to these review articles, Bollapragada and Morton (1999) describe efficient myopic heuristics for periodic review inventory problems. Rajaram and Karmarkar (2002) consider the issue of supply uncertainty in the process industry.

The opportunity to obtain additional fruit in our paper resembles the setting in Jones et al. (2001). In their paper, the hybrid seed corn producer gets a second chance of production in a different region of the world and experiences supply uncertainty. Our problem differs from the one presented in Jones et al. (2001) in two ways: (1) The sale price is endogenous to the model, therefore the producer uses the sale price as a mechanism to hedge against fluctuating supply. (2) When the producer purchases additional fruit from other growers, she does not experience another supply uncertainty. A similar setting with one unreliable and one reliable supply source is also examined in Kazaz (2004) and Tomlin (2009); the scenario with multiple unreliable suppliers is investigated in Tomlin and Wang (2005), Dada et al. (2007), and Federgruen and Yang (2008). However, these papers consider only the exogenous selling price, and are not concerned with insights that come as a result of the price-setting behavior.

Kazaz (2004) is the first study that considers the yield-dependent purchasing cost of fruit. There are several factors that distinguish our work from this study: (1) Kazaz (2004) assumes the selling price as an exogenous function of the yield random variable and is not influenced by the initial farm space investment. In our study, the selling price is endogenous and its optimal value is affected by the firm's farm space investment. (2) Our paper extends Kazaz (2004) by offering a risk-averse analysis and the influence of fruit futures. (3) We consider a yield-dependent selling price of fruit in the open market, whereas Kazaz (2004) assumes a constant selling price. (4) Whereas Kazaz (2004) focuses on the perfectly correlated supply and market prices, we offer an analysis where fruit prices in the open market are less than perfectly correlated with random yield. In our paper, the optimal second-period selling price is influenced by realized supply in the first period. There are other publications where the optimal second-period selling price is influenced by realized demand in the first period, e.g., Cachon and Kök (2007).

Price-setting behavior has been widely studied under demand uncertainty (see Petruzzi and Dada 1999, 2001; Federgruen and Heching 1999, 2002; Kocabıyıkoğlu and Popescu 2011), however, it has not been examined extensively under supply uncertainty. Li and Zheng (2006) is the first paper to consider endogenous pricing for inventory replenishment under supply and demand uncertainty. Their model differs from ours in several ways: (1) In their model, selling price is determined before supply uncertainty is realized, whereas in our model the firm has the opportunity to adjust its pricing and production-related decisions after supply uncertainty is revealed. (2) Their model provides the second-chance opportunity to purchase additional products after supply is realized, but their purchasing cost is static (i.e., independent of the yield), whereas our purchasing cost changes with the realized supply. (3) Whereas their model is risk-neutral, we offer both a risk-averse and a risk-neutral analysis. Tang and Yin (2007) also study pricing under supply uncertainty. Their demand function is linear in price. Supply uncertainty in their paper is restricted to a discrete uniform distribution, and therefore, their results are not generalizable. This paper extends their work by (1) incorporating continuous and arbitrary distributions that define supply uncertainty, (2) generalizing the price-dependent demand function, (3) considering the firm's opportunity to sell the crop in the open market, (4) featuring the yield-dependent nature of the purchasing cost and revenue, and (5) incorporating risk aversion and the use of fruit futures. Tomlin and Wang (2008) consider price-setting behavior under supply uncertainty for a firm that produces multiple products with an emphasis on downward substitution. As will be demonstrated later, our paper differs from these papers as it shows that the influence of the yield-dependent trading costs is significant in the initial investment and the profitability of the firm.

The challenges in agricultural businesses, different than a repetitive manufacturing environment, are also receiving attention in recent literature. Although the problem settings and the decisions investigated are distinctly different than our model, the interested reader is referred to Burer et al. (2008) and Huh and Lall (2008) for other challenging problems in the agricultural industry.

3. The Model

This section presents the modeling approach used for the pricing and production planning problem of an agricultural business that leases farm space and experiences supply uncertainty. The model is a two-stage stochastic program, where the first stage corresponds to the growing season of the fruit, and after the production takes place, the second stage is the selling season of the final product.

At the beginning of a growing season, the firm determines the amount of farm space to be leased, denoted by Q, at a unit cost of leasing c_l. Randomness in supply is represented with a stochastically proportional yield (or multiplicative random error term), denoted with $\tilde{u}$, where u is a realization, and $g(u)$ is the probability density function (pdf) defined on a support $[u_l, u_h]$ with a mean $\bar{u} = \mathrm{E}[\tilde{u}]$ and a variance σ^2.

At the end of the growing season, the firm realizes crop yield in the amount of Qu. Given the realized supply at the beginning of the selling season, corresponding to the second stage of our model, the firm makes four decisions to maximize its profit: (1) the selling price, denoted by p; (2) the amount of realized supply from the leased farm space (internal growth) to be converted to final product, denoted by q_i; (3) the amount of fruit to buy from other growers, denoted by q_b; and (4) the amount of fruit to sell in the open market, denoted by q_s. The sum of the converted supply and the fruit sold in the open market is restricted to be no more than the realized crop yield, i.e., $q_i + q_s \leq Qu$. In citrus juice, olive oil, and wine production, the firm presses the fruit to obtain the final product, and we let c_p denote the unit pressing (processing) cost. The firm incurs a processing cost of $c_p(q_i + q_b)$.

Before the selling season begins, the firm may choose to increase its production by purchasing additional fruit from other growers at a unit cost that depends on the realized yield. We denote the yield-dependent unit cost of buying additional fruit as $b(u)$, and the firm incurs $b(u)q_b$. The firm also has the flexibility to sell its fruit in the open market at a positive unit revenue that changes with the realized yield. The yield-dependent unit revenue from sales of the fruit is denoted by $s(u)$, and the firm earns $s(u)q_s$. There is a positive spread, defined as $\delta(u) = b(u) - s(u) > 0$ for all u, between the unit purchasing cost and the unit selling price of the fruit in the open market. This prohibits an arbitrage opportunity, i.e., the firm cannot make profits by simply buying and selling crop at the same time in the open market. It should be emphasized here that we make no assumptions regarding the shape of the yield-dependent costs except that both $b(u)$ and $s(u)$ are decreasing in yield u (we use the terms increasing/decreasing and positive/negative in their weak sense throughout the paper). For example, we do not require that expected trading costs are equal to their costs at the expected yield; specifically, we allow $\mathrm{E}[b(\tilde{u})] \neq b(\bar{u})$ and $\mathrm{E}[s(\tilde{u})] \neq s(\bar{u})$. The conclusions of the study are robust as the results hold under convex, concave, and other forms of yield-dependent fruit trading cost functions. Demand $d(p)$ is decreasing in price, and therefore, there exists a unique inverse $p(d)$. We assume that revenue $p(d)d$ is strictly concave, i.e., $2p'(d) + p''(d)d < 0$.

The model is a two-stage stochastic program with recourse, where the firm chooses the optimal amount of farm space to be leased in Stage 1. Given the realized crop, at the beginning of Stage 2, the firm determines the selling price, the amount of final product to be produced from internally grown and externally purchased fruit, as well as the amount of fruit to be sold in the open market without converting to the final product. Leftovers of finished product at the end of the selling season are salvaged at a unit revenue of s_2, where $s_2 \leq c_p + s(u_h)$. The model can be expressed as follows:

Stage 1.

$$\max_{Q \geq 0} \mathrm{E}[\Pi(Q)] = -c_l Q + \int_{u_l}^{u_h} P(Q, u) g(u)\, du;$$

Stage 2. Given Q and u,

$$P(Q, u) = \max_{\substack{(p, q_i, q_b, q_s) \geq 0 \\ q_i + q_s \leq Qu}} \pi(p, q_i, q_b, q_s \mid Q, u),$$

where

$$\pi(p, q_i, q_b, q_s \mid Q, u)$$
$$= p\min\{(q_i+q_b), d(p)\} - c_p(q_i+q_b) - b(u)q_b + s(u)q_s + s_2((q_i+q_b)-d(p))^+. \quad (1)$$

From $s_2 \le c_p + s(u_h)$ and $p'(d) < 0$, it follows that $q_i + q_b = d(p)$, and from $\partial\pi/\partial q_s = s(u) > 0$, it follows that $q_i + q_s = Qu$ is an optimal solution to the second-stage problem. Thus, the second-stage problem can be restated as

$$P(Q,u) = \max_{\substack{(p,q_i)\ge 0\\ q_i \le \min\{d(p),Qu\}}} \big\{\pi(p,q_i \mid Q,u) = (p-c_p-b(u))d(p) + (b(u)-s(u))q_i + s(u)Qu\big\}.$$

4. Model Analysis

4.1. The Case of No Trading of the Fruit

We first analyze the variant of the problem with deterministic supply by replacing the supply random variable $\tilde{u}$ with its mean $\bar{u}$. The firm leases Q units of farm space, and realizes crop yield of $Q\bar{u}$. In the second stage, because of the lack of a trading option (buying or selling) of the fruit (i.e., $q_b = q_s = 0$), the firm is restricted to convert the entire crop to the final product. In this setting, the selling price clears the production, and therefore, $d(p) = q_i = Q\bar{u}$. The first-stage objective function can be expressed as follows:

$$\Psi(Q) = (p(Q\bar{u}) - ((c_l/\bar{u}) + c_p))Q\bar{u}.$$

Remark 1. (a) The optimal amount of farm space to be leased, denoted by Q^0, under deterministic supply satisfies

$$p(Q^0\bar{u})\bar{u} + p'(Q^0\bar{u})Q^0(\bar{u})^2 = c_l + c_p\bar{u};$$

(b) the optimal deterministic profit, denoted by $\Psi(Q^0)$, is

$$\Psi(Q^0) = -p'(Q^0\bar{u})(Q^0\bar{u})^2.$$

We next analyze the firm's objective function under supply uncertainty:

$$\mathrm{E}[\Pi(Q)] = -(c_l + c_p\bar{u})Q + \int_{u_l}^{u_h} p(Qu)Qug(u)\,du$$
$$= \Psi(Q) - \int_{u_l}^{u_h} [p(Q\bar{u}) - p(Qu)]Qug(u)\,du. \quad (2)$$

Proposition 1. (a) *The first-stage objective function is concave in Q, and the optimal amount of farm space to be leased satisfies*

$$\int_{u_l}^{u_h} [p(Qu)u + p'(Qu)Qu^2]g(u)\,du = c_l + c_p\bar{u}; \quad (3)$$

(b) *the optimal profit is*

$$\mathrm{E}[\Pi(Q^*)] = -\int_{u_l}^{u_h} p'(Q^*u)(Q^*u)^2 g(u)\,du, \quad (4)$$

and is less than its deterministic equivalent;

(c) *if $3p''(d) + p'''(d)d \le 0$, then the optimal amount of farm space to be leased is less than that of the deterministic supply, i.e., $Q^* < Q^0$.*

The above proposition provides general results regarding the behavior of the optimal amount of farm space to be leased and the optimal profit expression under deterministic and stochastic supply. Because the demand function is not described by a specific function, a closed-form expression is not provided for the selling price. Under the linear demand function $d(p) = a - bp$, the optimal amount of farm space to be leased and the optimal profit expressions under deterministic and stochastic supply can be expressed as follows (derivations provided in Appendix A in the online supplement):

$$Q^* = \frac{[a - b((c_l/\bar{u}) + c_p)]\bar{u}}{2(\bar{u}^2 + \sigma^2)} = \frac{[a - b((c_l/\bar{u}) + c_p)]}{2\bar{u}(1 + cv^2)} = \frac{Q^0}{1 + cv^2} < Q^0,$$

$$\mathrm{E}[\Pi(Q^*)] = \frac{[a - b((c_l/\bar{u}) + c_p)]^2\bar{u}^2}{4b(\bar{u}^2 + \sigma^2)} = [Q^*\bar{u}]^2\left(\frac{1 + cv^2}{b}\right) = \frac{\Psi(Q^0)}{(1 + cv^2)} < \Psi(Q^0),$$

where $cv = \sigma/\bar{u}$ is the coefficient of variation of supply uncertainty. It should be highlighted that the above expressions do not depend on the form of the pdf of supply uncertainty. Moreover, both the mean and the variance terms are influencing the optimal amount of farm space to be leased and the optimal profit. Preserving the mean supply, the above inequality shows that both the optimal amount of farm space to be leased and the optimal expected profit are decreasing in cv.

4.2. Incorporating the Trading Option

We next incorporate the flexibility to trade the fruit crop in the open market. The firm now has the option of buying additional fruit to be converted to final product (i.e., $q_b \ge 0$) and the option of selling its fruit supply without converting to the final product (i.e., $q_s \ge 0$).

We first present the analysis regarding the buying and selling options independently. Let us begin with the selling option by setting $q_b = 0$ in (1). Recall that $s(u)$ is the yield-dependent unit sales revenue from selling the fruit in the open market. The function $s(u)$ resembles the salvage revenue in traditional modeling approaches; however, its value decreases with higher crop realization. The firm would consider the selling option of the fruit when the realized crop supply is high. When the yield realization is low, the pricing flexibility enables the firm to set higher prices to compensate for the lack of production. As a result, the firm sets a threshold for a maximum production level for each realization of u, where the remaining crop would be sold in the open market. We denote this threshold that determines the firm's switching decision from utilizing the fruit for production to the option of selling in the open market by $TS(u)$. This threshold can be determined by ignoring the effect of the realized supply, i.e., by setting $Qu = 0$. This leaves (1) as a maximization over a single variable q_i, and the optimal value of q_i provides $TS(u)$.

Proposition 2. *The threshold for the amount of final product to be produced is*

$$TS(u) = -[p^* - c_p - s(u)]\, d'(p^*) \tag{5}$$

and is increasing in u.

The first consequence of yield-dependent revenue of selling the fruit becomes clear in the firm's desired level of production, $TS(u)$. In the traditional modeling approaches where the salvage revenue is static and does not change with the realized yield, the firm's desired level of production would be constant for all realizations of the yield parameter u. Proposition 2 shows that the desired level of production changes with the supply realization, and specifically, increases in the yield fraction. When the realized supply is below the desired level of production, the firm produces all of its crops and charges a market-clearing price, and when the realized supply is above the desired level of production, it produces up to the threshold and the remaining fruit is sold in the open market. Optimal selling price and quantity decisions and the corresponding profit are provided in Proposition A2 in the online supplement.

We next present the analysis of the buying option by setting $q_s = 0$ in (1). Recall that $b(u)$ is the yield-dependent unit purchasing cost of the fruit from the open market. The firm would consider the buying option when the realized crop supply is low because of the pricing flexibility that compensates for the expensive purchasing option. The threshold for the buying option, denoted $TB(u)$, can be determined by ignoring the internal production option, i.e., $q_i = 0$. This is equivalent to the scenario when the firm does not lease any farm space and acquires all fruit on the open market. Once again, (1) becomes a maximization over a single variable q_b, and the optimal value of q_b provides $TB(u)$.

Proposition 3. *The threshold for the amount of fruit to be purchased in the open market is*

$$TB(u) = -[p^* - c_p - b(u)]d'(p^*)$$

and is increasing in u.

The above proposition demonstrates the influence of yield-dependent unit purchasing cost on the amount of fruit acquired from the open market. The threshold for the amount of fruit to be purchased from the open market increases in the yield fraction u. In the traditional modeling approaches that consider static purchasing costs from a secondary source, this threshold would be a constant; however, it changes according to the realization of the yield parameter for the agricultural businesses. When the realized supply is below the threshold for purchasing, the firm acquires additional crops from the open market and increases its level of production beyond the realized supply. In this case, the selling price reflects the influence of the purchasing cost. When the realized supply is above the threshold, however, the firm does not engage in additional purchasing, and converts only the internally grown fruit to the final product. In this event, the firm charges a market-clearing price. Optimal selling price, quantity decisions, and the corresponding profit are expressed in Proposition A3 in the online supplement.

We next combine the buying and selling conditions. Note that $TB(u) < TS(u)$, which is a consequence of $b(u) > s(u)$.

Proposition 4. *For a given realized yield of* Qu, *the optimal decisions for the selling price, the amount of crop yield to be converted to finished product, the amount to purchase from other growers, and the amount to sell in the open market are*

$$p^* = \begin{cases} p(TB(u)) & \text{if } Qu \le TB(u), \\ p(Qu) & \text{if } TB(u) \le Qu \le TS(u), \\ p(TS(u)) & \text{if } Qu \ge TS(u); \end{cases}$$

$$(q_i^*, q_b^*, q_s^*) = \begin{cases} (Qu, TB(u) - Qu, 0) & \text{if } Qu \le TB(u), \\ (Qu, 0, 0) & \text{if } TB(u) \le Qu \le TS(u), \\ (TS(u), 0, Qu - TS(u)) & \text{if } Qu \ge TS(u), \end{cases}$$

and the optimal second-stage profit is

$$\pi(p^*, q_i^*, q_b^*, q_s^* \mid Q, u) = \begin{cases} [p(TB(u)) - c_p - b(u)]\, TB(u) + b(u)\, Qu \\ \quad \text{if } Qu \le TB(u), \\ [p(Qu) - c_p]Qu \\ \quad \text{if } TB(u) \le Qu \le TS(u), \\ [p(TS(u)) - c_p - s(u)]\, TS(u) + s(u)\, Qu \\ \quad \text{if } Qu \ge TS(u). \end{cases}$$

We next consider the first-stage objective function. We define

$$B(Q) = \{u\colon\ TB(u) > Qu,\ u \in [u_l, u_h]\},$$
$$N(Q) = \{u\colon\ TB(u) \le Qu \le TS(u),\ u \in [u_l, u_h]\},$$
$$S(Q) = \{u\colon\ TS(u) < Qu,\ u \in [u_l, u_h]\}.$$

Note that $B(Q)$, $N(Q)$, and $S(Q)$ is a partition of $[u_l, u_h]$, i.e., $B(Q) \cup N(Q) \cup S(Q) = [u_l, u_h]$ and $B(Q) \cap N(Q) = N(Q) \cap S(Q) = B(Q) \cap S(Q) = \varnothing$. The set $B(Q)$ contains values of u where it is optimal to buy fruit in the open market, $N(Q)$ contains values of u where it is optimal to not trade fruit in the open market, and $S(Q)$ contains values of u where it is optimal to sell fruit in the open market. The structure of buy, no action, and sell resembles optimal policies that arise in different settings with random demand and exogenous prices (e.g., Taylor 2001, Kazaz 2004). It is important to make the distinction here that our structure is an outcome of the price-setting flexibility, rather than stemming from demand uncertainty. In the deterministic demand version of Kazaz (2004) with its yield-dependent exogenous price model, the "no trade" region of $N(Q)$ does not exist, limiting the optimal policy structure only to buy and sell policies.

Incorporating the optimal second-stage decisions, (2) can be rewritten as follows:

$$\mathrm{E}[\Pi(Q)] = -c_l Q + \begin{cases} \displaystyle\int_{B(Q)} [[p(TB(u)) - c_p - b(u)]TB(u) + b(u)Qu]g(u)\,du \\ \displaystyle\quad + \int_{N(Q)} [(p(Qu) - c_p)Qu]g(u)\,du \\ \displaystyle\quad + \int_{S(Q)} \big[[p(TS(u)) - c_p - s(u)]TS(u) \\ \qquad\qquad + s(u)Qu\big]g(u)\,du. \end{cases} \tag{6}$$

Proposition 5. *The first-stage objective function in* (6) *is concave in* Q.

The expected profit in (6) can be interpreted as a convex combination of the expected profits from three policies: buy additional fruit from the open market, do not trade fruit, and sell fruit in the open market. The concavity of the objective function is assured without enforcing any limitations on the pdf of supply uncertainty.

The value gained from the trading option is the difference between optimal expected profits with and without the option of trading in the open market. The next proposition shows that the value of the trading option is decreasing in the spread between the open market buying and selling functions. When the spread is sufficiently large, the firm does not engage in any buying or selling activity, and the two profit expressions become equal.

Before introducing the proposition, we require additional notation. For any given open market buying and selling functions $b(u)$ and $s(u)$, let $\hat{b}(u) = b(u) + \Delta/2$ and $\hat{s}(u) = s(u) - \Delta/2$. We replace $b(u)$ and $s(u)$ in (5) and (6) with $\hat{b}(u)$ and $\hat{s}(u)$, respectively, and we let $\mathrm{E}[\Pi^*(\Delta)]$ denote the optimal expected profit with the option of trading in the open market and $\mathrm{E}[\Pi^*_{NT}]$ denote the optimal expected profit without the option of trading in the open market. The value gained from the trading option is

$$V_T^*(\Delta) = \mathrm{E}[\Pi^*(\Delta)] - \mathrm{E}[\Pi^*_{NT}].$$

Proposition 6. *The value of trading in the open market,* $V_T^*(\Delta)$, *is decreasing in* Δ.

4.3. The Impact of Yield-Dependent Trading Costs

In this section, we illustrate the impact of yield-dependent cost and revenue structure on a firm's decisions and profits. Our cost and demand functions are based on data provided by two of the leading olive oil producers of Turkey. Figure 2 provides two firms' estimates for the unit purchasing cost of olives as a function of yield. Whereas one firm uses a convex-decreasing yield-dependent purchasing cost, the other firm uses a linearly decreasing yield-dependent cost. We use the data provided by Firm 1 as the primary source of the parameters used in the analysis in this section.

To illustrate the impact of the various forms of yield-dependent trading costs, we consider the following open market trading cost functions in our study: $s(u) = \alpha_\gamma - \beta_\gamma u^\gamma - \delta/2$ and $b(u) = \alpha_\gamma - \beta_\gamma u^\gamma + \delta/2$, where $\gamma \in \{0, 1, 0.5, 0.25\}$ and $\delta \in \{2, 3, 4\}$. The case of $\gamma = 0$ corresponds to static open market purchase and selling cost, representing the traditional modeling approach utilized in literature (e.g., Li and Zheng 2006, Tang and Yin 2007). The case of $\gamma = 1$ represents yield-dependent buying cost and selling

Figure 2 **The Purchasing Cost Functions Provided by Two of the Leading Olive Oil Producers in Turkey**

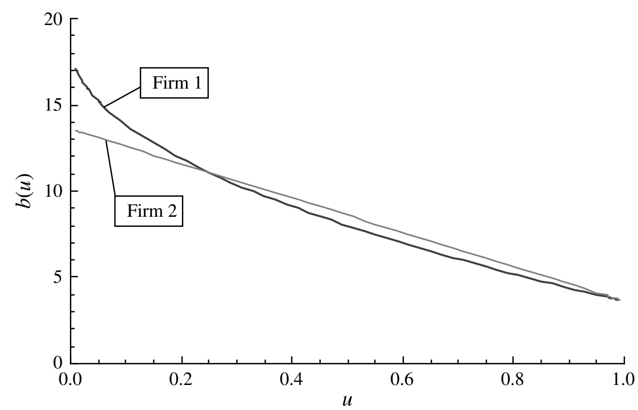


Table 1 **Yield-Dependent Function Parameters**

γ	α_γ	β_γ
0.00	7.09	0.00
1.00	12.07	9.96
0.50	17.05	14.94
0.25	27.01	24.90

revenue functions that are linearly decreasing in the yield. The other two values of γ, 0.50 and 0.25, reflect different degrees of convexity in $b(u)$ and $s(u)$, where convexity increases as γ approaches zero. In our computational experiments, we assume $\tilde{u}$ is uniform on $[0, 1]$.

The values of α_γ and β_γ are listed in Table 1. We set the values of α_γ and β_γ to be consistent with the firms' data while maintaining consistent expected unit revenue and unit cost, i.e., $\mathrm{E}[s(\tilde{u})] + \delta/2 = \mathrm{E}[b(\tilde{u})] - \delta/2 = 7.09$ for all γ. We use the demand function and the cost parameters provided by the firm: the demand is defined as $d(p) = 270{,}000 - 9{,}000\, p$, the leasing cost per unit is $c_l = 2.93$, and the processing cost per unit is $c_p = 2.97$.

As noted before, when the trading costs of the fruit are static, as in the traditional modeling approaches, these thresholds do not change with the realized yield. However, under a yield-dependent purchasing cost and selling revenue, these thresholds depend on the realized yield. Figure 3 exemplifies this behavior by comparing the thresholds under static versus yield-dependent trading costs.

4.3.1. The Impact on Expected Profit. A yield-dependent cost structure is detrimental for the profits of agricultural businesses. Table 2 provides the firm's optimal decision for the amount of farm space to be leased and the corresponding expected profit under various values of the spread (δ) and different forms of trading cost functions (static, linear, and convex). These computational results illustrate that

Figure 3 **The Comparison of the Two Threshold Expressions $TB(u)$ and $TS(u)$ Under a Static Cost ($\gamma = 0, \delta = 3$) and a Convex Decreasing Yield-Dependent Trading Cost ($\gamma = 0.50, \delta = 3$)**

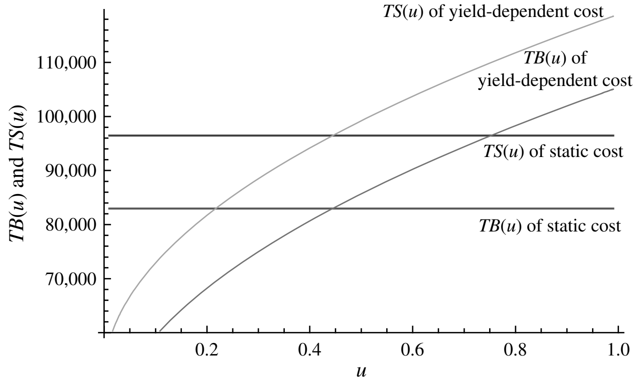


Table 2 **The Optimal Amount of Farm Space to Be Leased and the Optimal Expected Profit Under Various Values of Spread and Different Trading Cost Functions**

δ	γ	Q^*	$\mathrm{E}[\Pi(Q^*)]$
2	0.00	∞	∞
2	1.00	127,212	862,831
2	0.50	119,533	853,834
2	0.25	114,555	851,709
3	0.00	302,250	955,312
3	1.00	131,223	851,308
3	0.50	126,017	841,678
3	0.25	122,375	838,768
4	0.00	206,881	927,348
4	1.00	133,529	840,930
4	0.50	129,879	831,096
4	0.25	127,140	827,782

agribusinesses experience a lower expected profit when the trading costs are defined as yield dependent. This can be seen when the expected profit under $\gamma \in \{1, 0.5, 0.25\}$ is compared with that under $\gamma = 0$ using the same spread, i.e., $\mathrm{E}[\Pi(Q^* \mid \delta, 0 < \gamma \leq 1)] < \mathrm{E}[\Pi(Q^* \mid \delta, \gamma = 0)]$. The reasons for this result are twofold. First, when the realized supply is low, the firm prefers to purchase additional fruit from the open market. Under a yield-dependent cost structure, however, the unit purchasing cost is higher under low realizations, and therefore, the firm pays more for the additional fruit. Second, when the realized supply is high, the firm prefers to sell some of its crop in the open market. Under a yield-dependent cost structure, the unit revenue is smaller at higher realizations of the yield. Thus, the firm sells the fruit in the open market at a lower return. This also explains why the optimal amount of farm space to be leased under a yield-dependent cost structure is less than that under static costs, i.e., $Q^*_{\delta,\, 0<\gamma\leq 1} < Q^*_{\delta,\, \gamma=0}$. Given the trading cost implications (i.e., higher purchasing costs and lower selling revenues), the firm's optimal decision corresponds to a smaller investment in the farm space to reduce the associated risks from supply uncertainty.

Table 2 also exposes a weakness in the assumption of static trading costs in the form of a questionable prediction. Arguably, the expected revenue from selling the fruit in the open market should be greater than the (expected) cost of leasing farm space, i.e., $\mathrm{E}[s(\tilde{u})] > c_l/\bar{u}$. Without this condition, a business that leases farm space to grow fruit and sell in the open market would not be economically viable because expected profit would be negative. However, given $\mathrm{E}[s(\tilde{u})] > c_l/\bar{u}$ and static trading costs, the expected profit is increasing in leased farm space, resulting in an infinite amount of initial investment. Such a scenario exists in Table 2 when $\delta = 2$ (and $\gamma = 0$), where the initial investment in farm space and the expected profit go to infinity. In contrast, under a yield-dependent cost structure, the condition $E[s(\tilde{u})] > c_l/\bar{u}$ does not lead to infinite amount of farm space to be leased as long as the lowest selling revenue is lower than the (expected) cost of leasing, i.e., $s(u_h) < c_l/\bar{u}$.

4.3.2. The Impact on Pricing and Production Decisions. We next compare the impact of yield-dependent and static costs on the second-stage pricing and production decisions. Figure 4 depicts the optimal selling price and Figure 5 demonstrates the optimal total production quantity choices under static and yield-dependent costs, for a given Q in part (a), and under their respective optimal investments in the initial farm space, Q^*, in part (b). As can be seen from the static cost lines in Figures 4 and 5, the optimal price and the optimal production quantity are a constant when the firm engages in buying or selling. Specifically, the firm's pricing and production decisions are not influenced by supply uncertainty when the firm is in the trading mode. These two decisions are only influenced by the yield, and are decreasing in the yield, when the firm is not trading in the open market.

Figure 4 **Optimal Pricing Decisions Under a Static and a Yield-Dependent Cost: (a) Under $Q = 200{,}000$ for Both Cost Functions, and (b) Under Their Respective Optimal Initial Investments, $Q^*_{3,0} = 302{,}250$ Under Static Cost and $Q^*_{3,0.50} = 126{,}017$ Under Yield-Dependent Cost**

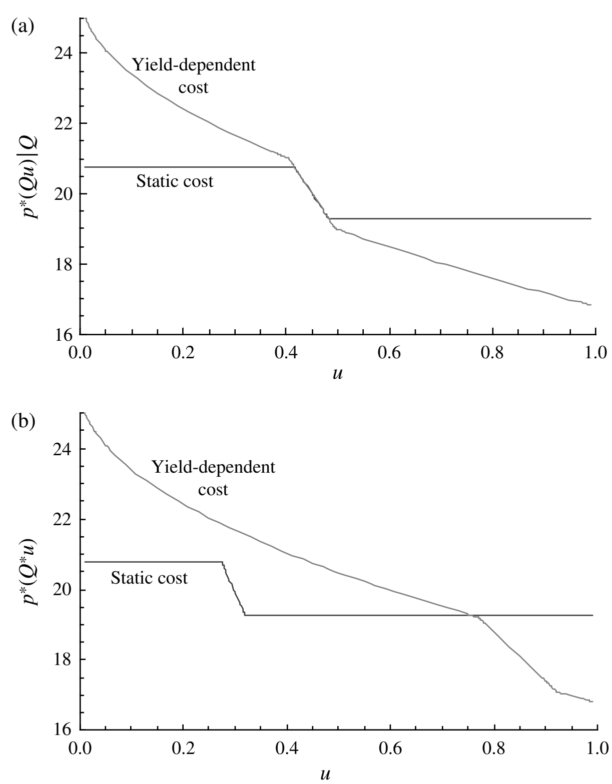
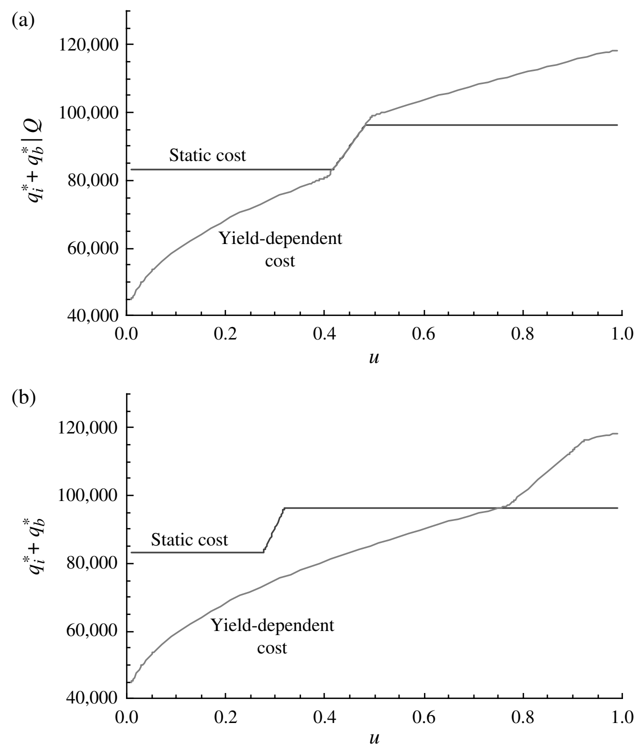


Figure 5 **Optimal Production Amount of the Finished Product Under a Static and a Yield-Dependent Cost: (a) Under $Q = 200{,}000$ for Both Cost Functions, and (b) Under Their Respective Optimal Initial Investments, $Q^*_{3,0} = 302{,}250$ for the Static Cost and $Q^*_{3,0.50} = 126{,}017$ for the Yield-Dependent Cost**



Under yield-dependent costs, Figures 4(b) and 5(b) show that the optimal pricing and production decisions differ characteristically from those developed under static costs. Under static cost, the firm is buying additional fruit from the open market for yield realization between 0 and 0.27, not trading fruit for yield realizations between 0.27 and 0.32, and is selling its fruit when the yield is between 0.32 and 1. At a spread of $\delta = 3$, the region of u values where the optimal price is influenced by the yield is $[0.27, 0.32]$. The region for not trading fruit shrinks with smaller spread values, reducing the difference between the two threshold functions $TB(u)$ and $TS(u)$. When the trading costs are yield dependent, however, the optimal selling price is decreasing and the optimal production quantity is increasing continuously. The firm is buying additional fruit for yield realizations between 0 and 0.77; this is a significantly larger region than that of the static cost. The reason for this result is twofold. First, the firm is now leasing much less farm space. Second, the buying cost is convex decreasing in the yield, and therefore, there is a significant reduction in the purchasing cost after the lowest values of u. As a result, for a large section of u values, the buying cost is cheaper under the yield-dependent cost structure that has its expected cost value equal to the static buying cost. The firm is not trading when the yield parameter is between 0.77 and 0.92, and this is again larger than that of the static cost. Finally, the firm is only selling in the open market when the yield is between 0.92 and 1 under the yield-dependent cost structure, and the firm is in the selling mode for a much larger interval under the static cost function. Aside from leasing a smaller amount of farm space, this result can be explained by the fact that the selling revenue is significantly larger for the static cost than the yield-dependent cost curve in this region. Under the yield-dependent cost structure, rather than selling its fruit in the open market, the firm relies heavily on buying additional fruit from the open market (for realizations in the region of $[0, 0.77]$).

4.3.3. The Cost of Ignoring the Yield-Dependent Cost Structure. As noted in §4.3.1, the use of static open market cost and revenue (set to expected value) in the model exaggerates the firm's profitability when the true cost structure is yield dependent. In this section, we illustrate the impact of decision making under an assumption of static costs when costs are yield dependent.

Consider a firm that applies a static model to determine the amount of farm space to lease when the true cost structure is yield dependent. With spread $\delta = 3$, the firm leases $Q = 302{,}250$ and expects to generate a profit of \$955,312 (see row 5 in Table 2). If the yield-dependent cost structure is convex, say with parameter $\gamma = 0.5$, then the optimal space to lease is $Q^* = 126{,}017$ with an expected profit of \$841,678. Furthermore, leasing $Q = 302{,}250$ under the same yield-dependent cost structure ($\gamma = 0.50$ and $\delta = 3$) reduces the firm's profitability by 15.64% by generating an expected profit of only \$710,034.

Figure 6 provides the expected profit curves under static costs and under yield-dependent trading costs ($\gamma = 0.50$ and $\delta = 3$). The figure illustrates that the static cost assumption exaggerates the amount of initial farm space investment and the profit expectations from this investment. Under a yield-dependent cost structure, the firm invests in less farm space and obtains a smaller expected profit.

Figure 6 **The Comparison of Expected Profit Under a Static Cost ($\gamma = 0$, $\delta = 3$) and a Convex Decreasing Yield-Dependent Trading Cost ($\gamma = 0.50$, $\delta = 3$)**

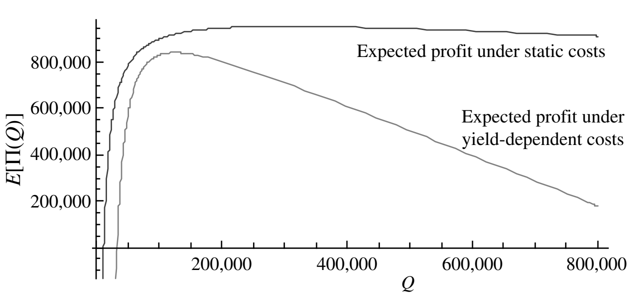


Table 3 **Percentage Reduction in Expected Profit When the Lease Decision Is Based on $\gamma = 0$ (Static Cost Structure) and When the True Cost Structure Is Yield-Dependent with Parameter $\gamma = 1.00$, 0.50, or 0.25**

δ	γ	$1 - \mathrm{E}[\Pi(Q^*_{\delta,\gamma=0}) \mid 0 < \gamma \le 1]/\mathrm{E}[\Pi(Q^*_{\delta,\gamma}) \mid 0 < \gamma \le 1]$ (%)
3	1.00	12.44
3	0.50	15.64
3	0.25	17.77
4	1.00	4.67
4	0.50	5.81
4	0.25	6.60

Table 3 shows the percentage reduction in expected profit by using a static cost decision-making model when the true cost structure is yield dependent. It is necessary to point out that the error is unbounded when the spread is equal to 2, because the optimal amount of farm space to be leased is infinite under static costs and is finite under yield-dependent costs. Consequently, we do not report results for the case of $\delta = 2$ in Table 3.

4.3.4. The Impact of Less Than Perfect Correlation Between Trading Costs and Yield. In the preceding analysis, the fruit trading costs are perfectly correlated with the random yield $\tilde{u}$, i.e., the trading cost is solely determined by the realization of $\tilde{u}$. What if the fruit trading costs are not perfectly correlated with $\tilde{u}$? As shown in Appendix B in the online supplement, the basic structure of the first-stage profit function remains the same. However, the optimal expected profit increases as the model shifts from perfect correlation to less than perfect correlation (see Proposition B1). The impact on the optimal lease quantity, on the other hand, is parameter dependent, i.e., the optimal lease quantity may increase, stay the same, or decrease when the open market trading cost is not perfectly correlated with yield.

5. Value of Fruit Futures

This section investigates the value of fruit futures in mitigating supply risk. Presently, there is no futures market for fruit (e.g., olives, oranges, grapes), and agricultural firms cannot reduce their risk exposure to uncertain trading costs using futures. Suppose that a futures market exists for fruit at a price c_f equal to the expected buying cost per unit, i.e., $c_f = \mathrm{E}[b(\tilde{u})]$. A futures price of $\mathrm{E}[b(\tilde{u})]$ is arguably a lower limit on the price in a viable futures market (i.e., corresponding to the case where expected profit of selling futures short is zero). We introduce Q_f as a first-stage decision variable representing the amount of fruit futures purchased at the beginning of the growing season at price c_f. The revised model is as follows:

Stage 1.

$$\max_{Q,\, Q_f \geq 0} \mathrm{E}[\Pi(Q, Q_f)] = \int_{u_l}^{u_h} (P(Q, Q_f, u) - c_l Q - c_f Q_f) g(u)\, du; \quad (7)$$

Stage 2. Given Q, Q_f, and u,

$$P(Q, Q_f, u) = \max_{\substack{(p, q_i, q_b, q_s) \geq 0 \\ q_i + q_s \leq Qu + Q_f \\ q_i + q_b \leq d(p)}} \pi(p, q_i, q_b, q_s \mid Q, Q_f, u),$$

where

$$\pi(p, q_i, q_b, q_s \mid Q, Q_f, u) = (p - c_p)(q_i + q_b) - b(u) q_b + s(u) q_s. \quad (8)$$

The amount of fruit supply available at the beginning of second stage is $Qu + Q_f$. It is important to note that Propositions 2–4 continue to hold for the second-stage problem, except that Qu is replaced with $Qu + Q_f$. We next prove that the revised objective function is jointly concave in its variables and that a risk-neutral firm cannot increase its expected profit by purchasing fruit futures.

Proposition 7. *The first-stage objective function in* (7) *is jointly concave in Q and Q_f.*

Proposition 8. $\mathrm{E}[\Pi(Q, 0)] \geq \mathrm{E}[\Pi(Q, Q_f)]$ *for any* Q, $Q_f \geq 0$.

Although fruit futures do not improve the firm's profitability in a risk-neutral setting, it can be beneficial under a risk-averse objective function. We next incorporate risk aversion into the model. For the risk-averse model, we assume a concave utility function; the firm gains utility $U(x)$ from profit x, where $U'(x) > 0$ and $U''(x) \leq 0$. We note that the second-stage problem is unaffected by the introduction of a risk-averse utility function (i.e., utility maximization requires that the deterministic second-stage profit be maximized). The risk-averse model can be described as follows:

Stage 1.

$$\max_{Q,\, Q_f \geq 0} \mathrm{E}[U(\Pi(Q, Q_f))] = \int_{u_l}^{u_h} U(P(Q, Q_f, u) - c_l Q - c_f Q_f) g(u)\, du; \quad (9)$$

Stage 2. As defined in (8).

The first-stage objective function in (9) is also jointly concave in the lease quantity and the futures amount.

Proposition 9. *The objective function in* (9) *is jointly concave in Q and Q_f.*

The following proposition shows that, under static trading costs, the firm does not benefit from fruit futures regardless of the firm's degree of risk aversion.

Proposition 10. *If $b(u)$ and $s(u)$ are static, then* $\mathrm{E}[U(\Pi(Q, 0))] \geq \mathrm{E}[U(\Pi(Q, Q_f))]$ *for any* Q, $Q_f \geq 0$.

We next show that fruit futures can add value under a yield-dependent cost structure using numerical illustrations. We employ a constant absolute risk-aversion (CARA) utility function, defined as $U(x) = 1 - e^{-rx}$ with the risk-aversion coefficient $r = 0.1$. For purposes of scaling, we compute the profit in our utility function in currency units of 100,000 (e.g., the firm realizes 50% of the maximum possible utility with realized profit of approximately 700,000). We use the same cost parameters described in §4.3.

Table 4 provides the optimal amount of farm space to be leased Q^*, the optimal amount of futures Q_f^* to be purchased, the optimal fruit commitment at time zero, and the corresponding value of the utility function both in the presence and absence of fruit futures. The following three factors influence the firm's optimal choices:

1. *The shape of yield-dependent cost curve* (γ): When the yield-dependent cost definition switches from linear to a convex form (e.g., from $\gamma = 1$ to $\gamma < 1$), the firm invests more in futures and reduces its investment in the farm space. Note that, for a given value of spread δ, whereas the cost of buying futures remains the same, the trading costs of fruit are different as γ changes. Moreover, the fruit cost in the open market is much higher at significantly low yield realizations under a convex cost function than a linear cost function. As a result, the firm that wants to reduce the negative effects of yield-dependent purchasing cost of the fruit would benefit from an increase in futures. Thus, the firm trades off its leased farm space for an increase in fruit futures.
2. *Spread* (δ): An increase in the spread between the buying cost and selling revenue of fruit results in a higher investment in farm space and lower investment in futures. This is because, as spread increases, the relative cost of buying fruit in the open market becomes more expensive, triggering an increase in the unit cost of futures. Higher values of spread make the leasing option relatively more preferable over the futures and fruit trading options.
3. *Degree of risk aversion* (r): Futures do not always add value under risk aversion and yield-dependent trading costs. A firm operating under yield-dependent trading costs must be sufficiently risk averse to benefit from fruit futures. For example, if the risk-aversion coefficient is $r \leq 0.04$ instead of $r = 0.1$ (i.e., the firm is less risk averse), then the values of Q_f^*

Table 4 The Optimal Investments in the Farm Space to Be Leased, Fruit Futures, Fruit Commitment at Time Zero, and the Utility Function Value in the Presence vs. Absence of Fruit Futures Under Various Values of Spread and Different Trading Cost Functions

δ	γ	Q^*	Q_f^*	$Q^*\bar{u}+Q_f^*$	$U[\Pi(Q^*, Q_f^*)]$	$Q^* \mid Q_f=0$	$U[\Pi(Q^* \mid Q_f=0)]$	$Q^*\bar{u} \mid Q_f=0$
3	0.00	143,143	0	71,572	0.56822	143,143	0.56822	71,572
3	1.00	65,248	50,201	82,826	0.55296	117,034	0.54220	58,517
3	0.50	57,163	56,308	84,890	0.55370	79,079	0.53334	39,539
4	0.00	141,869	0	70,935	0.57872	141,869	0.57872	70,935
4	1.00	81,906	37,646	78,599	0.54188	122,605	0.53512	61,302
4	0.50	67,081	49,425	82,965	0.54157	116,820	0.52451	58,410

in Table 4 are all zero—it is never optimal to purchase futures.

We next investigate how the presence of a futures market affects the expected fruit commitment at time zero and the effect of risk aversion on the optimal farm lease quantity.

5.1. Early Fruit Commitment

The firm's expected fruit commitment is higher in the presence of a fruit futures market. A comparison of $Q^*\bar{u}+Q_f^*$ in the presence of fruit futures versus $Q^*\bar{u} \mid Q_f=0$ in the absence of fruit futures in Table 4 provides this observation. Whereas increased convexity in the yield-dependent cost structure causes a reduction in the expected fruit commitment in the absence of fruit futures, it causes an increase in the initial expected fruit commitment in the presence of fruit futures.

5.2. Farm Lease Quantity

A comparison of column 7 in Table 4 ($Q^* \mid Q_f=0$) with column 3 of Table 2 illustrates the impact of risk aversion on the optimal lease quantity. In all instances in this example, the risk-averse lease quantity is less than the risk-neutral lease quantity. The results mirror the behavior of the classic newsvendor model where it is well known that risk aversion will lead to lower order quantities relative to the optimal risk-neutral quantity (see Eeckhoudt et al. 1995). Agrawal and Seshadri (2000) find the same behavior for a price-setting newsvendor under multiplicative random demand, which is akin to our model of multiplicative random supply. However, in contrast with the newsvendor literature, we find that the optimal lease quantity for a risk-averse firm can be larger than the optimal lease quantity for a risk-neutral firm. For example, at a spread of $\delta=3$ and a linear trading function ($\gamma=1$) under a uniform random yield, the risk-averse model yields $Q_{\mathrm{RA}}^* > Q_{\mathrm{RN}}^*$ when $u_l > 0.15$, e.g., at $u_l=0.3$, $Q_{\mathrm{RA}}^*=217{,}740$ and $Q_{\mathrm{RN}}^*=212{,}662$. When u_l is low (e.g., $u_l \le 0.15$ in the illustrative example), the benefits of trading, and in particular, the opportunity to purchase additional fruit from the open market puts sufficient pressure on the optimal lease amount, and therefore, the firm reduces its initial investment in the farm space quantity. Unlike the demand uncertainty setting, for the firm that operates under supply risk, the initial investment amount does not follow a monotone behavior of decrease or increase when the objective function switches from a risk-neutral definition to a risk-averse perspective.

In sum, we conclude with the following observations: (1) The yield-dependent trading costs present a riskier environment for agricultural business than businesses that operate under static costs. (2) The agricultural firm's supply risk increases with convexity in the yield-dependent trading costs. (3) A sufficiently risk-averse firm would benefit if a futures market were to exist to mitigate the risk of purchasing fruit at a high unit cost at lower yield realizations. (4) Consumers, as well as the firm, benefit from fruit futures because the larger expected fruit commitment translates into a lower expected selling price. (5) In contrast to the newsvendor literature, a risk-averse firm can make a larger initial lease than a risk-neutral firm.

6. Conclusions

This paper investigates the impact of a yield-dependent cost structure on the pricing and production planning decisions of an agricultural firm that operates under supply uncertainty. It is common in agricultural industry to lease farm space to grow fruit, but the harvested amount is random. After collecting its crop supply, the firm retains secondary trading options, corresponding to purchasing additional fruit and selling some (or all of its fruit supply) in the open market before converting it to the finished product. The problem considers the firm's pricing and quantity decisions (buy, produce, and sell).

Traditional modeling approaches are generally inspired by repetitive manufacturing activities where the unit cost of acquiring additional raw materials/ingredients is not influenced by the randomness in supply. However, agricultural businesses operate under a different scheme. The unit purchasing cost of fruit from other growers, for example, changes from year to year, depending on the supply realization in the region. Therefore, the buying cost of the fruit and the selling revenue in the open market depends on the realization of the crop supply (specifically, increases with lower yield). The manuscript provides examples of this cost structure from various agricultural industries.

The analysis shows that expected profit is concave in the amount of farm space to be leased. The results are robust as they are derived under general price-demand functions, without limitations on the pdf of supply uncertainty. Moreover, the unit buying cost and the unit selling revenue are assumed to be decreasing, but there are no assumptions made regarding the shape of these cost functions, i.e., neither convex nor concave.

The paper makes three contributions. First, the paper shows that incorporating the yield-dependent cost structure into the problem has a profound impact on the optimal amount of the initial investment in the farm space and on expected profit. We find that agricultural businesses operate under a higher supply risk because of the yield-dependent trading costs than firms that operate under static costs. Using data available from a Turkish olive oil producer, we show that an agricultural firm would be leasing significantly less farm space under a yield-dependent cost structure compared with static costs (approximately half the size). Moreover, the firm's expected profit is lower under the yield-dependent cost structure. Compared with the yield-dependent cost structure, the benefits from the secondary options of buying and selling fruit in the open market for agricultural firms are exaggerated under a static cost scheme. The firm purchases additional fruit when it realizes a lower supply, and under a yield-dependent cost structure, these costs are higher than that of the static cost parameters. Similarly, the firm prefers to sell some of its fruit when it realizes a high crop supply. In this case again, the yield-dependent revenue is lower than the static costs, leading to smaller returns from selling the fruit in the open market. Thus, the actual value an agricultural firm gains from trading its fruit is smaller because of the yield-dependent cost structure. The analysis demonstrates that the cost of ignoring the yield-dependent cost structure increases tremendously with decreasing values of the spread.

Second, the paper identifies conditions under which an agricultural firm may, and may not, benefit from fruit futures. A risk-neutral firm cannot increase expected profit by purchasing fruit futures. However, a risk-averse firm can use fruit futures to mitigate the risk of high unit purchasing costs with lower yield realizations. Although there are futures for the final product such as orange juice, wine, and olive oil, there are no futures for the fruit used in the making of these finished product. If firms are sufficiently risk averse, then establishing fruit futures can help agricultural firms mitigate the supply risks. The paper shows that the same risk-averse firm would not purchase fruit futures when operating under static costs; thus, fruit futures can add value only when the firm is operating under a yield-dependent cost structure. Moreover, the paper demonstrates that with increasing convexity in the yield-dependent cost structure, the firm increases its investment in fruit futures, and reduces its investment in the leased farm space.

It is well known that under demand uncertainty in the newsvendor problem, a risk-averse firm commits to a smaller initial quantity than a risk-neutral firm. Our third result proves that, under supply uncertainty, the initial investment in quantity (e.g., leased farm space) does not follow a monotone behavior, and moreover, a risk-averse firm may commit to a larger quantity than a risk-neutral firm. The flexibility to trade fruit in the open market puts a downward pressure on the initial quantity, reducing the need to rely on leased farm space. However, as demonstrated in the numerical analysis, a risk-averse firm may still commit to a higher leased farm space than a risk-neutral firm even in the presence of the trading flexibility.

Electronic Companion

An electronic companion to this paper is available on the *Manufacturing & Service Operations Management* website (http://msom.pubs.informs.org/ecompanion.html).

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