Production Planning under Supply and Quality Uncertainty with Two Customer Segments and Downward Substitution

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This paper examines the interrelationships among three forms of operational flexibilities—downward substitution, price setting, and fruit trading—that are valuable to an agricultural firm, specifically to a winemaker, operating under supply and quality uncertainty. The firm leases farm space to grow fruit and obtains two grades of a fruit that are used in making two different end products of differing quality sold to two customer segments. The high-grade fruit is downward substitutable—it can be used in the production of the low-quality end product.

This study makes three sets of contributions to the field of supply chain planning under random supply and quality. First, we show the interrelationships between these three flexibilities. Contradicting earlier findings, pricing flexibility can play a complementary role to the downward-substitution flexibility, increasing its utilization beyond the levels of exogenous price models. Second, we characterize the impact of these flexibilities on the firm's vineyard lease. The addition of fruit-trading flexibility reduces the amount of vineyard lease, however, the complementary behavior of pricing and downward substitution can create an incentive for a higher initial investment. Third, the paper demonstrates the influence of the variation in supply and quality and their correlation on the amount of vineyard lease, expected profit, expected amount and probability of downward substitution. For example, variation in quality does not influence the probability of fruit trading. The firm benefits most from downward substitution in the presence of limited supply variation and significant quality variation.

Keywords: *downward substitution, co-production, supply uncertainty, quality uncertainty, pricing, fruit trading*

1. Introduction

This paper investigates the interactions between three forms of operational flexibility—downward-substitution, pricing and trading flexibilities—for an agricultural firm that faces supply and quality uncertainty. Our work finds motivation from a boutique winery located in the State of New York, and is gaining popularity for its Pinot Noir wines among wine connoisseurs. The firm leases vineyard in order to grow its fruit. Leasing farm space is common among agricultural businesses (see Kazaz 2004, Şaşmaz and Bilgiç 2010, Kazaz and Webster 2011), particularly among wine producers. Unlike owning the land, leasing farm space is economical for an agro-business because it requires a smaller initial capital investment. As explained by an executive at one of the largest wine producers (and distributors) in the world, leasing farm space reduces the potential negative effects of supply and quality problems on the financial performance of the business. For example, when the firm obtains a smaller amount of crop, or experiences quality problems in its grapes, its return on equity is less affected. Thus, leasing farm space is less risky for the operating environment of the wine producer.

We investigate the impact of quality uncertainty, which along with supply uncertainty, is one of the most common challenges faced by a winemaker. Specifically, we examine the decisions made by the winemaker who obtains two grades of fruit crops (grapes) at the end of a growing season: high-quality fruit and low-quality fruit. The amount of these two grades of crops is uncertain for two reasons. First, *supply uncertainty* influences the overall amount of crop obtained, i.e., the sum of high-quality and low-quality crops is not known prior to the growing season. Second, *quality uncertainty* changes the proportion of high-quality vs. low-quality grades of fruit in the amount of total grape supply. Thus, we formulate the problem using two random variables: one variable represents the randomness in supply, corresponding to the random yield of the total crop, and another random variable represents the randomness in the proportion of high-quality versus low-quality grapes. We make no assumptions regarding the distribution of these two random variables. Moreover, we do not require these two random variables to be independent, and allow them to be correlated in our model.

Quality uncertainty in the fruit supply creates a natural segmentation for the wine producer. At the beginning of each growing season, this firm leases vineyard to grow its grapes; for the winemaker motivating our problem, this would be Pinot Noir grapes. At the end of the growing season, the firm obtains two grades of fruit: high-quality and low-quality grapes. The winemaker then produces two different types of end-product (wine). A premium wine is produced by using solely high-quality grapes, and is marketed towards a customer segment with a higher willingness to pay. We refer to this customer segment as the high-end market segment. One key characteristic of this market segment is that the price-elasticity of the demand function is significantly lower.[1] A regular wine is produced for the general public, populated with similar products with a lower selling price. We describe these consumers as the low-end market segment. The regular wine for the low-end market is generally produced by using low-quality grapes.

Our study investigates the influence of and the interrelationship between the following three forms of flexibilities that are present in the life of a winemaker:

1. Downward substitution flexibility: The firm can use some of its high-quality grapes in the making of the low-end wine. The main emphasis of the paper is the use of downward substitution, and therefore, the study focuses on identifying the conditions under which the firm benefits from this flexibility. In the analysis, we report on the expected amount of high-quality crops used for the making of low-end product as well as the probability of downward substitution.
2. Pricing flexibility: The high-end customer segment exhibits a low price-elasticity in its demand function, and the firm determines its selling price for its premium wine sold in the high-end customer segment. Reserve wines are generally considered as premium products as they have unique tastes. For the high-end products such as reserve wines, winemakers can influence the demand by appropriately choosing the selling price. The firm does not have the same price-

[1] Several factors contribute to the creation of high-end segment that has consumers with low price elasticity for this winery: recent wins at several blind-tasting competitions nationwide, a CBS Morning Show coverage for its outstanding Pinot Noir, a positive review from the second-most influential critic, Eric Asimov of the *New York Times*, and a book entitled "Summer in Glass" by Dawson (2011).

setting flexibility for its regular wine targeted for the low-end market segment, which is populated with many similar products at a lower price level. We specifically examine the influence of the price-setting flexibility in the high-end segment on the downward-substitution flexibility. We compare our results from an endogenous price model with those developed under a model that uses exogenous prices.

3. Fruit-trading flexibility: The firm can purchase additional fruit from the open market, or sell its excess fruit in the open market. This implies that, in the event of low crop realizations, the firm can obtain additional high-quality and low-quality grapes from other growers. Alternatively, in the event of excess fruit supply, the firm can sell its high-quality and low-quality grapes in the open market.[2] We consider the influence of the fruit-trading flexibility on the firm's downward substitution decisions.

The paper makes three sets of main contributions. First, we show the interactions between these three forms of flexibilities. While earlier research reports that pricing and downward-substitution flexibilities play a substitutable role, our study proves that these two flexibilities show a complementary behavior. Pricing encourages the firm to downward substitute a greater amount of its high-quality fruit and exercise it more often. Second, our study shows the impact of these three flexibilities on the firm's choice of initial vineyard lease. While fruit-trading flexibility generally reduces the amount of vineyard lease, the pricing and downward substitution flexibilities can create an incentive for a larger initial investment. Third, the paper demonstrates the influence of the variance in supply and quality and the correlation between these two uncertainties on the firm's initial vineyard lease investment, expected profits, expected amount and probability of downward substitution. We show that variation in quality does not influence the probability of fruit trading, and that the firm benefits more from downward substitution under significant variation in quality and limited variation in supply.

The paper is organized as follows: Section 2 presents a literature review. Section 3 introduces the model. Section 4 examines the relationship between the downward substitution and fruit-trading flexibilities with exogenous prices in both market segments. Section 5 demonstrates the influence of the price-setting flexibility, Section 6 shows the impact of the three forms of operational flexibilities on vineyard lease. Section 7 demonstrates the influence of quality and supply uncertainty and their correlation using numerical illustrations. Section 8 compares our model with price-setting in the high-end

[2] Participating wineries help establish the fruit-trading costs through the support of the Cornell University Cooperative Extension prior to the growing season. For example, the 2010 fruit trading costs for popular grapes are established as follows: High-quality Riesling grapes can be purchased at $1900/ton, sold at $1100/ton, whereas low-quality Riesling grapes can be purchased at $1500/ton and sold at $700/ton; high-quality Chardonnay grapes can be purchased at $1450/ton, sold at $1050/ton, and low-quality Chardonnay grapes can be purchased at $1200/ton, and sold at $900/ton; high-quality Cabernet Franc grapes can be purchased at $1500/ton, sold at $800/ton, and low-quality Cabernet Franc grapes can be purchased at $1200/ton, and sold at $750/ton .

segment to previous literature that allows for price-setting in both segments. Section 9 provides conclusions. All proofs are derivations are presented in an online supplement of Appendices A and B.

2. Literature Review

Earlier research in the area of production planning has given particular interest to solving the optimal production problem under supply uncertainty. Yano and Lee (1995) provide an extensive review on lot sizing problem with random yield. Gerchak et al. (1988) and Henig and Gerchak (1990) consider a periodic review production model with random yield and demand. They provide a detailed analysis of a single-period problem and show that the optimal production policy is not affected by yield variability.

In addition to the above publications, many studies have focused on the notion of using pricing and production recourse to mitigate supply and demand uncertainty. Van Mieghem and Dada (1999), Petruzzi and Dada (1999), Dana and Petruzzi (2001), Federgruen and Heching (1999, 2002) and Kocabıyıkoğlu and Popescu (2011) show that the producer uses production and pricing decisions to mitigate demand risk under deterministic supply. Furthermore, Van Mieghem and Dada (1999) demonstrate that, under postponed pricing, production postponement has little benefits to the producer.

While many have studied the price-setting problem under demand uncertainty, few have investigated the problem under supply uncertainty. Li and Zheng (2006) is the first to consider the price-setting problem under supply uncertainty. They investigate a single-product periodic-review model, where price is set at the beginning of each period, and excess demand is not lost, but backlogged. Tang and Yin (2007) also examine a firm's pricing decisions under supply uncertainty, but limit the analysis to a linear demand function in a single market and a discrete uniform distribution representing random supply. Our study departs from these two studies in four ways: (1) our model features co-production that leads to the making of two different end-products and market segmentation; (2) we incorporate quality uncertainty and emphasize downward substitution; (3) unlike the backlogged demand feature of Li and Zheng (2006), our formulation considers lost sales; and (4) we do not make restrictive assumptions regarding the demand function and distribution of uncertainty in our technical derivations. Moreover, we limit the firm's ability to set price in one segment alone in order to reflect the real-world scenario of limited number of consumers with low price elasticity.

In recent times, there has been an emergence of research that considers the option of utilizing a secondary source of supply that allows the firm to adjust its production level. Jones et al. (2001) investigate the production planning decisions for the hybrid seed corn production under random yield and demand; they allow the firm to use an external supply source after the yield is realized. Kazaz (2004) extends this work by incorporating a yield-dependent cost and selling price in the olive oil industry. Kazaz and Webster (2011) incorporate the price-setting and the fruit-trading flexibilities under a yield-dependent cost structure. Our paper departs from these studies as it features: (1) a co-production system that leads to market segmentation, (2) quality uncertainty, and (3) downward substitution.

There is a considerable amount of studies that investigate co-production systems. Bitran and Dasu (1992) investigate the ordering policies for multiple items with stochastic yield and substitutable demand using a dynamic programming formulation. Bitran and Gilbert (1994) extend this work by considering the production decisions in the semiconductor industry, and provide several practical heuristics with conditions for downward substitution decisions. Nahmias and Moinzadeh (1997) also investigate the problem of downward substitution of randomly-graded yield by formulating a continuous review EOQ-type model. Bassok et al. (1999) consider the production planning problem under downward-substitutable random demand in a single period. Their study shows that a greedy allocation policy is optimal, and demonstrates the conditions under which downward substitution is beneficial. Hsu and Bassok (1999) examine a similar problem by incorporating random yield. Their study shows that optimal solutions can be achieved by using several methods, and, computationally, the greedy algorithm is the most efficient solution approach. One main characteristic that is common among these works in the area of co-production is that prices are exogenous. Moreover, they ignore the influence of a secondary source of supply.

Other studies in the area of co-production include the work of Gerchak et al. (1996), which investigates a parallel production process, where one process produces randomly-graded yield, while the other produces only low-grade yield. Öner and Bilgiç (2008) consider products that cannot be substituted, but extend the economic lot scheduling model to include uncontrolled co-production. Motivated from the beef industry, Boyabatli et al. (2011), study the procurement problem with fixed proportions technology, i.e., the proportion of high-quality vs. low-quality output is fixed. They characterize optimal sourcing strategies based on long-term contracts and procuring from the spot market. Boyabatli (2011) extends this study on fixed proportions technology and investigates the procurement problem with multiple quantity-flexible contracts, demonstrating the benefits of dual sourcing. Our paper differs from these papers as it features pricing flexibility and random proportions.

Beyond the realm of exogenous price, Bish and Wang (2004) investigate the joint quantity and price-setting problem under perfect supply and uncertain demand for two products, and show that the firm can benefit from investment in flexible resources. The closest match for our study is Tomlin and Wang (2008) who also examine the pricing and operational recourse in a co-production system. They show that the producer benefits more from adopting recourse pricing policy, i.e., delaying the pricing decision until after all uncertainty is realized, than from adopting a downward substitution policy. Our current work studies a similar problem to Tomlin and Wang (2008), but differs from their work in the following ways:

1. We study a production planning problem with co-production that allows for the utilization of the open market. We also investigate the impact of trading flexibilities on the optimal investment, downward substitution, and pricing decisions;
2. Our work resembles the real-world scenario that the firm has the ability to set the selling price only in the high-end segment of the market as the consumers tend to be less sensitive to changes in price.
3. Tomlin and Wang (2008) examine only the influence of quality uncertainty, whereas our study investigates the influence both supply and quality uncertainty, and shows the influence of both supply and quality variation on the optimal downward substitution and fruit trading decisions.
4. We do not consider the problem of demand uncertainty as it has been shown in Tomlin and Wang (2008) that pricing and operational recourse dominate advance pricing and allocation decisions.

3. Problem Definition and the Model

This section presents the modeling approach used in the agricultural firm that experiences supply and quality uncertainty, and produces two different products to serve its two customer segments. The problem is formulated as a two-stage stochastic program. In the first stage, corresponding to the growing season, the firm determines the amount of farm space to be leased, denoted Q, at a unit cost of c_l in order to maximize expected profit in the presence of supply and quality uncertainty. At the end of the growing season, the firm realizes two grades of fruit influenced by two separate random variables. Randomness in the total crop supply is represented with a stochastically proportional random variable $\tilde{u}$, and its realization is denoted with u defined on a support $[u_l, u_h]$. Randomness in quality refers to the proportion of high-grade versus low-grade fruit obtained from the leased farm space, and is described by a stochastically proportional variable $\tilde{\alpha}$ defined on a support $[\alpha_l, \alpha_h]$, where α is the realized proportion of the high-quality fruit crop and $(1-\alpha)$ is the proportion of low-quality fruit crop. Our model allows for correlation to exist between the supply and quality random variables as they follow a joint probability density function (pdf) $g(u, \alpha)$ and a cumulative distribution function (cdf) $G(u, \alpha)$. Thus, the first-stage objective function can be written as follows:

$$\max_{Q\geq 0} E\left[\Pi\left(Q\right)\right] = -c_l Q + E\left[PA\left(Q,\tilde{u},\tilde{\alpha}\right)\right] \tag{1}$$

where $PA(Q, u, \alpha)$ is the optimal profit from the second stage given realizations u and α.

At the end of the first stage (growing season), the firm collects two grades of fruit supply; the realized amount of high-quality fruit crop is $Qu\alpha$ and the realized amount of low-quality fruit crop is $Qu(1-\alpha)$. Quality uncertainty creates this natural market segmentation for the winemaker where the firm produces two versions of the final product in order to serve two customer segments classified as high-end and low-end segments. A premium wine is produced from higher quality grapes, targeting a high-end customer segment that is less sensitive to the selling price. A regular wine is produced from the low-quality grapes, targeting a more price-sensitive low-end market segment. The pressing cost of high-quality fruit to obtain premium wine is defined as c_{pH} and the pressing cost of low-quality fruit to make regular wine as c_{pL}.

At the beginning of the second stage, the winemaker makes five sets of decisions: the optimal values of (1) the selling price of high-quality final product p_H, (2) the amount fruit crop (realized supply of high- and low-quality fruit supply) to be used in the production of high- and low-quality final products, denoted q_{IH} and q_{IL}, respectively, (3) the amount of additional high- and low-quality fruit to be purchased from other growers in the open market denoted q_{BH} and q_{BL}, at unit costs of b_H and b_L, respectively, (4) the amount of high- and low-quality fruit supply to be sold in the open market without being converted to the final product denoted q_{SH} and q_{SL}, at unit selling prices of s_H and s_L, respectively, and (5) the amount of high-quality fruit to be downward substituted for the production of low-end product, denoted w. It is important to note that the values of b_H, s_H, b_L and s_L are available to the firm prior to the growing season (see footnote 2). Due to the differences in fruit quality, we have $s_H > s_L$ and $b_H > b_L$. In addition, $b_H > s_H$, and $b_L > s_L$, which reflects the fact that the firm cannot make profit from buying the fruit in the open market and immediately selling it in the same market (i.e., no arbitrage). As a consequence of the inequalities in open market buying and selling prices, we have the following constraints:

$$q_{IH} + q_{SH} + w = Qu\alpha, \tag{2}$$

$$q_{IL} + q_{SL} = Qu(1 - \alpha). \tag{3}$$

Constraint (2) states that realized high-quality fruit yield is allocated among internal production, open market selling, and downward substitution (i.e., it is never more profitable to simply discard fruit rather than selling in the open market, and it is never profitable to buy high-quality fruit for the purposes of downward substitution). Similarly, constraint (3) states that realized low-quality fruit yield is allocated among internal production and open market selling.

The demand in each customer segment is represented by $D_H(p_H)$ and D_L, respectively. In the high-end customer segment, we assume that the demand is price-sensitive, and is decreasing in p_H. We denote the inverse of demand function $p_H(D_H)$, and assume that the revenue function in the high-end customer segment (i.e., $p_H(D_H)D_H)$) is concave, i.e., $2p_H'(D_H) + p_H''(D_H)D_H \leq 0$.

The second-stage problem can be described as maximizing profit from the production and sale of the two end products for a given realization of high- and low-quality fruit, $Qu\alpha$ and $Qu(1 - \alpha)$, respectively.

$$PA(Q, u, \alpha) = \max_{\substack{p_H, q_{IH}, q_{IL}, q_{BH}, \\ q_{BL}, q_{SH}, q_{SL}, w \geq 0}} \left\{ \begin{array}{l|l} p_H \min\left\{\left(q_{IH} + q_{BH}\right), D_H\left(p_H\right)\right\} - & \\ c_{pH}\left(q_{IH} + q_{BH}\right) - b_H q_{BH} + s_H q_{SH} + & \\ p_L \min\left\{\left(q_{IL} + q_{BL} + w\right), D_L\right\} - c & \text{s.t. (2) \& (3)} \\ {}_{pL}\left(q_{IH} + q_{BH} + w\right) - b_L q_{BL} + s_L q_{SL} & \end{array} \right\}$$

$$= \max_{\substack{p_H, q_{IH}, q_{IL}, w \geq 0 \\ q_{IH} \leq \min\{D_H(p_H), Qu\alpha\} \\ q_{IL} \leq \min\{D_L, Qu(1-\alpha)\} \\ w \leq Qu\alpha}} \left\{ \begin{array}{l} (p_H - c_{pH} - b_H) D_H(p_H) + (b_H - s_H) q_{IH} + s_H Qu\alpha + \\ (p_L - c_{pL} - b_L) D_L + (b_L - s_L) q_{IL} + s_L Qu(1-\alpha) + (b_L - s_H) w \end{array} \right\}. \qquad (4)$$

We develop and analyze eleven variants of the problem in order to identify the interactions among the three forms of flexibility. We make the following assumption regarding profit margins.

A1: The firm makes profit from buying low-quality fruit, converting it into final product, and selling the final product, i.e., $p_L - c_{pL} - b_L > 0$. Similarly, for models in which the high-end price is exogenous, $p_H - c_{pH} - b_H > 0$.

Table 1 provides the list of flexibilities included in each of these eleven models.

Flexibility \ Model	M1	M2	M3	M4	M5	M6	M7	M8	M9	M10	M11
Downward substitution		●		●		●		●		●	
Fruit trading			●	●			●	●			●
Pricing in high-end					●	●	●	●	●	●	●
Pricing in low-end									●	●	●

Table 1. Flexibilities included in each of the eleven model variants.

M1 does not feature any of the three flexibilities, and M8 is the model described in (1) – (4). M2 and M4 feature the downward substitution flexibility under exogenous prices, and M6 and M8 under the pricing flexibility in the high-end segment. M3 and M4 feature the fruit-trading flexibility under exogenous prices, and M7 and M8 under the pricing flexibility in the high-end segment. M9, M10 and M11 are developed in Section 8 in order to provide a comparison of the pricing flexibility present only in the high-end segment, representing the life of a winemaker, with the hypothetical scenario when the firm can set prices in both segments; M10 corresponds to the model of Tomlin and Wang (2008).

Before proceeding with the analysis of the stochastic supply and quality problem presented in (1) – (4), we briefly examine the properties of the problem with deterministic supply and quality. In the deterministic variant of the problem, we replace the supply random variable $\tilde{u}$ with its mean $\bar{u}$ and the quality random variable $\tilde{\alpha}$ with its mean $\bar{\alpha}$. The firm leases Q units of farm space, and realizes high-quality crop yield of $Q\bar{u}\bar{\alpha}$ and low-quality crop yield of $Q\bar{u}(1-\bar{\alpha})$. Eliminating the trading and downward substitution flexibilities, i.e., $q_{BH} = q_{SH} = q_{BL} = q_{SL} = w = 0$, the firm converts its entire crop to the final products. Assuming no demand restriction in the low-end segment, the selling price in the high-end clears the production, i.e., $Q\bar{u}\bar{\alpha} = q_{IH} = D_H(p_H)$, and the firm converts its entire crop of the low-quality fruit to the low-end product to be sold in the low-end segment, i.e., $Q\bar{u}(1-\bar{\alpha}) = q_{IL} = D_L$.

Appendix B provides derivations for the optimal amount of farm space to be leased and the corresponding profit under deterministic supply and quality. The analysis leads to the following observations: (1) Expected profit under stochastic supply and quality is less than that of the deterministic supply and quality; (2) Closed-form expressions can be provided when a demand function is defined. When demand in each market segment is linear, for example, the optimal amount of farm space and the corresponding profit under stochastic supply and quality decreases in the coefficient of variation, denoted $cv[u\alpha]$.

Under deterministic supply and quality, the firm engages in the lease opportunity only when the unit leasing cost is less than the expected fruit purchasing cost.

Remark 1. *a) If the unit cost of leasing is greater than or equal to the expected buying cost of high and low-quality fruit from other growers, i.e.,* $c_l \geq b_H E[\tilde{u}\tilde{\alpha}] + b_L E[\tilde{u}(1-\tilde{\alpha})]$*, then the firm relies solely on fruit purchasing and does not lease vineyard space* ($Q^* = 0$)*. b) If the unit cost of leasing is smaller than or equal to the expected fruit selling revenue in the open market, i.e.,* $c_l \leq s_H E[\tilde{u}\tilde{\alpha}] + s_L E[\tilde{u}(1-\tilde{\alpha})]$*, then the firm leases as much as it can because the optimal value of* Q^* *approaches infinity. c) If* $s_H E[\tilde{u}\tilde{\alpha}] + s_L E[\tilde{u}(1-\tilde{\alpha})] < c_l < b_H E[\tilde{u}\tilde{\alpha}] + b_L E[\tilde{u}(1-\tilde{\alpha})]$*, then* $Q^* > 0$ *and is finite.*

Under stochastic supply and quality, however, the firm can invest in vineyard lease even if the expected cost of buying fruit is greater than the unit cost of leasing. We next proceed with the analysis of stochastic supply and quality.

4. Fruit-Trading Flexibility and Downward Substitution (with Exogenous Pricing)

In this section, we treat price in both market segments as exogenous in order to identify the relationship between fruit-trading and downward substitution flexibilities in the presence of supply and quality uncertainty. This is accomplished with the comparison of M1 through M4.

4.1 The Case of No Trading (Buying or Selling) of Fruit

To create a benchmark for the benefits of additional flexibilities, we begin by investigating a classic production planning problem under supply and quality uncertainty, where the firm does not have the flexibility to downward substitute or trade once the fruit yield is realized (M1). We define the following regions of supply and quality random realizations for a given lease amount:

$R1(Q) = \{(u, \alpha) : Qu\alpha \leq D_H \text{ and } Qu(1-\alpha) < D_L\}$

$R2(Q) = \{(u, \alpha) : Qu\alpha \leq D_H \text{ and } Qu(1-\alpha) \geq D_L\}$

$R3(Q) = \{(u, \alpha) : Qu\alpha > D_H \text{ and } Qu(1-\alpha) < D_L\}$

$R4(Q) = \{(u, \alpha) : Qu\alpha > D_H \text{ and } Qu(1-\alpha) \geq D_L\}$

The firm converts its entire crop yield into the final product when the realized high- or low-quality crop is less than their respective demand, i.e., when $Qu\alpha \leq D_H$ or $Qu(1-\alpha) \leq D_L$. This situation is represented by regions $R1(Q)$ and $R2(Q)$ for the high-end fruit and $R1(Q)$ and $R3(Q)$ for the low-end fruit. On the other hand, when the realized yield of high- or low-quality crop is high and is greater than the demand, i.e., $Qu\alpha > D_H$ or $Qu(1-\alpha) > D_L$, the firm converts only the portion of the crop that would satisfy the demand to the final product; these are represented by regions R3(Q) and R4(Q) for the high-end fruit and R2(Q) and R4(Q) for the low-end fruit.

Using the above definition of four regions of realized crop supply, the optimal second-stage decisions for M1 can be expressed as follows:

$$\left(q_{IH}{}^{*}, q_{IL}{}^{*}\right) = \begin{cases} \left(Qu\alpha, Qu(1-\alpha)\right) & \text{if } (u,\alpha) \in \text{R1}(Q) \\ \left(Qu\alpha, D_L\right) & \text{if } (u,\alpha) \in \text{R2}(Q) \\ \left(D_H, Qu(1-\alpha)\right) & \text{if } (u,\alpha) \in \text{R3}(Q) \\ \left(D_H, D_L\right) & \text{if } (u,\alpha) \in \text{R4}(Q) \end{cases}$$

We next analyze M2, which adds downward substitution flexibility to M1. Downward substitution is beneficial only in region R3(Q) where the firm experiences an excess amount of high-quality fruit and an insufficient amount of low-quality fruit. We denote the shortage in the low-end as Δ, i.e.,

$$\Delta = D_L - Qu(1-\alpha),$$

and divide region R3(Q) into the following sub-regions:

R3a(Q) = $\{(u, \alpha) : D_H < Qu\alpha \le D_H + \Delta \text{ and } Qu(1-\alpha) < D_L\}$

R3b(Q) = $\{(u, \alpha) : D_H + \Delta < Qu\alpha \text{ and } Qu(1-\alpha) < D_L\}$

Region R3a(Q) represents a situation in which the excess yield of high-quality fruit is not sufficient to cover the shortages of the low-end final product and thus the firm converts all the excess high-quality fruit into low-end final product, i.e., $w^* = Qu\alpha - D_H$. Region R3b(Q) represents the scenario in which there is a high yield realization of high-quality crop and thus the firm converts a portion of the remaining high-quality fruit to satisfy the demand of low-end final product, i.e., $w^* = \Delta = D_L - Qu(1-\alpha)$. Figure 1 illustrates the uses of the high-end fruit with the boundary between R3a and R3b at $(D_H + \Delta)/Q$.

Using the above four regions of realized crop supply, the optimal second-stage decisions for M2 are:

$$\left(q_{IH}{}^{*}, w^{*}, q_{IL}{}^{*}\right) = \begin{cases} \left(Qu\alpha, 0, Qu(1-\alpha)\right) & \text{if } (u,\alpha) \in \text{R1}(Q) \\ \left(Qu\alpha, 0, D_L\right) & \text{if } (u,\alpha) \in \text{R2}(Q) \\ \left(D_H, Qu\alpha - D_H, Qu(1-\alpha)\right) & \text{if } (u,\alpha) \in \text{R3a}(Q). \\ \left(D_H, \Delta, Qu(1-\alpha)\right) & \text{if } (u,\alpha) \in \text{R3b}(Q) \\ \left(D_H, 0, D_L\right) & \text{if } (u,\alpha) \in \text{R4}(Q) \end{cases} \tag{5}$$

4.3 Incorporating Fruit-Trading Flexibility (Buying $q_{BH}, q_{BL} \ge 0$ and Selling $q_{SH}, q_{SL} \ge 0$)

We next incorporate the flexibility for the firm to trade fruit in the open market without downward substitution, as featured in M3. In this scenario, it follows from assumption A1 that the firm buys fruit from the open market when the realized amount of internally grown fruit is less than the demand, i.e., $q_{BH}{}^{*}$

$= D_H - Qu\alpha \geq 0$ and $q_{BL}{}^* = D_L - Qu(1-\alpha) = \Delta \geq 0$. Alternatively, when the realized amount of fruit crop exceeds the desired demand level, then the firm sells the unused crop in the open market, i.e., $q_{SH}{}^* = Qu\alpha - D_H \geq 0$ and $q_{SL}{}^* = Qu(1-\alpha) - D_L = -\Delta \geq 0$. Accordingly, the optimal second-stage decisions for M3 are:

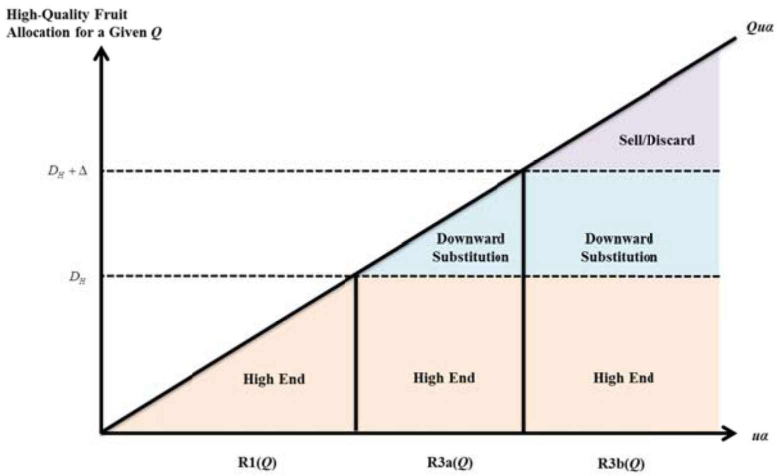

Figure 1. Optimal downward substitution quantity under exogenous pricing.

$$\begin{pmatrix} q_{IH}{}^*, q_{BH}{}^*, q_{SH}{}^*, \\ q_{IL}{}^*, q_{BL}{}^*, q_{SL}{}^* \end{pmatrix} = \begin{cases} \begin{pmatrix} Qu\alpha, D_H - Qu\alpha, 0 \\ Qu(1-\alpha), \Delta, 0 \end{pmatrix} & \text{if } (u,\alpha) \in \mathrm{R1}(Q) \\ \begin{pmatrix} Qu\alpha, D_H - Qu\alpha, 0 \\ D_L, 0, -\Delta \end{pmatrix} & \text{if } (u,\alpha) \in \mathrm{R2}(Q) \\ \begin{pmatrix} D_H, 0, Qu\alpha - D_H, \\ Qu(1-\alpha), \Delta, 0 \end{pmatrix} & \text{if } (u,\alpha) \in \mathrm{R3}(Q) \\ \begin{pmatrix} D_H, 0, Qu\alpha - D_H, \\ D_L, 0, -\Delta \end{pmatrix} & \text{if } (u,\alpha) \in \mathrm{R4}(Q) \end{cases}.$$

Next, we analyze M4 where the firm has both the flexibility to downward substitute and trade fruit in the open market. In this model, the downward substitution option is only viable when savings from the utilization of high-quality fruit crop in the making of the low-end product outweighs the selling price of high-quality crop in the open market, i.e., $w^* > 0$ if and only if $s_H < b_L$. Otherwise, downward substitution does not occur as it is more beneficial for the firm to sell the excess crop in the open market. It is important to note that $s_H < b_L$ for the winemakers motivating our study (see popular grapes prices in footnote 2). Therefore, to investigate the benefit from downward substitution, for the remainder of this paper, we assume $s_H < b_L$. The objective function in (4) can be rewritten as:

$$PA(Q,u,\alpha)=\max_{\substack{p_H,q_{IH},q_{IL}\ge 0\\ q_{IH}\le\min\{D_H(p_H),Qu\alpha\}\\ q_{IL}\le\min\{D_L,Qu(1-\alpha)\}}}\left\{\begin{array}{l}(p_H-c_{pH}-b_H)D_H+(b_H-s_H)q_{IH}+s_HQu\alpha+\\(p_L-c_{pL}-b_L)D_L+(b_L-s_L)q_{IL}+s_LQu(1-\alpha)+\\(b_L-s_H)\min\{(Qu\alpha-D_H),D_L-Qu(1-\alpha)\}\end{array}\right\}$$

Similar to the case where there is no trading option, the firm benefits from downward substitution when the realization of high-quality crop is high and there is an insufficient amount of low-quality fruit. In region R3a(Q), the excess amount of high-quality crop is smaller than the shortage in the low-quality fruit, and thus, the firm benefits from downward substitution, i.e., $w^* = Qu\alpha - D_H$, and saves $(b_L - s_H)(Qu\alpha - D_H)$ from purchasing additional low-quality fruit from the open market. On the other hand, in region R3b(Q), the supply of high-quality crop is sufficiently high to cover the shortage in the low-quality fruit; specifically, $w^* = D_L - Qu(1-\alpha) = \Delta$ with a resulting savings of $(b_L - s_H)\Delta$. Accordingly, the optimal second-stage decisions for M4 can be expressed as follows:

$$\begin{pmatrix}q_{IH}^*,q_{BH}^*,w^*,q_{SH}^*,\\q_{IL}^*,q_{BL}^*,q_{SL}^*\end{pmatrix}=\begin{cases}\begin{pmatrix}Qu\alpha,D_H-Qu\alpha,0,0,\\Qu(1-\alpha),\Delta,0\end{pmatrix} & \text{if }(u,\alpha)\in \mathrm{R1}(Q)\\ \begin{pmatrix}Qu\alpha,D_H-Qu\alpha,0,0,\\D_L,0,-\Delta\end{pmatrix} & \text{if }(u,\alpha)\in \mathrm{R2}(Q)\\ \begin{pmatrix}D_H,Qu\alpha-D_H,0,0,\\D_L,D_L-Qu\alpha+D_H,0\end{pmatrix} & \text{if }(u,\alpha)\in \mathrm{R3}a(Q)\\ \begin{pmatrix}D_H,0,\Delta,0,\\Qu(1-\alpha),0,0\end{pmatrix} & \text{if }(u,\alpha)\in \mathrm{R3}b(Q)\\ \begin{pmatrix}D_H,0,0,Qu\alpha-D_H,\\D_L,0,-\Delta\end{pmatrix} & \text{if }(u,\alpha)\in \mathrm{R4}(Q)\end{cases}. \qquad (6)$$

It is common wisdom that the introduction of an additional form of flexibility, as in the form of fruit-trading flexibility, would reduce the utilization of other forms of flexibility (e.g., downward substitution) present in the environment (e.g., Van Mieghem and Dada 1999, Jones et al. 2001, Kazaz 2004, and Tomlin and Wang 2008). However, as shown in the following proposition, the additional flexibility to trade fruit in the open market does not influence the probability of downward substitution and the expected amount of downward substitution. Thus, in the absence of the pricing flexibility, these two forms of flexibility neither present a substitutable role, nor play a complementary role to each other.

Proposition 1. *In the absence of pricing flexibility, for a given Q, the probability of downward substitution and the expected amount of downward substitution does not change with the additional flexibility of fruit-trading in the open market.*

5. The Combination of Downward Substitution, Pricing, and Fruit-Trading Flexibilities

In this section, we develop the structural properties of M5, M6, M7 and M8, where the firm has the pricing flexibility in the high-end segment.

5.1 Price-Setting Flexibility in the High-End Segment and Downward Substitution

We begin our analysis by assuming that the firm does not have the ability to acquire or sell fruit in the open market, or downward substitute its high-quality fruit for the production of its low-end product, which corresponds to M5, i.e. $q_{BH} = q_{SH} = q_{BL} = q_{SL} = w = 0$. Under the price-setting flexibility in the high-end market segment, the amount of high-quality fruit realization influences the pricing and quantity decisions. When the realized amount of high-quality fruit is high, the firm has the ability to set the profit-maximizing price and convert only the amount of fruit that corresponds to the demand at the profit-maximizing price. On the other hand, when the high-quality fruit realization is limited, the firm converts all the realized supply into the final product and sells at the market clearing price. In the case of low-end product, the optimal production decision follows our analysis of the case presented in Section 4.1.

In the following proposition, we define a threshold for the production amount in the high-end segment. The threshold, denoted TP_H, is the optimal amount of high-end product to produce when there is no constraint on the supply of high-quality fruit.

Proposition 2. *The threshold for the amount of high-end product to be produced from the internal resource for* M5 *is* $TP_H = -\left[p_H^* - c_{pH}\right] D_H'\left(p_H^*\right)$.

We use the threshold amount to define the following regions of supply and quality random realizations for a given lease amount and high-end selling price:

R1(Q) = $\{(u, \alpha) : Qu\alpha \le TP_H \text{ and } Qu(1-\alpha) < D_L\}$

R2(Q) = $\{(u, \alpha) : Qu\alpha \le TP_H \text{ and } Qu(1-\alpha) \ge D_L\}$

R3(Q) = $\{(u, \alpha) : Qu\alpha > TP_H \text{ and } Qu(1-\alpha) < D_L\}$

R4(Q) = $\{(u, \alpha) : Qu\alpha > TP_H \text{ and } Qu(1-\alpha) \ge D_L\}$

In regions R1(Q) and R2(Q), the firm sets the high-end price to clear the market, $p_H(Qu\alpha)$. In regions R3(Q) and R4(Q), the firm has excess supply of high-quality fruit and sets the high-end price to sell the threshold quantity. The optimal second-stage quantity decisions for M5 are

$$\left(q_{IH}^*, q_{IL}^*\right) = \begin{cases} \left(Qu\alpha, Qu(1-\alpha)\right) & \text{if } (u,\alpha) \in \mathrm{R1}(Q) \\ \left(Qu\alpha, D_L\right) & \text{if } (u,\alpha) \in \mathrm{R2}(Q) \\ \left(TP_H, Qu(1-\alpha)\right) & \text{if } (u,\alpha) \in \mathrm{R3}(Q) \\ \left(TP_H, D_L\right) & \text{if } (u,\alpha) \in \mathrm{R4}(Q) \end{cases}$$

and the optimal high-end price is $p_H^* = p_H(q_{IH}^*)$.

We next investigate M6 which incorporates downward substitution in addition to the pricing flexibility in the high-end segment. The second-stage problem in M6 can be rewritten as:

$$PA(Q,u,\alpha) = \max_{\substack{p_H, q_{IH}, q_{IL} \geq 0 \\ q_{IH} \leq \min\{D_H(p_H), Qu\alpha\} \\ q_{IL} \leq \min\{D_L, Qu(1-\alpha)\}}} \left\{ \left(p_H - c_{pH}\right) q_{IH} + \left(p_L - c_{pL}\right) \left[q_{IL} + \min\left\{ \left(Qu\alpha - q_{IH}\right)^+, \left(D_L - q_{IL}\right)^+ \right\} \right] \right\}$$

When $Qu(1 - \alpha) \geq D_L$, which corresponds to regions R2(Q) and R4(Q) above, the low-end market has sufficient supply; there is no downward substitution and the optimal decisions that apply in R2(Q) and R4(Q) for M5 are also optimal for M6.

To consider the case of $Qu(1 - \alpha) < D_L$, we require a threshold quantity. Recall that TP_H is the optimal high-end production quantity for M5 when there is no limit on high-end supply. We similarly define a threshold production amount for M6. In particular, TP_H^D denotes the optimal high-end production amount when high-quality fruit that is not used for high-end production gains unit profit $p_L - c_{PL}$ through downward substitution. The threshold in the presence of downward substitution is smaller than the threshold without downward substitution.

Proposition 3. *The threshold for the amount of high-end product to be produced from the internal resource for* M6 *is* $TP_H^D = -\left(p_H^* - c_{pH} - \left(p_L - c_{pL}\right)\right) D_H{}'\left(p_H^*\right) < TP_H$.

Recall that $\Delta = D_L - Qu(1 - \alpha)$ is the low-end shortage amount. We replace regions R1(Q) and R3(Q) with the following sub-regions:

R1a(Q) = $\{(u, \alpha) : Qu\alpha \leq TP_H^D \text{ and } Qu(1 - \alpha) < D_L\}$

R1b(Q) = $\{(u, \alpha) : TP_H^D < Qu\alpha \leq TP_H^D + \Delta \text{ and } Qu(1 - \alpha) < D_L\}$

R3a(Q) = $\{(u, \alpha) : TP_H^D + \Delta < Qu\alpha \leq TP_H + \Delta \text{ and } Qu(1 - \alpha) < D_L\}$

R3b(Q) = $\{(u, \alpha) : TP_H + \Delta < Qu\alpha \text{ and } Qu(1 - \alpha) < D_L\}$

An interesting transition occurs between region R1b(Q) and R3a(Q). In region R1b(Q) the firm is able to produce the optimal high-end threshold quantity under downward substitution (TP_H^D), then downward substitute the balance to satisfy a portion of the shortage in the low-end segment (Δ). In region R3a(Q), the firm has more than enough to cover TP_H^D and the shortage Δ. However, once the firm has allocated TP_H^D to high-end production and has downward substituted the quantity Δ, the change in profit associated with allocating more volume to the high end segment is positive, and thus the firm allocates the balance of high-quality fruit to high-end production (up to TP_H; see Figure 2).[3] Accordingly, the optimal second-stage quantity decisions for M6 are

[3] This is the optimal allocation because, in the event that the total allocated to the high end is less than TP_H, the firm would lose profit if a portion of the downward substitution amount is shifted to high-end production (follows from the definition of TP_H^D).

$$\left(q_{IH}^{\ *}, w^{*}, q_{IL}^{\ *}\right) = \begin{cases} \left(Qu\alpha, 0, Qu(1-\alpha)\right) & \text{if } (u,\alpha) \in \text{R1a}(Q) \\ \left(TP_H^D, Qu\alpha - TP_H^D, Qu(1-\alpha)\right) & \text{if } (u,\alpha) \in \text{R1b}(Q) \\ \left(Qu\alpha, 0, D_L\right) & \text{if } (u,\alpha) \in \text{R2}(Q) \\ \left(Qu\alpha - \Delta, \Delta, Qu(1-\alpha)\right) & \text{if } (u,\alpha) \in \text{R3a}(Q) \\ \left(TP_H, \Delta, Qu(1-\alpha)\right) & \text{if } (u,\alpha) \in \text{R3b}(Q) \\ \left(TP_H, 0, D_L\right) & \text{if } (u,\alpha) \in \text{R4}(Q) \end{cases} \tag{7}$$

and the optimal high-end price is $p_H^{\ *} = p_H(q_{IH}^{\ *})$.

In order to assess the impact of pricing flexibility on downward substitution, we compare M6 where the firm is free to set the high-end product price with M2 where price is exogenous. To isolate the effect of pricing flexibility, we set the exogenous high-end product price to the price that maximizes the high-end product profit when the low-end is ignored, i.e., the exogenous high-end product price for M2 is $p_H = p_H(TP_H)$. The following proposition shows that pricing flexibility in the high-end segment leads to a higher probability of downward substitution and a higher expected amount of fruit utilized in the making of the low-end product.

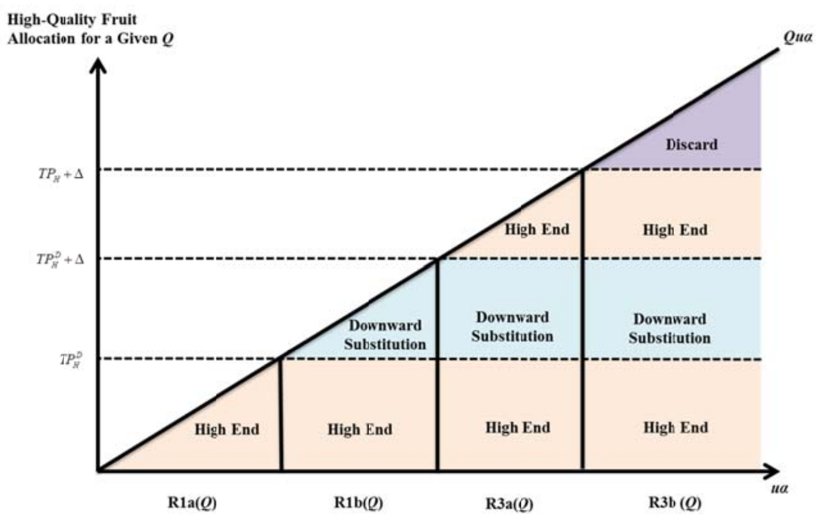

Figure 2. Optimal downward substitution quantity under endogenous pricing.

Proposition 4. *For a given Q, the price-setting flexibility in the high-end segment increases the probability of downward substitution and the expected amount of fruit downward substituted.*

5.2 Price-Setting Flexibility in the High-End Segment and the Fruit-Trading Flexibility

M7 features the fruit-trading flexibility in the presence of pricing flexibility in the high-end segment. We begin our analysis by analyzing the firm's ability to buy and sell fruit in the open market independently. Similar to the exogenous model, the firm would benefit from buying additional fruit from the open market when the fruit supply of high- or low-quality crop is low. On the other hand, when the supply of the high- or low-quality fruit is high, the firm can use the open market to gain additional revenue from selling its excess fruit crop. It should be noted here that, because the firm does not set price in the low-end segment, the structural properties pertaining to this segment decisions remain the same with those developed under exogenous price. The following proposition establishes a threshold for selling high-quality fruit in the open market, denoted TS_H, and another threshold for buying high-quality fruit from the open market, denoted TB_H.

Proposition 5. *The threshold for the amount of high-quality fruit to be sold in the open market is*

$$TS_H = -\left[p_H^* - c_{pH} - s_H\right] D_H{}'\left(p_H^*\right)$$

and the threshold for the amount of high-quality fruit to be purchased in the open market is

$$TB_H = -\left[p_H^* - c_{pH} - b_H\right] D_H{}'\left(p_H^*\right) < TS_H .$$

When high-end fruit supply is below TB_H, the firm purchases up to TB_H in the open market. When high-end fruit supply is above TS_H, the firm sells the excess in the open market. As noted above, the rules for open market buying and selling of low-quality fruit follow the rules for M3. This leads to six regions of supply and random quality random realizations (see Figure 3).

R1(Q) = $\{(u, \alpha) : Qu\alpha \le TB_H$ and $Qu(1-\alpha) < D_L\}$

R2(Q) = $\{(u, \alpha) : Qu\alpha \le TB_H$ and $Qu(1-\alpha) \ge D_L\}$

R3(Q) = $\{(u, \alpha) : TB_H < Qu\alpha \le TS_H$ and $Qu(1-\alpha) < D_L\}$

R4(Q) = $\{(u, \alpha) : TB_H < Qu\alpha \le TS_H$ and $Qu(1-\alpha) \ge D_L\}$

R5(Q) = $\{(u, \alpha) : Qu\alpha > TS_H$ and $Qu(1-\alpha) < D_L\}$

R6(Q) = $\{(u, \alpha) : Qu\alpha > TS_H$ and $Qu(1-\alpha) \ge D_L\}$.

Accordingly, the optimal second-stage quantity decisions for M7 are

$$\left(q_{IH}^*, q_{BH}^*, q_{SH}^*\right) = \begin{cases} \left(Qu\alpha, TB_H - Qu\alpha, 0\right) & \text{if } Qu\alpha \le TB_H \\ \left(Qu\alpha, 0, 0\right) & \text{if } TB_H < Qu\alpha \le TS_H \\ \left(TS_H, 0, Qu\alpha - TS_H\right) & \text{if } Qu\alpha > TS_H \end{cases}$$

$$\left(q_{IL}^*, q_{BL}^*, q_{SL}^*\right) = \begin{cases} \left(Qu\left(1-\alpha\right), \Delta, 0\right) & \text{if } Qu\left(1-\alpha\right) < D_L \\ \left(D_L, 0, -\Delta\right) & \text{if } Qu\left(1-\alpha\right) \ge D_L \end{cases}$$

and the optimal high-end price is $p_H^* = p_H(q_{IH}^* + q_{BH}^*)$.

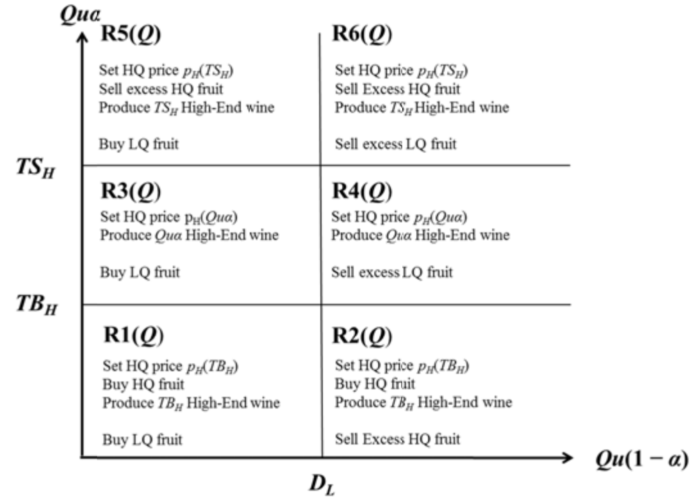

Figure 3. Different regions of *u, α* realization under endogenous pricing and trading in M7 (HQ in the figure refers to high-quality fruit and LQ refers to low-quality fruit).

We next compare M7 and M3. To provide a fair comparison, we set the exogenous price of M3 in the high-end segment to be in the interval of $p_H(TB_H)$ and $p_H(TS_H)$. The following proposition shows that the firm engages in fruit trading less frequently in the presence of the price-setting flexibility in the high-end segment. The result indicates that price-setting flexibility in the high-end segment and the fruit-trading flexibility play a substitutable role in the life of a winemaker.

Proposition 6. *For a given Q, the price-setting flexibility in the high-end segment decreases the probability of fruit trading and the expected amount of fruit trading.*

5.3 Price-Setting in the High-End Segment, Fruit-Trading and Downward-Substitution Flexibilities

M8, as presented in (1) – (4), features all three flexibilities: price-setting in the high-end segment, fruit-trading, and downward substitution. The second-stage objective function in (4) can be rewritten as:

$$PA(Q,u,\alpha) = \max_{\substack{p_H, q_{IH}, q_{IL} \ge 0 \\ q_{IH} \le \min\{D_H(p_H), Qu\alpha\} \\ q_{IL} \le \min\{D_L, Qu(1-\alpha)\}}} \left\{ \begin{array}{l} \left(p_H - c_{pH} - s_H\right) q_{IH} + s_H Qu\alpha + \\ \left(p_L - c_L\right)\left[q_{IL} + \min\left\{\left(Qu\alpha - q_{IH}\right)^+, \left(D_L - q_{IL}\right)^+\right\}\right] \\ + \left(b_L - s_H\right) \min\left\{\left(Qu\alpha - q_{IH}\right)^+, \left(D_L - q_{IL}\right)^+\right\} \end{array} \right\}$$

When $Qu(1 - \alpha) \ge D_L$, which corresponds to regions R2(*Q*), R4(*Q*), and R6(*Q*) above, the low-end market has sufficient supply; there is no downward substitution and the optimal decisions that apply in R2(*Q*), R4(*Q*), and R6(*Q*) for M7 are also optimal for M8.

To consider the case of $Qu(1 - \alpha) < D_L$, we require a threshold quantity. Recall that TP_H^D is the optimal high-end production amount when high-quality fruit that is not used for high-end production gains unit profit $p_L - c_{PL}$ through downward substitution. We similarly define a threshold production amount for M8. In particular, TP_H^{DT} denotes the optimal high-end production amount when high-quality fruit that is not used for high-end production saves the open market purchase cost b_L through downward substitution (the firm prefers to downward substitute over selling high-quality fruit in the open market because $s_H < b_L$). The production threshold in the presence of downward substitution TP_H^{DT} is smaller than the threshold without downward substitution (TS_H).

Proposition 7. *The threshold for the amount of high-end product to be produced from the internal resource for* M8 *is*

$$TP_H^{DT} = -\left(p_H^* - c_{pH} - b_L\right) D_H{}'\left(p_H^*\right) < TP_H,$$

and

$$TB_H < TP_H^{DT} < TS_H < TP_H,$$

and

$$TP_H^D < TP_H^{DT}.$$

Recall that $\Delta = D_L - Qu(1-\alpha)$ is the low-end shortage amount. We replace regions R3(Q) and R5(Q) with the following sub-regions:

R3a(Q) = $\{(u, \alpha) : TB_H < Qu\alpha \le TP_H^{DT}$ and $Qu(1-\alpha) < D_L\}$

R3b(Q) = $\{(u, \alpha) : TP_H^{DT} < Qu\alpha \le TP_H^{DT} + \Delta$ and $Qu(1-\alpha) < D_L\}$

R5a(Q) = $\{(u, \alpha) : TP_H^{DT} + \Delta < Qu\alpha \le TS_H + \Delta$ and $Qu(1-\alpha) < D_L\}$

R5b(Q) = $\{(u, \alpha) : Qu\alpha > TS_H + \Delta$ and $Qu(1-\alpha) < D_L\}$

Similar to M5, an interesting transition occurs between region R3b(Q) and R5a(Q) (see Figure 4). In region R3b(Q) the firm is able to produce the optimal high-end threshold quantity under downward substitution with trading flexibility (TP_H^{DT}), then downward substitute the balance to satisfy a portion of the shortage in the low-end segment (Δ). In region R5a(Q), the firm has more than enough to cover TP_H^{DT} and the shortage Δ. However, once the firm has allocated TP_H^{DT} to high-end production and has downward substituted the quantity Δ, the change in profit associated with allocating more volume to the high end is positive, and thus the firm allocates the balance of high- quality fruit to high-end production. Accordingly, the optimal second-stage quantity decisions for M8 are

$$\begin{pmatrix} q_{IH}^{\ *}, q_{BH}^{\ *}, w^{*}, q_{SH}^{\ *}, \\ q_{IL}^{\ *}, q_{BL}^{\ *}, q_{SL}^{\ *} \end{pmatrix} = \begin{cases} \begin{pmatrix} Qu\alpha, TB_H - Qu\alpha, 0, 0, \\ Qu(1-\alpha), \Delta, 0 \end{pmatrix} & \text{if } (u,\alpha) \in \mathrm{R}1(Q) \\ \begin{pmatrix} Qu\alpha, TB_H - Qu\alpha, 0, 0, \\ D_L, 0, -\Delta \end{pmatrix} & \text{if } (u,\alpha) \in \mathrm{R}2(Q) \\ \begin{pmatrix} Qu\alpha, 0, 0, 0, \\ Qu(1-\alpha), \Delta, 0 \end{pmatrix} & \text{if } (u,\alpha) \in \mathrm{R}3a(Q) \\ \begin{pmatrix} TP_H^{DT}, 0, Qu\alpha - TP_H^{DT}, 0, \\ Qu(1-\alpha), \Delta - Qu\alpha + TP_H^{DT}, 0 \end{pmatrix} & \text{if } (u,\alpha) \in \mathrm{R}3b(Q) \\ \begin{pmatrix} Qu\alpha, 0, 0, 0, \\ D_L, 0, -\Delta \end{pmatrix} & \text{if } (u,\alpha) \in \mathrm{R}4(Q) \\ \begin{pmatrix} Qu\alpha - \Delta, 0, \Delta, 0, \\ Qu(1-\alpha), 0, 0 \end{pmatrix} & \text{if } (u,\alpha) \in \mathrm{R}5a(Q) \\ \begin{pmatrix} TS_H, 0, \Delta, Qu\alpha - TS_H - \Delta, \\ Qu(1-\alpha), 0, 0 \end{pmatrix} & \text{if } (u,\alpha) \in \mathrm{R}5b(Q) \\ \begin{pmatrix} TS_H, 0, 0, Qu\alpha - TS_H - \Delta, \\ Qu(1-\alpha), 0, -\Delta \end{pmatrix} & \text{if } (u,\alpha) \in \mathrm{R}6(Q) \end{cases} \tag{8}$$

and the optimal high-end price is $p_H^{\ *} = p_H(q_{IH}^{\ *} + q_{BH}^{\ *})$.

We next compare M4 and M8 in order to examine the effect of the price-setting flexibility on the two other flexibilities. Similar to the comparison between M2 and M6, we fix the selling price in the high-end segment for M4 to be equal to the profit maximizing price $p_H(TP_H)$. The following proposition shows that in the presence of fruit-trading flexibility, pricing flexibility in the high-end segment leads to a higher probability of downward substitution and a higher expected amount of fruit utilized in the making of the low-end product.

Proposition 8. *For a given Q, the price-setting flexibility in the high-end segment increases the probability of downward substitution and the expected amount of fruit downward substituted.*

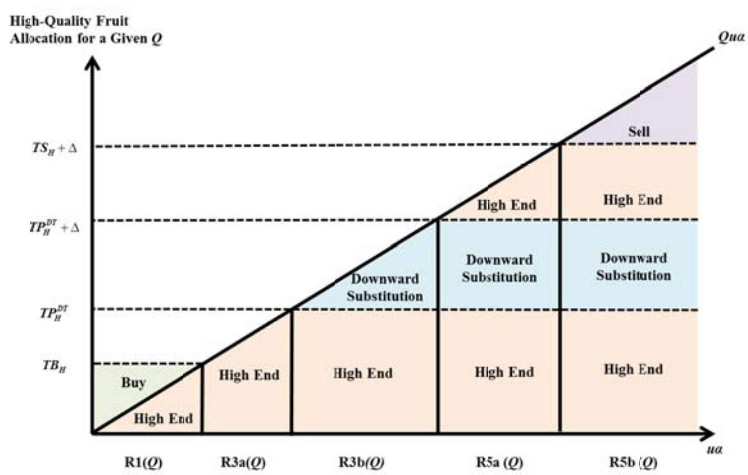

Figure 4. Optimal downward substitution quantity under endogenous pricing and fruit-trading.

Recall that when a firm does not have pricing flexibility, fruit-trading and downward-substitution flexibilities are neither complements nor substitutes (i.e., the amount and likelihood of downward substitution does not change when fruit-trading flexibility is introduced; see Proposition 1). To assess the relationship between fruit-trading and downward-substitution flexibilities in the presence of pricing flexibility in the high-end segment, we next compare M6 and M8.

Proposition 9. *a) For a given Q, in the presence of price-setting flexibility in the high-end segment, the downward substitution threshold with fruit-trading flexibility is higher than the downward substitution threshold without fruit-trading flexibility; b) For a given Q, in the presence of the price-setting flexibility in the high-end segment, fruit-trading flexibility decreases the probability of downward substitution and the expected amount of fruit downward substituted.*

The above proposition shows that the winemaker benefits more from downward substitution in the absence of fruit-trading flexibility. Because the firm engages in downward substitution at an earlier realization of high-quality fruit in the absence of fruit-trading flexibility, it experiences a higher probability of downward substitution and utilizes a greater (expected) amount of grapes for downward substitution. Furthermore, the above proposition shows that with the presence of the price-setting flexibility, fruit-trading and downward-substitution flexibility play a substitutable role. This result contradicts the earlier finding in the absence of the price-setting flexibility. Proposition 1 has shown that the fruit-trading flexibility does not influence the probability of downward substitution and the expected amount of downward substitution in the absence of price-setting flexibility. However, when the price-setting flexibility is included in the high-end segment, Proposition 9 shows that fruit trading and downward substitution flexibilities play a substitutable role.

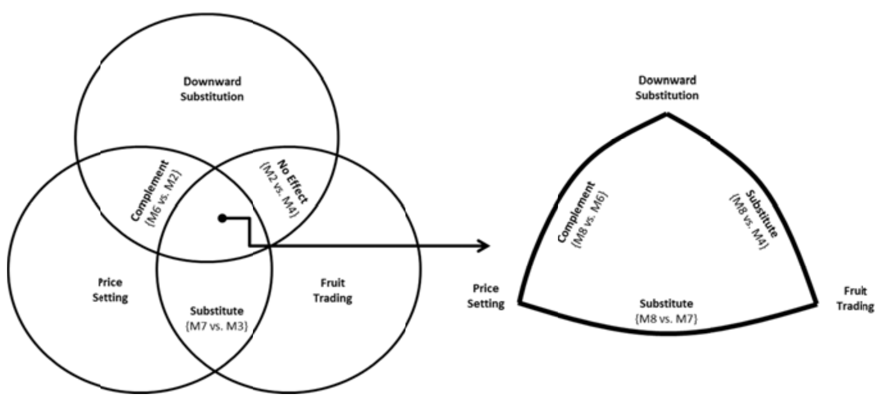

Figure 5. The relationship between downward-substitution, fruit-trading and price-setting flexibilities.

Figure 5 provides a summary of the relationship between the three forms of flexibilities presented in this study. From Figure 5, and our analysis in this section, it is clear that price-setting flexibility in the high-end segment and downward substitution flexibility play a complementary role with or without the fruit-trading flexibility. Our study proves that the winemaker benefits more by engaging in downward substitution at an earlier high-quality crop realization in the presence of the price-setting flexibility. In

Section 8, we compare our model to a model that allows for price setting in both market segments, and analytically demonstrate its effect on downward substitution.

5.4 The Impact of Quality and Supply Uncertainty

This section investigates the impact of increasing variance in supply or quality uncertainty on the probability of downward substitution and fruit trading. We begin our discussion with downward substitution. Proposition 8 has established that price-setting flexibility increases the likelihood of downward substitution. In order for the firm to engage in downward substitution, the high-quality fruit realization has to be greater than D_H in models M2 and M4, TP_H^{D} in M6, and TP_H^{DT} in M8. Let us denote TDS_j the threshold point for downward substitution in model $j \in \{$M2, M4, M6, M8$\}$. The following proposition describes how the probability of downward substitution changes with increasing variance in either $\tilde{u}$ or $\tilde{\alpha}$ when the other random variable is fixed at its mean. The proposition applies to any probability distribution that can be *standardized*, i.e., random variable X with mean μ and standard deviation σ can be written as $X = \mu + Z\sigma$ where Z is the corresponding standardized random variable with mean 0 and standard deviation 1. The class of distributions that can be standardized includes distributions such as normal (standardized pdf = $\phi(z) = \frac{1}{\sqrt{2\pi}} e^{-0.5z^2}$, $z \in (-\infty, \infty)$), truncated normal (standardized pdf = $\phi(z) = \frac{1}{\sqrt{2\pi}} e^{-0.5z^2} \Big/ \int_{-a}^{a} \phi(z)\,dz$, $z \in [-a, a]$), and uniform (standardized pdf = $\phi(z) = \frac{1}{2\sqrt{3}}$, $z \in \left[-\sqrt{3}, \sqrt{3}\right]$).

We let σ_u and σ_α denote the standard deviation of $\tilde{u}$ and $\tilde{\alpha}$, respectively.

Proposition 10. *For a probability distribution that can be standardized: a) When* $\sigma_u = \sigma_\alpha = 0$, $\bar{u}\bar{\alpha} > TDS_j/Q$, *and* $\bar{u}\left(1-\bar{\alpha}\right) < D_L/Q$ *for* $j \in \{$M2, M4, M6, M8$\}$*, the probability of downward substitution is equal to* 1*, and the probability of downward substitution is non-increasing in* σ_u (*with* $\sigma_\alpha = 0$) *and in* σ_α (*with* $\sigma_u = 0$)*. b) When* $\sigma_u = \sigma_\alpha = 0$, $\bar{u}\bar{\alpha} < TDS_j/Q$ *or* $\bar{u}\left(1-\bar{\alpha}\right) > D_L/Q$ *for* $j \in \{$M2, M4, M6, M8$\}$*, the probability of downward substitution is equal to* 0*, and the probability of downward substitution is non-decreasing in* σ_α (*with* $\sigma_u = 0$)*.*

Let us denote the probability that the firm engages in fruit trading as $P(FT > 0)$ when at least one of the four decision variables related with fruit trading q_{BH}, q_{SH}, q_{BL}, or q_{SL} takes a positive value. It is important to remind that, when quality uncertainty is ignored as in earlier publications (e.g. Kazaz 2004, Kazaz and Webster 2011), the firm does not engage in fruit trading with probability 1 under significant supply uncertainty. Considering the high-end fruit as the only product in the model, this means that fruit trading does not occur when $TB_H < Qu\bar{\alpha} < TS_H$; and, when the supply random variable shows significant variation, it is clear that $0 < P(FT > 0) < 1$. However, as shown in the following proposition, fruit trading occurs with probability 1, i.e., $P(FT > 0) = 1$ in M3, M4, and M7. Thus, the probability of fruit trading is not influenced by supply and quality variance. In the case of M3 and M4, this result is a consequence of the lack of price-setting flexibility, whereas in M7, the result is due to the lack of downward substitution flexibility. In M8, which includes both pricing and downward substitution flexibilities, changes in supply variation can affect the probability of fruit trading. However, the probability of fruit trading is unaffected by changes in quality variation.

Proposition 11. *a) In* M3, M4, *and* M7, *the probability of fruit trading always equals* 1; *b) For a probability distribution that can be standardized: In* M8, *when* $\sigma_u = 0$ *and* $(TP_H^{DT} + D_L)/Q < \bar{u} < (TS_H + D_L)/Q$, *the probability of fruit trading is* 0*, and its value is non-decreasing in* σ_u.

6. Impact of Flexibilities on Vineyard Lease

This section analyzes the firm's vineyard lease investment decisions. Incorporating the optimal second-stage decisions developed in Section 5 into the first-stage objective function, we first prove the concavity of the objective function of all models under supply and quality uncertainty; thus, each model has a unique optimal solution for its vineyard lease quantity that can be obtained from the first-order condition.

Proposition 12. *The first-stage objective functions in* M1 *through* M8 *are concave in* Q.

The following remark shows that uncertainty in supply and quality reduces the expected profit, but the addition of flexibilities increase the expected profit.

Remark 2. $E\left[\Pi^*_{M1}\right] \le \left\{ \begin{array}{l} \left\{E\left[\Pi^*_{M2}\right], E\left[\Pi^*_{M3}\right]\right\} \le E\left[\Pi^*_{M4}\right] \\ E\left[\Pi^*_{M5}\right] \le \left\{E\left[\Pi^*_{M6}\right], E\left[\Pi^*_{M7}\right]\right\} \end{array} \right\} \le E\left[\Pi^*_{M8}\right] \le \Pi^*_d$ *where* Π_d^* *and* $E[\Pi_j^*]$ *are the optimal profit under deterministic and stochastic supply and quality, respectively.*

We next present the analysis regarding how the initial vineyard lease investment decision, denoted Q_j^* for each model j = M1, …, M8, varies with the introduction of different flexibilities. We begin our discussion with the inclusion of the fruit-trading flexibility.

Proposition 13. *For any model with fruit-trading flexibility* (*i.e., j* ∈ {M3, M4, M7, M8}),

$$\frac{\partial Q_j^*}{\partial b_H} > 0,\ \frac{\partial Q_j^*}{\partial b_L} > 0,\ \frac{\partial Q_j^*}{\partial s_H} > 0,\ \frac{\partial Q_j^*}{\partial s_L} > 0\ .$$

The above proposition implies that the introduction of fruit-trading flexibility on the optimal vineyard lease is ambiguous. The reason is that a model without fruit-trading flexibility is equivalent to a model with fruit-trading flexibility but with a very high buying cost and a very low selling cost (i.e., not optimal to buy or sell in the open market). Thus, the introduction of fruit-trading flexibility can be viewed as a decrease in the buying cost, which puts downward pressure on the optimal vineyard lease, and an increase in the selling price, which puts upward pressure on the optimal vineyard lease. Depending on problem parameters, the optimal vineyard lease could increase or decrease when the fruit-trading flexibility is introduced. However, if the salvage values of excess fruit are sufficiently high, then it follows from Proposition 13 that the introduction of fruit-trading flexibility reduces the optimal lease.

Corollary 1. *If in models without fruit-trading flexibility* (*i.e.,* M1, M2, M5, M7), *the firm is able to salvage excess high-quality fruit at* s_H *and low-quality fruit at* s_L, *then the flexibility to buy fruit in the open market reduces the optimal lease, i.e.,* $Q_{M3}^* < Q_{M1}^*$, $Q_{M4}^* < Q_{M2}^*$, $Q_{M7}^* < Q_{M5}^*$, $Q_{M8}^* < Q_{M6}^*$.

The value gained from fruit trading decreases in the spread (or difference) between the buying cost and selling revenue from the open market, denoted δ_H and δ_L for the high-quality and low-quality fruit, respectively. Let us define m_H and m_L as reference prices, where $s_H = m_H - \delta_H/2$, $b_H = m_H + \delta_H/2$, $s_L = m_L - \delta_L/2$, and $b_L = m_L + \delta_L/2$.

Remark 3. *The optimal expected profit is decreasing in* δ_H *and* δ_L *in all models that feature fruit-trading flexibility, i.e.,* M3*,* M4*,* M7 *and* M8.

The above remark shows that the value from fruit trading diminishes with increasing spread between the buying cost of fruit and selling revenue from the fruit in the open market. The result follows from the fact that, at the optimal decision, a decrease in spread δ_H or δ_L will increase expected profit with no change in the decision variables (due to lower buying and high selling prices). And profit increases further when decisions are re-optimized at the new lower spread.

While the inclusion of the fruit-trading flexibility decreases the vineyard lease when the firm can salvage its excess fruit at the open market price, the introduction of downward substitution can both increase and decrease the optimal vineyard lease. We next consider the impact of pricing flexibility in the high-end segment on vineyard lease. In order to have a fair comparison of the exogenous and endogenous price models, we set the exogenous price in the high-end market in M1 to $p_H(TP_H)$. And for M3, which includes fruit trading flexibility, we consider the cases of exogenous price in high-end market at the buying and selling thresholds $p_H(TB_H)$ and $p_H(TS_H)$. In the presence of the fruit-trading flexibility, the following proposition states that the introduction of pricing flexibility decreases the optimal vineyard lease when the exogenous price is relatively low (i.e., at $p_H(TS_H)$), and increases the optimal vineyard lease when the exogenous price is relatively high (i.e., at $p_H(TB_H)$). In the absence of fruit-trading flexibility, the directional effect is ambiguous. However, the introduction of pricing flexibility decreases the optimal vineyard lease under the special case of linear demand and uniform demand.

Proposition 14. *a) When the exogenous price in the high-end segment in* M3 *is equal to* $p_H(TB_H)$, *pricing flexibility increases vineyard lease in the presence of fruit-trading flexibility, i.e.,* $Q_{M7}^* > Q_{M3}^*$; *b) When the exogenous price in the high-end segment in* M3 *is equal to* $p_H(TS_H)$, *pricing flexibility decreases vineyard lease in the presence of fruit-trading flexibility, i.e.,* $Q_{M7}^* < Q_{M3}^*$*; c) When exogenous price in the high-end segment in* M1 *is* $p_H(TP_H)$, *pricing flexibility reduces vineyard lease in the absence of fruit-trading flexibility,* $Q_{M5}^* < Q_{M1}^*$, *under linear demand and uniform distribution.*

The consequence of the above proposition is that, when compared to the exogenous price models, the pricing flexibility generally reduces the firm's vineyard lease investment regardless of the presence of the fruit-trading flexibility. However, when the exogenous price is high, and thus, the high-end demand is low, the addition of the pricing flexibility leads to an increase in the optimal vineyard lease decision.

Like the pricing flexibility, the inclusion of the downward substitution flexibility does not generate a definitive directional effect for an arbitrary pdf defining the randomness in supply and quality. Recall that Proposition 4 has shown that downward substitution and pricing flexibilities can play a complementary role, and can create the incentive for the firm to make a higher initial investment, despite the fact that the firm downward substitutes more units with higher probability. Thus, their combined effect is not unidirectional. Therefore, we next present numerical illustrations that demonstrate their influence.

7.1. Numerical Illustrations

This section presents numerical illustrations that demonstrate how quality and supply uncertainty, and their correlation, influence optimal vineyard lease, associated expected profit, expected amount of high-quality fruit downward substituted, and the probability of downward substitution in various models. We use the following cost parameters: $c_l = 10$, $c_H = 20$, $c_L = 15$, $b_H = 50$, $b_L = 45$, $s_H = 18$, $s_L = 13$. We consider linear demand functions $D_H(p_H) = 100{,}000 - 200p_H$ and $D_L = 120{,}000 - 300p_L$, which represent the demand characteristics in the wine industry: (1) the market size for high-end segment is lower than that of the low-end segment, and (2) consumers' price sensitivity is higher in the low-end segment. Given these parameters, we first establish the profit-maximizing price and quantity in each segment in Table 2.

	No Trading		Trading			
	p_H^*	TP_H	$p_H^*(TS_H)$	TB_H	$p_H^*(TS_H)$	TS_H
No Downward substitution	260	48,000	285	43,000	269	46,200
Downward substitution	356.25	28,750	-	-	282.5	43,500

Table 2. Profit-maximizing price and demand.

We use the profit-maximizing price as the exogenous price for the high-end segment in M1 – M4, and in the low-end segment in M1 – M8, i.e., $p_H = 260$, $p_L = 207.5$, $D_H(p_H) = 48{,}000$ and $D_L = 57{,}750$. Table 3 provides the comprehensive list of computational results, and reports the optimal vineyard lease, expected profit, expected amount of downward substitution (denoted $E[w^*]$), and the probability of downward substation (denoted $P(w^* > 0)$) in each model for various levels of supply and quality uncertainty.

Numerical illustrations confirm our earlier analytical results: (1) price-setting and downward substitution flexibilities play a complementary role; (2) fruit trading plays a substitutable role with pricing and downward substitution flexibilities; (3) fruit-trading flexibility reduces the optimal vineyard lease, and specifically, we have $Q_{M4}^* < \{Q_{M1}^*, Q_{M2}^*, Q_{M3}^*\}$, $Q_{M3}^* < Q_{M1}^*$, $Q_{M7}^* < Q_{M5}^*$; (4) pricing flexibility decreases the optimal vineyard lease, i.e., $Q_{M5}^* < Q_{M1}^*$, $Q_{M6}^* < Q_{M2}^*$, $Q_{M7}^* < Q_{M3}^*$, and $Q_{M8}^* < Q_{M4}^*$; (5)

vineyard lease when all flexibilities are present (M8) is not always smaller than M6 that features pricing and downward substitution flexibilities; indeed, for lower supply variations $Q_{\mathrm{M6}}{}^{*} < Q_{\mathrm{M8}}{}^{*}$, and for higher supply variations $Q_{\mathrm{M8}}{}^{*} < Q_{\mathrm{M6}}{}^{*}$.

Because our numerical illustrations support our earlier analytical results, the following discussion emphasizes the impact of supply and quality uncertainty on the three flexibilities. Focusing on the percentage change in the expected profit when a flexibility is added into the model, our numerical illustrations demonstrate that the inclusion of price-setting and downward substitution flexibilities provides the biggest impact. In the absence of fruit-trading flexibility, downward substitution can increase expected profit of a winemaker by as much 9.82% in the presence of pricing flexibility in the high-end segment. The results also demonstrate that downward substitution is most beneficial under high quality variation and limited supply variation, i.e., when α and u are uniformly distributed in [0.1, 0.9] and [0.4, 0.6], respectively. However, the impact of downward substitution is significantly reduced under the following conditions: (1) in the presence of fruit-trading flexibility due to the substitutable role these two flexibilities play, and (2) under limited quality and significant supply variances. We next summarize the findings regarding the impact of quality and supply variations, and their correlation in these numerical illustrations.

Supply Uncertainty	Quality Uncertainty	M1		M2				M3		M4			
u ~ Uniform	α ~ Uniform	Q^*	$E[\Pi(Q^*)]$	Q^*	$E[\Pi(Q^*)]$	$E[w^*]$	$P(w^* > 0)$	Q^*	$E[\Pi(Q^*)]$	Q^*	$E[\Pi(Q^*)]$	$E[w^*]$	$P(w^* > 0)$
[0.4, 0.6]	[0.4, 0.6]	266039	19.734	250060	20.041	1862.80	0.329	256056	20.387	245356	20.437	2025.41	0.358
[0.4, 0.6]	[0.3, 0.7]	299466	19.229	257878	19.775	3926.24	0.383	279930	20.289	251444	20.386	4278.67	0.408
[0.4, 0.6]	[0.2, 0.8]	342514	18.394	262937	19.243	6559.12	0.409	303179	20.139	254316	20.298	7051.54	0.431
[0.4, 0.6]	[0.1, 0.9]	365217	17.175	262583	18.585	9692.22	0.432	305490	19.948	252703	20.196	10172.70	0.452
[0.3, 0.7]	[0.4, 0.6]	299466	19.229	294555	19.417	988.94	0.184	279930	20.289	277164	20.318	1068.34	0.205
[0.3, 0.7]	[0.3, 0.7]	316438	18.840	298625	19.269	2361.83	0.249	290786	20.220	280129	20.290	2634.59	0.284
[0.3, 0.7]	[0.2, 0.8]	350153	18.133	309323	18.915	4541.68	0.314	309902	20.094	286630	20.225	5064.75	0.347
[0.3, 0.7]	[0.1, 0.9]	373316	16.991	313027	18.313	7549.73	0.356	312265	19.915	285902	20.129	8130.12	0.386
[0.2, 0.8]	[0.4, 0.6]	342514	18.394	339272	18.506	582.40	0.113	303179	20.139	300514	20.157	657.52	0.127
[0.2, 0.8]	[0.3, 0.7]	350153	18.133	341902	18.415	1485.05	0.168	309902	20.094	303160	20.139	1674.84	0.190
[0.2, 0.8]	[0.2, 0.8]	368661	17.605	348889	18.186	3114.29	0.240	323151	20.003	309405	20.096	3508.84	0.271
[0.2, 0.8]	[0.1, 0.9]	389889	16.615	356783	17.698	5808.04	0.300	326128	19.847	310622	20.015	6290.59	0.327
[0.1, 0.9]	[0.4, 0.6]	365217	17.175	361760	17.253	409.65	0.079	305490	19.948	302805	19.961	489.41	0.095
[0.1, 0.9]	[0.3, 0.7]	373316	16.991	364564	17.190	1044.56	0.118	312265	19.915	305472	19.948	1246.62	0.141
[0.1, 0.9]	[0.2, 0.8]	389889	16.615	371673	17.028	2200.97	0.173	326128	19.847	311991	19.916	2621.92	0.206
[0.1, 0.9]	[0.1, 0.9]	415045	15.842	384136	16.657	4364.64	0.239	336172	19.718	317351	19.850	4957.05	0.264

Supply Uncertainty	Quality Uncertainty	M5		M6				M7		M8			
u ~ Uniform	α ~ Uniform	Q^*	$E[\Pi(Q^*)]$	Q^*	$E[\Pi(Q^*)]$	$E[w^*]$	$P(w^* > 0)$	Q^*	$E[\Pi(Q^*)]$	Q^*	$E[\Pi(Q^*)]$	$E[w^*]$	$P(w^* > 0)$
[0.4, 0.6]	[0.4, 0.6]	261799	19.750	227655	20.241	4269.96	0.562	252563	20.407	240386	20.465	2660.53	0.421
[0.4, 0.6]	[0.3, 0.7]	284633	19.312	229519	20.149	6420.86	0.525	272992	20.316	244867	20.426	4900.48	0.443
[0.4, 0.6]	[0.2, 0.8]	310476	18.641	234270	19.873	8705.33	0.500	294617	20.176	248319	20.348	7555.72	0.452
[0.4, 0.6]	[0.1, 0.9]	330083	17.650	237493	19.383	11316.70	0.491	296863	19.993	247235	20.252	10613.00	0.466
[0.3, 0.7]	[0.4, 0.6]	284633	19.312	265483	19.707	3234.35	0.349	272992	20.316	269494	20.350	1575.30	0.264
[0.3, 0.7]	[0.3, 0.7]	296442	19.008	266228	19.664	4637.67	0.392	284131	20.252	272714	20.328	3192.15	0.322
[0.3, 0.7]	[0.2, 0.8]	319575	18.420	270607	19.483	6911.77	0.417	301151	20.132	278480	20.271	5640.50	0.373
[0.3, 0.7]	[0.1, 0.9]	337721	17.480	275965	19.066	9598.48	0.428	303447	19.961	278396	20.182	8664.31	0.404
[0.2, 0.8]	[0.4, 0.6]	310476	18.641	301157	18.922	2587.46	0.303	294617	20.176	291475	20.197	983.68	0.170
[0.2, 0.8]	[0.3, 0.7]	319575	18.420	302712	18.893	3517.35	0.307	301151	20.132	293743	20.182	2090.72	0.225
[0.2, 0.8]	[0.2, 0.8]	337879	17.968	307547	18.774	5319.34	0.349	313921	20.044	299370	20.143	4025.18	0.300
[0.2, 0.8]	[0.1, 0.9]	353342	17.133	315236	18.449	7942.77	0.378	316907	19.895	301533	20.069	6811.81	0.349
[0.1, 0.9]	[0.4, 0.6]	330083	17.650	321525	17.848	1855.84	0.229	296863	19.993	293697	20.009	732.18	0.127
[0.1, 0.9]	[0.3, 0.7]	337721	17.480	323074	17.819	2577.00	0.240	303447	19.961	295982	19.997	1556.18	0.168
[0.1, 0.9]	[0.2, 0.8]	353342	17.133	327810	17.721	3963.28	0.282	316907	19.895	301786	19.969	3017.84	0.230
[0.1, 0.9]	[0.1, 0.9]	373282	16.440	338025	17.460	6291.10	0.329	325886	19.771	307110	19.908	5389.58	0.285

Table 3. Summary of numerical results for M1-M8 correlation (expected profits are in 10^6).

Influence of Quality Uncertainty: (1) Higher variation in quality decreases expected profits in all models; (2) Higher variation in quality generally increases vineyard lease. This result is consistent in M1 – M3 under exogenous price, and in M5 – M7 in the presence of pricing flexibility. However, vineyard lease exhibits both an increasing and decreasing behavior in quality variance in M4 and M8 due to the complementary behavior between pricing and downward substitution flexibilities. At limited supply variances, increasing quality variance initially increases the optimal vineyard lease, but with higher quality variations, it starts decreasing the optimal vineyard lease; (3) Variation in quality increases the expected amount of downward substitution in all models.

Influence of Supply Uncertainty: (1) Higher supply variation reduces expected profit in all models; (2) Vineyard lease increases in supply variation; (3) Both expected amount of high-quality fruit downward substituted and the probability of downward substitution decrease in supply variation. While the result might appear to be surprising at a first look, it can be explained by the fact that, with higher supply variation, there is more of the crop for both high-quality and low-quality fruit, diminishing the need for downward substitution; (4) The firm leases a smaller vineyard when supply variation is low under downward substitution flexibility than it does under fruit-trading flexibility (i.e., $Q_{M2}^* < Q_{M3}^*$); it leases a greater vineyard under downward substitution flexibility when supply variation is high than it does under fruit-trading flexibility (i.e., $Q_{M2}^* > Q_{M3}^*$).

We investigate the impact of correlation between supply and quality uncertainty, denoted with ρ. In our analysis, we restrict the conditional variance of quality for a given u, denoted $Var[\alpha \mid u]$, to be constant for a given u; this allows the overall quality variance, denoted $Var[\alpha]$, to change with respect to ρ (technical details of our derivations are provided in Appendix B of the online supplement). In the wine industry, supply and quality can typically have a positive correlation[4]; therefore, we restrict our numerical illustrations to the various levels of positive correlations. Table 4 presents the results of the numerical illustrations with various values of the correlation coefficient. In these calculations, supply random variable is distributed uniformly on [0.25, 0.75], but correlation changes the distribution of α.

Influence of Correlation between supply and quality: (1) Expected profit decreases with higher values of correlation. The result is a consequence of a "distributional effect" which stems from the expansion in the tails of the distribution for quality uncertainty. With higher correlation, the overall quality variance increases, resulting in a higher quality risk and a lower the expected profit; (2) Without downward substitution flexibility, an increase in correlation causes an increase in vineyard lease, which can be explained again by the same distributional effect. Downward substitution, however, can cause a decrease

[4] The positive correlation between supply (abundance of grapes) and quality (high scores) can be exemplified by the 2005 vintage in Bordeaux wines. The unusually high number of sunny and warm days resulted in the highest amount of fruit crop with the highest ratings achieved from the two most influential publications: the Wine Spectator and the Wine Advocate.

in the amount of vineyard lease with higher values of the correlation coefficient. This is because downward substitution flexibility takes advantage of the expansion in tails of quality uncertainty distribution, and negates the detrimental consequences of distributional effect; (3) In the presence of downward substitution flexibility, an increase in correlation has similar effects with those presented for quality uncertainty. Specifically, while the probability of high-quality fruit downward substituted increases in correlation (due to higher quality variation), the expected amount of downward substitution can exhibit a decreasing behavior with higher values of correlation (see models M2 and M6). The latter has the same characteristics of downward substitution behavior.

	M1		M2				M3		M4			
ρ	**Q^***	**$E[\Pi(Q^*)]$**	**Q^***	**$E[\Pi(Q^*)]$**	**$E[w^*]$**	**$P(w^*>0)$**	**Q^***	**$E[\Pi(Q^*)]$**	**Q^***	**$E[\Pi(Q^*)]$**	**$E[w^*]$**	**$P(w^*>0)$**
0.25	343634	18.365	320124	18.702	1929.72	0.226	310954	20.145	292740	20.205	2405.78	0.262
0.375	347603	18.313	318223	18.621	1877.90	0.228	314081	20.141	290951	20.201	2470.78	0.265
0.5	351191	18.203	315448	18.513	2015.53	0.232	316996	20.131	288401	20.194	2726.79	0.271
	M5		**M6**				**M7**		**M8**			
ρ	**Q^***	**$E[\Pi(Q^*)]$**	**Q^***	**$E[\Pi(Q^*)]$**	**$E[w^*]$**	**$P(w^*>0)$**	**Q^***	**$E[\Pi(Q^*)]$**	**Q^***	**$E[\Pi(Q^*)]$**	**$E[w^*]$**	**$P(w^*>0)$**
0.25	319967	18.679	284720	19.235	4093.85	0.359	303970	20.182	284631	20.249	2903.42	0.298
0.375	323700	18.665	283174	19.194	4010.78	0.363	307133	20.180	283015	20.247	2949.20	0.300
0.5	327217	18.602	281003	19.131	4087.78	0.368	310078	20.172	280800	20.242	3177.14	0.304

Table 4. Summary of numerical results for models under increasing correlation (expected profits are in 10^6).

8. Discussion on Price-Setting in Both Segments and Downward-Substitution Flexibilities

Earlier analysis has shown that the pricing flexibility in the high-end segment increases the level of downward substitution. If the firm has the pricing flexibility in the low-end segment as well, does this additional flexibility lead to another increase in downward substitution? We next investigate the impact of pricing flexibility in the low-end segment on the conditions for downward substitution using models M9 and M10. It should be stated here that, in the motivating application of this study, the winemaker cannot set a selling price for its low-end product. However, such a comparison sheds light into the similarities and differences of our model with an earlier model established in Tomlin and Wang (2008) where the firm has the price-setting flexibility in both segments.

With price-setting flexibility in both the high- and low-end segments, downward substitution occurs when the marginal revenues from the two market segments are equal. The following proposition defines the threshold for the production amount in the low-end segment.

Proposition 15. *The threshold for the amount of low-quality fruit to be produced from the internal resource for* M9 *is* $TP_L = -\left[p_L^* - c_{pL}\right]D_L'\left(p_L^*\right)$.

When the realized amount of low-quality grapes is less than the production threshold (i.e., $Qu(1-\alpha) \leq TP_L$), the firm charges a market-clearing price $p_L(Qu(1-\alpha))$. However, when there is excess amount of low-quality fruit (i.e., $Qu(1–\alpha) > TP_L$), the firm sells TP_L at price $p_L(TP_L)$. Adapting the regions for the low-end threshold quantity, we have the following regions for M9:

$R1(Q) = \{(u, \alpha) : Qu\alpha \leq TP_H \text{ and } Qu(1 - \alpha) < TP_L \}$

$R2(Q) = \{(u, \alpha) : Qu\alpha \leq TP_H \text{ and } Qu(1 - \alpha) \geq TP_L \}$

$R3(Q) = \{(u, \alpha) : Qu\alpha > TP_H \text{ and } Qu(1 - \alpha) < TP_L \}$

$R4(Q) = \{(u, \alpha) : Qu\alpha > TP_H \text{ and } Qu(1 - \alpha) \geq TP_L \}$

The optimal second-stage quantity decisions for M9 are

$$\left(q_{IH}{}^*, q_{IL}{}^*\right) = \begin{cases} \left(Qu\alpha, Qu\left(1-\alpha\right)\right) & \text{if } \left(u,\alpha\right) \in \mathrm{R1}\left(Q\right) \\ \left(Qu\alpha, TP_L\right) & \text{if } \left(u,\alpha\right) \in \mathrm{R2}\left(Q\right) \\ \left(TP_H, Qu\left(1-\alpha\right)\right) & \text{if } \left(u,\alpha\right) \in \mathrm{R3}\left(Q\right) \\ \left(TP_H, TP_L\right) & \text{if } \left(u,\alpha\right) \in \mathrm{R4}\left(Q\right) \end{cases}$$

and the optimal prices are $p_H^* = p_H(q_{IH}^*)$ and $p_L^* = p_H(q_{IL}^*)$.

In the analysis of M10, equivalent to the model of Tomlin and Wang (2008), we follow a similar approach, and equate the marginal revenues from the two market segments in order to obtain the critical quality realization that would trigger the downward substitution decision. When there is insufficient low-quality fruit to fulfill the threshold, i.e., $Qu(1–\alpha) < TP_L$, the revenue and the marginal revenue become:

$$\pi_L\left(\cdot\middle|Q,u,\alpha\right) = \left(p_L\left(Qu\left(1-\alpha\right)\right) - c_{pL}\right)Qu\left(1-\alpha\right).$$

Moreover, the critical quality realization for downward substitution can be obtained by setting the marginal return from the low-end segment to the marginal return in the high-end segment

$$\partial\pi_L\left(\cdot\middle|Q,u,\alpha\right)/\partial\alpha = \partial\pi_H\left(\cdot\middle|Q,u,\alpha\right)/\partial\alpha .$$

Considering the case when the price in the low-end segment of M6 is equal to the profit-maximizing price of M10, we show that the probability of downward substitution in M10 is always greater than or equal to that of M6. This is because the profit-maximizing price in the low-end segment, denoted p_L^* is smaller than or equal to the market-clearing price of $p_L(Qu(1- \alpha))$, and thus, TP_H^D of M10 is smaller than that of M6. Downward substitution continues to take place until the high-quality fruit exceeds TP_H.

Proposition 16. *For a given* Q, $P(w^* > 0)$ *in* M10 *is greater than or equal to that of* M6.

Section 5.4 has shown that the firm always engages in fruit trading in M3, M4, and M7 with probability 1 under supply and quality uncertainty, and in M8 with probability 1 under quality uncertainty alone. The following proposition shows that: (1) quality uncertainty influences the probability of fruit trading, (2) the probability of fruit trading is not always equal to 1 under significant supply and quality variation when the firm can set prices in both segments as in M11.

Proposition 17. *For a probability distribution that can be standardized: a) When* $\sigma_u = \sigma_\alpha = 0$ *in* M11, *and*

$(TB_H/Q_{M11}) < \bar{u}\bar{\alpha} < (TS_H/Q_{M11})$ *and* $(TB_L/Q_{M11}) < \bar{u}\left(1-\bar{\alpha}\right) < (TS_L/Q_{M11})$*, the probability of fruit trading is equal to* 1*, and is non-increasing in* σ_u *(with* $\sigma_\alpha = 0$*) and in* σ_α *(with* $\sigma_u = 0$*). b) When* $\sigma_u = \sigma_\alpha = 0$, $\bar{u}\bar{\alpha} < (TB_H/Q_{M11})$*, or* $\bar{u}\bar{\alpha} > (TS_H/Q_{M11})$*, or* $\bar{u}\left(1-\bar{\alpha}\right) < (TB_L/Q_{M11})$*, or* $\bar{u}\left(1-\bar{\alpha}\right) > (TS_L/Q_{M11})$ *in* M11*, the probability of fruit trading is equal to* 0*, and is non-decreasing in* σ_α *(with* $\sigma_u = 0$*).*

9. Conclusions

The paper examines the interactions between the three forms of operational flexibility available to agricultural firms in mitigating supply and quality uncertainty. These flexibilities are: (1) downward substitution, where high-quality fruit can be used in the making of a low-end product, (2) price-setting, where the firm can influence the demand of the high-end product by appropriately selecting the selling price in the high-end segment (in which consumers are less price-elastic); and (3) fruit trading flexibility, where the firm can purchase additional fruit in the event of lower supply realizations, or sell some of its excess fruit in the open market for revenue. The paper provides a comprehensive analysis that demonstrates the interrelationships between these three forms of operational flexibilities.

The paper makes three sets of main contributions. First, the study identifies the interrelationships between the above three forms of flexibilities; not all flexibilities exhibit a substitutable role. Our paper proves that pricing and downward substitution flexibilities play a complementary role. Pricing flexibility enables the firm to engage in downward substitution early and frequently, yielding higher expected amount and probability of downward substitution. Fruit trading flexibility, on the other hand, does not influence downward substitution in the absence of pricing flexibility, but plays a substitutable role to downward substitution in the presence of pricing flexibility. It also exhibits a substitutable role to pricing flexibility.

Second, our results provide insight into how these three forms of flexibilities influence the winemaker's initial vineyard investment. The inclusion of fruit trading generally decreases the optimal amount of vineyard lease. Pricing and downward substitution flexibilities (and their combination), however, can lead to both an increase and a decrease in the optimal vineyard lease. The latter occurs under limited supply and significant quality variances.

Our third contribution relates to the impact of the variation in supply and quality uncertainty on vineyard lease, expected profits, expected amount and probability of downward substitution. Variation in quality uncertainty does not influence the probability of fruit trading, always decreases the expected profits, and increases the expected amount of downward substitution. Higher variations in quality generally increases vineyard lease, but can also show a decreasing behavior in the presence of downward substitution flexibility. Variation in supply generally increases the firm's vineyard lease, reduces expected profits, and decreases the expected amount and probability of downward substitution in all models.

Significant variation in quality and limited variation in supply makes downward substitution more attractive; they reduce the need to rely on vineyard lease. While increasing quality variation generally increases the probability of downward substitution, the likelihood of a crop supply and demand mismatch is reduced at lower supply realizations; therefore, the probability of downward substitution can exhibit a decreasing behavior in quality variance in the presence of low supply variations. The correlation coefficient mimics the reactions observed under increasing quality variation.

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Appendix A

Proof of Proposition 1: In M2, with the absence of price-setting and fruit-trading flexibility, the winemaker engages in downward substitution when there high-quality fruit realization is higher than the high-end demand and the low-quality fruit realization is below the low-end demand i.e. $Qu\alpha > D_H$ and $Qu(1-\alpha) \leq D_L$. Therefore it is possible to see that downward substitution occurs when $D_H/Q\alpha < u \leq D_L/Qu(1-\alpha)$.

In M4, with the absence of price-setting flexibility, due to the fact that downward substitution can only occurs when $s_H \leq b_L$, the winemaker is better off when downward substituting excess high-quality fruit for the production of low-end wine comparing to selling the high-quality fruit in the open market. Therefore, in M4 downward substitution occurs when $Qu\alpha > D_H$ and $Qu(1-\alpha) \leq D_L$ or $D_H/Q\alpha < u \leq D_L/Qu(1-\alpha)$. This is equivalent to the M2. From this analysis it is possible to say that the introduction of fruit-trading flexibility does not change the probability of downward substitution or the expected amount of downward substitution. □

Proof of Proposition 2: The high-end threshold production quantity is obtained by maximizing

$$\pi(p_H) = (p_H - c_{pH}) D_H(p_H)$$

(i.e., it is optimal for the firm to set the production quantity equal to demand). The profit function is concave because $p_H D_H(p_H)$ is concave by assumption. Thus the optimal price and threshold production quantity are given by the first-order condition

$$\pi'(p_H) = (p_H - c_{pH}) D_H'(p_H) + D_H(p_H) = 0,$$

which can be rewritten as

$$TP_H = -(p_H^* - c_{pH}) D_H'(p_H^*). \quad \square$$

Proof of Proposition 3: The proof is similar to the proof of Proposition 2, but the profit function now includes the margin from the low-end segment, $p_L - c_{pL}$. The high-end profit function is

$$\pi(p_H) = (p_H - c_{pH}) D_H(p_H) + (p_L - c_{pL})(Qu\alpha - D_H(p_H)).$$

The profit function is concave because $p_H D_H(p_H)$ is concave by assumption. Thus the optimal price and threshold production quantity are given by the first-order condition

$$\pi'(p_H) = (p_H - c_{pH} - (p_L - c_{pL})) D_H'(p_H) + D_H(p_H) = 0,$$

which can be rewritten as

$$TP_H^D = -(p_H^* - c_{pH} - (p_L - c_{pL})) D_H'(p_H^*).$$

From the proof of Proposition 2, the optimal price for M5 (without downward substitution) satisfies

$$\pi_{M5}'(p_H^{M5}) = (p_H^{M5} - c_{pH}) D_H'(p_H^{M5}) + D_H(p_H^{M5}) = 0.$$

As shown above, the optimal price for M6 (with downward substitution) satisfies

$$\pi_{\mathrm{M6}}'\left(p_H^{\mathrm{M6}}\right)=\left(p_H^{\mathrm{M6}}-c_{pH}\right)D_H'\left(p_H^{\mathrm{M6}}\right)+D_H\left(p_H^{\mathrm{M6}}\right)-\left(p_L-c_{pL}\right)D_H'\left(p_H^{\mathrm{M6}}\right)=0\,.$$

Thus, from $D_H'(p) < 0$, it follows that

$$\pi_{\mathrm{M6}}'\left(p_H^{\mathrm{M5}}\right)=\left(p_H^{\mathrm{M5}}-c_{pH}\right)D_H'\left(p_H^{\mathrm{M5}}\right)+D_H\left(p_H^{\mathrm{M5}}\right)-\left(p_L-c_{pL}\right)D_H'\left(p_H^{\mathrm{M5}}\right)>0\,,$$

which implies $p_H^{\mathrm{M6}} > p_H^{\mathrm{M5}}$ and $TP_H^D = D_H\left(p_H^{\mathrm{M6}}\right) < D_H\left(p_H^{\mathrm{M5}}\right) = TP_H$. □

Proof of Proposition 4: Recall that $\Delta = D_L - Qu\alpha$. For M2 at exogenous high-end product price $p_H = p_H(TP_H)$ and high-end product demand $D_H = TP_H$, the optimal quantity decisions are

$$\left(q_{IH}{}^{*},w^{*},q_{IL}{}^{*}\right)=\begin{cases}\left(Qu\alpha,0,Qu(1-\alpha)\right) & \text{if } Qu\alpha\le TP_H \text{ and } Qu(1-\alpha)<D_L\\ \left(Qu\alpha,0,D_L\right) & \text{if } Qu\alpha\le TP_H \text{ and } Qu(1-\alpha)\ge D_L\\ \left(TP_H,Qu\alpha-TP_H,Qu(1-\alpha)\right) & \text{if } TP_H<Qu\alpha\le TP_H+\Delta \text{ and } Qu(1-\alpha)<D_L\\ \left(TP_H,\Delta,Qu(1-\alpha)\right) & \text{if } TP_H+\Delta<Qu\alpha \text{ and } Qu(1-\alpha)<D_L\\ \left(TP_H,0,D_L\right) & \text{if } Qu\alpha>TP_H \text{ and } Qu(1-\alpha)\ge D_L\end{cases}$$

(see (5)). For M6, the optimal quantity decisions are

$$\left(q_{IH}{}^{*},w^{*},q_{IL}{}^{*}\right)=\begin{cases}\left(Qu\alpha,0,Qu(1-\alpha)\right) & \text{if } Qu\alpha\le TP_H^D \text{ and } Qu(1-\alpha)<D_L\\ \left(TP_H^D,Qu\alpha-TP_H^D,Qu(1-\alpha)\right) & \text{if } TP_H^D<Qu\alpha\le TP_H^D+\Delta \text{ and } Qu(1-\alpha)<D_L\\ \left(Qu\alpha,0,D_L\right) & \text{if } Qu\alpha\le TP_H \text{ and } Qu(1-\alpha)\ge D_L\\ \left(Qu\alpha-\Delta,\Delta,Qu(1-\alpha)\right) & \text{if } TP_H^D+\Delta<Qu\alpha\le TP_H+\Delta \text{ and } Qu(1-\alpha)<D_L\\ \left(TP_H,\Delta,Qu(1-\alpha)\right) & \text{if } TP_H+\Delta<Qu\alpha \text{ and } Qu(1-\alpha)<D_L\\ \left(TP_H,0,D_L\right) & \text{if } Qu\alpha>TP_H \text{ and } Qu(1-\alpha)\ge D_L\end{cases}$$

(see (7)). We see that the optimal quantity decisions for M2 and M6 are identical when $Qu(1-\alpha) \ge D_L$. However, when $Qu(1-\alpha) < D_L$, we see that $w^* > 0$ iff $Qu\alpha > TP_H$ for M2 and that $w^* > 0$ iff $Qu\alpha > TP_H^D$ for M6. From $TP_H^D < TP_H$ (see Proposition 3), it follows that the probability of downward substitution is higher for M6 than for M2. Furthermore, for any Q, u, and α, we see that $w_{M6}^* \ge w_{M2}^*$ (with strict inequality for some parameter values), and thus the expected amount of fruit downward substituted is greater with price flexibility (M6) and without price flexibility (M2). □

Proof of Proposition 5: The proof is similar to the proofs of propositions 2 and 3. Given excess high-quality fruit supply, the high-end profit function is

$$\pi(p_H)=\left(p_H-c_{pH}\right)D_H(p_H)+s_H\left(Qu\alpha-D_H(p_H)\right).$$

The first-order condition yields TS_H. Given a shortage of high-quality fruit supply, the high-end profit function is

$$\pi(p_H)=(p_H-c_{pH})Qu\alpha+(p_H-c_{pH}-b_H)(D_H(p_H)-Qu\alpha),$$

The first-order condition yields TB_H. The inequality $TB_H < TS_H$, follows from $b_H > s_H$. □

Proof of Proposition 6: For M3, the firm buys quantity $D_H - Qu\alpha$ of fruit in the open market iff $Qu\alpha < D_H$ and sells quantity $Qu\alpha - D_H$ of fruit in the open market iff $Qu\alpha > D_H$. For M7, the firm buys quantity $TB_H - Qu\alpha$ of fruit in the open market iff $Qu\alpha < TB_H$ and sells quantity $Qu\alpha - TS_H$ of fruit in the open market iff $Qu\alpha > TS_H$. The result follows from $TB_H < D_H < TS_H$. □

Proof of Proposition 7: The proof is similar to the proof of Proposition 3, but the profit function replaces the margin from the low-end segment ($p_L - c_{pL}$) with the cost of purchasing low-quality fruit in the open market (b_L). The high-end profit function is

$$\pi(p_H)=(p_H-c_{pH})D_H(p_H)+b_L(Qu\alpha-D_H(p_H)).$$

The profit function is concave because $p_H D_H(p_H)$ is concave by assumption. Thus the optimal price and threshold production quantity are given by the first-order condition

$$\pi'(p_H)=(p_H-c_{pH}-b_L)D_H'(p_H)+D_H(p_H)=0,$$

which can be rewritten as

$$TP_H^{DT}=-(p_H^*-c_{pH}-b_L)D_H'(p_H^*).$$

The inequality, $TB_H < TP_H^{DT} < TS_H < TP_H = -\left[p_H^*-c_{pH}-s_H\right]D_H'(p_H^*)$, follows from $b_H > b_L > s_H > 0$.

Recall that $TP_H^D=-(p_H^*-c_{pH}-(p_L-c_{pL}))D_H'(p_H^*)$ (see Proposition 3). From $b_L < p_L - c_{pL}$ (see assumption (A1)), it follows that $TP_H^{DT} > TP_H^D$. □

Proof of Proposition 8: Recall that $\Delta = D_L - Qu\alpha$. For M4 at exogenous high-end product price $p_H = p_H(TP_H)$ and high-end product demand $D_H = TP_H$, the optimal quantity decisions are

$$\begin{pmatrix} q_{IH}^*, q_{BH}^*, w^*, q_{SH}^*, \\ q_{IL}^*, q_{BL}^*, q_{SL}^* \end{pmatrix} = \begin{cases} \begin{pmatrix} Qu\alpha, TP_H-Qu\alpha, 0, 0, \\ Qu(1-\alpha), \Delta, 0 \end{pmatrix} & \text{if } Qu\alpha \le TP_H \text{ and } Qu(1-\alpha) < D_L \\ \begin{pmatrix} Qu\alpha, TP_H-Qu\alpha, 0, 0, \\ D_L, 0, -\Delta \end{pmatrix} & \text{if } Qu\alpha \le TP_H \text{ and } Qu(1-\alpha) \ge D_L \\ \begin{pmatrix} TP_H, Qu\alpha-TP_H, 0, 0, \\ D_L, D_L-Qu\alpha+TP_H, 0 \end{pmatrix} & \text{if } TP_H < Qu\alpha \le TP_H+\Delta \text{ and } Qu(1-\alpha) < D_L \\ \begin{pmatrix} TP_H, 0, \Delta, 0, \\ Qu(1-\alpha), 0, 0 \end{pmatrix} & \text{if } Qu\alpha > TP_H+\Delta \text{ and } Qu(1-\alpha) < D_L \\ \begin{pmatrix} TP_H, 0, 0, Qu\alpha-TP_H, \\ D_L, 0, -\Delta \end{pmatrix} & \text{if } Qu\alpha > TP_H \text{ and } Qu(1-\alpha) \ge D_L \end{cases}$$

(see (6)). For M8, the optimal quantity decisions are

$$\begin{pmatrix} q_{IH}{}^*, q_{BH}{}^*, w^*, q_{SH}{}^*, \\ q_{IL}{}^*, q_{BL}{}^*, q_{SL}{}^* \end{pmatrix} = \begin{cases} \begin{pmatrix} Qu\alpha, TB_H - Qu\alpha, 0, 0, \\ Qu(1-\alpha), \Delta, 0 \end{pmatrix} & \text{if } Qu\alpha \le TB_H \text{ and } Qu(1-\alpha) < D_L \\ \begin{pmatrix} Qu\alpha, TB_H - Qu\alpha, 0, 0, \\ D_L, 0, -\Delta \end{pmatrix} & \text{if } Qu\alpha \le TB_H \text{ and } Qu(1-\alpha) \ge D_L \\ \begin{pmatrix} Qu\alpha, 0, 0, 0, \\ Qu(1-\alpha), \Delta, 0 \end{pmatrix} & \text{if } TB_H < Qu\alpha \le TP_H^{DT} \text{ and } Qu(1-\alpha) < D_L \\ \begin{pmatrix} TP_H^{DT}, 0, Qu\alpha - TP_H^{DT}, 0, \\ Qu(1-\alpha), \Delta - Qu\alpha + TP_H^{DT}, 0 \end{pmatrix} & \text{if } TP_H^{DT} < Qu\alpha \le TP_H^{DT} + \Delta \text{ and } Qu(1-\alpha) < D_L \\ \begin{pmatrix} Qu\alpha, 0, 0, 0, \\ D_L, 0, -\Delta \end{pmatrix} & \text{if } Qu\alpha > TB_H \text{ and } Qu(1-\alpha) \ge D_L \\ \begin{pmatrix} Qu\alpha - \Delta, 0, \Delta, 0, \\ Qu(1-\alpha), 0, 0 \end{pmatrix} & \text{if } TP_H^{DT} < Qu\alpha \le TS_H + \Delta \text{ and } Qu(1-\alpha) < D_L \\ \begin{pmatrix} TS_H, 0, \Delta, Qu\alpha - TS_H - \Delta, \\ Qu(1-\alpha), 0, 0 \end{pmatrix} & \text{if } Qu\alpha > TS_H + \Delta \text{ and } Qu(1-\alpha) < D_L \end{cases}$$

(see (8)). We see that the optimal quantity decisions for M4 and M8 are identical when $Qu(1-\alpha) \ge D_L$. However, when $Qu(1-\alpha) < D_L$, we see that $w^* > 0$ iff $Qu\alpha > TP_H$ for M4 and that $w^* > 0$ iff $Qu\alpha > TP_H^{DT}$ for M8. From $TP_H^{DT} < TP_H$ (see Proposition 7), it follows that the probability of downward substitution is higher for M8 than for M4. Furthermore, for any Q, u, and α, we see that $w_{\text{M8}}^* \ge w_{\text{M4}}^*$ (with strict inequality for some parameter values), and thus the expected amount of fruit downward substituted is greater with price flexibility (M8) than without price flexibility (M4). □

Proof of Proposition 9: For M6, the production threshold (and downward substitution threshold) is

$$TP_H^D = -\left(p_H^* - c_{pH} - \left(p_L - c_{pL}\right)\right) D_H{}'\left(p_H^*\right)$$

(see Proposition 3). For M8, the production threshold (and downward substitution threshold) is TP_H^{DT}, and for M6, the production threshold is TP_H^D. From $TP_H^{DT} > TP_H^D$ (see Proposition 7), it follows that the probability of downward substitution is greater without fruit-trading flexibility (M6) and with fruit-trading flexibility (M8). Furthermore, for any Q, u, and α, $w_{\text{M6}}^* \ge w_{\text{M8}}^*$ (with strict inequality for some parameter values), and thus the expected amount of fruit downward substituted is greater without fruit-trading flexibility (M6) than with fruit-trading flexibility (M8). □

Proof of Proposition 10: a) If $\sigma_u = \sigma_\alpha = 0$, then by the definition of TDS_j, there is downward substitution when $\bar{u}\bar{\alpha} > TDS_j/Q$ and $\bar{u}(1-\bar{\alpha}) < D_L/Q$, i.e., the probability of downward substitution is equal to 1.

We will now show that the probability of downward substitution is non-increasing in σ_u (with $\sigma_\alpha = 0$). Because the probability distribution of $\tilde{u}$ can be standardized, the cdf of $\tilde{u}$ can be written as

$\Phi_u\left(\frac{u-\bar{u}}{\sigma_u}\right)$ where $\Phi_u(z)$ is the corresponding standardized cdf. Let $u_1 = \frac{TDS_j}{Q\bar{\alpha}}$ and $u_2 = \frac{D_L}{Q(1-\bar{\alpha})}$ and note that $u_1 < \bar{u} < u_2$. Accordingly, the probability of downward substitution is $P(\tilde{u} \in [u_1, u_2])$ = $\Phi_u\left(\frac{u_2-\bar{u}}{\sigma_u}\right) - \Phi_u\left(\frac{u_2-\bar{u}}{\sigma_u}\right)$. From $\partial\left(\frac{u_2-\bar{u}}{\sigma_u}\right)/\partial\sigma_u < 0$ (due to $u_2 > \bar{u}$) and $\partial\left(\frac{u_1-\bar{u}}{\sigma_u}\right)/\partial\sigma_u > 0$ (due to $u_1 < \bar{u}$), it follows that $\partial P(\tilde{u} \notin [u_1, u_2])/\partial\sigma_u \leq 0$, and thus the probability of downward substitution is non-increasing in σ_u. Similar arguments, which are also illustrated in part b) below, can be used to show that the probability of downward substitution is non-increasing in σ_α (with $\sigma_u = 0$).

b) If $\sigma_u = \sigma_\alpha = 0$, then by the definition of TDS_j, there is no downward substitution when $\bar{u}\bar{\alpha} < TDS_j/Q$ or $\bar{u}(1-\bar{\alpha}) > D_L/Q$, i.e., the probability of downward substitution is equal to 0.

We will now show that the probability of downward substitution is non-decreasing in σ_α (with σ_u = 0). Because the probability distribution of $\tilde{\alpha}$ can be standardized, the cdf of $\tilde{\alpha}$ can be written as $\Phi_\alpha\left(\frac{\alpha-\bar{\alpha}}{\sigma_\alpha}\right)$ where $\Phi_\alpha(z)$ is the corresponding standardized cdf. Let $\alpha_1 = \max\left\{\frac{TDS_j}{Q\bar{u}}, 1-\frac{D_L}{Q\bar{u}}\right\}$ and note that $\alpha_1 > \bar{\alpha}$. Accordingly, the probability of downward substitution is $P(\tilde{\alpha} \geq \alpha_1) = 1 - \Phi_\alpha\left(\frac{\alpha_1-\bar{\alpha}}{\sigma_\alpha}\right)$. From $\partial\left(\frac{\alpha_1-\bar{\alpha}}{\sigma_\alpha}\right)/\partial\sigma_\alpha < 0$ (due to $\alpha_1 > \bar{\alpha}$), it follows that $\partial P(\tilde{\alpha} \geq \alpha_1)/\partial\sigma_\alpha \geq 0$, and thus the probability of downward substitution is non-decreasing in σ_α. □

Proof of Proposition 11: The proof of part a) follows from the definition of the trading threshold values. The proof of b) is similar to the proof of Proposition 10. We omit the details. □

Proof of Proposition 12: a) The expected profit for model M8 can be written as:

$$
E\left[\Pi_{\mathrm{M8}}(Q)\right]=-c_l Q+\left\{\begin{array}{l}
\iint\limits_{\mathrm{R1}(Q)}\left[\begin{array}{l}\left(p_H\left(TB_H\right)-c_{pH}-b_H\right)TB_H+b_H Qu\alpha\\+\left(p_L-c_{pL}-b_L\right)D_L+b_L Qu(1-\alpha)\end{array}\right]g(u,\alpha)\,du\,d\alpha\\
+\iint\limits_{\mathrm{R2}(Q)}\left[\begin{array}{l}\left(p_H\left(TB_H\right)-c_{pH}-b_H\right)TB_H+b_H Qu\alpha\\+\left(p_L-c_{pL}-s_L\right)D_L+s_L Qu(1-\alpha)\end{array}\right]g(u,\alpha)\,du\,d\alpha\\
+\iint\limits_{\mathrm{R3a}(Q)}\left[\begin{array}{l}\left(p_H\left(Qu\alpha\right)-c_{pH}\right)Qu\alpha\\+\left(p_L-c_{pL}-b_L\right)D_L+b_L Qu(1-\alpha)\end{array}\right]g(u,\alpha)\,du\,d\alpha\\
+\iint\limits_{\mathrm{R3b}(Q)}\left[\begin{array}{l}\left(p_H\left(TP_H^{DT}\right)-c_{pH}\right)TP_H^{DT}\\\left(p_L-c_{pL}\right)D_L-b_L\left(D_L+TP_H^{DT}-Qu\right)\end{array}\right]g(u,\alpha)\,du\,d\alpha\\
+\iint\limits_{\mathrm{R4}(Q)}\left[\begin{array}{l}\left(p_H\left(Qu\alpha\right)-c_{pH}\right)Qu\alpha\\+\left(p_L-c_{pL}-s_L\right)D_L+s_L Qu(1-\alpha)\end{array}\right]g(u,\alpha)\,du\,d\alpha\\
+\iint\limits_{\mathrm{R5a}(Q)}\left[\begin{array}{l}\left(p_H\left(Qu-D_L\right)-c_{pH}\right)\left(Qu-D_L\right)\\+\left(p_L-c_{pL}\right)D_L\end{array}\right]g(u,\alpha)\,du\,d\alpha\\
+\iint\limits_{\mathrm{R5b}(Q)}\left[\begin{array}{l}\left(p_H\left(TS_H\right)-c_{pH}\right)TS_H+s_H\left(Qu-TS_H-D_L\right)\\+\left(p_L-c_{pL}\right)D_L\end{array}\right]g(u,\alpha)\,du\,d\alpha\\
+\iint\limits_{\mathrm{R6}(Q)}\left[\begin{array}{l}\left(p_H\left(TS_H\right)-c_{pH}-s_H\right)TS_H+s_H Qu\alpha\\+\left(p_L-c_{pL}-s_L\right)D_L+s_L Qu(1-\alpha)\end{array}\right]g(u,\alpha)\,du\,d\alpha
\end{array}\right\}
$$

Let us define the boundary points: $u_1(Q,\alpha) = TB_H/Q\alpha$, $u_2(Q,\alpha) = TS_H/Q\alpha$, $u_3(Q,\alpha) = TP_H^{DT}/Q\alpha$, $u_4(Q) = (TP_H^{DT}+D_L)/Q$, $u_5(Q) = (TS_H+D_L)/Q$ and $\alpha_1(Q,u) = 1 - (D_L/Qu)$

Note that: $u_1'(Q,\alpha) = \partial u_1(Q,\alpha)/\partial Q \leq 0$, $u_2'(Q,\alpha) = \partial u_2(Q,\alpha)/\partial Q \leq 0$, $u_3'(Q,\alpha) = \partial u_3(Q,\alpha)/\partial Q \leq 0$, $u_4'(Q) = \partial u_4(Q)/\partial Q \leq 0$, $u_5'(Q) = \partial u_5(Q)/\partial Q \leq 0$ and $\alpha_1'(Q,u) = \partial \alpha_1(Q,u)/\partial Q > 0$.

Taking the first-order derivative of the first-stage objective function $E[\Pi_{\mathrm{M8}}(Q)]$ gives:

$$\frac{\partial E\left[\Pi_{\mathrm{M8}}(Q)\right]}{\partial Q} = -c_l + \left\{ \begin{array}{l} \int_{u_l}^{u_1(Q,\alpha)} \int_{\alpha_1(Q,u)}^{\alpha_h} \left[b_H u\alpha + b_L u(1-\alpha)\right] g(u,\alpha)\, d\alpha du \\ + \int_{u_l}^{u_1(Q,\alpha)} \int_{\alpha_l}^{\alpha_1(Q,u)} \left[b_H u\alpha + s_L u(1-\alpha)\right] g(u,\alpha)\, d\alpha du \\ + \int_{u_1(Q,\alpha)}^{u_3(Q,\alpha)} \int_{\alpha_1(Q,u)}^{a_h} \left[\left(p_H^{'}(Qu\alpha)Qu\alpha + p_H(Qu\alpha) - c_{pH}\right)u\alpha + b_L u(1-\alpha)\right] g(u,\alpha)\, d\alpha du \\ + \int_{u_3(Q,\alpha)}^{u_4(Q,\alpha)} \int_{\alpha_1(Q,u)}^{a_h} \left[b_L u\right] g(u,\alpha)\, d\alpha du \\ + \int_{u_1(Q,\alpha)}^{u_2(Q,\alpha)} \int_{\alpha_l}^{\alpha_1(Q,u)} \left[\left(p_H^{'}(Qu\alpha)Qu\alpha + p_H(Qu\alpha) - c_{pH}\right)u\alpha + s_L u(1-\alpha)\right] g(u,\alpha)\, d\alpha du \\ + \int_{u_4(Q,\alpha)}^{u_5(Q,\alpha)} \int_{\alpha_1(Q,u)}^{\alpha_h} \left[\left(p_H^{'}(Qu - D_L)(Qu - D_L) + p_H(Qu - D_L) - c_{pH}\right)u\right] g(u,\alpha)\, d\alpha du \\ + \int_{u_5(Q,\alpha)}^{u_h} \int_{\alpha_1(Q,u)}^{\alpha_h} \left[s_H u\right] g(u,\alpha)\, d\alpha du \\ + \int_{u_2(Q,\alpha)}^{u_h} \int_{\alpha_l}^{\alpha_1(Q,u)} \left[s_H u\alpha + s_L u(1-\alpha)\right] g(u,\alpha)\, d\alpha du \end{array} \right\}$$

From the first-order condition at boundary points: $u_1(Q,\alpha)$, $u_2(Q,\alpha)$, $u_3(Q,\alpha)$, $u_4(Q)$ and $u_5(Q)$, the optimal price must satisfies:

$$p_H^{'}\left(Qu_1(Q,\alpha)\alpha\right)Qu_1(Q,\alpha)\alpha + p_H\left(Qu_1(Q,\alpha)\alpha\right) = c_{pH} + b_H$$
$$p_H^{'}\left(Qu_2(Q,\alpha)\alpha\right)Qu_2(Q,\alpha)\alpha + p_H\left(Qu_2(Q,\alpha)\alpha\right) = c_{pH} + s_H$$
$$p_H^{'}\left(Qu_3(Q,\alpha)\alpha\right)Qu_3(Q,\alpha)\alpha + p_H\left(Qu_3(Q,\alpha)\alpha\right) = c_{pH} + b_L$$
$$p_H^{'}\left(Qu_4(Q) - D_L\right)\left(Qu_4(Q) - D_L\right) + p_H\left(Qu_4(Q) - D_L\right) = c_{pH} + b_L$$
$$p_H^{'}\left(Qu_5(Q) - D_L\right)\left(Qu_5(Q) - D_L\right) + p_H\left(Qu_5(Q) - D_L\right) = c_{pH} + s_H$$

Taking the second-order derivative of the first-stage objective function $E[\Pi_{\mathrm{M8}}(Q)]$ gives:

$$\frac{\partial^2 E\left[\Pi_{M8}(Q)\right]}{\partial Q^2}=\left\{\begin{array}{l}
u_1'(Q,\alpha)\int_{\alpha_1(Q,u_1(Q,\alpha))}^{\alpha_h}\left[b_H u_1(Q,\alpha)\alpha+b_L u_1(Q,\alpha)(1-\alpha)\right]g\left(u_1(Q,\alpha),\alpha\right)d\alpha \\
+u_1'(Q,\alpha)\int_{\alpha_l}^{\alpha_1(Q,u_1(Q,\alpha))}\left[b_H u_1(Q,\alpha)\alpha+s_L u_1(Q,\alpha)(1-\alpha)\right]g\left(u_1(Q,\alpha),\alpha\right)d\alpha \\
+\int_{u_1(Q,\alpha)}^{u_3(Q,\alpha)}\int_{\alpha_1(Q,u)}^{a_h}\left[\left(2p_H'(Qu\alpha)+p_H''(Qu\alpha)Qu\alpha\right)(u\alpha)^2\right]g(u,\alpha)\,d\alpha du \\
-u_1'(Q,\alpha)\int_{\alpha_1(Q,u_1(Q,\alpha))}^{\alpha_h}\left[b_H u_1(Q,\alpha)\alpha+b_L u_1(Q,\alpha)(1-\alpha)\right]g\left(u_1(Q,\alpha),\alpha\right)d\alpha \\
+u_3'(Q,\alpha)\int_{\alpha_1(Q,u_3(Q,\alpha))}^{\alpha_h}\left[b_L u_3(Q,\alpha)\alpha+b_L u_3(Q,\alpha)(1-\alpha)\right]g\left(u_3(Q,\alpha),\alpha\right)d\alpha \\
-u_3'(Q,\alpha)\int_{\alpha_1(Q,u_3(Q,\alpha))}^{a_h}\left[b_L u_3(Q,\alpha)\right]g\left(u_3(Q,\alpha),\alpha\right)d\alpha du \\
+u_4'(Q)\int_{\alpha_1(Q,u_4(Q,\alpha))}^{a_h}\left[b_L u_4(Q)\right]g\left(u_4(Q),\alpha\right)d\alpha du \\
+\int_{u_1(Q,\alpha)}^{u_2(Q,\alpha)}\int_{\alpha_l}^{\alpha_1(Q,u)}\left[\left(2p_H'(Qu\alpha)+p_H''(Qu\alpha)Qu\alpha\right)(u\alpha)^2\right]g(u,\alpha)\,d\alpha du \\
-u_1'(Q,\alpha)\int_{\alpha_l}^{\alpha_1(Q,u_1(Q,\alpha))}\left[b_H u_1(Q,\alpha)\alpha+s_L u_1(Q,\alpha)(1-\alpha)\right]g\left(u_1(Q,\alpha),\alpha\right)d\alpha \\
+u_2'(Q,\alpha)\int_{\alpha_l}^{\alpha_1(Q,u_2(Q,\alpha))}\left[s_H u_2(Q,\alpha)\alpha+s_L u_2(Q,\alpha)(1-\alpha)\right]g\left(u_2(Q,\alpha),\alpha\right)d\alpha \\
+\int_{u_4(Q)}^{u_5(Q)}\int_{\alpha_1(Q,u)}^{\alpha_h}\left[\left(2p_H'(Qu-D_L)+p_H''(Qu-D_L)(Qu-D_L)\right)u^2\right]g(u,\alpha)\,d\alpha du \\
-u_4'(Q)\int_{\alpha_1(Q,u_4(Q,\alpha))}^{a_h}\left[b_L u_4(Q)\right]g\left(u_4(Q),\alpha\right)d\alpha du \\
+u_5'(Q)\int_{\alpha_1(Q,u_5(Q,\alpha))}^{a_h}\left[s_H u_5(Q)\right]g\left(u_5(Q),\alpha\right)d\alpha du \\
-u_5'(Q)\int_{\alpha_1(Q,u_5(Q,\alpha))}^{a_h}\left[s_H u_5(Q)\right]g\left(u_5(Q),\alpha\right)d\alpha du \\
-u_2'(Q,\alpha)\int_{\alpha_l}^{\alpha_1(Q,u_2(Q,\alpha))}\left[s_H u_2(Q,\alpha)\alpha+s_L u_2(Q,\alpha)(1-\alpha)\right]g\left(u_2(Q,\alpha),\alpha\right)d\alpha
\end{array}\right\}$$

Because the derivatives at the boundary point of each region, the second-order derivatives cancel out, yielding the following expression:

$$\frac{\partial^2 E\left[\Pi_{\mathrm{M8}}(Q)\right]}{\partial Q^2}=\left\{\begin{aligned}&+\int_{u_1(Q,\alpha)}^{u_3(Q,\alpha)}\int_{\alpha_1(Q,u)}^{\alpha_h}\left[\left(2p_H'(Qu\alpha)+p_H''(Qu\alpha)Qu\alpha\right)(u\alpha)^2\right]g(u,\alpha)\,d\alpha\,du\\&+\int_{u_1(Q,\alpha)}^{u_2(Q,\alpha)}\int_{\alpha_l}^{\alpha_1(Q,u)}\left[\left(2p_H'(Qu\alpha)+p_H''(Qu\alpha)Qu\alpha\right)(u\alpha)^2\right]g(u,\alpha)\,d\alpha\,du\\&+\int_{u_4(Q)}^{u_5(Q)}\int_{\alpha_1(Q,u)}^{\alpha_h}\left[\left(2p_H'(Qu-D_L)+p_H''(Qu-D_L)(Qu-D_L)\right)u^2\right]g(u,\alpha)\,d\alpha\,du\end{aligned}\right\}$$

The above is negative because of the assumption that $2p_i'(q_i) + p_i''(q_i) \le 0$, and $\partial^2 E[\Pi_{\mathrm{M8}}(Q)]/\partial Q^2 < 0$. Thus, M8 first-stage objective function is continuous and concave in the amount of vineyard lease Q.

b) In model M7, the optimal second stage decision can be divided into the following sets:

R1(Q) = $\{(u, \alpha) : Qu\alpha \le TB_H$ and $Qu(1-\alpha) < D_L\}$

R2(Q) = $\{(u, \alpha) : Qu\alpha \le TB_H$ and $Qu(1-\alpha) \ge D_L\}$

R3(Q) = $\{(u, \alpha) : TB_H < Qu\alpha \le TS_H$ and $Qu(1-\alpha) < D_L\}$

R4(Q) = $\{(u, \alpha) : TB_H < Qu\alpha \le TS_H$ and $Qu(1-\alpha) \ge D_L\}$

R5(Q) = $\{(u, \alpha) : Qu\alpha > TS_H$ and $Qu(1-\alpha) < D_L\}$

R6(Q) = $\{(u, \alpha) : Qu\alpha > TS_H$ and $Qu(1-\alpha) \ge D_L\}$.

Accordingly, the optimal second-stage quantity decisions for M7 are:

$$\left(q_{IH}^*, q_{BH}^*, q_{SH}^*\right)=\begin{cases}\left(Qu\alpha, TB_H - Qu\alpha, 0\right) & \text{if } Qu\alpha \le TB_H\\ \left(Qu\alpha, 0, 0\right) & \text{if } TB_H < Qu\alpha \le TS_H\\ \left(TS_H, 0, Qu\alpha - TS_H\right) & \text{if } Qu\alpha > TS_H\end{cases}$$

$$\left(q_{IL}^*, q_{BL}^*, q_{SL}^*\right)=\begin{cases}\left(Qu(1-\alpha), \Delta, 0\right) & \text{if } Qu(1-\alpha) < D_L\\ \left(D_L, 0, -\Delta\right) & \text{if } Qu(1-\alpha) \ge D_L\end{cases}$$

Therefore, the first-stage objective function can be written as:

$$E\left[\Pi_{\mathrm{M7}}(Q)\right]=-c_l Q+\left\{\begin{array}{l}\displaystyle\iint_{\mathrm{R1}(Q)}\left[\begin{array}{l}\left(p_H\left(TB_H\right)-c_{pH}-b_H\right)TB_H+b_H Qu\alpha\\+\left(p_L-c_{pL}-b_L\right)D_L+b_L Qu(1-\alpha)\end{array}\right]g(u,\alpha)\,dud\alpha\\
\displaystyle+\iint_{\mathrm{R2}(Q)}\left[\begin{array}{l}\left(p_H\left(TB_H\right)-c_{pH}-b_H\right)TB_H+b_H Qu\alpha\\+\left(p_L-c_{pL}-s_L\right)D_L+s_L Qu(1-\alpha)\end{array}\right]g(u,\alpha)\,dud\alpha\\
\displaystyle+\iint_{\mathrm{R3}(Q)}\left[\begin{array}{l}\left(p_H\left(Qu\alpha\right)-c_{pH}\right)Qu\alpha\\+\left(p_L-c_{pL}-b_L\right)D_L+b_L Qu(1-\alpha)\end{array}\right]g(u,\alpha)\,dud\alpha\\
\displaystyle+\iint_{\mathrm{R4}(Q)}\left[\begin{array}{l}\left(p_H\left(Qu\alpha\right)-c_{pH}\right)Qu\alpha\\+\left(p_L-c_{pL}-s_L\right)D_L+s_L Qu(1-\alpha)\end{array}\right]g(u,\alpha)\,dud\alpha\\
\displaystyle+\iint_{\mathrm{R5}(Q)}\left[\begin{array}{l}\left(p_H\left(TS_H\right)-c_{pH}-s_H\right)TS_H+s_H Qu\alpha\\+\left(p_L-c_{pL}-b_L\right)D_L+b_L Qu(1-\alpha)\end{array}\right]g(u,\alpha)\,dud\alpha\\
\displaystyle+\iint_{\mathrm{R6}(Q)}\left[\begin{array}{l}\left(p_H\left(TS_H\right)-c_{pH}-s_H\right)TS_H+s_H Qu\alpha\\+\left(p_L-c_{pL}-s_L\right)D_L+s_L Qu(1-\alpha)\end{array}\right]g(u,\alpha)\,dud\alpha\end{array}\right\}$$

Rewriting the first-stage objective function according to the returns from high and low-end segments:

$$E\left[\Pi_{\mathrm{M7}}(Q)\right]=-c_l Q+\left\{\begin{array}{l}\displaystyle\iint_{\mathrm{R1}(Q)\cup\mathrm{R2}(Q)}\left[\left(p_H\left(TB_H\right)-c_{pH}-b_H\right)TB_H+b_H Qu\alpha\right]g(u,\alpha)\,dud\alpha\\
\displaystyle+\iint_{\mathrm{R3}(Q)\cup\mathrm{R4}(Q)}\left[\left(p_H\left(Qu\alpha\right)-c_{pH}\right)Qu\alpha\right]g(u,\alpha)\,dud\alpha\\
\displaystyle+\iint_{\mathrm{R5}(Q)\cup\mathrm{R6}(Q)}\left[\left(p_H\left(TS_H\right)-c_{pH}-s_H\right)TS_H+s_H Qu\alpha\right]g(u,\alpha)\,dud\alpha\\
\displaystyle+\iint_{\mathrm{R1}(Q)\cup\mathrm{R3}(Q)\cup\mathrm{R5}(Q)}\left[\left(p_L-c_{pL}-b_L\right)D_L+b_L Qu(1-\alpha)\right]g(u,\alpha)\,dud\alpha\\
\displaystyle+\iint_{\mathrm{R2}(Q)\cup\mathrm{R4}(Q)\cup\mathrm{R6}(Q)}\left[+\left(p_L-c_{pL}-s_L\right)D_L+s_L Qu(1-\alpha)\right]g(u,\alpha)\,dud\alpha\end{array}\right\}$$

Taking the first-order derivative of the first-stage objective function provides:

$$\frac{\partial E\left[\Pi_{\mathrm{M7}}(Q)\right]}{\partial Q}=-c_l+\left\{\begin{array}{l}\displaystyle\iint_{\mathrm{R1}(Q)\cup\mathrm{R2}(Q)}\left[b_H u\alpha\right]g(u,\alpha)\,dud\alpha\\
\displaystyle+\iint_{\mathrm{R3}(Q)\cup\mathrm{R4}(Q)}\left[\left(p_H'\left(Qu\alpha\right)Qu\alpha+p_H\left(Qu\alpha\right)-c_{pH}\right)u\alpha\right]g(u,\alpha)\,dud\alpha\\
\displaystyle+\iint_{\mathrm{R5}(Q)\cup\mathrm{R6}(Q)}\left[s_H u\alpha\right]g(u,\alpha)\,dud\alpha\\
\displaystyle+\iint_{\mathrm{R1}(Q)\cup\mathrm{R3}(Q)\cup\mathrm{R5}(Q)}\left[b_L u(1-\alpha)\right]g(u,\alpha)\,dud\alpha\\
\displaystyle+\iint_{\mathrm{R2}(Q)\cup\mathrm{R4}(Q)\cup\mathrm{R6}(Q)}\left[s_L u(1-\alpha)\right]g(u,\alpha)\,dud\alpha\end{array}\right\}$$

Similar to part a) let us define the boundary points: $\alpha_1(Q,u) = 1 - (D_L/Qu)$, $\alpha_2(Q,u) = TB_H/Qu$ and $\alpha_3(Q,u) = TS_H/Qu$; and note that $\alpha_1^{'}(Q,u) = \partial\alpha_1(Q,u)/\partial Q > 0$, $\alpha_2^{'}(Q,u) = \partial\alpha_2(Q,u)/\partial Q \le 0$ and $\alpha_3^{'}(Q,u) = \partial\alpha_3(Q,u)/\partial Q \le 0$.

From the first-order condition, at the boundary point $\alpha_2(Q,u)$ and $\alpha_3(Q,u)$, the optimal price satisfies:

$$p_H^{'}\left(Qu\alpha_2(Q,u)\right)Qu\alpha_2(Q,u) + p_H\left(Qu\alpha_2(Q,u)\right) = c_{pH} + b_H$$

$$p_H^{'}\left(Qu\alpha_3(Q,u)\right)Qu\alpha_3(Q,u) + p_H\left(Qu\alpha_3(Q,u)\right) = c_{pH} + s_H$$

Therefore, taking the second-order derivative of M7 objective function gives:

$$\frac{\partial^2 E\left[\Pi_{\mathrm{M7}}(Q)\right]}{\partial Q^2} = \left\{\begin{array}{l} \alpha_2^{'}(Q,u)\int_{u_l}^{u_h}\left[b_H u\alpha_2(Q,u)\right]g\left(u,\alpha_2(Q,u)\right)du \\ +\iint_{R3(Q)\cup R4(Q)}\left[\left(2p_H^{'}(Qu\alpha) + p_H^{''}(Qu\alpha)Qu\alpha\right)(u\alpha)^2\right]g(u,\alpha)\,dudga \\ -\alpha_2^{'}(Q,u)\int_{u_l}^{u_h}\left[b_H u\alpha_2(Q,u)\right]g\left(u,\alpha_2(Q,u)\right)du \\ +\alpha_3^{'}(Q,u)\int_{u_l}^{u_h}\left[s_H u\alpha_3(Q,u)\right]g\left(u,\alpha_3(Q,u)\right)du \\ -\alpha_3^{'}(Q,u)\int_{u_l}^{u_h}\left[s_H u\alpha_3(Q,u)\right]g\left(u,\alpha_3(Q,u)\right)du \\ -\alpha_1^{'}(Q,u)\int_{u_l}^{u_h}\left[b_L u\left(1-\alpha_1(Q,u)\right)\right]g\left(u,\alpha_1(Q,u)\right)du \\ +\alpha_1^{'}(Q,u)\int_{u_l}^{u_h}\left[s_L u\left(1-\alpha_1(Q,u)\right)\right]g\left(u,\alpha_1(Q,u)\right)du \end{array}\right\}$$

The above expression is negative because $2p_i'(q_i) + p_i''(q_i) \le 0$ and because $b_L > s_L$; thus $\partial^2 E[\Pi_{M7}(Q)]/\partial Q^2 < 0$ and the first-stage objective function in M7 is continuous and concave the amount of vineyard lease Q.

c) The optimal second-stage decisions for model M6 can be divided into the following sets:

R1a(Q) = $\{(u, \alpha) : Qu\alpha \le TP_H^D$ and $Qu(1-\alpha) < D_L\}$

R1b(Q) = $\{(u, \alpha) : TP_H^D < Qu\alpha \le TP_H^D + \Delta$ and $Qu(1-\alpha) < D_L\}$

R2(Q) = $\{(u, \alpha) : Qu\alpha \le TP_H$ and $Qu(1-\alpha) \ge D_L\}$

R3a(Q) = $\{(u, \alpha) : TP_H^D + \Delta < Qu\alpha \le TP_H + \Delta$ and $Qu(1-\alpha) < D_L\}$

R3b(Q) = $\{(u, \alpha) : TP_H + \Delta < Qu\alpha$ and $Qu(1-\alpha) < D_L\}$

R4(Q) = $\{(u, \alpha) : Qu\alpha > TP_H$ and $Qu(1-\alpha) \ge D_L\}$

As a result of this the first-stage objective function of model M6 can be written as:

$$E\left[\Pi_{\text{M6}}(Q)\right]=-c_l Q+\left\{\begin{array}{l}
+\iint\limits_{\text{R1a}(Q)}\left[\left(p_H(Qu\alpha)-c_{pH}\right)Qu\alpha+\left(p_L-c_{pL}\right)D_L\right]g(u,\alpha)\,du\,d\alpha\\
+\iint\limits_{\text{R1b}(Q)}\left[\left(p_H\left(TP_H^D\right)-c_{pH}\right)TP_H^D+\left(p_L-c_{pL}\right)\left(D_L+Qu-TP_H^{DT}\right)\right]g(u,\alpha)\,du\,d\alpha\\
+\iint\limits_{\text{R2}(Q)}\left[\left(p_H(Qu\alpha)-c_{pH}\right)Qu\alpha+\left(p_L-c_{pL}\right)D_L\right]g(u,\alpha)\,du\,d\alpha\\
+\iint\limits_{\text{R3a}(Q)}\left[\left(p_H(Qu-D_L)-c_{pH}\right)(Qu-D_L)+\left(p_L-c_{pL}\right)D_L\right]g(u,\alpha)\,du\,d\alpha\\
+\iint\limits_{\text{R3b}(Q)}\left[\left(p_H(TP_H)-c_{pH}\right)TP_H+\left(p_L-c_{pL}\right)D_L\right]g(u,\alpha)\,du\,d\alpha\\
+\iint\limits_{\text{R4}(Q)}\left[\left(p_H(TP_H)-c_{pH}\right)TP_H+\left(p_L-c_{pL}\right)D_L\right]g(u,\alpha)\,du\,d\alpha
\end{array}\right\}.$$

The proof can be completed by setting the cost of purchasing fruit in the open market to be infinitely high and the revenue from selling the fruit in the open market to be 0 in model M8, i.e. $b_H=b_L=\infty$ and $s_H=s_L=0$. From proposition 5 and proposition 7, it is possible to show that $p_H(TP_H)=p_H(TS_H)$, $p_H(TB_H)=\infty$ and $p_H(TP_H^{D})=p_H(TP_H^{DT})$, resulting in $TP_H=TS_H$, $TB_H=0$ and $TP_H^{D}=TP_H^{DT}$. Furthermore, as the cost of buying fruit in the low-end segment is infinitely high, and the selling price of the fruit is zero, the fruit-trading flexibility in the low-end segment becomes unattractive for the winemaker, resulting in q_{BL} and q_{SL} to equal 0. As a result of this analysis, regions R1(Q) and R2(Q) from model M8 collapses to 0 while regions R3a(Q), R3b(Q), R4(Q), R5a(Q) R5b(Q) and R6(Q) in model M8 are equivalent to regions R1a(Q), R1b(Q), R2(Q), R3a(Q) R3b(Q) and R4(Q) of model M6 respectively. From this analysis the first-stage objective function of model M8 can be written as:

$$E\left[\Pi_{\text{M8}}(Q)\right]=E\left[\Pi_{\text{M6}}(Q)\right]=-c_l Q+\left\{\begin{array}{l}
\iint\limits_{\text{R1a}(Q)}\left[\left(p_H(Qu\alpha)-c_{pH}\right)Qu\alpha+\left(p_L-c_{pL}\right)D_L\right]g(u,\alpha)\,du\,d\alpha\\
+\iint\limits_{\text{R1b}(Q)}\left[\begin{array}{l}\left(p_H\left(TP_H^D\right)-c_{pH}\right)TP_H^D\\+\left(p_L-c_{pL}\right)\left(D_L+Qu-TP_H^{DT}\right)\end{array}\right]g(u,\alpha)\,du\,d\alpha\\
+\iint\limits_{\text{R2}(Q)}\left[\left(p_H(Qu\alpha)-c_{pH}\right)Qu\alpha+\left(p_L-c_{pL}\right)D_L\right]g(u,\alpha)\,du\,d\alpha\\
+\iint\limits_{\text{R3a}(Q)}\left[\begin{array}{l}\left(p_H(Qu-D_L)-c_{pH}\right)(Qu-D_L)\\+\left(p_L-c_{pL}\right)D_L\end{array}\right]g(u,\alpha)\,du\,d\alpha\\
+\iint\limits_{\text{R3b}(Q)}\left[\left(p_H(TP_H)-c_{pH}\right)TP_H+\left(p_L-c_{pL}\right)D_L\right]g(u,\alpha)\,du\,d\alpha\\
+\iint\limits_{\text{R4}(Q)}\left[\left(p_H(TP_H)-c_{pH}\right)TP_H+\left(p_L-c_{pL}\right)D_L\right]g(u,\alpha)\,du\,d\alpha
\end{array}\right\}$$

As the first stage objective function of both models are now identical, i.e. $E[\Pi^{\text{M6}}(Q)]=E[\Pi^{\text{M8}}(Q)]$, the proof that model M8 is continuous and concave in the vineyard lease Q holds true for model M6.

d) The optimal second-stage decisions for model M5 can be divided into the following sets:

R1(Q) = $\{(u, \alpha) : Qu\alpha \leq TP_H$ and $Qu(1-\alpha) < D_L\}$

R2(Q) = $\{(u, \alpha) : Qu\alpha \leq TP_H$ and $Qu(1-\alpha) \geq D_L\}$

R3(Q) = $\{(u, \alpha) : Qu\alpha > TP_H$ and $Qu(1-\alpha) < D_L\}$

R4(Q) = $\{(u, \alpha) : Qu\alpha > TP_H$ and $Qu(1-\alpha) \geq D_L\}$

The first-stage objective function of model M5 can be written as:

$$E\left[\Pi_{\mathrm{M5}}(Q)\right] = -c_l Q + \left\{ \begin{array}{l} \iint\limits_{\mathrm{R1}(Q)} \left[\left(p_H(Qu\alpha) - c_{pH}\right)Qu\alpha + \left(p_L - c_{pL}\right)Qu(1-\alpha)\right] g(u,\alpha)\,du\,d\alpha \\ + \iint\limits_{\mathrm{R2}(Q)} \left[\left(p_H(Qu\alpha) - c_{pH}\right)Qu\alpha + \left(p_L - c_{pL}\right)D_L\right] g(u,\alpha)\,du\,d\alpha \\ + \iint\limits_{\mathrm{R3}(Q)} \left[\left(p_H(TP_H) - c_{pH}\right)TP_H + \left(p_L - c_{pL}\right)Qu(1-\alpha)\right] g(u,\alpha)\,du\,d\alpha \\ + \iint\limits_{\mathrm{R4}(Q)} \left[\left(p_H(TP_H) - c_{pH}\right)TP_H + \left(p_L - c_{pL}\right)D_L\right] g(u,\alpha)\,du\,d\alpha \end{array} \right\}$$

The proof can be completed by setting the cost of purchasing fruit in the open market to be infinitely high and the selling price of the fruit in the open market to be 0 in model M7, i.e. $b_H = b_L = \infty$ and $s_H = s_L = 0$. From proposition 5, it is possible to show that $p_H(TP_H) = p_H(TS_H)$, $p_H(TB_H) = \infty$ resulting in $TP_H = TS_H$ and $TB_H = 0$. Furthermore as the cost of buying fruit in the low-end segment is infinitely high and the selling price of the fruit is 0, the fruit-trading flexibility in the low-end segment becomes unattractive for the winemaker, resulting in q_{BL} and q_{SL} to equal 0. As a result, regions R1(Q) and R2(Q) from model M7 collapses to 0 awhile region while regions R3(Q), R4(Q), R5a(Q) R5b(Q) and R6(Q) in model M7 are equivalent to regions R1(Q), R2(Q), R3(Q) and R4(Q) of model M5 respectively. The first-stage objective function can then be written as:

$$E\left[\Pi_{\mathrm{M7}}(Q)\right] = E\left[\Pi_{\mathrm{M5}}(Q)\right] = -c_l Q + \left\{ \begin{array}{l} \iint\limits_{\mathrm{R1}(Q)} \left[\left(p_H(Qu\alpha) - c_{pH}\right)Qu\alpha + \left(p_L - c_{pL}\right)Qu(1-\alpha)\right] g(u,\alpha)\,du\,d\alpha \\ + \iint\limits_{\mathrm{R2}(Q)} \left[\left(p_H(Qu\alpha) - c_{pH}\right)Qu\alpha + \left(p_L - c_{pL}\right)D_L\right] g(u,\alpha)\,du\,d\alpha \\ + \iint\limits_{\mathrm{R3}(Q)} \left[\left(p_H(TP_H) - c_{pH}\right)TP_H + \left(p_L - c_{pL}\right)Qu(1-\alpha)\right] g(u,\alpha)\,du\,d\alpha \\ + \iint\limits_{\mathrm{R4}(Q)} \left[\left(p_H(TP_H) - c_{pH}\right)TP_H + \left(p_L - c_{pL}\right)D_L\right] g(u,\alpha)\,du\,d\alpha \end{array} \right\}$$

Because the first-stage objective functions of both model are identical, i.e. $E[\Pi_{\mathrm{M5}}(Q)] = E[\Pi_{\mathrm{M7}}(Q)]$, the proof that model M7 is continuous and concave in the vineyard lease Q holds true for model M5.

e) The expected profit for model M4 can be written as:

$$E\left[\Pi_{\mathrm{M4}}(Q)\right]=-c_l Q+\left\{\begin{array}{l}\iint\limits_{\mathrm{R1}(Q)}\left[\begin{array}{l}\left(p_H-c_{pH}-b_H\right)D_H+b_H Qu\alpha\\+\left(p_L-c_{pL}-b_L\right)D_L+b_L Qu(1-\alpha)\end{array}\right]g(u,\alpha)\,du\,d\alpha\\+\iint\limits_{\mathrm{R2}(Q)}\left[\begin{array}{l}\left(p_H-c_{pH}-b_H\right)D_H+b_H Qu\alpha\\+\left(p_L-c_{pL}-s_L\right)D_L+s_L Qu(1-\alpha)\end{array}\right]g(u,\alpha)\,du\,d\alpha\\+\iint\limits_{\mathrm{R3a}(Q)}\left[\begin{array}{l}\left(p_H-c_{pH}\right)D_H\\+\left(p_L-c_{pL}\right)D_L-b_L\left(D_L+D_H-Qu\right)\end{array}\right]g(u,\alpha)\,du\,d\alpha\\+\iint\limits_{\mathrm{R3b}(Q)}\left[\begin{array}{l}\left(p_H-c_{pH}\right)D_H+s_H\left(Qu-D_H-D_L\right)\\\left(p_L-c_{pL}\right)D_L\end{array}\right]g(u,\alpha)\,du\,d\alpha\\+\iint\limits_{\mathrm{R4}(Q)}\left[\begin{array}{l}\left(p_H-c_{pH}-s_H\right)D_H+s_H Qu\alpha\\+\left(p_L-c_{pL}-s_L\right)D_L+s_L Qu(1-\alpha)\end{array}\right]g(u,\alpha)\,du\,d\alpha\end{array}\right\}$$

Let us define the boundary points: $u_6(Q,\alpha) = D_H/Q\alpha$, $u_7(Q) = (D_H+D_L)/Q$ and $\alpha_1(Q,u) = 1 - (D_L/Qu)$

Note that: $u_6^{'}(Q,\alpha) = \partial u_6(Q,\alpha)/\partial Q \leq 0$, $u_7^{'}(Q) = \partial u_7(Q)/\partial Q \leq 0$ and $\alpha_1^{'}(Q,u) = \partial\alpha_1(Q,u)/\partial Q > 0$.

Taking the first-order derivative of the first-stage objective function $E[\Pi_{\mathrm{M4}}(Q)]$ gives:

$$\frac{\partial E\left[\Pi_{\mathrm{M4}}(Q)\right]}{\partial Q}=-c_l+\left\{\begin{array}{l}\int\limits_{u_l}^{u_6(Q,\alpha)}\int\limits_{\alpha_1(Q,u)}^{\alpha_h}\left[b_H u\alpha+b_L u(1-\alpha)\right]g(u,\alpha)\,d\alpha\,du\\+\int\limits_{u_l}^{u_6(Q,\alpha)}\int\limits_{\alpha_l}^{\alpha_1(Q,u)}\left[b_H u\alpha+s_L u(1-\alpha)\right]g(u,\alpha)\,d\alpha\,du\\+\int\limits_{u_6(Q,\alpha)}^{u_7(Q,\alpha)}\int\limits_{\alpha_1(Q,u)}^{a_h}\left[b_L u\right]g(u,\alpha)\,d\alpha\,du\\+\int\limits_{u_7(Q,\alpha)}^{u_h}\int\limits_{\alpha_1(Q,u)}^{\alpha_h}\left[s_H u\right]g(u,\alpha)\,d\alpha\,du\\+\int\limits_{u_6(Q,\alpha)}^{u_h}\int\limits_{\alpha_l}^{\alpha_1(Q,u)}\left[s_H u\alpha+s_L u(1-\alpha)\right]g(u,\alpha)\,d\alpha\,du\end{array}\right\}$$

Taking the second-order derivative of the first-stage objective provides:

$$\frac{\partial^2 E\left[\Pi_{M4}(Q)\right]}{\partial Q^2}=\left\{\begin{array}{l} u_6'(Q,\alpha)\displaystyle\int_{\alpha_1(Q,u_6(Q,\alpha))}^{\alpha_h}\left[b_H u_6(Q,\alpha)\alpha+b_L u_6(Q,\alpha)(1-\alpha)\right]g\left(u_6(Q,\alpha),\alpha\right)d\alpha \\ +u_6'(Q,\alpha)\displaystyle\int_{\alpha_l}^{\alpha_1(Q,u_6(Q,\alpha))}\left[b_H u_6(Q,\alpha)\alpha+s_L u_6(Q,\alpha)(1-\alpha)\right]g\left(u_6(Q,\alpha),\alpha\right)d\alpha \\ -u_6'(Q,\alpha)\displaystyle\int_{\alpha_1(Q,u_6(Q,\alpha))}^{\alpha_h}\left[b_L u_6(Q,\alpha)\right]g\left(u_6(Q,\alpha),\alpha\right)d\alpha \\ +u_7'(Q,\alpha)\displaystyle\int_{\alpha_1(Q,u_7(Q,\alpha))}^{\alpha_h}\left[b_L u_7(Q,\alpha)\right]g\left(u_7(Q,\alpha),\alpha\right)d\alpha \\ -u_7'(Q,\alpha)\displaystyle\int_{\alpha_1(Q,u_7(Q,\alpha))}^{\alpha_h}\left[s_H u_7(Q,\alpha)\right]g\left(u_7(Q,\alpha),\alpha\right)d\alpha \\ -u_6'(Q,\alpha)\displaystyle\int_{\alpha_l}^{\alpha_1(Q,u_6(Q,\alpha))}\left[s_H u_6(Q,\alpha)\alpha+s_L u_6(Q,\alpha)(1-\alpha)\right]g\left(u_6(Q,\alpha),\alpha\right)d\alpha \end{array}\right\}$$

Observe that, because $b_H > s_H$, $b_L > s_L$ and $b_H > b_L$, while $u_6'(Q,\alpha)$ and $u_7'(Q,\alpha)$ are negative, the second-order derivative of model M4 first stage objective function is negative, i.e. $\partial^2 E[\Pi_{M4}(Q)]/\partial Q^2 < 0$, and thus M4 first stage objective function is continuous and concave in the amount of vineyard lease Q.

f) In model M3, the optimal second stage decision can be divided into the following sets:

$R1(Q) = \{(u, \alpha) : Qu\alpha \le D_H \text{ and } Qu(1-\alpha) < D_L\}$

$R2(Q) = \{(u, \alpha) : Qu\alpha \le D_H \text{ and } Qu(1-\alpha) \ge D_L\}$

$R3(Q) = \{(u, \alpha) : Qu\alpha > D_H \text{ and } Qu(1-\alpha) < D_L\}$

$R4(Q) = \{(u, \alpha) : Qu\alpha > D_H \text{ and } Qu(1-\alpha) \ge D_L\}$

Accordingly, the optimal second-stage quantity decisions for M7 are:

$$\begin{pmatrix} q_{IH}^*, q_{BH}^*, q_{SH}^*, \\ q_{IL}^*, q_{BL}^*, q_{SL}^* \end{pmatrix}=\left\{\begin{array}{ll} \begin{pmatrix} Qu\alpha, D_H-Qu\alpha, 0 \\ Qu(1-\alpha), \Delta, 0 \end{pmatrix} & \text{if } (u,\alpha)\in R1(Q) \\ \begin{pmatrix} Qu\alpha, D_H-Qu\alpha, 0 \\ D_L, 0, -\Delta \end{pmatrix} & \text{if } (u,\alpha)\in R2(Q) \\ \begin{pmatrix} D_H, 0, Qu\alpha-D_H, \\ Qu(1-\alpha), \Delta, 0 \end{pmatrix} & \text{if } (u,\alpha)\in R3(Q) \\ \begin{pmatrix} D_H, 0, Qu\alpha-D_H, \\ D_L, 0, -\Delta \end{pmatrix} & \text{if } (u,\alpha)\in R4(Q) \end{array}\right.$$

Therefore, the first-stage objective function can be written as:

$$E\left[\Pi_{\mathrm{M3}}(Q)\right]=-c_l Q+\left\{\begin{array}{l}\displaystyle\iint_{\mathrm{R1}(Q)}\left[\begin{array}{l}\left(p_H-c_{pH}-b_H\right)D_H+b_H Qu\alpha\\+\left(p_L-c_{pL}-b_L\right)D_L+b_L Qu(1-\alpha)\end{array}\right]g(u,\alpha)\,du\,d\alpha\\+\displaystyle\iint_{\mathrm{R2}(Q)}\left[\begin{array}{l}\left(p_H-c_{pH}-b_H\right)D_H+b_H Qu\alpha\\+\left(p_L-c_{pL}-s_L\right)D_L+s_L Qu(1-\alpha)\end{array}\right]g(u,\alpha)\,du\,d\alpha\\+\displaystyle\iint_{\mathrm{R3}(Q)}\left[\begin{array}{l}\left(p_H-c_{pH}-s_H\right)D_H+s_H Qu\alpha\\+\left(p_L-c_{pL}-b_L\right)D_L+b_L Qu(1-\alpha)\end{array}\right]g(u,\alpha)\,du\,d\alpha\\+\displaystyle\iint_{\mathrm{R4}(Q)}\left[\begin{array}{l}\left(p_H-c_{pH}-s_H\right)D_H+s_H Qu\alpha\\+\left(p_L-c_{pL}-s_L\right)D_L+s_L Qu(1-\alpha)\end{array}\right]g(u,\alpha)\,du\,d\alpha\end{array}\right\}$$

Rewriting the first-stage objective function according to the returns from high and low-end segments:

$$E\left[\Pi_{\mathrm{M3}}(Q)\right]=-c_l Q+\left\{\begin{array}{l}\displaystyle\iint_{\mathrm{R1}(Q)\cup\mathrm{R2}(Q)}\left[\left(p_H-c_{pH}-b_H\right)D_H+b_H Qu\alpha\right]g(u,\alpha)\,du\,d\alpha\\+\displaystyle\iint_{\mathrm{R3}(Q)\cup\mathrm{R4}(Q)}\left[\left(p_H-c_{pH}-s_H\right)D_H+s_H Qu\alpha\right]g(u,\alpha)\,du\,d\alpha\\+\displaystyle\iint_{\mathrm{R1}(Q)\cup\mathrm{R3}(Q)}\left[\left(p_L-c_{pL}-b_L\right)D_L+b_L Qu(1-\alpha)\right]g(u,\alpha)\,du\,d\alpha\\+\displaystyle\iint_{\mathrm{R2}(Q)\cup\mathrm{R4}(Q)}\left[+\left(p_L-c_{pL}-s_L\right)D_L+s_L Qu(1-\alpha)\right]g(u,\alpha)\,du\,d\alpha\end{array}\right\}$$

Taking the first-order derivative of the above expression gives:

$$\frac{\partial E\left[\Pi_{\mathrm{M3}}(Q)\right]}{\partial Q}=-c_l+\left\{\begin{array}{l}\displaystyle\iint_{\mathrm{R1}(Q)\cup\mathrm{R2}(Q)}\left[b_H u\alpha\right]g(u,\alpha)\,du\,d\alpha\\+\displaystyle\iint_{\mathrm{R3}(Q)\cup\mathrm{R4}(Q)}\left[s_H u\alpha\right]g(u,\alpha)\,du\,d\alpha\\+\displaystyle\iint_{\mathrm{R1}(Q)\cup\mathrm{R3}(Q)}\left[b_L u(1-\alpha)\right]g(u,\alpha)\,du\,d\alpha\\+\displaystyle\iint_{\mathrm{R4}(Q)\cup\mathrm{R6}(Q)}\left[s_L u(1-\alpha)\right]g(u,\alpha)\,du\,d\alpha\end{array}\right\}$$

Let us define the boundary points: $\alpha_1(Q,u) = 1 - (D_L/Qu)$, $\alpha_4(Q,u) = D_H/Qu$ and note that, $\alpha_1^{'}(Q,u) = \partial\alpha_1(Q,u)/\partial Q > 0$ and $\alpha_4^{'}(Q,u) = \partial\alpha_4(Q,u)/\partial Q \leq 0$.

Therefore taking the second-order derivative of M3 first-stage objective function gives:

$$\frac{\partial^2 E\left[\Pi_{\text{M3}}(Q)\right]}{\partial Q^2}=\left\{\begin{array}{l}\alpha_6^{'}(Q,u)\int_{u_l}^{u_h}\left[b_H u\alpha_6(Q,u)\right]g\left(u,\alpha_6(Q,u)\right)du \\ +\alpha_6^{'}(Q,u)\int_{u_l}^{u_h}\left[s_H u\alpha_6(Q,u)\right]g\left(u,\alpha_6(Q,u)\right)du \\ -\alpha_1^{'}(Q,u)\int_{u_l}^{u_h}\left[b_L u\left(1-\alpha_1(Q,u)\right)\right]g\left(u,\alpha_1(Q,u)\right)du \\ +\alpha_1^{'}(Q,u)\int_{u_l}^{u_h}\left[s_L u\left(1-\alpha_1(Q,u)\right)\right]g\left(u,\alpha_1(Q,u)\right)du\end{array}\right\}$$

Observe that, because $b_H > s_H$, $b_L > s_L$, $\alpha_1^{'}(Q,u) > 0$ and $\alpha_6^{'}(Q,u) < 0$, the second-order derivative of model M3 first stage objective function is negative, i.e. $\partial^2 E[\Pi_{\text{M3}}(Q)]/\partial Q^2 < 0$, and thus, the first-stage objective function of M3 is continuous and concave in the amount of vineyard lease Q.

g) In model M2, the optimal second stage decision can be divided into the following sets:

R1(Q) = $\{(u, \alpha) : Qu\alpha \leq D_H$ and $Qu(1-\alpha) < D_L\}$

R2(Q) = $\{(u, \alpha) : Qu\alpha \leq D_H$ and $Qu(1-\alpha) \geq D_L\}$

R3a(Q) = $\{(u, \alpha) : D_H < Qu\alpha \leq D_H + \Delta$ and $Qu(1-\alpha) < D_L\}$

R3b(Q) = $\{(u, \alpha) : D_H + \Delta < Qu\alpha$ and $Qu(1-\alpha) < D_L\}$

R4(Q) = $\{(u, \alpha) : Qu\alpha > D_H$ and $Qu(1-\alpha) \geq D_L\}$

The proof can be completed by setting the cost of purchasing fruit in the open market to be infinitely high and the selling price of the fruit in the open market to be 0 in model M4, i.e. $b_H=b_L=\infty$ and $s_H=s_L=0$. It is possible to show that the fruit-trading flexibility in the low-end segment becomes unattractive for the winemaker, resulting in q_{BL} and q_{SL} to equal 0. As a result, the returns from all five regions in model M4 become identical to the returns in the five regions of model M2. Therefore, the first-stage objective function of the model can be written as:

$$E\left[\Pi_{\text{M4}}(Q)\right]=E\left[\Pi_{\text{M2}}(Q)\right]-c_l Q+\left\{\begin{array}{l}\iint_{\text{R1}(Q)}\left[\left(p_H-c_{pH}\right)Qu\alpha+\left(p_L-c_{pL}\right)Qu(1-\alpha)\right]g(u,\alpha)\,du\,d\alpha \\ +\iint_{\text{R2}(Q)}\left[\left(p_H-c_{pH}\right)Qu\alpha+\left(p_L-c_{pL}\right)D_L\right]g(u,\alpha)\,du\,d\alpha \\ +\iint_{\text{R3a}(Q)}\left[\left(p_H-c_{pH}\right)D_H+\left(p_L-c_{pL}\right)\left(Qu-D_H\right)\right]g(u,\alpha)\,du\,d\alpha \\ +\iint_{\text{R3b}(Q)}\left[\left(p_H-c_{pH}\right)D_H+\left(p_L-c_{pL}\right)D_L\right]g(u,\alpha)\,du\,d\alpha \\ +\iint_{\text{R4}(Q)}\left[\left(p_H-c_{pH}\right)D_H+\left(p_L-c_{pL}\right)D_L\right]g(u,\alpha)\,du\,d\alpha\end{array}\right\}$$

Therefore, as the first stage objective function of both model are equivalent, i.e. $E[\Pi_{M4}(Q)] = E[\Pi_{M2}(Q)]$, the proof that model M4 is continuous and concave in the vineyard lease Q holds true for model M2.

h) In model M1, the optimal second stage decision can be divided into the following sets:

$R1(Q) = \{(u, \alpha) : Qu\alpha \leq D_H \text{ and } Qu(1-\alpha) < D_L\}$

$R2(Q) = \{(u, \alpha) : Qu\alpha \leq D_H \text{ and } Qu(1-\alpha) \geq D_L\}$

$R3(Q) = \{(u, \alpha) : Qu\alpha > D_H \text{ and } Qu(1-\alpha) < D_L\}$

$R4(Q) = \{(u, \alpha) : Qu\alpha > D_H \text{ and } Qu(1-\alpha) \geq D_L\}$

The proof can be completed by setting the cost of purchasing fruit in the open market to be infinitely high and the selling price of the fruit in the open market to be 0 in model M3, i.e. $b_H = b_L = \infty$ and $s_H = s_L = 0$. In this case, fruit-trading in the low-end segment becomes unattractive, resulting in $q_{BL} = q_{SL} = 0$. As a result, the returns from all four regions in model M3 become identical to the returns in the four regions of model M1. Therefore the first-stage objective function can be written as:

$$E\left[\Pi_{M3}(Q)\right] = E\left[\Pi_{M1}(Q)\right] = -c_l Q + \left\{ \begin{array}{l} \iint\limits_{R1(Q)} \left[\left(p_H - c_{pH}\right)Qu\alpha + \left(p_L - c_{pL}\right)Qu(1-\alpha)\right] g(u,\alpha)\,du\,d\alpha \\ + \iint\limits_{R2(Q)} \left[\left(p_H - c_{pH}\right)Qu\alpha + \left(p_L - c_{pL}\right)D_L\right] g(u,\alpha)\,du\,d\alpha \\ + \iint\limits_{R3(Q)} \left[\left(p_H - c_{pH}\right)D_H(p_H) + \left(p_L - c_{pL}\right)Qu(1-\alpha)\right] g(u,\alpha)\,du\,d\alpha \\ + \iint\limits_{R4(Q)} \left[\left(p_H - c_{pH}\right)D_H(p_H) + \left(p_L - c_{pL}\right)D_L\right] g(u,\alpha)\,du\,d\alpha \end{array} \right\}$$

Because the first-stage objective function of both models are equivalent, i.e. $E[\Pi^{M3}(Q)] = E[\Pi^{M1}(Q)]$, the proof that model M3 is continuous and concave in the vineyard lease Q holds true for model M1.□

Proof of Proposition 13: By the implicit function theorem, for parameter $a \in \{b_H, b_L, s_H, s_L\}$,

$$\frac{\partial Q_j^*}{\partial a} = -\frac{\partial^2 E\left[\Pi\left(Q_j^*\right)\right] / \partial Q \partial a}{\partial^2 E\left[\Pi\left(Q_j^*\right)\right] / \partial Q^2},\ j \in \{\text{M3, M4, M7, M8}\} \tag{9}$$

From Proposition 12, $\partial^2 E\left[\Pi\left(Q_j^*\right)\right] / \partial Q^2 < 0$, so the sign of (9) is determined by the sign of $\partial^2 E\left[\Pi\left(Q_j^*\right)\right] / \partial Q \partial a$. Consider the case of $a = b_H$. At the optimal lease quantity for model j, there exists a set of realizations of $(\tilde{u}, \tilde{\alpha})$, denoted B_H, where the firm buys high-quality fruit from the open market. The optimal expected profit can be decomposed into two terms—one term that includes parameter b_H and another term, denoted $A^*(Q^*_j)$, that does not include b_H (i.e., $\partial^2 A^*(Q^*_j)/\partial Q \partial a = 0$) and thus

$$E\left[\Pi\left(Q_j^*\right)\right] = \iint\limits_{B_H} \left[\left(p_H^* - c_{pH}\right)D_H\left(p_H^*\right) - b_H\left(D_H\left(p_H^*\right) - Q_j^* u\alpha\right)\right] g(u,\alpha)\,du\,d\alpha + A^*.$$

Accordingly,

$$\frac{\partial E\left[\Pi\left(Q_j^*\right)\right]}{\partial Q} = b_H \iint_{B_H} u\alpha g(u,\alpha)\,du\,d\alpha + \frac{\partial A^*\left(Q_j^*\right)}{\partial Q}$$

$$\frac{\partial^2 E\left[\Pi\left(Q_j^*\right)\right]}{\partial Q \partial b_H} = \iint_{B_H} u\alpha g(u,\alpha)\,du\,d\alpha > 0.$$

A similar approach can be used to show $\frac{\partial Q_j^*}{\partial b_L} > 0$, $\frac{\partial Q_j^*}{\partial s_H} > 0$, $\frac{\partial Q_j^*}{\partial s_L} > 0$. We omit the details. □

Proof of Corollary 1: i) The first-stage objective function for model M1 when the firm can sell its fruit in the open market can be written as follows:

$$E\left[\Pi_{\mathrm{M1}}(Q)\right] = -c_l Q + \iint_{\mathrm{R1}(Q)} \left[\left(p_H - c_{pH}\right)Qu\alpha + \left(p_L - c_{pL}\right)Qu(1-\alpha)\right] g(u,\alpha)\,du\,d\alpha$$

$$+ \iint_{\mathrm{R2}(Q)} \left[\left(p_H - c_{pH}\right)Qu\alpha + \left(p_L - c_{pL} - s_L\right)D_L + s_L Qu(1-\alpha)\right] g(u,\alpha)\,du\,d\alpha$$

$$+ \iint_{\mathrm{R3}(Q)} \left[\left(p_H - c_{pH} - s_H\right)D_H + s_H Qu\alpha + \left(p_L - c_{pL}\right)Qu(1-\alpha)\right] g(u,\alpha)\,du\,d\alpha$$

$$+ \iint_{\mathrm{R4}(Q)} \left[\left(p_H - c_{pH} - s_H\right)D_H + s_H Qu\alpha + \left(p_L - c_{pL} - s_L\right)D_L + s_L Qu(1-\alpha)\right] g(u,\alpha)\,du\,d\alpha$$

The first-order derivative of the first-stage objective function in model M1 is equal to:

$$\partial E\left[\Pi_{\mathrm{M1}}(Q)\right] / \partial Q = -c_l + \iint_{\mathrm{R1}(Q)} \left[\left(p_H - c_{pH}\right)u\alpha + \left(p_L - c_{pL}\right)u(1-\alpha)\right] g(u,\alpha)\,du\,d\alpha$$

$$+ \iint_{\mathrm{R2}(Q)} \left[\left(p_H - c_{pH}\right)u\alpha + s_L u(1-\alpha)\right] g(u,\alpha)\,du\,d\alpha + + \iint_{\mathrm{R3}(Q)} \left[s_H u\alpha + \left(p_L - c_{pL}\right)u(1-\alpha)\right] g(u,\alpha)\,du\,d\alpha$$

$$+ \iint_{\mathrm{R4}(Q)} \left[s_H u\alpha + s_L u(1-\alpha)\right] g(u,\alpha)\,du\,d\alpha$$

Equating the above first-order derivative to zero provides $Q_{M1}{}^*$ where at $Q = Q_{M1}{}^*$, we have

$$c_l = \iint_{\mathrm{R1}\left(Q_{\mathrm{M1}}^*\right)} \left[\left(p_H - c_{pH}\right)u\alpha + \left(p_L - c_{pL}\right)u(1-\alpha)\right] g(u,\alpha)\,du\,d\alpha$$

$$+ \iint_{\mathrm{R2}\left(Q_{\mathrm{M1}}^*\right)} \left[\left(p_H - c_{pH}\right)u\alpha + s_L u(1-\alpha)\right] g(u,\alpha)\,du\,d\alpha + \iint_{\mathrm{R3}\left(Q_{\mathrm{M1}}^*\right)} \left[s_H u\alpha + \left(p_L - c_{pL}\right)u(1-\alpha)\right] g(u,\alpha)\,du\,d\alpha$$

$$+ \iint_{\mathrm{R4}\left(Q_{\mathrm{M1}}^*\right)} \left[s_H u\alpha + s_L u(1-\alpha)\right] g(u,\alpha)\,du\,d\alpha$$

We next consider the objective function of model M3:

$$E\left[\Pi_{M3}(Q)\right] = -c_l Q + \iint_{R1(Q)} \begin{bmatrix} \left(p_H - c_{pH} - b_H\right) D_H + b_H Q u\alpha \\ +\left(p_L - c_{pL} - b_L\right) D_L + b_L Q u(1-\alpha) \end{bmatrix} g(u,\alpha)\,du\,d\alpha$$

$$+ \iint_{R2(Q)} \left[\left(p_H - c_{pH} - b_H\right) D_H + b_H Q u\alpha + \left(p_L - c_{pL} - s_L\right) D_L + s_L Q u(1-\alpha)\right] g(u,\alpha)\,du\,d\alpha$$

$$+ \iint_{R3(Q)} \left[\left(p_H - c_{pH} - s_H\right) D_H + s_H Q u\alpha + \left(p_L - c_{pL} - b_L\right) D_L + b_L Q u(1-\alpha)\right] g(u,\alpha)\,du\,d\alpha$$

$$+ \iint_{R4(Q)} \left[\left(p_H - c_{pH} - s_H\right) D_H + s_H Q u\alpha + \left(p_L - c_{pL} - s_L\right) D_L + s_L Q u(1-\alpha)\right] g(u,\alpha)\,du\,d\alpha$$

The first-order derivative of model M3 is:

$$\partial E\left[\Pi_{M3}(Q)\right] / \partial Q = -c_l + \iint_{R1(Q)} \left[b_H u\alpha + b_L u(1-\alpha)\right] g(u,\alpha)\,du\,d\alpha + \iint_{R2(Q)} \left[b_H u\alpha + s_L u(1-\alpha)\right] g(u,\alpha)\,du\,d\alpha$$

$$+ \iint_{R3(Q)} \left[s_H u\alpha + b_L u(1-\alpha)\right] g(u,\alpha)\,du\,d\alpha + \iint_{R4(Q)} \left[s_H u\alpha + s_L u(1-\alpha)\right] g(u,\alpha)\,du\,d\alpha$$

Evaluating the first-order derivative at $Q = Q_{M1}{}^{*}$ and substituting the above expression for c_l provides:

$$\partial E\left[\Pi_{M3}(Q)\right] / \partial Q\Big|_{Q=Q^{*}_{M1}} = -\iint_{R1(Q^{*}_{M1})} \left[\left(p_H - c_{pH} - b_H\right) u\alpha + \left(p_L - c_{pL} - b_L\right) u(1-\alpha)\right] g(u,\alpha)\,du\,d\alpha$$

$$- \iint_{R2(Q^{*}_{M1})} \left[\left(p_H - c_{pH} - b_H\right) u\alpha\right] g(u,\alpha)\,du\,d\alpha - \iint_{R3(Q^{*}_{M1})} \left[\left(p_L - c_{pL} - b_L\right) u(1-\alpha)\right] g(u,\alpha)\,du\,d\alpha < 0$$

because $p_H - c_{pH} - b_H > 0$ and $p_L - c_{pL} - b_L > 0$. This implies that $Q_{M3}{}^{*}$ is less than $Q_{M1}{}^{*}$.

ii) The first-stage objective function for M2 can be written as:

$$E\left[\Pi_{M2}(Q)\right] = -c_l Q + \iint_{R1(Q)} \left[\left(p_H - c_{pH}\right) Q u\alpha + \left(p_L - c_{pL}\right) Q u(1-\alpha)\right] g(u,\alpha)\,du\,d\alpha$$

$$+ \iint_{R2(Q)} \left[\left(p_H - c_{pH}\right) Q u\alpha + \left(p_L - c_{pL} - s_L\right) D_L + s_L Q u(1-\alpha)\right] g(u,\alpha)\,du\,d\alpha$$

$$+ \iint_{R3a(Q)} \left[\left(p_H - c_{pH}\right) D_H + \left(p_L - c_{pL}\right)\left(Qu - D_H\right)\right] g(u,\alpha)\,du\,d\alpha$$

$$+ \iint_{R3b(Q)} \left[\left(p_H - c_{pH}\right) D_H + s_H\left(Qu - D_H - D_L\right) + \left(p_L - c_{pL}\right) D_L\right] g(u,\alpha)\,du\,d\alpha$$

$$+ \iint_{R4(Q)} \left[\left(p_H - c_{pH} - s_H\right) D_H + s_H Q u\alpha + \left(p_L - c_{pL} - s_L\right) D_L + s_L Q u(1-\alpha)\right] g(u,\alpha)\,du\,d\alpha$$

and the first-order derivative is equal to:

$$\partial E\left[\Pi_{M2}(Q)\right] / \partial Q = -c_l + \iint_{R1(Q)} \left[\left(p_H - c_{pH}\right) u\alpha + \left(p_L - c_{pL}\right) u(1-\alpha)\right] g(u,\alpha)\,du\,d\alpha$$

$$+ \iint_{R2(Q)} \left[\left(p_H - c_{pH}\right) u\alpha + s_L u(1-\alpha)\right] g(u,\alpha)\,du\,d\alpha + \iint_{R3a(Q)} \left[\left(p_L - c_{pL}\right) u\right] g(u,\alpha)\,du\,d\alpha$$

$$+ \iint_{R3b(Q)} \left[s_H u\right] g(u,\alpha)\,du\,d\alpha + \iint_{R4(Q)} \left[s_H u\alpha + s_L u(1-\alpha)\right] g(u,\alpha)\,du\,d\alpha$$

Equating the above first-order derivative to zero provides Q_{M2}^* where at $Q = Q_{M2}^*$, we have

$$c_l = \iint_{R1(Q_{M2}^*)} \left[\left(p_H - c_{pH}\right)u\alpha + \left(p_L - c_{pL}\right)u(1-\alpha)\right]g(u,\alpha)\,du\,d\alpha$$
$$+ \iint_{R2(Q_{M2}^*)} \left[\left(p_H - c_{pH}\right)u\alpha + s_L u(1-\alpha)\right]g(u,\alpha)\,du\,d\alpha + \iint_{R3a(Q_{M2}^*)} \left[\left(p_L - c_{pL}\right)u\right]g(u,\alpha)\,du\,d\alpha$$
$$+ \iint_{R3b(Q_{M2}^*)} \left[s_H u\right]g(u,\alpha)\,du\,d\alpha + \iint_{R4(Q_{M2}^*)} \left[s_H u\alpha + s_L u(1-\alpha)\right]g(u,\alpha)\,du\,d\alpha$$

We next consider the objective function of model M4:

$$E\left[\Pi_{M4}(Q)\right] = -c_l Q + \iint_{R1(Q)} \begin{bmatrix} \left(p_H - c_{pH} - b_H\right)D_H + b_H Qu\alpha \\ +\left(p_L - c_{pL} - b_L\right)D_L + b_L Qu(1-\alpha) \end{bmatrix} g(u,\alpha)\,du\,d\alpha$$
$$+ \iint_{R2(Q)} \left[\left(p_H - c_{pH} - b_H\right)D_H + b_H Qu\alpha + \left(p_L - c_{pL} - s_L\right)D_L + s_L Qu(1-\alpha)\right]g(u,\alpha)\,du\,d\alpha$$
$$+ \iint_{R3a(Q)} \left[\left(p_H - c_{pH} - s_H\right)D_H + \left(p_L - c_{pL} - b_L\right)\left(Qu - D_H\right)\right]g(u,\alpha)\,du\,d\alpha$$
$$+ \iint_{R3b(Q)} \left[\left(p_H - c_{pH} - s_H\right)D_H + s_H\left(Qu - D_H - D_L\right) + \left(p_L - c_{pL} - b_L\right)D_L\right]g(u,\alpha)\,du\,d\alpha$$
$$+ \iint_{R4(Q)} \left[\left(p_H - c_{pH} - s_H\right)D_H + s_H Qu\alpha + \left(p_L - c_{pL} - s_L\right)D_L + s_L Qu(1-\alpha)\right]g(u,\alpha)\,du\,d\alpha$$

The first-order derivative of model M4 is:

$$\partial E\left[\Pi_{M4}(Q)\right]/\partial Q = -c_l + \iint_{R1(Q)} \left[b_H u\alpha + b_L u(1-\alpha)\right]g(u,\alpha)\,du\,d\alpha$$
$$+ \iint_{R2(Q)} \left[b_H u\alpha + s_L u(1-\alpha)\right]g(u,\alpha)\,du\,d\alpha + \iint_{R3a(Q)} \left[\left(p_L - c_{pL}\right)u\right]g(u,\alpha)\,du\,d\alpha$$
$$+ \iint_{R3b(Q)} \left[s_H u\right]g(u,\alpha)\,du\,d\alpha + + \int_{R4(Q)} \left[s_H u\alpha + s_L u(1-\alpha)\right]g(u)\,du$$

Evaluating the first-order derivative at $Q = Q_{M2}^*$ and substituting the above expression for c_l provides:

$$\partial E\left[\Pi_{M4}(Q)\right]/\partial Q\Big|_{Q=Q_{M2}^*} = -\iint_{R1(Q_{M2}^*)} \left[\left(p_H - c_{pH} - b_H\right)u\alpha + \left(p_L - c_{pL} - b_L\right)u(1-\alpha)\right]g(u,\alpha)\,du\,d\alpha$$
$$- \iint_{R2(Q_{M2}^*)} \left[\left(p_H - c_{pH} - b_H\right)u\alpha\right]g(u,\alpha)\,du\,d\alpha < 0$$

because $p_H - c_{pH} - b_H > 0$ and $p_L - c_{pL} - b_L > 0$. This implies that Q_{M4}^* is less than Q_{M2}^*.□

iii) The first-stage objective function for models M5 and M7 can be written as:

$$E\left[\Pi_{M5}(Q)\right] = -c_l Q + \iint_{R1(Q)\cup\ldots\cup R4(Q)} PA(Q,u,\alpha)g(u,\alpha)\,du\,d\alpha\,,$$

and $E\left[\Pi_{M7}(Q)\right] = -c_l Q + \iint_{R1(Q)\cup\ldots\cup R6(Q)} PA(Q,u,\alpha)g(u,\alpha)\,du\,d\alpha\,.$

Similar to the proof of Proposition 13 i), for the purpose of comparison, we allow for the winemaker to sell (salvage) fruits in model M5. Therefore, with the ability to sell excess fruit in the open market, the winemaker sets the profit maximizing price, $p_H = p_H(TS_H)$ and sells TS_H amount of high-end wine. The first-derivative of the first-stage objective function then can be written as:

$$\partial E\left[\Pi_{\mathrm{M5}}(Q)\right]/\partial Q = -c_l + \iint_{\mathrm{R1}(Q)} \begin{Bmatrix} \left(p_H'(Qu\alpha)Qu\alpha + p_H(Qu\alpha) - c_{pH}\right)u\alpha \\ +\left(p_L - c_{pL}\right)u(1-\alpha) \end{Bmatrix} g(u,\alpha)\,du\,d\alpha$$

$$+ \iint_{\mathrm{R2}(Q)} \left\{\left(p_H'(Qu\alpha)Qu\alpha + p_H(Qu\alpha) - c_{pH}\right)u\alpha + s_L u(1-\alpha)\right\} g(u,\alpha)\,du\,d\alpha$$

$$+ \iint_{\mathrm{R3}(Q)} \left\{s_H u\alpha + \left(p_L - c_{pL}\right)u(1-\alpha)\right\} g(u,\alpha)\,du\,d\alpha + \iint_{\mathrm{R4}(Q)} \left\{s_H u\alpha + s_L u(1-\alpha)\right\} g(u,\alpha)\,du\,d\alpha$$

As the production threshold is TS_H in model M5, the bounds on regions R3(Q) and R4(Q) are equivalent to the bounds on regions R5(Q) and R6(Q) in model M7. Furthermore, due to continuity of the model, it is possible to split regions R1(Q) and R2(Q) in model M5 into four separate regions with bounds that correspond to regions R1(Q), R2(Q), R3(Q) and R4(Q) of M7. Therefore we can re-write the first-derivative of the M5 objective function as:

$$\partial E\left[\Pi_{\mathrm{M5}}(Q)\right]/\partial Q = -c_l + \iint_{\mathrm{R1}(Q)\cup\mathrm{R3}(Q)} \begin{Bmatrix} \left(p_H'(Qu\alpha)Qu\alpha + p_H(Qu\alpha) - c_{pH}\right)u\alpha \\ +\left(p_L - c_{pL}\right)u(1-\alpha) \end{Bmatrix} g(u,\alpha)\,du\,d\alpha$$

$$+ \iint_{\mathrm{R2}(Q)\cup\mathrm{R4}(Q)} \left\{\left(p_H'(Qu\alpha)Qu\alpha + p_H(Qu\alpha) - c_{pH}\right)u\alpha + s_L u(1-\alpha)\right\} g(u,\alpha)\,du\,d\alpha$$

$$+ \iint_{\mathrm{R5}(Q)} \left\{s_H u\alpha + \left(p_L - c_{pL}\right)u(1-\alpha)\right\} g(u,\alpha)\,du\,d\alpha + \iint_{\mathrm{R6}(Q)} \left\{s_H u\alpha + s_L u(1-\alpha)\right\} g(u,\alpha)\,du\,d\alpha$$

and,

$$\partial E\left[\Pi_{\mathrm{M7}}(Q)\right]/\partial Q = -c_l + \iint_{\mathrm{R1}(Q)} \left\{b_H u\alpha + b_L u(1-\alpha)\right\} g(u,\alpha)\,du\,d\alpha$$

$$+ \iint_{\mathrm{R2}(Q)} \left\{b_H u\alpha + s_L u(1-\alpha)\right\} g(u,\alpha)\,du\,d\alpha$$

$$+ \iint_{\mathrm{R3}(Q)} \left\{\left(p_H'(Qu\alpha)Qu\alpha + p_H(Qu\alpha) - c_{pH}\right)u\alpha + \left(p_L - c_{pL}\right)u(1-\alpha)\right\} g(u,\alpha)\,du\,d\alpha$$

$$+ \iint_{\mathrm{R4}(Q)} \left\{\left(p_H'(Qu\alpha)Qu\alpha + p_H(Qu\alpha) - c_{pH}\right)u\alpha + s_L u(1-\alpha)\right\} g(u,\alpha)\,du\,d\alpha$$

$$+ \iint_{\mathrm{R5}(Q)} \left\{s_H u\alpha + b_L u(1-\alpha)\right\} g(u,\alpha)\,du\,d\alpha$$

$$+ \iint_{\mathrm{R6}(Q)} \left\{s_H u\alpha + s_L u(1-\alpha)\right\} g(u,\alpha)\,du\,d\alpha$$

To see the relationship between models M5 and M7, we evaluate the first-derivative of the objective function in model M7 at the optimal Q for model M5. Let Q_{M5}^* be the value of Q that maximizes the expected profit in model M5, i.e. $\partial E\left[\Pi_{M5}\left(Q\right)\right]/\partial Q = 0$.

$$\partial E\left[\Pi_{M7}\left(Q\right)\right]/\partial Q\Big|_{Q=Q_{M5}^*} = \iint_{R1(Q_{M5}^*)} \left\{ \begin{array}{l} \left[b_H - \left(p_H'\left(Qu\alpha\right)Qu\alpha + p_H\left(Qu\alpha\right) - c_{pH}\right)\right]u\alpha \\ + \left[b_L - \left(p_L - c_{pL}\right)\right]u\left(1-\alpha\right) \end{array} \right\} g\left(u,\alpha\right)dud\alpha$$

$$+ \iint_{R2(Q_{M5}^*)} \left\{\left[b_H - \left(p_H'\left(Qu\alpha\right)Qu\alpha + p_H\left(Qu\alpha\right) - c_{pH}\right)\right]u\alpha\right\} g\left(u,\alpha\right)dud\alpha$$

$$+ \iint_{R5(Q_{M5}^*)} \left\{\left[b_L - \left(p_L - c_{pL}\right)\right]u\left(1-\alpha\right)\right\} g\left(u,\alpha\right)dud\alpha$$

As, $p_H'(Qu\alpha) + p_H(Qu\alpha) - c_{pH} \geq p_H(TS_H) - c_{pH} > b_H$ and $p_L - c_{pL} > b_L$, the first-derivative of the objective function in model M7 evaluated at $Q = Q_{M5}^*$ is negative, i.e. $\partial E\left[\Pi_{M7}\left(Q\right)\right]/\partial Q\Big|_{Q=Q_{M5}^*} < 0$. Therefore, at optimal vineyard lease in model M5, the objective function of model M7 is decreasing. Due to the concavity of model M7, the optimal solution for model M7 must have already been reached, and thus $Q_{M5}^* < Q_{M7}^*$.

iv) The proof is similar to the one presented in part ii) when models M2 and M4 are compared. First, observe that when the firm can sell its excess fruit, TP_H of model M6 becomes equivalent to TS_H of model M8. Second, the downward substitution thresholds become equivalent in models M6 and M8 when $p_L - c_{pL} - b_L = 0$, i.e, $TP_H^D = TP_H^{DT}$. Let us begin with model M8. The optimal second-stage decisions can be divided into the following sets:

R1(Q) = $\{(u, \alpha) : Qu\alpha \leq TB_H$ and $Qu(1-\alpha) < D_L\}$

R2(Q) = $\{(u, \alpha) : Qu\alpha \leq TB_H$ and $Qu(1-\alpha) \geq D_L\}$

R3a(Q) = $\{(u, \alpha) : TB_H < Qu\alpha \leq TP_H^{DT}$ and $Qu(1-\alpha) < D_L\}$

R3b(Q) = $\{(u, \alpha) : TP_H^{DT} < Qu\alpha \leq TP_H^{DT} + D_L$ and $Qu(1-\alpha) < D_L\}$

R4(Q) = $\{(u, \alpha) : TB_H < Qu\alpha \leq TS_H$ and $Qu(1-\alpha) \geq D_L\}$

R5a(Q) = $\{(u, \alpha) : TP_H^{DT} + D_L < Qu\alpha \leq TS_H + D_L$ and $Qu(1-\alpha) < D_L\}$

R5b(Q) = $\{(u, \alpha) : Qu\alpha > TS_H + D_L$ and $Qu(1-\alpha) < D_L\}$

R6(Q) = $\{(u, \alpha) : Qu\alpha > TS_H$ and $Qu(1-\alpha) \geq D_L\}$.

The optimal second-stage quantity decisions for model M8 are:

$$\begin{pmatrix} q_{IH}^*, q_{BH}^*, w^*, q_{SH}^*, \\ q_{IL}^*, q_{BL}^*, q_{SL}^* \end{pmatrix} = \begin{cases} \begin{pmatrix} Qu\alpha, TB_H - Qu\alpha, 0, 0, \\ Qu(1-\alpha), D_L - Qu(1-\alpha), 0 \end{pmatrix} & \text{if } (u,\alpha) \in \text{R1}(Q) \\ \begin{pmatrix} Qu\alpha, TB_H - Qu\alpha, 0, 0, \\ D_L, 0, Qu(1-\alpha) - D_L \end{pmatrix} & \text{if } (u,\alpha) \in \text{R2}(Q) \\ \begin{pmatrix} Qu\alpha, 0, 0, 0, \\ Qu(1-\alpha), D_L - Qu(1-\alpha), 0 \end{pmatrix} & \text{if } (u,\alpha) \in \text{R3}a(Q) \\ \begin{pmatrix} TP_H^{DT}, 0, Qu\alpha - TP_H^{DT}, 0, \\ Qu(1-\alpha), D_L - Qu + TP_H^{DT}, 0 \end{pmatrix} & \text{if } (u,\alpha) \in \text{R3}b(Q) \\ \begin{pmatrix} Qu\alpha, 0, 0, 0, \\ D_L, 0, Qu(1-\alpha) - D_L \end{pmatrix} & \text{if } (u,\alpha) \in \text{R4}(Q) \\ \begin{pmatrix} Qu - D_L, 0, D_L - Qu(1-\alpha), 0, \\ Qu(1-\alpha), 0, 0 \end{pmatrix} & \text{if } (u,\alpha) \in \text{R5}a(Q) \\ \begin{pmatrix} TS_H, 0, D_L - Qu(1-\alpha), Qu - TS_H - D_L, \\ Qu(1-\alpha), 0, 0 \end{pmatrix} & \text{if } (u,\alpha) \in \text{R5}b(Q) \\ \begin{pmatrix} TS_H, 0, 0, Qu\alpha - TS_H, \\ Qu(1-\alpha), 0, Qu(1-\alpha) - D_L \end{pmatrix} & \text{if } (u,\alpha) \in \text{R6}(Q) \end{cases}$$

The optimal second-stage quantity decisions for model M6 are:

$$\begin{pmatrix} q_{IH}^*, q_{BH}^*, w^*, q_{SH}^*, \\ q_{IL}^*, q_{BL}^*, q_{SL}^* \end{pmatrix} = \begin{cases} \begin{pmatrix} Qu\alpha, 0, 0, 0, \\ Qu(1-\alpha), 0, 0 \end{pmatrix} & \text{if } (u,\alpha) \in \text{R1}(Q) \\ \begin{pmatrix} Qu\alpha, 0, 0, 0, \\ D_L, 0, Qu(1-\alpha) - D_L \end{pmatrix} & \text{if } (u,\alpha) \in \text{R2}(Q) \\ \begin{pmatrix} Qu\alpha, 0, 0, 0, \\ Qu(1-\alpha), 0, 0 \end{pmatrix} & \text{if } (u,\alpha) \in \text{R3}a(Q) \\ \begin{pmatrix} TP_H^{DT}, 0, Qu\alpha - TP_H^{DT}, 0, \\ Qu(1-\alpha), 0, 0 \end{pmatrix} & \text{if } (u,\alpha) \in \text{R3}b(Q) \\ \begin{pmatrix} Qu\alpha, 0, 0, 0, \\ D_L, 0, Qu(1-\alpha) - D_L \end{pmatrix} & \text{if } (u,\alpha) \in \text{R4}(Q) \\ \begin{pmatrix} Qu - D_L, 0, D_L - Qu(1-\alpha), 0, \\ Qu(1-\alpha), 0, 0 \end{pmatrix} & \text{if } (u,\alpha) \in \text{R5}a(Q) \\ \begin{pmatrix} TS_H, 0, D_L - Qu(1-\alpha), Qu - TS_H - D_L, \\ Qu(1-\alpha), 0, 0 \end{pmatrix} & \text{if } (u,\alpha) \in \text{R5}b(Q) \\ \begin{pmatrix} TS_H, 0, 0, Qu\alpha - TS_H, \\ Qu(1-\alpha), 0, Qu(1-\alpha) - D_L \end{pmatrix} & \text{if } (u,\alpha) \in \text{R6}(Q) \end{cases}$$

The only difference in the expected profit expressions of models M8 and M6 are in regions R1(Q) and R2(Q). The first-stage objective function for model M6 is:

$$E\left[\Pi_{\mathrm{M6}}(Q)\right]=-c_l Q+\left\{\begin{array}{l}\iint\limits_{\mathrm{R1}(Q)}\left[\begin{array}{l}\left(p_H(Qu\alpha)-c_{pH}\right)Qu\alpha\\+\left(p_L-c_{pL}\right)Qu(1-\alpha)\end{array}\right]g(u,\alpha)\,du\,d\alpha\\+\iint\limits_{\mathrm{R2}(Q)}\left[\begin{array}{l}\left(p_H(Qu\alpha)-c_{pH}\right)Qu\alpha\\+\left(p_L-c_{pL}-s_L\right)D_L+s_L Qu(1-\alpha)\end{array}\right]g(u,\alpha)\,du\,d\alpha\\+\iint\limits_{\mathrm{R3a}(Q)}\left[\begin{array}{l}\left(p_H(Qu\alpha)-c_{pH}\right)Qu\alpha\\+\left(p_L-c_{pL}\right)Qu(1-\alpha)\end{array}\right]g(u,\alpha)\,du\,d\alpha\\+\iint\limits_{\mathrm{R3b}(Q)}\left[\begin{array}{l}\left(p_H\left(TP_H^{DT}\right)-c_{pH}\right)TP_H^{DT}\\\left(p_L-c_{pL}\right)\left(Qu-TP_H^{DT}\right)\end{array}\right]g(u,\alpha)\,du\,d\alpha\\+\iint\limits_{\mathrm{R4}(Q)}\left[\begin{array}{l}\left(p_H(Qu\alpha)-c_{pH}\right)Qu\alpha\\+\left(p_L-c_{pL}-s_L\right)D_L+s_L Qu(1-\alpha)\end{array}\right]g(u,\alpha)\,du\,d\alpha\\+\iint\limits_{\mathrm{R5a}(Q)}\left[\begin{array}{l}\left(p_H(Qu-D_L)-c_{pH}\right)(Qu-D_L)\\+\left(p_L-c_{pL}\right)D_L\end{array}\right]g(u,\alpha)\,du\,d\alpha\\+\iint\limits_{\mathrm{R5b}(Q)}\left[\begin{array}{l}\left(p_H(TS_H)-c_{pH}\right)TS_H+s_H\left(Qu-TS_H-D_L\right)\\+\left(p_L-c_{pL}\right)D_L\end{array}\right]g(u,\alpha)\,du\,d\alpha\\+\iint\limits_{\mathrm{R6}(Q)}\left[\begin{array}{l}\left(p_H(TS_H)-c_{pH}-s_H\right)TS_H+s_H Qu\alpha\\+\left(p_L-c_{pL}-s_L\right)D_L+s_L Qu(1-\alpha)\end{array}\right]g(u,\alpha)\,du\,d\alpha\end{array}\right\}$$

The difference in the first-order derivatives for models M8 and M6, evaluated at Q_{M6}^* is:

$$\frac{\partial E\left[\Pi_{\mathrm{M8}}(Q)\right]}{\partial Q}\Big|_{Q=Q_{\mathrm{M6}}^*}-\frac{\partial E\left[\Pi_{\mathrm{M6}}(Q)\right]}{\partial Q}\Big|_{Q=Q_{\mathrm{M6}}^*}=\iint\limits_{\mathrm{R1}(Q)\cup\mathrm{R2}(Q)}\left[\left(b_H-p_H^{'}(Qu\alpha)Qu\alpha-p_H(Qu\alpha)\right)u\alpha\right]g(u,\alpha)\,du\,d\alpha<0$$

Therefore, $Q_{\mathrm{M8}}^* < Q_{\mathrm{M6}}^*$. □

Proof of Proposition 14: The optimal second-stage decisions in model M7 can be classified in the following regions:

$R1(Q) = \{(u, \alpha) : Qu\alpha \le TB_H \text{ and } Qu(1-\alpha) < D_L\}$

$R2(Q) = \{(u, \alpha) : Qu\alpha \le TB_H \text{ and } Qu(1-\alpha) \ge D_L\}$

$R3(Q) = \{(u, \alpha) : TB_H < Qu\alpha \le TS_H \text{ and } Qu(1-\alpha) < D_L\}$

$R4(Q) = \{(u, \alpha) : TB_H < Qu\alpha \le TS_H \text{ and } Qu(1-\alpha) \ge D_L\}$

$R5(Q) = \{(u, \alpha) : Qu\alpha > TS_H \text{ and } Qu(1-\alpha) < D_L\}$

$R6(Q) = \{(u, \alpha) : Qu\alpha > TS_H \text{ and } Qu(1-\alpha) \ge D_L\}$.

It must be noted that the price-setting flexibility is only available for the high-end segment, and the expected revenue from the low-end segment remains the same for both models. Thus, for the purposes of simplification, the low-end revenue function can be omitted from this proof.

a) Let $p_H = p_H(TB_H)$ and therefore $D_H = TB_H$. It is possible to rewrite the first-stage objective function of model M3 as:

$$E\left[\Pi_{\mathrm{M3}}(Q)\right] = -c_l Q + \iint_{\mathrm{R1}(Q)\cup \mathrm{R2}(Q)} \left[\left(p_H(TB_H) - c_{pH} - b_H\right)TB_H + b_H Qu\alpha\right] g(u,\alpha)\,du\,d\alpha$$
$$+ \iint_{\mathrm{R3}(Q)\cup \mathrm{R4}(Q)} \left[\left(p_H(TB_H) - c_{pH} - s_H\right)TB_H + s_H Qu\alpha\right] g(u,\alpha)\,du\,d\alpha$$
$$+ \iint_{\mathrm{R5}(Q)\cup \mathrm{R6}(Q)} \left[\left(p_H(TB_H) - c_{pH} - s_H\right)TB_H + s_H Qu\alpha\right] g(u,\alpha)\,du\,d\alpha$$

And the first-stage objective function of model M7 can be written as:

$$E\left[\Pi_{\mathrm{M7}}(Q)\right] = -c_l Q + \iint_{\mathrm{R1}(Q)\cup \mathrm{R2}(Q)} \left[\left(p_H(TB_H) - c_{pH} - b_H\right)TB_H + b_H Qu\alpha\right] g(u,\alpha)\,du\,d\alpha$$
$$+ \iint_{\mathrm{R3}(Q)\cup \mathrm{R4}(Q)} \left[\left(p_H(Qu\alpha) - c_{pH}\right)Qu\alpha\right] g(u,\alpha)\,du\,d\alpha$$
$$+ \iint_{\mathrm{R5}(Q)\cup \mathrm{R6}(Q)} \left[\left(p_H(TS_H) - c_{pH} - s_H\right)TS_H + s_H Qu\alpha\right] g(u,\alpha)\,du\,d\alpha$$

Because the demand in model M3 is $D_H = TB_H$ of model M7, the firm buys additional fruit in regions R1(Q) and R2(Q) in both models. In regions R3(Q) and R4(Q), the return from the high-end segment in model M3 is $(p_H(TB_H) - c_{pH} - s_H)\, TB_H + s_H Qu\alpha$, whereas in model M7, the return in the high-end segment is $(p_H(Qu\alpha) - c_{pH})Qu\alpha$. Lastly, optimal second-stage decisions and the corresponding returns are equal in regions R5(Q) and R6(Q) in both models. Therefore, let $\Lambda_{M7\text{-}M3}$ be the difference between the expected returns in models M7 and M3, i.e. $\Lambda_{\mathrm{M7\text{-}M3}} = E[\Pi_{\mathrm{M7}}(Q)] - E[\Pi_{\mathrm{M3}}(Q)]$, where:

$$\Lambda_{\mathrm{M7-M3}} = \iint_{\mathrm{R3}(Q)\cup \mathrm{R4}(Q)} \begin{bmatrix} \left(p_H(Qu\alpha) - c_{pH}\right)Qu\alpha \\ -\left(p_H(TB_H) - c_{pH} - s_H\right)TB_H - s_H Qu\alpha \end{bmatrix} g(u,\alpha)\,du\,d\alpha$$
$$+ \iint_{\mathrm{R5}(Q)\cup \mathrm{R6}(Q)} \begin{bmatrix} \left(p_H(TS_H) - c_{pH} - s_H\right)TS_H \\ -\left(p_H(TB_H) - c_{pH} - s_H\right)TB_H \end{bmatrix} g(u,\alpha)\,du\,d\alpha$$

Taking the first-order derivative of $\Lambda_{\mathrm{M7\text{-}M3}}$ w.r.t. Q provides:

$$\frac{\partial \Lambda_{\mathrm{M7-M3}}}{\partial Q} = \iint_{\mathrm{R3}(Q)\cup \mathrm{R4}(Q)} \left[\left(p_H'(Qu\alpha)Qu\alpha + \left(p_H(Qu\alpha) - c_{pH}\right) - s_H\right)u\alpha\right] g(u,\alpha)\,du\,d\alpha \,\text{.}$$

From Proposition 5, it is possible to show that the first-order condition $\partial\Pi(q)/\partial q = p_H'(q)q + p_H(q) - c_{pH} - s_H = 0$, yields the production threshold TS_H. Therefore as $p_H'(Qu\alpha) < 0$, and in region R3(Q) and R4(Q) where $Qu\alpha \le TS_H$, it is clear that $p_H'(Qu\alpha)Qu\alpha + p_H(Qu\alpha) - c_{pH} - s_H \ge 0$. As a result, the first-order derivate of $\Lambda_{\mathrm{M7\text{-}M3}}$ must be positive, i.e. $\partial\left[E\left[\Pi_{\mathrm{M7}}(Q)\right] - E\left[\Pi_{\mathrm{M3}}(Q)\right]\right]/\partial Q \ge 0$. This implies that

$\left.\frac{\partial E\left[\Pi^{\mathrm{M7}}(Q)\right]}{\partial Q}\right|_{Q=Q^*_{\mathrm{M3}}} - \left.\frac{\partial E\left[\Pi^{\mathrm{M3}}(Q)\right]}{\partial Q}\right|_{Q=Q^*_{\mathrm{M3}}} \ge 0$. Therefore, Q_{M7}^* is greater than Q_{M3}^*.

b) Let $p_H = p_H(TS_H)$ and therefore $D_H = TS_H$. It is possible to rewrite the first-stage objective function of model M3 as:

$$E\left[\Pi_{\mathrm{M3}}(Q)\right] = -c_l Q + \iint_{\mathrm{R1}(Q)\cup\mathrm{R2}(Q)} \left[\left(p_H(TS_H) - c_{pH} - b_H\right)TS_H + b_H Qu\alpha\right] g(u,\alpha)\,du\,d\alpha$$
$$+ \iint_{\mathrm{R3}(Q)\cup\mathrm{R4}(Q)} \left[\left(p_H(TS_H) - c_{pH} - b_H\right)TS_H + b_H Qu\alpha\right] g(u,\alpha)\,du\,d\alpha$$
$$+ \iint_{\mathrm{R5}(Q)\cup\mathrm{R6}(Q)} \left[\left(p_H(TS_H) - c_{pH} - s_H\right)TS_H + s_H Qu\alpha\right] g(u,\alpha)\,du\,d\alpha$$

The proof is similar to the one presented for part a). Let us compare the optimal second-stage decisions in both models. Because the demand in model M3 is $D_H = TS_H$ of model M7, the firm buys additional fruit in regions R1(Q), R2(Q), R3(Q) and R4(Q) in model M3, but purchases additional fruit only in regions R1(Q) and R2(Q) in model M7. In regions R1(Q) and R2(Q) of model M3, the return in the high-end segment is $(p_H(TS_H) - c_{pH} - b_H)\, TS_H + b_H Qu\alpha$, whereas in model M7, the return in the high-end segment is $(p_H(TB_H) - c_{pH} - b_H)\, TB_H + b_H Qu\alpha$. In regions R3($Q$) and R4($Q$), the return from the high-end segment in model M3 is $(p_H(TS_H) - c_{pH} - b_H)\, TS_H + b_H Qu\alpha$, whereas in model M7, the return in the high-end segment is $(p_H(Qu\alpha) - c_{pH})Qu\alpha$. Lastly, the optimal second-stage decisions and the corresponding returns are equal in regions R5(Q) and R6(Q) in both models. Therefore, let $\Lambda_{\mathrm{M7\text{-}M3}}$ be the difference between the expected returns in models M7 and M3, i.e. $\Lambda_{\mathrm{M7\text{-}M3}} = E[\Pi_{\mathrm{M7}}(Q)] - E[\Pi_{\mathrm{M3}}(Q)]$, where:

$$\Lambda_{\mathrm{M7-M3}} = \iint_{\mathrm{R1}(Q)\cup\mathrm{R2}(Q)} \begin{bmatrix} \left(p_H(TB_H) - c_{pH} - b_H\right)TB_H \\ -\left(p_H(TS_H) - c_{pH} - b_H\right)TS_H \end{bmatrix} g(u,\alpha)\,du\,d\alpha$$
$$+ \iint_{\mathrm{R3}(Q)\cup\mathrm{R4}(Q)} \begin{bmatrix} \left(p_H(Qu\alpha) - c_{pH}\right)Qu\alpha \\ -\left(p_H(TS_H) - c_{pH} - b_H\right)TS_H - b_H Qu\alpha \end{bmatrix} g(u,\alpha)\,du\,d\alpha$$

Taking the first-order derivative of $\Lambda_{M7\text{-}M3}$ w.r.t. Q provides:

$$\frac{\partial \Lambda_{\mathrm{M7-M3}}}{\partial Q} = \iint_{\mathrm{R3}(Q)\cup\mathrm{R4}(Q)} \left[\left(p_H'(Qu\alpha)Qu\alpha + \left(p_H(Qu\alpha) - c_{pH}\right) - b_H\right)u\alpha\right] g(u,\alpha)\,du\,d\alpha$$

From Proposition 5, it is possible to show that the first order condition $\partial\Pi(q)/\partial q = p_H'(q)q + p_H(q) - c_{pH} - b_H = 0$, yields the production threshold TB_H. Therefore as $p_H'(Qu\alpha) < 0$ and in region R3(Q) and R4(Q) where $Qu\alpha > TB_H$, it is clear that $p_H'(Qu\alpha)Qu\alpha + p_H(Qu\alpha) - c_{pH} - b_H < 0$. As a result, the first-order derivate of $\Lambda_{\mathrm{M7\text{-}M3}}$ must be negative, i.e. $\partial\left[E\left[\Pi_{\mathrm{M7}}(Q)\right] - E\left[\Pi_{\mathrm{M3}}(Q)\right]\right]/\partial Q < 0$. This implies that

$$\left.\frac{\partial E\left[\Pi_{\mathrm{M7}}(Q)\right]}{\partial Q}\right|_{Q=Q_{\mathrm{M3}}^*} - \left.\frac{\partial E\left[\Pi_{\mathrm{M3}}(Q)\right]}{\partial Q}\right|_{Q=Q_{\mathrm{M3}}^*} < 0.$$ Therefore, Q_{M7}^* is smaller than Q_{M3}^*.

c) For the purposes of a fair comparison, let $p_H = p_H(TP_H)$ and therefore $D_H = TP_H$, and the first-stage objective function for model M1 can be written as follows:

$$E\left[\Pi_{\mathrm{M1}}(Q)\right]=-c_lQ+\iint_{\mathrm{R1}(Q)}\left[\left(p_H\left(TP_H\right)-c_{pH}\right)Qu\alpha+\left(p_L-c_{pL}\right)Qu(1-\alpha)\right]g(u,\alpha)\,du\,d\alpha$$
$$+\iint_{\mathrm{R2}(Q)}\left[\left(p_H\left(TP_H\right)-c_{pH}\right)Qu\alpha+\left(p_L-c_{pL}\right)D_L\right]g(u,\alpha)\,du\,d\alpha$$
$$+\iint_{\mathrm{R3}(Q)}\left[\left(p_H\left(TP_H\right)-c_{pH}\right)TP_H+\left(p_L-c_{pL}\right)Qu(1-\alpha)\right]g(u,\alpha)\,du\,d\alpha$$
$$+\iint_{\mathrm{R4}(Q)}\left[\left(p_H\left(TP_H\right)-c_{pH}\right)TP_H+\left(p_L-c_{pL}\right)D_L\right]g(u,\alpha)\,du\,d\alpha$$

Next, we consider the first-stage objective function of model M5, which can be expressed as follows:

$$E\left[\Pi_{\mathrm{M5}}(Q)\right]=-c_lQ+\iint_{\mathrm{R1}(Q)}\left[\left(p_H\left(Qu\alpha\right)-c_{pH}\right)Qu\alpha+\left(p_L-c_{pL}\right)Qu(1-\alpha)\right]g(u,\alpha)\,du\,d\alpha$$
$$+\iint_{\mathrm{R2}(Q)}\left[\left(p_H\left(Qu\alpha\right)-c_{pH}\right)Qu\alpha+\left(p_L-c_{pL}\right)D_L\right]g(u,\alpha)\,du\,d\alpha$$
$$+\iint_{\mathrm{R3}(Q)}\left[\left(p_H\left(TP_H\right)-c_{pH}\right)TP_H+\left(p_L-c_{pL}\right)Qu(1-\alpha)\right]g(u,\alpha)\,du\,d\alpha$$
$$+\iint_{\mathrm{R4}(Q)}\left[\left(p_H\left(TP_H\right)-c_{pH}\right)TP_H+\left(p_L-c_{pL}\right)D_L\right]g(u,\alpha)\,du\,d\alpha$$

Let $\Lambda_{\text{M5-M1}}$ be the difference between the expected returns in models M5 and M1, i.e. $\Lambda_{\text{M5-M1}} = E[\Pi_{\text{M5}}(Q)] - E[\Pi_{\text{M1}}(Q)]$, where:

$$\Lambda_{\mathrm{M5-M1}}=\iint_{\mathrm{R1}(Q)\cup\mathrm{R2}(Q)}\left[\left(p_H\left(Qu\alpha\right)-c_{pH}\right)Qu\alpha-\left(p_H\left(TP_H\right)-c_{pH}\right)Qu\alpha\right]g(u,\alpha)\,du\,d\alpha$$

Assuming demand in the high-end segment is linear, let $D_H = a_H - \beta_H p_H$. The winemaker can set a profit maximizing price in the high-end segment as $p_H = (a_H + \beta_H c_{pH})/2\beta_H$, while setting a production target $TP_H = (a_H - \beta_H c_{pH})/2$. When high-quality crop realization is below the production target, i.e. $Qu\alpha < TP_H$, the firm sets the market-clearing price $p_H(Qu\alpha) = (a_H - Qu\alpha)/\beta_H$.

Rewriting $\Lambda_{\text{M5-M1}}$ with linear demand gives:

$$\Lambda_{\mathrm{M5-M1}}=\iint_{\mathrm{R1}(Q)\cup\mathrm{R2}(Q)}\left[\left(\frac{a_H-Qu\alpha}{\beta_H}-c_{pH}\right)Qu\alpha-\left(\frac{a_H+\beta_Hc_{pH}}{2\beta_H}-c_{pH}\right)Qu\alpha\right]g(u,\alpha)\,du\,d\alpha$$

Taking the first-order derivative of $\Lambda_{\text{M5-M1}}$ w.r.t. Q provides:

$$\frac{\partial\Lambda_{\mathrm{M5-M1}}}{\partial Q}=\iint_{\mathrm{R1}(Q)\cup\mathrm{R2}(Q)}\left[\left(\frac{a_H-Qu\alpha}{\beta_H}-c_{pH}\right)u\alpha-\left(\frac{u\alpha}{\beta_H}\right)Qu\alpha-\left(\frac{a_H+\beta_Hc_{pH}}{2\beta_H}-c_{pH}\right)u\alpha\right]g(u,\alpha)\,du\,d\alpha$$
$$=\iint_{\mathrm{R1}(Q)\cup\mathrm{R2}(Q)}\left[\left(\frac{a_H-2Qu\alpha}{\beta_H}-\frac{a_H+\beta_Hc_{pH}}{2\beta_H}\right)u\alpha\right]g(u,\alpha)\,du\,d\alpha$$
$$=\iint_{\mathrm{R1}(Q)\cup\mathrm{R2}(Q)}\left[\left(\frac{a_H-\beta_Hc_{pH}}{2\beta_H}-\frac{2Qu\alpha}{\beta_H}\right)u\alpha\right]g(u,\alpha)\,du\,d\alpha$$

As, $TP_H = (a_H - \beta_H c_{pH})/2$, it is possible to rewrite the first-order derivate of $\Lambda_{\text{M5-M1}}$ w.r.t. Q as follows:

$$\frac{\partial \Lambda_{\text{M5-M1}}}{\partial Q} = \iint_{\text{R1}(Q)\cup\text{R2}(Q)} \left[\left(TP_H - 2Qu\alpha\right)u\alpha\right] g\left(u,\alpha\right) du d\alpha$$

From the above expression, it is possible to show that the first-order derivate of $\Lambda_{\text{M5-M1}}$ is negative when $Qu\alpha > TP_H/2$. Furthermore with a uniformly distributed $g(u,\alpha)$, it is possible to see that $u\alpha$ has a higher value when $Qu\alpha > TP_H/2$ than when $Qu\alpha < TP_H/2$. As a result, $|TP_H - 2Qu\alpha|$ is higher when $Qu\alpha > TP_H/2$, resulting in the first-order derivate of $\Lambda_{\text{M5-M1}}$ to be negative, i.e. $\partial\left[E\left[\Pi_{\text{M5}}\left(Q\right)\right] - E\left[\Pi_{\text{M1}}\left(Q\right)\right]\right] / \partial Q < 0$. This implies that $\frac{\partial E\left[\Pi_{\text{M5}}\left(Q\right)\right]}{\partial Q}\Big|_{Q=Q^*_{\text{M1}}} - \frac{\partial E\left[\Pi_{\text{M1}}\left(Q\right)\right]}{\partial Q}\Big|_{Q=Q^*_{\text{M1}}} <0$.

Therefore, Q_{M5}^* is smaller than Q_{M1}^*. □

Proof of Proposition 15: The proof is identical to the proof of Proposition 2 except that high-end demand and processing cost is replaced with low-end demand and processing cost. □

Proof of Proposition 16: The result follows from the fact that the market-clearing price in in the low-end segment, denoted $p_L(Qu(1-\alpha)) > p_L^*$ when $Qu(1-\alpha) \leq TP_L$, and thus, $p_L(Qu(1-\alpha)) - c_{pL} \geq p_L^* - c_{pL}$, which leads to the fact that the threshold for downward substitution TP_H^D (for a given Q) in model M10 to be smaller than that of M6. Because the upper threshold for downward substitution remains to be the same TP_H point in both models, downward substitution occurs in a larger interval of $Qu\alpha$ values in model M10. For the same pdf, this implies that $P(w^* > 0)$ is greater than or equal to that of model M6.

Proof of Proposition 17:

The proof follows from Proposition 10 by replacing the downward substitution with the no fruit trading region. We omit the details. □

Appendix B

Notes on correlation analysis:

For our numerical analysis, we will want to change the variance of $\alpha|u$ without changing the variance of u. Random variables u and z are independent with mean normalized to 0.5, i.e., $E[u] = E[z] = 0.5$. However, we allow $\sigma_u^2 \neq \sigma_z^2$. Let $\tau = \frac{\sigma_Z}{\sigma_u}$ and define

$$\alpha = \gamma u + (1-\gamma)z$$

Due to independence, we have $E[uz] = 0.25$ (follows from $E\left[\left(u-\mu_u\right)\left(z-\mu_Z\right)\right] = 0$). Note that

$$\mu_\alpha = E[\gamma u + (1-\gamma)z] = 0.5$$

$$\sigma_\alpha = \left[\gamma^2\sigma_u^2 + \left(1-\gamma\right)^2\sigma_Z^2\right]^{1/2} = \left[\gamma^2 + \tau\left(1-\gamma\right)^2\right]^{1/2}\sigma_u$$

$\sigma_{\alpha u} = E\left[(\alpha-\mu_\alpha)(u-\mu_u)\right] = E\left[(u-0.5)(\gamma u+(1-\gamma)\alpha-0.25)\right] = \gamma E\left[u^2\right]+(1-\gamma)E[uz]-0.25 = \gamma\sigma_u^2$.

Thus, $\rho_{\alpha u} = \frac{\sigma_{\alpha u}}{\sigma_\alpha \sigma_u} = \frac{\gamma}{\left[\gamma^2+\tau(1-\gamma)^2\right]^{1/2}}$. Assuming that the correlation is nonnegative (i.e., $\rho_{\alpha u} \geq 0$, $\gamma \geq 0$), solving the above equation for γ yields $\gamma = \frac{\left[\tau^2 \rho_{u\alpha}{}^2\left(1-\rho_{u\alpha}{}^2\right)\right]^{1/2} - \tau^2\rho_{u\alpha}{}^2}{1-\left(\tau^2+1\right)\rho_{u\alpha}{}^2} = \frac{\tau\rho_{\alpha u}}{\left(1-\rho_{\alpha u}^2\right)+\tau\rho_{\alpha u}}$. The mean and variance of α given realization u are:

$E\left[\alpha|u\right] = \gamma u+(1-\gamma)\mu_\alpha = \gamma u+(1-\gamma)0.5$ and $Var\left[\alpha|u\right] = (1-\gamma)^2\sigma_Z^2 = \tau(1-\gamma)^2\sigma_u^2$.

Notes on deterministic quality and supply:

For the problem variant with deterministic supply and quality, we assume the firm converts all of its fruit crop into the final product, i.e., $q_{IH} = Q\bar{u}\bar{\alpha}$ and $q_{IL} = Q\bar{u}(1-\bar{\alpha})$. The first-stage objective function, denoted $\Psi(Q)$, can be expressed as follows:

$$\Psi(Q) = -c_l Q + \left(p_H\left(Q\bar{u}\bar{\alpha}\right)-c_{pH}\right)Q\bar{u}\bar{\alpha} + \left(p_L - c_{pL}\right)Q\bar{u}\left(1-\bar{\alpha}\right).$$

Remark B1. *a) The optimal amount of farm space to be leased, denoted by Q^0, under deterministic supply and quality satisfies*

$$p_H\left(Q^0\bar{u}\bar{\alpha}\right)\bar{u}\bar{\alpha} + p_H{}'\left(Q^0\bar{u}\bar{\alpha}\right)Q^0\left(\bar{u}\bar{\alpha}\right)^2 = c_l + c_{pH}\bar{u}\bar{\alpha} - \left(p_L - c_{pL}\right)\bar{u}\left(1-\bar{\alpha}\right); \tag{10}$$

b) the optimal deterministic profit, denoted by $\Psi(Q^0)$, is

$$\Psi\left(Q^0\right) = -p_H{}'\left(Q^0\bar{u}\bar{\alpha}\right)\left(Q^0\bar{u}\bar{\alpha}\right)^2, \tag{11}$$

We next analyze the firm's objective function under supply and quality uncertainty:

$$E\left[\Pi(Q)\right] = -\left(c_l + c_{pH}E[u\alpha]\right)Q + \int_{u_l}^{u_h}\int_{\alpha_l}^{\alpha_h} p_H(Qu\alpha)Qu\alpha g(u,\alpha)\,d\alpha du$$

$$+\int_{u_l}^{u_h}\int_{\alpha_l}^{\alpha_h}\left(p_L - c_{pL}\right)Qu(1-\alpha)g(u,\alpha)\,d\alpha du$$

$$= \Psi(Q) - \int_{u_l}^{u_h}\int_{\alpha_l}^{\alpha_h}\left[p_H\left(Q\bar{u}\bar{\alpha}\right) - p_H(Qu\alpha)\right]Qu\alpha g(u,\alpha)\,d\alpha du \tag{12}$$

Proposition B1. *a) The first-stage objective function in* (12) *is concave in Q, and the optimal amount of farm space to be leased satisfies*

$$\int_{u_l}^{u_h}\int_{\alpha_l}^{\alpha_h}\left[p_H(Qu\alpha)u\alpha + p_H{}'(Qu\alpha)Q(u\alpha)^2\right]g(u,\alpha)\,d\alpha du = c_l + c_{pH}E[u\alpha] - \left(p_L - c_{pL}\right)E\left[u(1-\alpha)\right]; \tag{13}$$

b) the optimal profit is

$$E\left[\Pi\left(Q^*\right)\right] = -\int_{u_l}^{u_h}\int_{\alpha_l}^{\alpha_h} p_H'(Q^*u\alpha)(Q^*u\alpha)^2 g\left(u,\alpha\right) du d\alpha \,, \tag{14}$$

and is less than its deterministic equivalent;

Proof of Proposition B1: a) Expected profit is concave in Q because the demand function is concave, i.e., $\partial^2 E\left[\Pi\left(Q\right)\right]/\partial Q^2 = \int_{u_l}^{u_h}\int_{\alpha_l}^{\alpha_h}\left[2p_H\left(Qu\alpha\right)\left(u\alpha\right)^2 + p_H''\left(Qu\alpha\right)Q\left(u\alpha\right)^3\right]g\left(u,\alpha\right)d\alpha du < 0$

and thus the first-order condition

$$\frac{\partial E\left[\Pi\left(Q\right)\right]}{\partial Q} = -\left(c_l + c_{pH}E\left[u\alpha\right] - \left(p_L - c_{pL}\right)E\left[u\left(1-\alpha\right)\right]\right) + \int_{u_l}^{u_h}\int_{\alpha_l}^{\alpha_h}\left[p_H\left(Qu\alpha\right)u\alpha + p_H'\left(Qu\alpha\right)Q\left(u\alpha\right)^2\right]g\left(u,\alpha\right)d\alpha du = 0$$

b) From the first-order condition, we have

$$\int_{u_l}^{u_h}\int_{\alpha_l}^{\alpha_h} p_H\left(Q^*u\alpha\right)u\alpha g\left(u,\alpha\right)d\alpha du = \left(c_l + c_{pH}E\left[u\alpha\right] - \left(p_L - c_{pL}\right)E\left[u\left(1-\alpha\right)\right]\right) - \int_{u_l}^{u_h}\int_{\alpha_l}^{\alpha_h} p_H'\left(Q^*u\alpha\right)Q^*\left(u\alpha\right)^2 g\left(u,\alpha\right)d\alpha du$$

Substituting this expression in (12) provides (14). From the fact that $p_H(Qu\alpha)Qu\alpha$ is concave in u and α, it follows from Jensen's inequality that

$$\Psi(Q) - E\left[\Pi\left(Q\right)\right] = \int_{u_l}^{u_h}\int_{\alpha_l}^{\alpha_h}[p_H\left(Q\bar{u}\bar{\alpha}\right) - p_H(Qu\alpha)]Qu\alpha g\left(u,\alpha\right)d\alpha\, du = p_H\left(Q\bar{u}\bar{\alpha}\right)Q\bar{u}\bar{\alpha} - E\left[p_H(Q\tilde{u}\tilde{\alpha})Q\tilde{u}\tilde{\alpha}\right]$$

> 0, and thus $\Psi(Q^0) - E[\Pi(Q^*)] \geq \Psi(Q^*) - E[\Pi(Q^*)] > 0$. □

The above proposition provides general results regarding the behavior of the optimal amount of farm space to be leased and the optimal profit expression under deterministic and stochastic supply and quality. Because the demand function is not described by a specific function, a closed-form expression is not provided for the optimal decisions; however, one can provide them for specific demand functions. The following analysis shows the optimal amount of farm space to be leased and the optimal profit of the firm under deterministic and stochastic supply and quality using linear demand, i.e., $D_H(p_H) = a_H - \beta_H p_H$.

Remark B2. *a) The optimal amount of farm space to be leased under deterministic supply and quality is*

$$Q^0 = \frac{\left[a_H - \beta_H\left(\left(c_l/\bar{u}\bar{\alpha}\right) + c_{pH} - \left(\left(p_L - c_{pL}\right)\bar{u}\left(1-\bar{\alpha}\right)/\bar{u}\bar{\alpha}\right)\right)\bar{u}\bar{\alpha}\right]}{2\bar{u}\bar{\alpha}}$$

; b) The optimal deterministic profit is

$$\Psi\left(Q^0\right) = \frac{1}{4b}\left[a_H - \beta_H\left(\left(c_l/\bar{u}\bar{\alpha}\right) + c_{pH} - \left(\left(p_L - c_{pL}\right)\bar{u}\left(1-\bar{\alpha}\right)/\bar{u}\bar{\alpha}\right)\right)\right]^2.$$

Proof of Remark B2: The deterministic objective function

$$\Psi(Q) = -c_l Q + \left(\frac{a_H - Q\bar{u}\bar{\alpha}}{\beta_H} - c_{pH}\right) Q\bar{u}\bar{\alpha} + \left(p_L - c_{pL}\right) Q\bar{u}\left(1-\bar{\alpha}\right)$$

$$= \frac{1}{\beta_H}\left(a_H - \beta_H\left(\left(\frac{c_l}{\bar{u}\bar{\alpha}}\right) + c_{pH} - \left(\frac{\left(p_L - c_{pL}\right)\bar{u}\left(1-\bar{\alpha}\right)}{\bar{u}\bar{\alpha}}\right)\right) - Q\bar{u}\bar{\alpha}\right) Q\bar{u}\bar{\alpha}$$

is concave in Q because

$$\frac{\partial \Psi(Q)}{\partial Q} = \frac{1}{\beta_H}\left(a_H - \beta_H\left(\left(\frac{c_l}{\bar{u}\bar{\alpha}}\right) + c_{pH} - \left(\frac{\left(p_L - c_{pL}\right)\bar{u}\left(1-\bar{\alpha}\right)}{\bar{u}\bar{\alpha}}\right)\right) - 2Q\bar{u}\bar{\alpha}\right)\bar{u}\bar{\alpha} \text{ and}$$

$\frac{\partial^2 \Psi(Q)}{\partial Q^2} = -\frac{2}{\beta_H}\left(\bar{u}\bar{\alpha}\right)^2 \leq 0$. The first-order condition provides the deterministic optimal amount of farm space to be leased: $Q^0 = \frac{\left[a_H - \beta_H\left(\left(c_l / \bar{u}\bar{\alpha}\right) + c_{pH} - \left(\left(p_L - c_{pL}\right)\bar{u}\left(1-\bar{\alpha}\right) / \bar{u}\bar{\alpha}\right)\right)\bar{u}\bar{\alpha}\right]}{2\bar{u}\bar{\alpha}}$. Substituting the deterministic optimal amount of farm space to be leased back into the objective function leads to

$$\frac{1}{\beta_H}\left(a_H - \beta_H\left(\left(\frac{c_l}{\bar{u}\bar{\alpha}}\right) + c_{pH} - \left(\frac{\left(p_L - c_{pL}\right)\bar{u}\left(1-\bar{\alpha}\right)}{\bar{u}\bar{\alpha}}\right)\right) - Q\bar{u}\bar{\alpha}\right) Q\bar{u}\bar{\alpha}. \ \square$$

Proposition B2. *Under stochastic supply and quality: a) The first-stage objective function is concave in Q, and the optimal amount of farm space to be leased is* $Q^* = \frac{\left[a_H - \beta_H\left(\left(c_l / E[u\alpha]\right) + c_{pH} - \left(\left(p_L - c_{pL}\right)E\left[u\left(1-\alpha\right)\right] / E[u\alpha]\right)\right)\right]}{2E[u\alpha]\left(1 + cv[u\alpha]^2\right)}$ *; b) The optimal amount of farm space to be leased is less than that of the deterministic supply and quality, i.e.,* $Q^* < Q^0$ *; c) The optimal profit is* $E\left[\Pi\left(Q^*\right)\right] = \frac{\left[a_H - \beta_H\left(\left(c_l / E[u\alpha]\right) + c_{pH} - \left(\left(p_L - c_{pL}\right)E\left[u\left(1-\alpha\right)\right] / E[u\alpha]\right)\right)\right]^2 E[u\alpha]^2}{4\beta_H\left(E[u\alpha]^2 + Var[ua]\right)}$*, and is less than its deterministic equivalent; d) The optimal amount of farm space to be leased and the optimal profit are both decreasing in the variance of supply and quality uncertainty.*

Proof of Proposition B2:

$$E\left[\Pi(Q)\right] = -\left(c_l + c_{pH}E[u\alpha]\right)Q + \int_{u_l}^{u_h}\int_{\alpha_l}^{\alpha_h} p_H\left(Qu\alpha\right)Qu\alpha g\left(u,\alpha\right)d\alpha du + \int_{u_l}^{u_h}\int_{\alpha_l}^{\alpha_h}\left(p_L - c_{pL}\right)Qu\left(1-\alpha\right)g\left(u,\alpha\right)d\alpha du$$

$$= \frac{\left[a_H - \beta_H\left(\frac{c_l}{E[u\alpha]} + c_{pH} - \frac{\left(p_L - c_{pL}\right)E\left[u\left(1-\alpha\right)\right]}{E[u\alpha]}\right)\right]Q\bar{u}\bar{\alpha} - Q^2\left(E[u\alpha]^2 + Var[u\alpha]^2\right)}{\beta_H}$$

$$\frac{\partial E\left[\Pi(Q)\right]}{\partial Q}=\frac{\left[a_H-\beta_H\left(\frac{c_l}{E[u\alpha]}+c_{pH}-\frac{(p_L-c_{pL})E\left[u(1-\alpha)\right]}{E[u\alpha]}\right)\right]\overline{u}\overline{\alpha}-2Q\left(E[u\alpha]^2+Var[u\alpha]^2\right)}{\beta_H},$$

$$\frac{\partial^2 E\left[\Pi(Q)\right]}{\partial Q^2}=-\frac{2}{\beta_H}\left(E[u\alpha]^2+Var[u\alpha]^2\right)\leq 0\,.$$

Therefore, the first-order condition, when equated to zero, provides the optimal amount of farm space to be leased:

$$Q^*=\frac{\left[a_H-\beta_H\left(\left(c_l/E[u\alpha]\right)+c_{pH}-\left(\left(p_L-c_{pL}\right)E\left[u(1-\alpha)\right]/E[u\alpha]\right)\right)\right]E[u\alpha]}{2\left(E[u\alpha]^2+Var[ua]\right)}$$

b) Observe that the above optimal amount of farm space to be leased can also be expressed as:

$$Q^*=\frac{\left[a_H-\beta_H\left(\frac{c_l}{E[u\alpha]}+c_{pH}-\frac{(p_L-c_{pL})E\left[u(1-\alpha)\right]}{E[u\alpha]}\right)\right]}{2\left(E[u\alpha]+\frac{Var[ua]}{E[u\alpha]}\right)}<Q^0=\frac{\left[a_H-\beta_H\left(\frac{c_l}{\overline{u}\overline{\alpha}}+c_{pH}-\frac{(p_L-c_{pL})\overline{u}(1-\overline{\alpha})}{\overline{u}\overline{\alpha}}\right)\overline{u}\overline{\alpha}\right]}{2\overline{u}\overline{\alpha}}$$

c) Substituting Q^* back into the objective function provides

$$E\left[\Pi(Q^*)\right]=\frac{\left[a_H-\beta_H\left(\frac{c_l}{E[u\alpha]}+c_{pH}-\frac{(p_L-c_{pL})E\left[u(1-\alpha)\right]}{E[u\alpha]}\right)\right]^2E[u\alpha]^2}{4\beta_H\left(E[u\alpha]^2+Var[u\alpha]\right)}$$

Moreover,

$$\begin{aligned}E\left[\Pi(Q^*)\right]&=\frac{\left[a_H-\beta_H\left(\frac{c_l}{E[u\alpha]}+c_{pH}-\frac{(p_L-c_{pL})E\left[u(1-\alpha)\right]}{E[u\alpha]}\right)\right]^2E[u\alpha]^2}{4\beta_H\left(E[u\alpha]^2+Var[u\alpha]\right)}\\&=\frac{\left[a_H-\beta_H\left(\frac{c_l}{E[u\alpha]}+c_{pH}-\frac{(p_L-c_{pL})E\left[u(1-\alpha)\right]}{E[u\alpha]}\right)\right]^2}{4\beta_H\left(1+cv[u\alpha]^2\right)}\\&=\frac{\Psi\left(Q^0\right)}{\left(1+cv[u\alpha]^2\right)}<\Psi\left(Q^0\right)\end{aligned}$$

d) Because $\frac{\partial Q^*}{\partial Var[u\alpha]]^2} = -\frac{\left[a_H - \beta_H\left(\frac{c_l}{E[u\alpha]} + c_{pH} - \frac{(p_L - c_{pL})E[u(1-\alpha)]}{E[u\alpha]}\right)\right]E[u\alpha]}{2\left(E[u\alpha]^2 + Var[u\alpha]\right)^2} \le 0$ and

$$\frac{\partial E\left[\Pi\left(Q^*\right)\right]}{\partial Var[u\alpha]^2} = -\frac{\left[a_H - \beta_H\left(\frac{c_l}{E[u\alpha]} + c_{pH} - \frac{(p_L - c_{pL})E[u(1-\alpha)]}{E[u\alpha]}\right)\right]^2 E[u\alpha]^2}{4\beta_H\left(E[u\alpha]^2 + Var[u\alpha]\right)^2}$$

, the optimal amount of farm space to be leased and the optimal profit are monotonically decreasing in the variance term of supply and quality uncertainty.

Denoting the coefficient of variation in supply uncertainty as $cv[u\alpha] = Var[u\alpha] / E[u\alpha]$, the optimal amount of farm space to be leased can also be expressed as follows:

$$\begin{aligned} Q^* &= \frac{\left[a_H - \beta_H\left(\left(c_l / E[u\alpha]\right) + c_{pH} - \left(\left(p_L - c_{pL}\right)E\left[u(1-\alpha)\right] / E[u\alpha]\right)\right)\right]E[u\alpha]}{2\left(E[u\alpha]^2 + Var[ua]\right)} \\ &= \frac{\left[a_H - \beta_H\left(\left(c_l / E[u\alpha]\right) + c_{pH} - \left(\left(p_L - c_{pL}\right)E\left[u(1-\alpha)\right] / E[u\alpha]\right)\right)\right]}{2E[u\alpha]\left(1 + cv[u\alpha]^2\right)} \\ &= \frac{Q^0}{1 + cv[u\alpha]^2} < Q^0 \end{aligned}$$

Therefore, the optimal amount of farm space to be leased is decreasing in coefficient of variation, and because we keep the mean fixed, it decreases in supply and quality variation under random supply and quality. Similarly, the optimal value of the objective function is decreasing in the coefficient of variation,

$$E\left[\Pi\left(Q^*\right)\right] = \frac{\Psi\left(Q^0\right)}{\left(1 + cv[u\alpha]^2\right)} < \Psi\left(Q^0\right)$$

and is less than its deterministic equivalent. □