Appendix S1[1]

Proof of Lemma 1. Taking first and second partial derivatives of the expected profit function, as expressed in Eq. (7), with respect to *l*:

$$\frac{\partial \Pi(z,\lambda,l)}{\partial z} = \left[-\frac{\theta}{\beta} + s_l - (s_l + h_l)\int_0^l g_\lambda(t)\,dt \right]\left[\lambda - \Omega(z)\right]$$

$$\frac{\partial^2 \Pi(z,\lambda,l)}{\partial z^2} = -(s_l + h_l)\, g_\lambda(l)\left[\lambda - \Omega(z)\right] < 0 .$$

Note that $\Omega(z) = \int_z^B (x-z) f(x)dx \le \int_A^B (x-A) f(x)dx = -A$. Therefore $\lambda - \Omega(z) \ge \lambda - A > 0$, and thus $\Pi(z, \lambda, l)$ is concave in *l*. The optimal solution follows directly from the first-order condition.

Proof of Lemma 2. From Eq. (9), it is easy to verify that $\partial\Pi(z, \lambda, l(\lambda))/\partial\lambda = \{(\alpha - \beta c) - (1 + \theta\delta + \beta\varphi)[2\lambda - \Omega(z)]\}/\beta$ and $\partial^2\Pi(z, \lambda, l(\lambda))/\partial\lambda^2 = -2(1 + \theta\delta + \beta\varphi)/\beta \le 0$. Therefore the concavity of $\Pi(z, \lambda, l(\lambda))$ in λ is established. From the first-order condition, we have

$$\lambda^* = \lambda^0 + \frac{1}{2}\Omega(z) ,$$

where $\lambda^0 = \dfrac{\alpha - \beta c}{2(1+\theta\delta+\beta\varphi)}$, which is the optimal mean demand when there is no demand uncertainty. It is obtained by substituting Eq. (6) and $l^* \equiv l(\lambda)$ into Eq. (4) and then optimizing over λ. p^0 is the price corresponding to this λ^0. The optimal price p^* is then obtained by substituting λ^* and $l(\lambda)$ into Eq. (6).

Proof of comparative statics in table 1. We only present an example here to show how price sensitivity β affects the average demand. The other results can be derived in a similar fashion.

We first note that

$$\frac{\partial \varphi}{\partial \delta} = h_l \int_0^\delta \phi(y)\,dy + s_l \int_\delta^\infty \phi(y)\,dy = (h_l + s_l)\Phi(\delta) - s_l = -\theta / \beta.$$

Also,

$$\frac{d(1+\theta\delta+\beta\varphi)}{d\beta} = \theta\frac{\partial\delta}{\partial\beta} + \beta\frac{\partial\varphi}{\partial\delta}\frac{\partial\delta}{\partial\beta} + \varphi = \varphi > 0 .$$

[1] Supplemental information for: Wu, Z., B. Kazaz, S. Webster, K.K. Yang. 2011. Ordering, pricing, and lead-time quotation under lead-time and demand uncertainty. *Production and Operations Management*, to appear.

Therefore, it is easy to see λ^0 decreases in β.

Proof of Theorem 1. The proof is similar to that of Theorem 1 in Petruzzi and Dada (1999). It is produced here for completeness. Recall that the expected profit function is given by Eq. (12). Taking the first-order derivative with respect to z, we have

$$\begin{aligned} R(z) &\equiv \frac{d\Pi\left(z,\lambda(z),l(z)\right)}{dz} \\ &= \frac{1-F(z)}{2\beta}\left[\alpha-\beta c+2\beta s-(1+\theta\delta+\beta\varphi)\Omega(z)\right]-(h+c)F(z) \\ &= -(h+c)+\left[1-F(z)\right]\left[h+c+s+\frac{\alpha-\beta c-(1+\theta\delta+\beta\varphi)\Omega(z)}{2\beta}\right] \end{aligned}$$

In order to identify values of z that satisfy the first-order necessary condition, we characterize the shape of $R(z)$ through the following:

$$\frac{dR(z)}{dz}=-\frac{f(z)}{2\beta}\left\{\alpha-\beta c+2\beta(h+c+s)-(1+\theta\delta+\beta\varphi)\left[\Omega(z)+\frac{1-F(z)}{r(z)}\right]\right\} ,$$

where $r(\cdot)\equiv f(\cdot)/[1-F(\cdot)]$ denotes the hazard rate (or failure rate). Also,

$$\begin{aligned} \frac{d^2R(z)}{dz^2} &= \frac{dR(z)/dz}{f(z)}\frac{df(z)}{dz} \\ &\quad -\frac{(1+\theta\delta+\beta\varphi)f(z)}{2\beta}\left\{1-F(z)+\frac{f(z)}{r(z)}+\frac{[dr(z)/dz][1-F(z)]}{r^2(z)}\right\} \\ \Rightarrow \left.\frac{d^2R(z)}{dz^2}\right|_{dR(z)/dz=0} &= -\frac{(1+\theta\delta+\beta\varphi)f(z)[1-F(z)]}{2\beta r^2(z)}\left[2r^2(z)+\frac{dr(z)}{dz}\right]. \end{aligned}$$

If $F(\cdot)$ is a distribution satisfying the condition $2r^2(z) + dr(z)/dz > 0$, then it follows that $R(z)$ is either monotone or unimodal, implying that $R(z) = 0$ has at most two roots. Further, $R(A) = s + [\alpha - \beta c + A(1 + \theta\delta + \beta\varphi)]/(2\beta) > 0$ according to condition (14), and $R(B) = -(h + c) < 0$. Therefore, $R(z) = 0$ can only have one root, indicating a change of sign for $R(z)$ from positive to negative, and thus it corresponds to a local maximum of $\Pi(z, \lambda(z), l(z))$. As a result, the unique root of $R(z) = 0$ is the optimal stocking factor z^* that maximizes $\Pi(z, \lambda(z), l(z))$.

Proof of Theorem 2. Part (a) and (c) are immediate from results in Lemma 2. Part (b) follows directly from part (a), because the optimal lead-time to quote is always proportional to the average demand. To prove part (d), note that $\Pi^0=\max_{(p,l)}\Psi(p,l)=\Psi(p^0,l^0)$, and

$$\begin{aligned}\Pi^* &= \max_{(z,p,l)}\left[\Psi(p,l) - L(z,p,l)\right] \\ &= \Psi(p^*,l^*) - L(z^*,p^*,l^*) \\ &< \Psi(p^*,l^*) \\ &\le \Psi(p^0,l^0) = \Pi^0.\end{aligned}$$

The last inequality follows from the fact that p^0 and l^0 are the global maximizers of $\Psi(p, l)$.

Proof of Theorem 3. Note that

$$\begin{aligned}\frac{\partial p^0}{\partial \rho} &= -\frac{1}{\beta}\left[\theta\tau\lambda^0 + (1+\theta\tau\rho)\frac{\partial \lambda^0}{\partial \rho}\right] \\ &= \frac{(\alpha-\beta c)\eta}{2(1+\theta\tau\rho+\beta\eta\rho)^2} > 0, \\ \frac{\partial \lambda^0}{\partial \rho} &= -\frac{(\alpha-\beta c)(\theta\tau+\beta\eta)}{2(1+\theta\tau\rho+\beta\eta\rho)^2} < 0.\end{aligned}$$

The associated optimal expected profit is given by

$$\begin{aligned}\Pi^0 &= \left(p^0 - c - \eta\rho\lambda^0\right)\lambda^0 \\ &= \frac{\alpha-\beta c}{2\beta}\lambda^0 \\ &= \frac{(\alpha-\beta c)^2}{4\beta(1+\theta\tau\rho+\beta\eta\rho)}.\end{aligned}$$

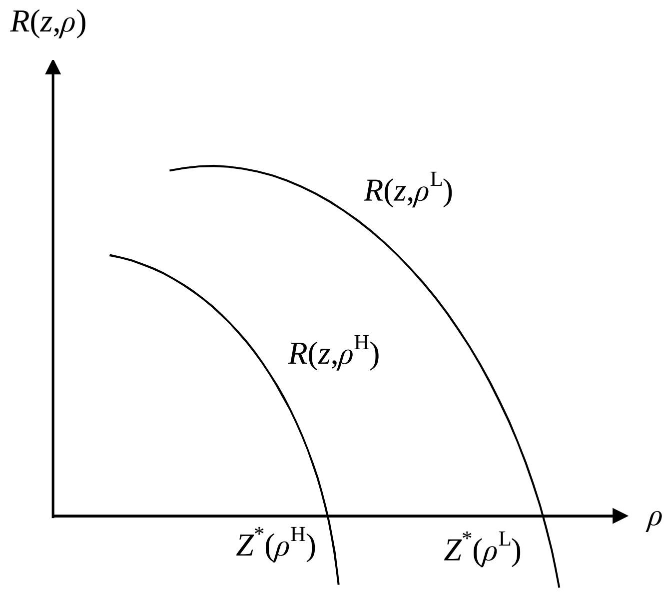
It is straightforward to see that $\partial\Pi^0/\partial\rho < 0$.

Proof of Lemma 3. Consider two arbitrary values of ρ, with $\rho^H > \rho^L \geq 0$. It is straightforward to see that the derivative of the expected profit function with respect to z, $R(z)$, is decreasing in ρ, i.e., $R(z, \rho^L) > R(z, \rho^H)$. Therefore we have $R(z^*(\rho^H), \rho^L) > R(z^*(\rho^H), \rho^H) = 0$, where $z^*(\rho^H)$ is the root of $R(z, \rho^H) = 0$. In other words, at $z^*(\rho^H)$, the curve $R(z, \rho^L)$ is still positive. Recall that we have established in proof of Theorem 1 that under conditions (13) and (14), $R(z)$ is either monotone or unimodal with a change of sign from positive to negative. Thus we conclude that $R(z, \rho^L)$ will cross the x-axis to the right of $z^*(\rho^H)$, namely, $z^*(\rho^L) > z^*(\rho^H)$. See the figure below for a sketch of the proof.

A simpler way to show $\partial z^*/\partial\rho < 0$ is to make use of the concept of supermodularity (Topkis 1998). It can be shown that

$$\frac{\partial^2 \Pi(z,\lambda(z),l(z))}{\partial z \partial \rho} = \frac{\partial R(z)}{\partial \rho} = -\frac{1-F(z)}{2\beta}(\theta\tau + \beta\eta)\Omega(z) < 0,$$

which implies that the expected profit function $\Pi(z,\lambda(z),l(z))$ is submodular in z and ρ, or supermodular in $-z$ and ρ. The fact that $\partial z^*/\partial \rho < 0$ then directly follows from Theorem 2.8.2 of Topkis (1998).



Taking derivative of l^* with respect to ρ yields

$$\frac{\partial l^*}{\partial \rho} = \frac{(\alpha - \beta c)\tau}{2(1+\theta\tau\rho + \beta\eta\rho)^2} + \frac{\tau}{2}\Omega(z) + \frac{\tau\rho}{2}[F(z)-1]\frac{\partial z^*}{\partial \rho} > 0.$$

Proof of Theorem 4. Note that Eq. (12), the expected profit as a function of the single variable z, can be simplified as follows.

$$\Pi^*(z^*) = \frac{1}{\beta}\left[\frac{\alpha-\beta c}{2} - \frac{1+\theta\tau\rho+\beta\eta\rho}{2}\Omega(z^*)\right]\left[\frac{\alpha-\beta c}{2(1+\theta\tau\rho+\beta\eta\rho)} - \frac{1}{2}\Omega(z^*)\right]$$
$$-(h+c)\Delta(z^*) - s\Omega(z^*)$$

From the envelope theorem,

$$\frac{\partial \Pi^*(z^*)}{\partial \rho} = -\frac{(\theta\tau+\beta\eta)\left\{(\alpha-\beta c)^2 - \left[(1+\theta\tau\rho+\beta\eta\rho)\Omega(z^*)\right]^2\right\}}{4\beta(1+\theta\tau\rho+\beta\eta\rho)^2} < 0,$$

where the inequality follows directly from our assumption (17).

$$l^0 = k\tau\rho\lambda + (1-k)\frac{\omega}{\omega+1}\rho\lambda \quad \text{(A1)}$$

$$\lambda^0 = \frac{\alpha - \beta c}{2\left(1 + k\theta\tau\rho + k\beta\eta\rho + (1-k)\frac{\omega}{\omega+1}\theta\rho\right)} \quad \text{(A2)}$$

$$p^0 = \frac{\alpha - \left(1 + k\theta\tau\rho + (1-k)\frac{\omega}{\omega+1}\theta\rho\right)\lambda^0}{\beta} \quad \text{(A3)}$$

$$\partial p^0/\partial k = \frac{(\alpha - \beta c)\left(1 + \frac{\omega}{\omega+1}\theta\right)\eta\rho^2}{2\left(1 + k\theta\tau\rho + k\beta\eta\rho + (1-k)\frac{\omega}{\omega+1}\theta\rho\right)^2} > 0. \quad \text{(A4)}$$

References

Petruzzi, C. N., M. Dada. 1999. Pricing and the newsvendor problem: A review with extensions. *Operations Research* **47**(2) 183-194.

Topkis, M. D. 1998. *Supermodularity and Complementarity*. Princeton University Press, Princeton, NJ.