

# Ordering, Pricing, and Lead-Time Quotation Under Lead-Time and Demand Uncertainty

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In this article, we study the newsvendor problem with endogenous setting of price and quoted lead-time. This problem can be observed in situations where a firm orders semi-finished product prior to the selling season and customizes the product in response to customer orders during the selling season. The total demand during the selling season and the lead-time required for customization are uncertain. The demand for the product depends not only on the selling price but also on the quoted lead-time. To set the quoted lead-time, the firm has to carefully balance the benefit of increasing demand as the quoted lead-time is reduced against the cost of increased tardiness. Our model enables the firm to determine the optimal selling price, quoted lead-time, and order quantity simultaneously, and provides a new set of insights to managers.

*Key words:* newsvendor problem; pricing; lead-time quotation; inventory; revenue management

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## 1. Introduction

In environments where customers are spoiled with choices, competing only on price is not enough. Customers also pay considerable attention to other dimensions of the product or service, such as quality, delivery speed, and other elements of customer service. Among the elements of customer service, it is well known that speed and consistency of delivery time are the two most important (Ballou 1998, Sterling and Lambert 1989).

In this article, we consider decisions surrounding a customized product with a short selling season and demand that is influenced by both price and quoted lead-time. The firm's production system has a purchase-to-stock element and a customize-to-order element, with a one-time order of semi-finished product prior to the selling season and customization during the selling season. A firm places an order for semi-finished product prior to the selling season with no opportunity for follow-up replenishment orders. The selling season begins and customer orders arrive over time. In response to each customer order, the firm finishes the product to customer specifications. The actual customization lead-time between order place-

ment and shipment is random. On the one hand, a short quoted lead-time not only helps increase demand but also increases the number of late shipments. On the other hand, a long quoted lead-time not only helps reduce the number of late shipments but also decreases demand. We study the questions of how to stock, price, and quote a lead-time for such a product.

Our study is motivated by settings with a single stocking cycle of semi-finished product for each generation of a customized product. Such settings arise in customized seasonal goods and customized products with a short life-cycle relative to the replenishment lead-time. For example, Bradfords has to order seasonal items ahead and then packs them into customized hampers according to customer orders received during the Christmas selling season (<http://www.bradfordsbakers.com>). Nike's custom shoe division orders components with colors and features that respond to frequently changing fashions and product innovations (<http://www.nikeid.nike.com/nikeid/>). More generally, our study applies to firms that must set price and lead-time for a product and market with the following characteristics: (i) short selling season (e.g., semi-finished stock is acquired prior to the

selling season with no future replenishment), (ii) a positive and random lead-time between the receipt and shipment of customer order (e.g., for product customization), (iii) demand that is influenced by both price and quoted lead-time, and (iv) a firm that does not dynamically adjust the price and quoted lead-time over the selling season.

The research questions that motivate the study and the resulting contributions of the paper can be summarized as follows:

1. How should the firm make optimal decisions regarding the amount of inventory to be ordered prior to the selling season, the selling price, and the quoted lead-time on customized delivery under lead-time and demand uncertainty?

As a consequence of the above research question, this article brings together two major research streams by introducing a model that combines features present in both. One research stream is the Price-Setting Newsvendor Problem (PSNP) wherein a firm must determine the price and order quantity of a short life-cycle product. The other research stream is the pricing and lead-time quotation literature wherein a firm must determine the price and lead-time of a long life-cycle product. In the first stream, the firm does not determine a quoted lead-time, and lead-time uncertainty is not considered in the formulation. In these models, demand is satisfied immediately from stock. In the second stream, there is no stocking decision and sufficient raw materials are assumed to be available. Our model enables the firm to determine simultaneously the optimal selling price, order quantity, and quoted lead-time.

2. Price-Setting Newsvendor Problem with an additive random error term describing demand uncertainty suggests that a firm should reduce its optimal selling price with increasing demand uncertainty, resulting in an increase in expected demand. The firm's reaction to increasing demand uncertainty hurts the firm but benefits consumers. Does this hold true when the firm has to quote a delivery lead-time and operates under both lead-time and demand uncertainty?

In PSNP, where lead-time uncertainty is omitted, the firm has the incentive to reduce its demand uncertainty as it results in an increase in its optimal selling price and expected profit. However, the reduction in demand uncertainty hurts consumers as the selling price increases. While we find the same directional effects regarding demand uncertainty, this article shows that the optimal quote for delivery lead-time decreases with reduced demand uncertainty. The consequence of this finding is that there is at least a segment of consumers—specifically, those who are lead-time sensitive but price insensitive—who benefit from reduced demand uncertainty.

3. Is the inclusion of lead-time uncertainty in PSNP equivalent to an increased level of demand uncertainty?

One might intuit that incorporating lead-time uncertainty into PSNP would yield similar results with increased level of demand uncertainty. The analysis in section 4.2.1 shows that the influence of lead-time uncertainty is different from that of demand uncertainty. More specifically, while an increase in demand uncertainty is known to cause a decrease in the optimal selling price in PSNP (with an additive random error term), an increase in lead-time uncertainty results in an increase in the optimal selling price in our problem, a lower average demand, and a lower expected profit. Thus, while consumers benefit from increasing levels of demand uncertainty, they are worse off under increasing levels of lead-time uncertainty.

4. How does the presence of demand uncertainty influence the impact of capacity on the firm's optimal decisions?

Consistent with the pricing and lead-time quotation literature, when demand uncertainty is omitted, we find that the optimal selling price and quoted lead-time are increasing in the mean service time. Thus, a decrease in capacity (equivalently, an increase in the mean service time) results in an increase in the selling price and quoted lead-time, hurting all consumers. However, when demand uncertainty is included, this notion is not consistently applicable. While the optimal quoted lead-time increases in the mean service time, the optimal selling price can be decreasing. This result occurs when the firm has a small customer base, so it cannot afford to increase the selling price and lose its customers.

5. In addition to addressing the above research questions, this article makes a technical contribution by introducing a general multiplicative random lead-time model that leads to analytical optimal solutions, enabling us to provide a comprehensive discussion on various factors influencing the firm's optimal decisions and expected profit.

In the next section, we outline the related literature. Section 3 presents the model and analysis. Section 4 presents managerial implications, and section 5 concludes the paper. All proofs and technical derivations are provided in Appendix S1.

## 2. Literature Review

As noted above, this article brings together two streams of literature. In one area of study, research has investigated the optimal selling price and inventory level decisions under demand uncertainty, namely the PSNP, ignoring the decision regarding a promised lead-time and the uncertainty associated with actual delivery time. In the other area of study, pricing

and lead-time decisions are made in the presence of lead-time uncertainty, but the influence of stocking decisions is not investigated. In this section, we review both streams of research and present how our work is related to each.

The modeling approach used in this article regarding the optimal selling price and inventory decisions is similar to the models that examine PSNP with lost sales. Specifically, when the firm does not have sufficient stock, excess demand is not backlogged, and the opportunity to generate revenues through sales is foregone. The operating environment described in our article is similar to those presented in Zabel (1970), Young (1978), Petruzzi and Dada (1999), Yao et al. (2006), Wu et al. (2011), and Kocabiyıkoğlu and Popescu (2011). Dana and Petruzzi (2001) present a variation of these papers where they incorporate the consumer's expected utility maximization into the definition of demand uncertainty, but still consider the case of lost sales. Like in these papers, the choice of the selling price in our article influences the expected shortage cost and the dynamics between the expected costs of shortages and leftovers. Petruzzi et al. (2009) extend this work by allowing demand to depend on the order quantity as well as price. Gavirneni and Isen (2010) apply a verbal protocol analysis to a newsvendor experiment to study the behavioral aspects of the decision-making process. However, the above studies assume that the firm operates under short and deterministic lead-times, and does not have to quote a lead-time to its consumers. The firm's decisions are price and order quantity. Our study expands their work by incorporating lead-time uncertainty into the model, and by including promised lead-time along with price and quantity as the decision variables.

Chatterjee et al. (2002) provide a detailed discussion on delivery guarantees and its relationship with marketing and operations. Li (1992) provides the optimal stocking decisions for make-to-order and make-to-stock firms in monopolistic and competition settings, where the demand is influenced by consumers' preference of price, quoted lead-time and quality. Li and Lee (1994) extend this model to the setting where firms compete on price and processing times. Webster (2002) studies pricing, lead-time, and capacity decisions for a make-to-order product. Liu et al. (2007) consider the pricing and lead-time decisions from a supplier–retailer perspective. Demand in their model is sensitive to price and lead-time decisions, but is deterministic. Zhao et al. (2011) compare a single price and lead-time quotation pair with a menu of price and lead-time quotes. Allon et al. (2011) examine strategic consumer behavior in settings where a firm posts price and information related to lead-time. The focus is on identifying conditions

for Nash equilibrium and on understanding the impact of the precision of lead-time information on profitability and market share. There is a rapidly growing literature on strategic consumer behavior (e.g., the reader can review Netessine and Tang [2009] for examples), although with the exception of Allon et al. (2011), decisions other than the combination of price and lead-time are considered.

Demand uncertainty in the price and lead-time literature is incorporated into the modeling approach by using queuing systems. In these studies, it is commonly assumed that the service facility follows a G/G/s queuing system. The sojourn time is used to model the actual delivery time. For high service levels, this can be approximated by an exponential function, resulting in M/M/1 queuing systems. Using this approach, So and Song (1998) and Palaka et al. (1998) determine price, quoted lead-time, and capacity simultaneously for a single firm. So (2000) analyzes the competition case and finds that the optimal price and lead-time decisions in an oligopolistic market are similar to those in the monopolistic setting. It is well reported that M/M/1 has its limitations in practice despite the convenience it provides in analytical derivations. Using a slightly different approach, Easton and Moodie (1999) also study pricing and lead-time decisions for make-to-order firms with an emphasis on the influence of contingent orders. Lederer and Li (1997) examine the pricing and lead-time decisions in a competition setting while focusing on the firm's choice of the production scheduling priority (e.g., FIFO). Our work differs from these studies in that we consider the optimal stocking decision along with the price and lead-time decisions.

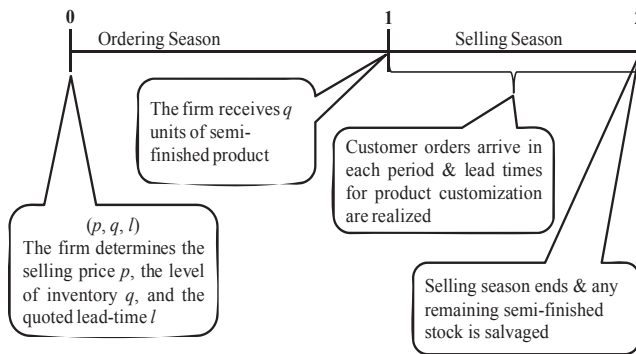
### 3. Model Analysis

The firm must determine the stocking quantity of a semi-finished product  $q$ , selling price  $p$ , and quoted lead-time  $l$  for a customized product to be sold during a short selling season. The firm knows how the probability distribution of total demand and the probability distribution of order lead-time are affected by the values of  $p$  and  $l$ . The probability distributions are a consequence of the price- and lead-time-dependent stochastic process of customer orders during the selling season.

The timing of events in our model is as follows (see Figure 1): (i) the firm selects the values of  $p$ ,  $q$ , and  $l$ ; (ii) an order of  $q$  semi-finished products arrives; (iii) the selling season begins and customer orders arrive over time; and (iv) the selling season ends and the firm salvages any remaining semi-finished product at a loss.

Let  $d_i$  denote random demand during period  $i$  of the selling season that lasts for  $n$  periods. We assume the following consumer choice model that determines

Figure 1 The Timing of Events in Our Model



$d_i$ . There are  $m$  consumers who consider purchasing a unit of the product in each period. Consumers have a standard waiting cost rate of  $c_w$  and will purchase the product only if the utility of ownership is more than  $p + c_w l$ . We assume that customers interpret the quoted lead-time as credible, which Allon et al. (2011) show is at Nash equilibrium under various models that incorporate strategic consumer behavior. Consumer utility in period  $i$  is uniformly distributed over  $[u_i - \zeta, u_i + \zeta]$ , and thus the fraction of consumers who purchase the product in period  $i$  is  $(u_i + \zeta - p - c_w l) / (2\zeta)$ . The mean utility  $u_i$  in period  $i$  is a random variable and  $u_i$  are i.i.d. Letting  $E[u_i] = \mu_u$ , the random demand during period  $i$  is

$$d_i = x(p, l) + \varepsilon_i,$$

where  $x(p, l) = m(\mu_u + \zeta - p - c_w l) / (2\zeta) = E[d_i | p, l]$  and  $\varepsilon_i = m(u_i - \mu_u) / (2\zeta)$  are i.i.d. with  $E[\varepsilon_i] = 0$ . Note that  $\varepsilon_i$  reflects the degree of randomness in mean consumer utility in a period and, consequently, the degree of uncertainty in demand. If  $u_i$  is fixed (i.e., deterministic), then  $d_i = x(p, l)$ .

Our model of consumer choice results in additive price- and lead-time-dependent model of demand that is linear in  $p$  and  $l$ . While not derived from an underlying consumer choice model, Rao et al. (2005) also employ additive price- and lead-time-dependent model of random demand.

The total demand during the selling season is

$$D = \sum_i d_i = \lambda(p, l) + \varepsilon,$$

where  $\lambda(p, l) = n \times x(p, l)$  and  $\varepsilon = \sum_i \varepsilon_i$ . Note that  $E[\varepsilon] = 0$ , and thus  $\lambda(p, l)$  is the expected demand during the selling season. Expected demand is linear in  $p$  and  $l$ , and we write it as

$$\lambda(p, l) = \alpha - \beta p - \theta l, \quad (1)$$

which is identical to the demand model in Liu et al. (2007). The value of  $\alpha$  is the market potential over the selling season, and the values of  $\beta$  and  $\theta$  are

price and lead-time sensitivity factors, respectively. The noise term  $\varepsilon$  is defined on the range  $[A, B]$  with mean 0 and standard deviation  $\sigma$ , where  $A < 0$  and  $B > 0$ . Let  $F(\cdot)$  and  $f(\cdot)$  represent the cumulative distribution function (cdf) and the probability density function (pdf) of  $\varepsilon$ , respectively.

Our model of demand during the selling season is similar in spirit to the model in Petruzzi et al. (2009) in the sense that random demand is also influenced by two decisions. In the case of Petruzzi et al. (2009), these decisions are price and order quantity, whereas the decisions in our model are price and quoted lead-time.

To describe the expected lead-time-related costs, we need to specify the probability distribution of lead-time associated with each sale. We emphasize that the lead-time distribution applies only when semi-finished product is in stock. As in Baron and Milner (2009), we assume that the probability distribution of lead-time during a fraction of the selling season is representative of the probability distribution of lead-time during the entire selling season (see also Steckley et al. 2009). Consequently, the value of the order quantity  $q$  does not affect the lead-time distribution associated with each sale. The probability distribution of the lead-time  $T$  required for customization depends on the customization process and on the demand process, and the demand process is affected by the price and quoted lead-time decisions through the average demand  $\lambda$ . Let  $G_\lambda(\cdot)$  and  $g_\lambda(\cdot)$  denote the lead-time cdf and pdf, respectively.

The firm incurs inventory-related costs and lead-time related costs, which are described below.

*Inventory-related costs:* If the firm holds too much semi-finished inventory, then there is a per-unit cost  $h$  for leftovers (e.g., increase in per unit holding cost on an unsold unit vs. a sold unit, less the end-of-season salvage value). While we allow the possibility of negative  $h$ , we assume  $c + h > 0$  (i.e., there is a cost of over-ordering), where  $c$  is the unit purchasing cost. If shortages occur, there is a non-negative loss of goodwill cost  $s$  per unit short.

*Lead-time-related costs:* There are lead-time-related costs that include the echelon holding cost  $h_l$  per unit-period if  $t < l$ , or tardiness (penalty) cost  $s_l$  per unit-period if  $t > l$ . We emphasize that the lead-time-related tardiness cost  $s_l$  only applies when there is positive stock of semi-finished product, whereas the inventory-related goodwill cost  $s$  only applies after stock is depleted. The echelon holding cost  $h_l$  is the difference in the costs of holding a finished product and a semi-finished product, which is non-negative due to the value added during customization. (Hereafter, we refer to  $h_l$  as simply holding cost.) In settings where it is acceptable to ship an order early, then  $h_l = 0$  (e.g., there is no holding cost because the firm

ships as soon as an order is complete). The expected lead-time cost associated with each unit of sales during the selling season is

$$C(l, \lambda) = h_l \int_0^l (l-t)g_\lambda(t)dt + s_l \int_l^\infty (t-l)g_\lambda(t)dt, \quad (2)$$

which depends on  $\lambda$  through lead-time distribution  $G_\lambda(\cdot)$ .

The firm's objective is to determine  $q$ ,  $p$ , and  $l$  to maximize expected profit. It is useful to apply a transformation of variables to facilitate the optimization procedure. We define  $z = q - \lambda$ , which is known as the stocking factor (Petruzzi and Dada 1999). The expected profit of the firm can then be expressed as:

$$\begin{aligned} \Pi(z, p, l) &= -c(\lambda + z) \\ &+ \int_A^z [(p - C(l, \lambda))(\lambda + x) - h(z - x)]f(x)dx \\ &+ \int_z^B [(p - C(l, \lambda))(\lambda + z) - s(x - z)]f(x)dx \\ &= (p - c - C(l, \lambda))\lambda - (h + c) \int_A^z (z - x)f(x)dx \\ &- (s + p - c - C(l, \lambda)) \int_z^B (x - z)f(x)dx \\ &= \Psi(p, l) - L(z, p, l), \end{aligned} \quad (3)$$

where

$$\Psi(p, l) = (p - c - C(l, \lambda))\lambda, \quad (4)$$

$$L(z, p, l) = (h + c)\Delta(z) + [s + p - c - C(l, \lambda)]\Omega(z), \quad (5)$$

$$\Delta(z) = \int_A^z (z - x)f(x)dx,$$

$$\Omega(z) = \int_z^B (x - z)f(x)dx.$$

Equation (4) represents the firm's profit function when  $\varepsilon$  is replaced by 0, that is, when total demand during the selling season is equal to its expected value. Equation (5) is the loss function, which assesses an overage cost  $h + c$  and an underage cost  $s + p - c - C(l, \lambda)$ . Note that the firm's expected profit, as expressed in Equation (3), is analogous to the newsvendor profit in the absence of demand uncertainty  $\Psi(p, l)$ , less the expected loss that occurs as a result of the presence of uncertainty  $L(z, p, l)$  (Petruzzi and Dada 1999).

We have  $\partial^2 \Pi(z, p, l) / \partial z^2 = -[h + s + p - C(l, \lambda)]f(z) \leq 0$ , which implies that  $\Pi(z, p, l)$  is concave in  $z$  for given  $p$  and  $l$ . Thus, we can reduce the original optimization problem over three variables to one over two variables  $p$  and  $l$  by first solving for the optimal

value of  $z$  as a function of  $p$  and  $l$  and then substituting the result back into  $\Pi(z, p, l)$ . However, the resulting expected profit function is neither concave in  $p$  (for a given  $l$ ), nor concave in  $l$  (for a given  $p$ ), which precludes us from obtaining the optimal  $p$  and  $l$  analytically. To tackle this technical difficulty, we introduce another transformation of variables to treat  $\lambda$ , instead of  $p$ , as our decision variable. From Equation (1), we can write

$$p = \frac{(\alpha - \lambda - \theta l)}{\beta}. \quad (6)$$

Equation (3) becomes

$$\begin{aligned} \Pi(z, \lambda, l) &= \left[ \frac{1}{\beta}(\alpha - \lambda - \theta l) - c - C(l, \lambda) \right] \lambda - (h + c)\Delta(z) \\ &- \left[ \frac{1}{\beta}(\alpha - \lambda - \theta l) + s - c - C(l, \lambda) \right] \Omega(z). \end{aligned} \quad (7)$$

In what follows, we use a sequential procedure to first solve for optimal  $l$ , then optimal  $\lambda$ , and finally optimal  $z$ .

### 3.1. Optimal Lead-Time Quotation

We first show  $\Pi(z, \lambda, l)$  is concave in  $l$  for given  $\lambda$  and  $z$ . The following lemma provides the optimal lead-time quotation as a function of the average demand.

**LEMMA 1.** *The optimal lead-time to quote is independent of  $z$ . It is determined uniquely as a function of any given  $\lambda$  as follows:*

$$l^* \equiv l(\lambda) = G_\lambda^{-1}(v), \quad (8)$$

where  $v = (s_l - \theta/\beta)/(s_l + h_l)$  and  $G_\lambda^{-1}(\cdot)$  is the inverse of the lead-time distribution  $G_\lambda(\cdot)$ .

The value of  $v$  is the probability that actual lead-time will not exceed the quoted lead-time. It is immediate that  $v \leq 1$ . In addition,  $\theta/\beta$  can be interpreted as customers' waiting cost per unit of the quoted lead-time  $l$  (i.e.,  $\theta/\beta = c_w$ ). If the value of  $\theta/\beta$  is more than the firm's tardiness cost  $s_l$ , then it is optimal for the firm to quote a zero lead-time regardless of the mean demand. The above formula is analogous to the newsvendor critical fractile result, with underage cost  $s_l - \theta/\beta$  and overage cost  $\theta/\beta + h_l$ . Note that  $l^*$  is dependent on price  $p$  through  $\lambda$ , for example, Lemma 1 could be alternatively phrased as the optimal lead-time is determined uniquely as a function of  $p$ .

### 3.2. Lead-Time Distribution

Thus far, we have characterized the optimal lead-time quotation. It is, however, difficult to further solve for

the optimal price without knowing the functional form of the lead-time distribution  $G_\lambda(\cdot)$ , which depends on average demand  $\lambda$ . Most work on similar topics in the literature assume an M/M/1 production system (e.g., Boyaci and Ray 2003, Palaka et al. 1998, So 2000), and consequently the lead-time is the steady-state customer sojourn time, with distribution being given by

$$G_\lambda(t) = 1 - e^{-(\mu-\lambda)t},$$

where  $\mu$  is the mean service rate, and  $\lambda$  is the mean demand (see, e.g., Wolff 1989). Liu et al. (2007), for instance, show that the optimal  $\lambda$  can be uniquely determined given M/M/1 and some other assumptions.

Limitations of the M/M/1 lead-time distribution, arising from its restrictive technical requirements, have long been recognized by researchers. In reality, hardly any practical production environment is consistent with the stringent assumptions. The widespread use of M/M/1 is mainly due to its analytical tractability (Palaka et al. 1998).

We propose a different lead-time model, which takes the multiplicative form. In particular, the random lead-time can be expressed as the product of the expected demand during the selling season  $\lambda$  and a random variable that is independent of  $\lambda$ , that is, the random lead-time can be expressed as

$$T = \lambda\xi,$$

wherein random variable  $\xi$  has cdf  $\Phi(\xi)$  and pdf  $\phi(\xi)$ . From basic statistics, we know that  $G_\lambda(t) = \Phi(t/\lambda)$  and  $g_\lambda(t) = \phi(t/\lambda)/\lambda$ .

The above modeling approach draws on Liu et al. (2007), who consider a more general (i.e., a combination of multiplicative and additive) relationship between  $T$  and  $\xi$ . For the most commonly used G/M/1 queue, their general model reduces to our multiplicative case. Therefore, our class of multiplicative lead-time distributions is general enough to capture the settings in most studies. Moreover, our class of multiplicative lead-time distributions provides a technical advantage where it is possible to obtain a complete characterization of the optimal decisions. We note that the linear relationship between the expected lead-time and expected demand is a key property that enables the expressions for the optimal decisions and costs in our model. And a linear relationship between lead-time and demand arises, in approximation, in the classical M/M/1 system when system utilization is low (e.g., follows from a Taylor series expansion of the expected lead-time function).

### 3.3. Optimal Price

We now proceed to solve for the optimal price under multiplicative lead-time distribution assumption. Letting  $\delta = \Phi^{-1}(\nu)$  and substituting  $G_\lambda^{-1}(\nu) = \Phi^{-1}(\nu)$   $\lambda = \delta\lambda$  into Equation (8) yields

$$l^* \equiv l(\lambda) = \delta\lambda.$$

Analogous to the optimal stocking factor in the PSNP, the value of  $\delta$  can be defined as the optimal *timing factor*. If  $s_l \leq \theta/\beta$ , for example, then the optimal timing factor is zero and  $l^* = 0$ .

The above result shows that  $l^*$  is increasing in the mean demand  $\lambda$ . We now offer an interpretation to see why this is so. It is commonly understood that in the newsvendor problem, when mean and variance of uncertain demand increase, the optimal order quantity also increases (provided the optimal service level is reasonably high, which is usually the case in practice). We observe that the mean and variance of the lead-time distribution are  $E[T] = \lambda E[\xi]$  and  $V[T] = \lambda^2 V[\xi]$ , both of which are increasing in  $\lambda$ . Therefore, it makes sense that  $l^*$  is also increasing in  $\lambda$ , due to the newsvendor-like trade-off involved in determining the lead-time cost  $C(l, \lambda)$ , for example, quoting the optimal lead-time essentially involves balancing the risks of underage vs. overage costs.

Substituting  $l^*$  into Equation (2) yields

$$\begin{aligned} C(l^*, \lambda) &= h_l \int_0^{\delta\lambda} (\delta\lambda - t) \phi\left(\frac{t}{\lambda}\right) d\frac{t}{\lambda} \\ &\quad + s_l \int_{\delta\lambda}^\infty (t - \delta\lambda) \phi\left(\frac{t}{\lambda}\right) d\frac{t}{\lambda} \\ &= \lambda \left[ h_l \int_0^\delta (\delta - y) \phi(y) dy + s_l \int_\delta^\infty (y - \delta) \phi(y) dy \right] \\ &= \varphi\lambda, \end{aligned}$$

where  $\varphi = h_l \int_0^\delta (\delta - y) \phi(y) dy + s_l \int_\delta^\infty (y - \delta) \phi(y) dy$ , which is the optimal lead-time cost per unit of expected demand and is totally independent of  $\lambda$ .

The linearity of the optimal lead-time quotation  $l^*$  and the expected lead-time cost  $C(l^*, \lambda)$  in  $\lambda$ , as a result of our multiplicative lead-time distribution assumption, greatly facilitates the derivation of the optimal  $\lambda$ , which allows us to derive some qualitative insights from our model. The problem can be viewed as a PSNP with a convex cost structure, that is, expected cost is  $[c + C(l^*, \lambda)][\lambda + z - \Delta(z)] + c\Delta(z)$ , which is quadratic in expected demand  $\lambda$ . Furthermore, the convex cost structure is unaffected by whether or not the optimal quoted lead-time is zero or positive ( $\varphi$  is positive even if  $l^* = 0$ ).

Substituting  $l^*$  into  $\lambda$ , the expected demand is

$$\lambda = \left( \frac{\alpha}{1 + \theta\delta} \right) - \left( \frac{\beta}{1 + \theta\delta} \right) p,$$

and substituting  $l^*$  and  $C(l^*, \lambda)$  into Equation (7), we get

$$\begin{aligned} \Pi(z, \lambda) &= (p - c - \varphi\lambda)(\lambda - \Omega(z)) - (c + h)\Delta(z) - s\Omega(z) \\ &= \left[ \frac{1}{\beta}(\alpha - \lambda - \theta\delta\lambda) - c - \varphi\lambda \right] \lambda - (h + c)\Delta(z) \\ &\quad - \left[ \frac{1}{\beta}(\alpha - \lambda - \theta\delta\lambda) + s - c - \varphi\lambda \right] \Omega(z). \end{aligned} \tag{9}$$

The following lemma provides the optimal price.

LEMMA 2. For a fixed  $z$ , the optimal  $\lambda$  and the optimal price are

$$\lambda^* \equiv \lambda(z) = \lambda^0 + \frac{1}{2}\Omega(z), \tag{10}$$

$$p^* \equiv p(z) = p^0 - \left( \frac{1 + \theta\delta}{2\beta} \right) \Omega(z), \tag{11}$$

where  $\lambda^0 = \frac{\alpha - \beta c}{2(1 + \theta\delta + \beta\varphi)}$  and  $p^0 = \frac{\alpha - (1 + \theta\delta)\lambda^0}{\beta}$ .

The term  $\lambda^0$  is the corresponding average demand that maximizes  $\Psi(p, l)$ , the deterministic portion of expected profit. Similarly, the term  $p^0$  is the price that maximizes  $\Psi(p, l)$ . We note that  $p^0$  can be alternatively expressed as

$$p^0 = \frac{\alpha + \beta c}{2\beta} + C(l^*, \lambda^0).$$

The expression says that the optimal price is the optimal price in a model without lead-time considerations (e.g.,  $\theta = 0$ ) plus the unit lead-time cost.

Compared with past results that have looked at price–time dependent demand, but have not included the newsvendor ordering problem, the insights associated with the expressions for  $l^0 = l(\lambda^0)$ ,  $p^0$ , and  $\lambda^0$  are consistent with those reported in earlier publications. Palaka et al. (1998) assume the lead-time distribution conforms to the M/M/1 model. They find that  $p^0 = (\alpha - \lambda^0 - \theta l^0) / \beta$  (i.e., when demand is deterministic, the intercept of the demand function, called  $a$  in Palaka et al., is  $\alpha$ ). Since  $l^0 = l(\lambda^0) = \delta\lambda^0$ , we have

$$p^0 = \frac{(\alpha - \lambda^0 - \theta l^0)}{\beta},$$

confirming the selling price result of Palaka et al. (1998). The expressions for  $l^0$  and  $\lambda^0$  in our article vary from those presented in Palaka et al. (1998)

Table 1 Comparative Statics

	Average demand ( $\lambda^0$ )	Optimal quoted lead-time ( $l^0$ )	Probability of meeting quoted lead-time ( $v$ )
Price sensitivity ( $\beta$ )	↓	↑	↑
Lead-time sensitivity ( $\theta$ )	↓	↓	↓
Unit holding cost ( $h$ )	↓	↓	↓
Unit tardiness cost ( $s$ )	↓	↑	↑

because our study employs a different lead-time distribution model. Table 1 summarizes some of the important comparative statics (a proof is provided in Appendix S1). In the table, the symbol  $\updownarrow$  means either increasing or decreasing.

Note that our model corresponds to the non-binding service level constraint case in Palaka et al. (1998). The results in Table 1 are consistent with their results with one exception. The quoted lead-time in our model affects the expected holding cost, and therefore, the probability of meeting quoted lead-time decreases in the unit holding cost (e.g., so as to reduce the fraction of orders held in inventory prior to shipment). In the results of Palaka et al. (1998), however, the unit holding cost does not have any impact on the probability of meeting quoted lead-time due to the assumption that the expected holding cost is not dependent on quoted lead-time directly. Our result seems to be more appealing, because intuitively one would expect, as unit holding cost  $h_l$  increases, a shorter lead-time to be quoted in order to reduce the chance of holding those completed products. We draw additional comparisons with results from the literature after characterizing the optimal stocking decision in the next section.

### 3.4. Optimal Inventory Decision

Substituting  $\lambda^*$  into Equation (9), the expected profit function  $\Pi(z, \lambda(z), l(z))$  becomes a function of the single variable  $z$  as follows:

$$\begin{aligned} \Pi(z) &= (p - c - C(l, \lambda))(\lambda - \Omega(z)) \\ &\quad - (h + c)\Delta(z) - s\Omega(z) \\ &= \frac{1}{\beta} \left[ \alpha - \beta c - (1 + \theta\delta + \beta\varphi) \left( \lambda^0 + \frac{\Omega(z)}{2} \right) \right] \\ &\quad \times \left[ \lambda^0 - \frac{\Omega(z)}{2} \right] - (h + c)\Delta(z) - s\Omega(z). \end{aligned} \tag{12}$$

Employing a similar approach presented in Petrucci and Dada (1999), we characterize the optimal solution in the following theorem.

**THEOREM 1.** *The optimal decision is to quote a lead-time  $l^*$  as stated in Lemma 1, and to stock  $q^* = \lambda^* + z^*$  to sell at the price  $p^*$ , where  $\lambda^*$  and  $p^*$  are specified in Lemma 2 and  $z^*$  is determined as follows:*

- (a) *If  $F(\cdot)$  is an arbitrary distribution, then an exhaustive search over all values of  $z$  in the region  $[A, B]$  will determine  $z^*$ , which maximizes  $\Pi(z, \lambda(z), l(z))$ .*
- (b) *If  $F(\cdot)$  is a distribution function satisfying the condition*

$$2r^2(z) + \frac{dr(z)}{dz} > 0, \quad (13)$$

for all  $z$  in the region  $[A, B]$ , where  $r(\cdot) \equiv f(\cdot)/[1 - F(\cdot)]$  is the hazard rate (or failure rate), and the following condition is met

$$s + \frac{[\alpha - \beta c + A(1 + \theta\delta + \beta\varphi)]}{2\beta} > 0, \quad (14)$$

then  $z^*$  is the unique  $z$  in the region  $[A, B]$  that satisfies the first-order condition  $d\Pi(z)/dz = 0$ , that is,

$$\begin{aligned} &-(h + c) + [1 - F(z)] \\ &\times \left[ h + c + s + \frac{\alpha - \beta c - (1 + \theta\delta + \beta\varphi)\Omega(z)}{2\beta} \right] \\ &= 0. \end{aligned} \quad (15)$$

Note that the condition  $2r^2(z) + dr(z)/dz > 0$  is weaker than the increasing failure rate condition (i.e.,  $dr(z)/dz > 0$ ), which is satisfied by many commonly used distributions, for example, normal, uniform, exponential (see Bagnoli and Bergstrom [2005] for a more complete list).

An intuitive interpretation of the first-order condition specified in Equation (15) is as follows. Note that it can be rewritten as

$$F(z) = \frac{s + [\alpha - \beta c - (1 + \theta\delta + \beta\varphi)\Omega(z)]/(2\beta)}{(h + c) + s + [\alpha - \beta c - (1 + \theta\delta + \beta\varphi)\Omega(z)]/(2\beta)},$$

which is the typical newsvendor critical fractile-like solution, with overage cost  $h + c$  and underage cost  $s + [\alpha - \beta c - (1 + \theta\delta + \beta\varphi)\Omega(z)]/(2\beta)$ . This is consistent with our previous explanation of Equation (5) where we note that underage cost is  $s + p - c - C(l, \lambda)$ , because after substituting the optimal solutions, we have

$$\begin{aligned} s + p - c - C(l, \lambda) &= s + \frac{\alpha - \lambda - \theta\delta\lambda}{\beta - c - \varphi\lambda} \\ &= s + \frac{\alpha - \beta c - \lambda(1 + \theta\delta + \beta\varphi)}{\beta} \\ &= s + \frac{\alpha - \beta c - (1 + \theta\delta + \beta\varphi)\Omega(z)}{2\beta}. \end{aligned} \quad (16)$$

Given the above interpretation, the physical meaning of condition (14) is now immediate. By

noting the fact that  $\Omega(z)$  is decreasing in  $z$ , and  $\Omega(A) = -A > 0$ , we know that condition (14) guarantees that the newsvendor underage cost is always positive.

Note that the profit margin  $p - c - C(l, \lambda)$  can be simplified to  $[\alpha - \beta c - (1 + \theta\delta + \beta\varphi)\Omega(z)]/(2\beta)$ . Thus, while condition (14) is technically correct (i.e., first-order condition returns the maximum expected profit given that the firm sells the product), we actually require a slightly stronger condition

$$\alpha - \beta c + A(1 + \theta\delta + \beta\varphi) > 0, \quad (17)$$

to exclude the trivial case where the expected profit is negative. We point out, however, that condition (17) is satisfied by all reasonable parameter sets (including all numerical examples in the next section).

Before concluding this section, we remark that when  $\theta = h_l = s_l = 0$  (i.e., when lead-time is not a factor), our problem reduces to the traditional PSNP reviewed by Petruzzi and Dada (1999).

## 4. Managerial Implications

In this section, we discuss some managerial insights implied by our modeling results, and relate them to real-life examples. As stated earlier, a main contribution of our article is that we bridge two streams of existing literature, namely, the pricing and lead-time quotation stream, and the newsvendor problem with endogenous pricing stream. Compared to the former, our problem has the additional dimension of a stocking decision. Compared to the latter, our problem is complicated by the additional lead-time quotation decision, which affects the demand. Therefore, in this section we compare our problem and its results with the two closely related research streams. We focus on understanding the effect of demand uncertainty and that of lead-time quotation. For the purpose of easy comparison, we assume both conditions (13) and (17) hold, if not otherwise specified, to ensure the uniqueness of the optimal solutions.

### 4.1. The Effect of Demand Uncertainty

In this subsection, we explore the impact of loss  $L(z, p, l)$  due to uncertain total demand on the optimal decisions and the associated optimal expected profit. Recall that the  $\lambda^0, l^0$ , and  $p^0$  maximize the “deterministic” portion of expected profit  $\psi(p, l)$ . Let  $\Pi^0 = \psi(p^0, l^0)$ .

The following theorem claims that consideration of  $L(z, p, l)$  in the optimization problem increases the optimal values of the quoted lead-time and the resulting average demand, but decreases the optimal selling price and the expected profit.



**THEOREM 2** *The following inequalities hold:*

- (a)  $\lambda^* > \lambda^0$ ,
- (b)  $l^* > l^0$ ,
- (c)  $p^* < p^0$ ,
- (d)  $\Pi^* < \Pi^0$ .

To understand the above comparison, we first note that price and lead-time are extra (i.e., in addition to inventory) means to guard against demand risk, which can be measured by its coefficient of variation  $\sigma/\lambda$ . Therefore, in the presence of demand uncertainty, it makes sense to price and quote lead-time in such a way that the resulting average demand  $\lambda$  increases, to lower the demand risk. This explains part (a) of Theorem 2.

It is easy to understand part (b) from a technical perspective because we know  $l^* = \delta\lambda^*$ . We now look at an intuitive explanation. From a queuing perspective, the lead-time becomes stochastically larger when the mean demand  $\lambda$  becomes larger. As a result, a longer lead-time should be quoted to reduce the penalty cost due to late deliveries. At a more detailed and technical level, when the mean demand  $\lambda$  increases, both the mean and variance of the lead-time distribution increase. Therefore, it is natural for  $l^*$  to be larger. Note that the unit lead-time cost  $C(l, \lambda) = \varphi\lambda$  is also higher, as it is proportional to  $\lambda$ . Thus, our results indicate that in consideration of  $L(z, p, l)$ , it is optimal to trade lead-time-related cost for reduced inventory cost.

Keep in mind that demand is decreasing in both price and quoted lead-time. Now that  $l^*$  is larger, in order for  $\lambda^*$  to be larger, the optimal price  $p^*$  should be smaller. It should decrease to the extent that the decrease in demand caused by a longer promised lead-time is more than offset. Note that the marginal profit  $p - c - C(l, \lambda)$  is reduced as a result.

As for the profit comparison, it is easy to understand that the expected profit is lower when demand uncertainty is present, due to the additional cost term  $L(z, p, l)$ .

The impact of demand uncertainty has interesting implications. It suggests that customers with high lead-time sensitivity but low price sensitivity benefit from cooperating with the firm to eliminate (or reduce) demand uncertainty. For example, cooperation could take the form of advance purchase commitment, which resolves the demand uncertainty. Given the reduced demand uncertainty, the firm will set a higher price but promise a shorter lead-time, leading to a smaller average demand  $\lambda$ . As the average service rate of the system remains unchanged, this lower average demand will in turn result in a stochastically shorter lead-time, thus benefiting the lead-time sensitive but price insensitive customers.

The impact of demand uncertainty on optimal decisions and profit noted above adds a dimension to interpretations from the PSNP literature. When lead-time does not affect demand, corresponding to the PSNP model, it is known that the introduction of demand uncertainty in the form of an additive error term causes the optimal price and profit to decrease, and expected demand to increase. A managerial insight is that there is a conflict of interest with respect to the elimination of demand uncertainty; that is, it benefits the firm (higher profit), but hurts the customer (higher price). In our model, we find the same directional effects with respect to price, profit, and mean demand, but we also find that the optimal quoted lead-time decreases with the elimination of demand uncertainty. Thus, in contrast to PSNP, we find that a segment of consumers (lead-time sensitive but price insensitive) benefit from reduced demand uncertainty. Thus, as noted above, there are potential opportunities for the firm and customers to work together to reduce demand uncertainty, which is an insight that does not come out in traditional PSNP models.

The essence of the above discussion is not lost in the launch of the Xbox game console. When many game fanatics who were eager to be among the first to experience the product placed pre-orders, Microsoft was able to take advantage of the reduced demand uncertainty to price the product higher (the price dropped £100 just 42 days after its initial launch in the United Kingdom) and catered to the pre-orders first (Europemedia 2002).

#### 4.2. The Effect of Lead-time

In this subsection, we explore the effect of lead-time. Specifically, we study how the optimal price, stocking decision, the resulting average demand, and the associated expected profit are affected by the existence and magnitude of lead-time. To examine these relationships at a detailed level, we require a specific pdf for the random variable  $\xi$ . (Recall that random lead-time is  $T = \lambda\xi$ .) We assume that  $\xi$  is a power function distribution random variable with support  $[0, \rho]$ . The cdf and pdf are given by

$$\Phi(x) = \left(\frac{x}{\rho}\right)^\omega, \quad (18)$$

$$\phi(x) = \frac{\omega x^{\omega-1}}{\rho^\omega}, \quad (19)$$

respectively, where both the shape parameter  $\omega$  and the scale parameter  $\rho$  are positive. The scale parameter  $\rho$  influences the maximum time to complete one job in the system. Therefore, the cdf and pdf of the lead-time  $T$  are

$$G_\lambda(t) = \Phi\left(\frac{t}{\lambda}\right) = \left(\frac{t}{\rho\lambda}\right)^\omega,$$

$$g_\lambda(t) = \phi\left(\frac{t/\lambda}{\lambda}\right) = \frac{\omega t^{\omega-1}}{(\rho\lambda)^\omega},$$

respectively. In this case,  $\rho\lambda$  represents the longest possible lead-time when mean demand is  $\lambda$  and  $1/\rho$  is analogous to the mean service rate in the queuing system. As a result, the power function lead-time distribution behaves, in spirit, similarly to the M/M/1 lead-time distribution—it is a function of the mean demand and service rate parameters.

The shape parameter  $\omega$  plays a critical role in both the definition of the power distribution function and the ensuing analytical results. Figure 2 provides exemplary forms of the pdf of the power function distribution (where the upper support  $\rho\lambda$  is 1) with different values of the shape parameter  $\omega$ .

Although the power function distribution is not derived from a specific model of a production system, and thus has the shortcoming of being an approximation, we believe this assumption is reasonable because of its flexibility and ability to capture the essence of lead-time in practical situations. The power function distribution has the following advantages. First, it is bounded, reflecting the reality that infinite lead-time is not possible. Second, it is consistent with the results in traditional queuing theory in the sense that the lead-time distribution can be described as a function of the mean demand rate and service rate. Third, it is applicable to a variety of settings. It can not only mimic the exponential distribution when M/M/1 is an appropriate approximation of reality but also captures features beyond M/M/1's capability, for example, lead-time is more likely to be long than short. Fourth, we find the approximation works well in numerical tests. For example, we let  $\varepsilon_i$  be a geometric random variable so that the total demand during the season is a negative binomial random variable, which

is commonly used to model retail demand and has proven to fit empirical data well (Smith and Agrawal 2000). The time to customize the product is an exponential random variable. Similar to the approximation model in Liu et al. (2007), we find that the approximation error tends to decrease in the critical fractile  $v$ . For instance, when the critical fractile is above 85%, the maximum error (i.e., maximum difference between the observed cdf and the power function distribution cdf) is <5%.

Given that power function distribution random variable with cdf and pdf defined in Equations (18) and (19), one can verify that the optimal timing factor  $\delta = \tau\rho$ , and the optimal lead-time cost per unit of expected demand  $\varphi = \eta\rho$ , where  $\tau = v^{1/\omega}$  and  $\eta = [h_1\tau^{\omega+1} + s_1(\tau^{\omega+1} + \omega - \tau - \tau\omega)]/(\omega + 1)$ .

We first analyze a simpler variant of the problem where we eliminate the demand uncertainty (e.g., random lead-time is solely a consequence of random service time).

#### 4.2.1. The Case Without Demand Uncertainty.

We begin with an observation that the optimal lead-time is closely related to the scale parameter,  $\rho$ , of the lead-time distribution. Recall that in the absence of demand uncertainty, the optimal lead-time is determined by  $l^0 = \tau\rho\lambda^0 = (\alpha - \beta c)\tau\rho/[2(1 + \theta\tau\rho + \beta\eta\rho)]$ , which can be easily verified to be increasing in  $\rho$ . Therefore, in this subsection, to study the effect of lead-time on the optimal decisions and the expected profit, it suffices to analyze the influence of  $\rho$ , which corresponds to the varying levels of lead-time. Recall that  $\rho$  can be interpreted as the average time to serve a customer. As its value increases, both the mean and the variance of the lead-time distribution increase. Thus, it is intuitive to quote a longer lead-time as  $\rho$  increases.

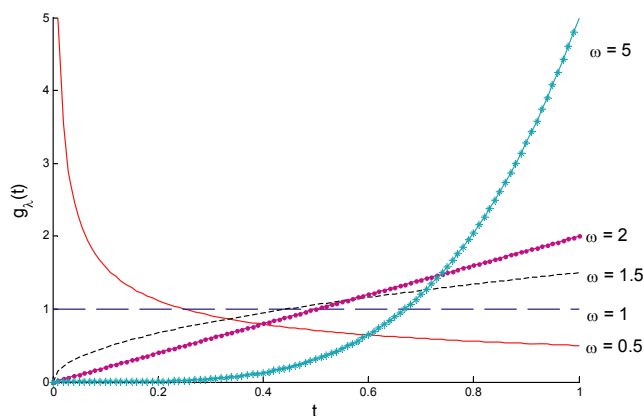
In the special case where  $\rho = 0$ , the optimal lead-time is 0. This actually becomes the make-to-stock situation where products are readily available when orders arrive. We point out that the power function distribution requires the scale parameter  $\rho$  to be positive. However, we can simply redefine its pdf as constant at 0 when  $\rho = 0$  to make it consistent for all non-negative  $\rho$ -values.

The following theorem states the effect of lead-time in the absence of demand uncertainty.

**THEOREM 3.** *When  $\rho$  increases so that it is optimal to quote a longer lead-time, the optimal price increases, resulting in a lower average demand. In addition, the associated optimal expected profit decreases, that is,  $\partial p^0/\partial\rho > 0$ ,  $\partial\lambda^0/\partial\rho < 0$ , and  $\partial\Pi^0/\partial\rho < 0$ .*

To see why the optimal price is increasing in  $\rho$ , note that the profit margin  $p^0 - c - C(l^0, \lambda^0)$  should

Figure 2 Probability Density Function of Power Function Distribution



remain constant at  $(\alpha - \beta c)/2\beta$  (see the proof of Theorem 3 for the expression of  $\Pi^0$ ) in order for the expected profit to be maximized. This observation suggests that the core interaction between the pricing and lead-time decisions is to maintain the profit margin. Since  $C(l^0, \lambda^0) = \eta\rho\lambda^0$  is increasing in  $\rho$ , the optimal price should also be increasing in  $\rho$  to hold the profit margin unchanged.

As both price and lead-time increase as a result of increased  $\rho$ , the mean demand decreases, for example, the customer base shrinks. Not surprisingly, the expected profit also decreases (the number of buying customers is declining, while the profit margin remains constant).

One might expect price and lead-time to play substitutable roles in shaping customers' utilities as capacity increases; for example, if a higher price is charged, a shorter lead-time is quoted to maintain customer utilities and not turn customers away, and vice versa. On further thought, however, our results stated above make sense if we look at it from the following angle. If the firm adopts some advanced manufacturing technology to reduce the processing time  $\rho$ , it can then afford to quote a shorter lead-time and lower price to attract more demand without overloading its capacity.

The positive relationship between  $p^*$  and  $\rho$  derives from a fundamental property of deterministic pricing models known as the Lerner index relation: the optimal profit margin as a percent of selling price is equal to the multiplicative inverse of the price-elasticity of demand (Lerner 1934). From this property, it follows that the optimal price is increasing in unit cost for any demand function with price-elasticity that is non-decreasing in price (e.g., as is the case for linear demand, iso-elastic demand, among others). Any such model in which an increase in service time forces an increase in the quoted lead-time also causes an increase in unit cost (i.e., through higher lead-time cost) and, by the Lerner index relation, an increase in the optimal price.

Past research on the PSNP model has shown that the optimal price under an additive form of demand uncertainty is lower than the optimal price for deterministic demand (Petruzzi and Dada 1999). The result in Theorem 2, which shows that optimal price decreases as demand uncertainty is introduced, is consistent with the PSNP model predictions. However, a result in Theorem 3 hints at meaningful differences in the effect of lead-time vs. demand uncertainty on optimal price. In particular, optimal price is increasing as the mean and standard deviation of lead-time increase, that is,  $\partial p^0/\partial\rho > 0$ . To prove that the directional effect of the standard deviation of lead-time continues to hold when mean lead-time is held constant, we use a mean-preserving transforma-

tion of the lead-time random variable  $T$  that was originally introduced in Gerchak and Mossman (1992):

$$T_k = kT + (1 - k)E[T] = kT + (1 - k)\frac{\omega}{\omega + 1}\rho\lambda, \quad (20)$$

where  $k \geq 0$ . Accordingly,  $E[T_k] = E[T]$  and  $V[T_k] = k^2V[T]$ . In this transformation, it can be seen that the variance is increasing with the value of  $k$  while the mean is kept constant. Using  $T_k$  in place of  $T$  in the analysis above, we see that the optimal price is increasing in lead-time uncertainty (see Equations (SA1)–(SA4) in Appendix S1). The effect of increasing lead-time uncertainty on optimal price is similar to the effect of increasing demand uncertainty on optimal price in the PSNP model with the multiplicative form of random demand (Petruzzi and Dada 1999). The intuition behind this similarity is that the standard deviation of the random variable (lead-time or demand) is increasing in the mean demand, which relative to the additive form of random demand where there is no linkage between the mean and standard deviation of demand, introduces pressure to reduce mean demand, for example, by increasing price. In sum, in our model, while lead-time uncertainty leads to an increase in the optimal selling price, the additive form of demand uncertainty causes a reduction in the optimal selling price. The consequence of this result is that the inclusion of lead-time uncertainty in the PSNP model cannot be simply treated as a further increase in demand uncertainty.

**4.2.2 The Case with Demand Uncertainty.** We now investigate the effect of lead-time in the presence of demand uncertainty. As in the case of deterministic demand, we establish the relationship between the optimal lead-time and the parameter  $\rho$  to facilitate our discussion.

**LEMMA 3.** *The optimal stocking factor  $z^*$  is decreasing in  $\rho$ , and the optimal lead-time  $l^*$  is increasing in  $\rho$ , that is,  $\partial z^*/\partial\rho < 0$ , and  $\partial l^*/\partial\rho > 0$ .*

As in the deterministic demand case, the optimal lead-time  $l^*$  is increasing in  $\rho$ , and the interpretation is the same. From the first-order condition specified in Equation (15), we can see that an increase in  $\rho$  causes a decrease in the underage cost, whereas the overage cost is unaffected. Therefore, the optimal stocking factor  $z^*$  is decreasing in  $\rho$ .

The effect of lead-time on the optimal expected profit is stated in the following theorem.

**THEOREM 4.** *In the presence of demand uncertainty, the optimal expected profit is decreasing in  $\rho$ , that is,  $\partial\Pi^*/\partial\rho < 0$ .*

While it is easy to understand the optimal expected profit decreases in  $\rho$ , it is not easy to see how the average demand  $\lambda^*$  and optimal price  $p^*$  change with  $\rho$ , as the comparative statics analysis is not tractable. We conducted extensive numerical experiments and found that as  $\rho$  increases, the average demand always decreases, but the optimal price  $p^*$  may either increase or decrease. A representative example is reported below in Figure 3. The parameters used in this example are  $\alpha = 530$ ,  $\beta = \theta = 5$ ,  $c = 50$ ,  $h = h_l = 0.5$ ,  $s = s_l = 2$ ,  $\omega = 1$ , and  $\varepsilon$  is assumed to follow a negative binomial distribution with parameters 5 and 0.2.

To see why the average demand  $\lambda^*$  decreases in  $\rho$ , recall that  $1/\rho$  can be interpreted as the mean service rate of the system. As  $\rho$  increases (or  $1/\rho$  decreases), it is natural to induce a smaller demand rate (average demand  $\lambda^*$ ) to avoid potentially high tardiness cost. As a result, a high lead-time related cost  $C(l, \lambda)$  can be avoided, which is desirable because the lead-time related cost erodes the profit margin (recall that the net profit margin is given by  $p - c - C(l, \lambda)$ ).

Unlike the deterministic demand case, the optimal profit margin  $p - c - C(l, \lambda)$ , which can be rewritten as  $[\alpha - \beta c - (1 + \theta\tau\rho + \beta\eta\rho)\Omega(z)]/(2\beta)$ , is not independent of  $\rho$  when demand is random. Instead, it now can be verified to be decreasing in  $\rho$ . Therefore, it is hard to predict how the optimal price  $p^*$  is affected by  $\rho$  even if  $C(l, \lambda)$  increases in  $\rho$ . Our numerical studies indicate that  $p^*$  increases in  $\rho$  in most cases, but there are also circumstances in which  $p^*$  decreases in  $\rho$ . While it is unclear what conditions exactly characterize the impact of  $\rho$ , a general observation is that when  $\rho$  is relatively small,  $p^*$  increases in  $\rho$ . When  $\rho$  is relatively large,  $p^*$  can be decreasing. A potential reason is that when  $\rho$  is relatively small, the service rate  $1/\rho$  is still large enough to allow for a relatively large average demand, so it is more important to maintain the profit margin. Although the profit margin always decreases in  $\rho$ , an increasing price can offset the eroding margin partially. When  $\rho$  is relatively large, only a

small number of customers can be served. An increase in  $\rho$  then further shrinks the customer base. Consequently, it is essential for the company to lower price in order to retain customers.

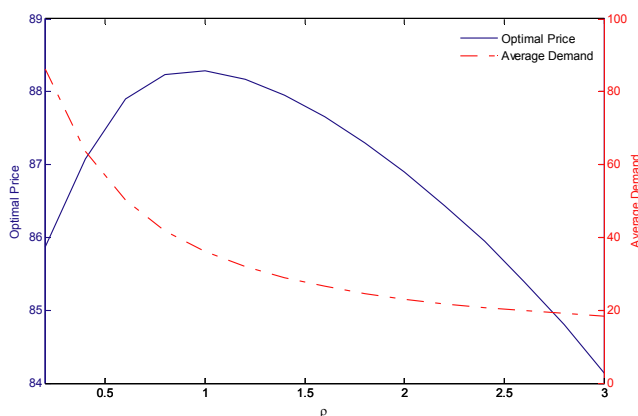
When demand is deterministic (i.e.,  $\sigma = 0$ ), we find that optimal price and quoted lead-time are increasing in the mean service time (in spirit)  $\rho$ , whereas profit is decreasing in  $\rho$ . This result is consistent with the pricing and lead-time quotation literature (e.g., similar result is implied by the centralized solution of Liu et al. 2007). A managerial implication, which comes from the existing literature, is that both the firm and customers benefit from reductions in service time (ignoring the sunk cost of reducing service time). It can be observed that the coefficient of variation of lead-time distribution is constant in both M/M/1 systems and our model, for a given  $\omega$ . This might help explain the similar effects of increasing mean service time that we see in our model and the M/M/1 models from the literature. However, we find that this managerial implication regarding the impact of the mean service time  $\rho$  on the optimal price and quoted lead-time from past research may be disqualified when  $\sigma > 0$ . While the firm always benefits from reductions in  $\rho$ , there are instances where price will increase as  $\rho$  is reduced, which can go against the interests of price-sensitive consumers. In other words, for a product with a small market, some customers can actually benefit, for example, those who care a lot about price but not about lead-time, as the firm's production capabilities diminish.

## 5. Conclusion

We formulate a price-setting newsvendor model with price and lead-time-dependent demand. The model combines and extends models in the price-setting newsvendor literature and the pricing and lead-time quotation literature. We derive expressions for the optimal quantity, price, and quoted lead-time. By examining these expressions and by conducting numerical analyses, we expose relationships between problem parameters, optimal decisions, and optimal expected profit. We show that while lead-time uncertainty leads to an increase in the optimal selling price, the demand uncertainty with an additive random error term causes a decrease in its value. As a result, the inclusion of lead-time uncertainty cannot be treated as simply an increased level of demand uncertainty, necessitating the type of analysis provided in this article.

We find that in settings with endogenous and lead-time-dependent demand, both the firm and some customers (i.e., lead-time sensitive and price insensitive) benefit from lower demand uncertainty, thus creating greater opportunities for collaborative efforts to reduce demand uncertainty. This is in contrast to

Figure 3 Impact of  $\rho$  on Average Demand ( $\lambda^*$ ) and Optimal Price



settings where lead-time is exogenous, as in the case of the traditional PSNP, for example, the firm benefits with higher profit whereas the consumer suffers from higher prices.

We examine how demand uncertainty influences the impact of capacity on the firm's optimal decisions. Consistent with literature, we find that, in the absence of demand uncertainty, a reduction in capacity causes an increase in the optimal selling price and quoted lead-time, hurting all consumers and the firm. However, when demand uncertainty is incorporated into the problem, the same notion does not apply consistently. Indeed, for a product with a small market size, we find that a reduction in capacity can cause the firm to decrease its optimal selling price (rather than increasing it) in order to retain some of its customer base. As a result, at least one segment of consumers (who are highly sensitive to price but less so to quoted lead-time) can benefit from a reduction in capacity.

On the technical side, our study utilizes two transformations of variables to turn an otherwise difficult to solve multivariate optimization problem into a tractable sequential optimization problem. These transformations enable us to determine the optimal solutions for all decision variables: order quantity, selling price, and the quoted lead-time. We introduce a general multiplicative lead-time model. The model is general enough to capture a majority of the settings described in literature and it enables us to obtain a complete characterization of the optimal decisions. For detailed investigation of lead-time effects, we use a power function distribution. This distribution accommodates real-world settings where work is more likely to get completed near the end of the quoted lead-time window, as well as settings where the reverse applies. Much of the pricing and lead-time quotation literature draws on an M/M/1 model where work is more likely to get completed near the beginning of the quoted lead-time window.

We suggest two issues for future research that stem from our assumptions. First, our analysis is based on a linear demand function with an additive form of demand uncertainty. Other demand functions (e.g., isoelastic) and forms of demand uncertainty (e.g., multiplicative) are worthy of investigation. Second, our model assumes a tardiness (penalty) cost that is independent of the selling price. While it is difficult to address the relationship between the tardiness cost and the selling price in our modeling framework, it can be investigated in a setting which considers multiple demand classes.

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