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Theory and Methodology

Optimization of printed circuit board manufacturing: Integrated modeling and algorithms

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Abstract

This paper focuses on an integrated optimization problem that is designed to improve productivity in printed circuit board (PCB) manufacturing. We examine the problems of allocating the components to feeders and sequencing the placement of these components on the PCBs, populated by a rotary head machine with surface mount technology. While previous research focuses on sequencing the placement and only considers this subproblem as part of an interrelated set of problems, we provide an integrated approach which tackles all subproblems simultaneously as a single problem. Given an ε-approximation algorithm for the vehicle routing problem we present a solution with an ε-error gap for the PCB problem.

Keywords: Optimization; ε-approximation; Modeling; Printed circuit board

1. Introduction

There has been an increasing interest regarding the modeling and analysis of manufacturing systems in the electronics and semiconductor industry. This interest has been influenced by the advancements made in the design of computerized numerical control (CNC) machines. The sophistication of these machines varies tremendously and, in this paper, we study the surface mount technology with a CNC machine that has a rotary head to perform the operations. A CNC functions as follows: a PCB is placed on a table and adjacent to it are multiple feeder locations with a different component type assigned to each. A head is used to grasp these components from their feeder locations and mount them on the PCB with the help of an arm (the head is usually located on top of the arm). Some heads are capable of picking up more than one component simultaneously, and these are called "rotary heads". These heads are widely used in the Surface Mount Technology. Our particular focus is on a CNC with a rotary single head that can pick up a certain number of components of the same type at a single time and mount them on the board (e.g. Quad 400 series). Fig. 1 is a generalized

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illustration of a CNC that is used for manufacturing a PCB. The components that are mounted on a PCB are discrete devices such as resistors, diodes, transistors, transformers, or connectors.

The manufacturing of a PCB consists of three basic steps or subproblems (see, e.g., Ball and Magazine, 1988):
1. allocation of component types to machines,
2. allocation of component types to feeders at each machine,
3. pick-and-placement sequencing.

Earlier studies have formulated the third subproblem, pick-and-placement sequencing, independent of the allocation decisions. However, the sequencing of components on a PCB is dependent on the feeder location of the component types. When the assignment of component types to feeders is not done carefully, even if pick-and-placement sequencing is solved for optimality, it can result in an extremely poor performance. The efficiency/inefficiency of such independent approaches can best be understood by presenting an error guarantee. The error guarantee can be defined as the ratio of the difference between the feasible solution and the lower bound values to the value of the lower bound.

In general, there is no guarantee for the quality of optimal solutions in the PCB manufacturing problem, and this study aims to enlighten the quality of solutions. As a result of the need for an integrated approach, we integrate the above-mentioned three subproblems into a single problem. We use Lagrangian Relaxation to decompose the integrated mathematical model into two subproblems and then propose an algorithm that attempts to solve both of these subproblems. Given the optimal costs of the vehicle routing problem, we propose a method which finds an optimal solution for the integrated PCB problem. We also show that our integrated algorithm has an error guarantee of ε if the error bound of each vehicle routing problem is ε.

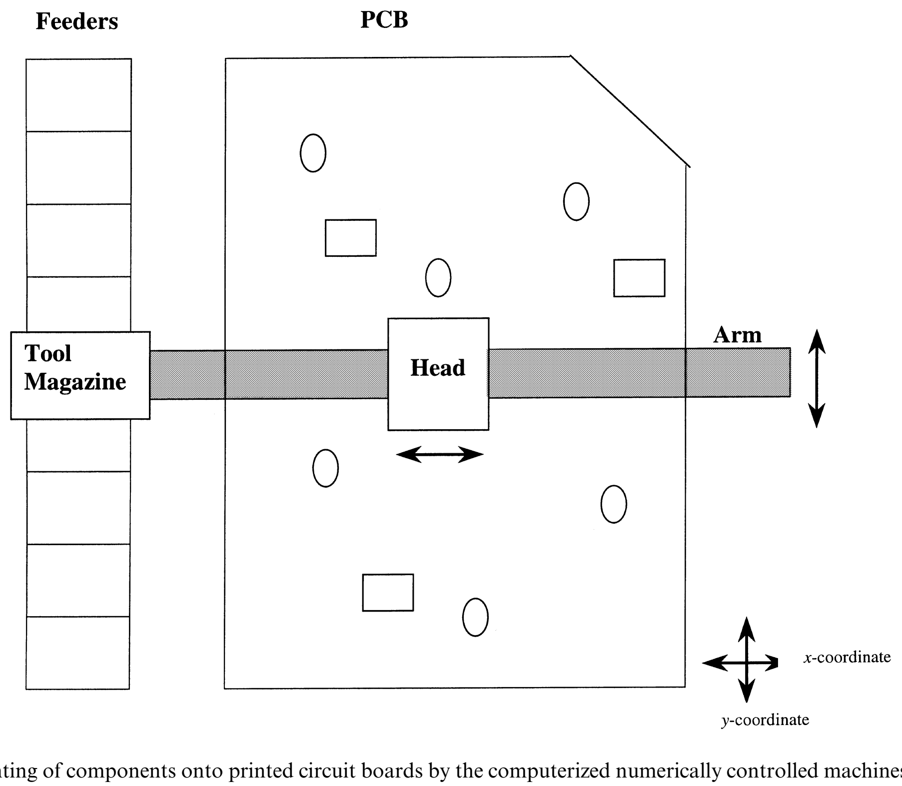

Fig. 1. The mounting of components onto printed circuit boards by the computerized numerically controlled machines. (*Note*: The arm moves in the y-axis whereas the head moves in the x-axis.)

2. Literature review

Among the many studies which examine the PCB manufacturing problem are Ball and Magazine (1988), Christopher et al. (1991), Drezner and Nof (1984), Gavish and Seidmann (1988), Leipala and Nevalainen (1989), McGinnis et al. (1992), McGinnis et al. (1993), and Or and Demirkol (1996). Ball and Magazine (1988) describe the three subproblems defined in the previous section and consider only a subset of our problem, sequencing the placement for insertion technology machines. These machines are generally designed for picking up one component at a time. While they formulate the problem as a Rural Postman Problem and provide a heuristic with a constant error, we extend it to include the feeder assignment decisions in our formulation. Christopher et al. (1991) show that the sequencing problem belongs to the NP-hard class of problems, and we extend their finding for our integrated problem. Similar to Ball and Magazine (1988), Drezner and Nof (1984) present an algorithmic solution to the component assignment problem where the machine again belongs to insertion technology. Gavish and Seidmann (1988), McGinnis et al. (1993) and Or and Demirkol (1996) state the importance of an integrated solution and Gavish and Seidmann (1988) consider the subproblems simultaneously for an insertion machine. Their technology can move the table, the feeder, and the head; it yields a distance measure that is the maximum of these three movements. Their formulation is a combination of the Assignment and the Traveling Salesman Problem (TSP) and they use the Quadratic Assignment Problem (QAP) to generate heuristic solutions. However, they do not present a performance analysis on their heuristic. Another combined QAP and TSP formulation is presented in Leipala and Nevalainen (1989) for an insertion technology CNC and the farthest insertion heuristic is presented by using the shortest Hamiltonian path problem. One motivating factor for our work comes from the fact that none of these studies can solve the problem for a rotary head that can pick up a certain number of components and mount them on the board in one tour.

Since the pick and placement sequencing problem resembles the Vehicle Routing Problem (VRP), we provide a comprehensive list of these studies as well. The reader is referred to Bodin et al. (1983) and Fisher and Jaikumar (1981) for the VRP, and Laporte et al. (1985), Fischetti et al. (1994) and Fisher (1994) for the Capacitated Vehicle Routing Problem (CVRP). Furthermore, heuristics using Lagrangian relaxation (Fisher, 1981; Geoffrion, 1974) to generate a lower bound have been developed for finding a set of routes for a fleet of capacitated vehicles to satisfy the cargo delivery requirements of customers (Altinkemer and Gavish, 1991).

Process planning in PCB assembly operations is another problem that has been studied extensively. Similar to our feeder assignment problem, McGinnis et al. (1992) propose an iterative procedure with TSP and QAP formulations for a sequencing of multiple cards in a multiple machine environment. Ammonds et al. (1997) extend this study to balance the workload of the assembly and provide an 8–10% improvement in their implementation. Askin et al. (1994) extend this problem to an open assembly shop and provide heuristics for component assignment and group formation with the original objective of minimizing makespan and a secondary objective of minimizing average flow time. While these studies aim at increasing productivity on the shop floor, we focus on the productivity of manufacturing a single PCB.

Other relevant studies include component allocation to feeders, partitioning, and feeder and fixture positioning. While Ahmadi et al. (1988), Ahmadi and Kouvelis (1994), and Ahmadi et al. (1995) study this problem on a dual delivery placement machine which has two heads mounting components sequentially, we examine single rotary head technology.

3. The integrated model and an algorithm

In this section, we provide a mathematical model that simultaneously considers the three subproblems defined earlier. We discuss the implications of two different technologies; one with the capability of moving the feeder locator, and the other with no feeder locator movement.

The first subproblem, allocation of component types to machines, can be reduced to the second subproblem, allocation of components to feeders at each machine, by indexing the feeder locations. For each component type, the machines that can process the component would be determined. The index number of the feeder locations at these machines is placed in a set for this component. As a result, each component type has a set, $L(k)$, that shows the indices of feeder locations to which the component type (type k) can be assigned. Thus, if a component type cannot be processed in a specific machine, then the indices corresponding to the feeder locations of this machine will not exist in this set. This simplifies the formulation of the original problem and now two major subproblems, one assignment-like and one vehicle routing-like problem, can be integrated into a single mathematical model.

The CNC machines that manufacture the PCBs have considerable variety in their design and manufacturing capability. The head of the CNC machines, which picks the components from the feeders, also have several designs. Newer technology permits more than one head in a machine through rotary heads. These machines have multiple heads and each of these heads has the capability of mounting the same component type in one trip on the board. The same variation can be observed with the number of feeders on each machine. The number of feeders at each machine varies from 4 to 256 (Prasad, 1989). Typically the average number is around 80. The capacity of each machine is determined by the capacity of feeder locations as well as the type of head, which can pick a certain number of components and then mount them onto the PCB. The head of a CNC is thus restricted in the number of components it can select. This capacity restriction is more significant than the capacity restriction introduced by the feeder locations, because the average number of components of one type that is mounted on the PCB usually exceeds the number of heads, but the average number of component types is smaller than the number of feeders at a machine. Furthermore, these feeders are usually fed with rolls of component types by the operator. If the components of one component type run out, the operator can place a new roll into the same feeder. Therefore the capacity of each feeder is not restrictive and is not included in this formulation. On the other hand, the head may have to make multiple tours as the number of components of the same type can be larger than the capacity of the head. The proposed formulation allows for multiple tours.

The distance measure is a Chebychev metric, because the arm can move in the vertical direction simultaneously while the head moves along the horizontal axis. The distance traveled between the two points on the board, i and j, for component type k is then the maximum of the distance traveled in either the horizontal axis or the vertical axis, which can be represented as

$$c_{ij}^k = \max\left\{\left|x_i - x_j\right|, \left|y_i - y_j\right|\right\},$$

where x_i, x_j represent the x-coordinates and y_i, y_j are the y-coordinates of points i and j, respectively. The head and the arm travel in the x and y directions simultaneously and the head positions itself on top of the point where the component will be mounted. Next, the head moves down and mounts the component on the board. This is referred to as the movement in the z-direction. The head does not move in the z-direction while moving in x and y directions simultaneously, in order to prevent potential errors in the precision of location. The location where the component will be mounted is required to be extremely precise in surface mount technology. Therefore, the head moves in the z-direction after completing the movement in the x and y directions. Hence, the total number of movements in the z-direction becomes constant and thus these movements can be neglected in the formulation.

As a result, one can describe the tour of the head in three steps:

1. a certain number of components are picked from the feeder without exceeding the capacity of the head,
2. the components are mounted onto predetermined points on the board,
3. the head is returned back to the feeder location.

After the completion of the mounting process for a particular component type, the head of the arm travels to the tool magazine in order to change the nozzle. The nozzle is the part with which the head grasps the components from the feeders. This is the set-up of the CNC before starting to select and place another component type.

As mentioned earlier, there are several different technologies in the PCB manufacturing process. Some allow the feeder locator to move and locate the feeder right next to the tool magazine while the head is traveling on the board. This enables the head to go to the tool magazine directly and grasp the components from the feeder. There are, however, technologies that do not allow the feeder locator to move. Both of these types are analyzed below.

Case 1: *The feeder locator moves*: In this particular technology, the feeder of the component type that will be processed next can move towards the tool magazine, so the distance traveled by the head will be shorter. Recall that the rotary head can pick a certain number of components initially. Then, while the head is mounting that component type, the feeder of the successor component type can, in the meantime, move towards the tool magazine. Therefore, the distance between the feeder locations and the points on the PCB can be measured from a fixed point next to the tool magazine.

The distance from the feeder locations to the points on the board can then be defined as follows:

$$c_{j1}^k = \max\{|x_j - x_1|, |y_j - y_1|\},$$

where point 1 stands for the location of the feeder when it is next to the tool magazine and j is a point on the PCB.

After completing the tour, the head turns back to the tool magazine in order to change the nozzle. Therefore, $c_{j1}^k = c_{1j}^k$ since the feeder was originally located next to the tool magazine. Here we assume that the feeder of the following tour can move next to the tool magazine before the head completes its tour on the PCB. This simultaneous movement saves time and enables each component type to have the same origin and destination points. This, in fact, motivates the following Lemma and allows the formulation to be an independent CVRP for each component type.

Lemma 1. *If the feeder can move next to the tool magazine, then finding the minimum number of tours minimizes the total travel cost for each component type separately.*

Proof. The traveling distances determine the cost of touring. The arm and head can move simultaneously both in the x and y axes. Therefore, the metric is Chebychev and the distance is the maximum of the movements in the x and y directions. Suppose that a certain group of points are already mounted on the board. The next decision is whether to include the last point in the tour or start a new tour for that component. The last point visited in the tour, point n, has coordinates (x_n, y_n) where the point in consideration, point $n+1$, has coordinates (x_{n+1}, y_{n+1}). The coordinate of the tool magazine is denoted as (x_1, y_1). Remember that in the same tour, the head will go directly to point $n+1$ from point n, then return to the tool magazine. In the second alternative, the head will return to the tool magazine and complete a round trip from the tool magazine to point $n+1$. The costs of these two alternatives are defined as the Same Tour Cost (STC) and New Tour Cost (NTC), respectively.

$$\begin{aligned}\text{NTC} &= \max\{|x_n - x_1|, |y_n - y_1|\} \\ &\quad + 2 \times \max\{|x_{n+1} - x_1|, |y_{n+1} - y_1|\}, \\ \text{STC} &= \max\{|x_n - x_{n+1}|, |y_n - y_{n+1}|\} \\ &\quad + \max\{|x_{n+1} - x_1|, |y_{n+1} - y_1|\}.\end{aligned}$$

In both of the above alternatives there is a common term,

$$\max\{|x_{n+1} - x_1|, |y_{n+1} - y_1|\}.$$

If we subtract this term from both of these costs, we get

$$\begin{aligned}\text{NTC}' &= \max\{|x_n - x_1|, |y_n - y_1|\} \\ &\quad + \max\{|x_{n+1} - x_1|, |y_{n+1} - y_1|\}, \\ \text{STC}' &= \max\{|x_n - x_{n+1}|, |y_n - y_{n+1}|\}.\end{aligned}$$

This yields a triangular inequality proof of the Chebychev distance.

$$\begin{aligned}\text{NTC}' &\geqslant \max\{|x_n - x_1|, |x_{n+1} - x_1|, \\ &\qquad |y_n - y_1|, |y_{n+1} - y_1|\}, \\ \text{NTC}' &\geqslant \max\{|x_n - x_1 - x_{n+1} + x_1|, \\ &\qquad |y_n - y_1 - y_{n+1} + y_1|\} \\ &= \max\{|x_n - x_{n+1}|, |y_n - y_{n+1}|\} = \text{STC}'.\end{aligned}$$

Then NTC $\geqslant$ STC. Since the new tour cost is always higher than the same tour cost, we conclude that the minimum number of tours is sufficient to minimize the total traveling distance for each component type. □

Since the distance between a point on the PCB and the feeder is independent of where the component type is located among feeders, the pick and placement sequencing is not dependent on the assignment decisions. Hence, the assignment problem does not need to be integrated with the pick and placement sequencing decisions. The formulation turns out to be a CVRP solved independently for each component type with the help of Lemma 1. This problem can be solved by using one of the recently published algorithms such as in Fisher (1994).

Case 2: *The feeder locator does not move*: This is a more complex environment where the pick and placement sequencing is not independent of assignment decisions. Therefore, this case requires an integrated formulation and the remainder of this paper will examine their interdependency. The formulation presented in this section has three assumptions that allow us to present a mathematical model.

Assumptions:
1. The head completes the tour by returning to the feeder location before going to the tool magazine.
2. All components of one type are assigned to the same feeder location.
3. The travel time between two coordinates is approximately linear in distance.

The second assumption eliminates the possibility of a complex task for manufacturers, the task of partitioning the components into smaller groups. Therefore, the manufacturer's optimal solution may have an objective function value less than or equal to that of the optimal solution obtained by the formulation presented in this section. Some manufacturers, however, already assign all components of the same type to a single feeder for practical reasons. They find it manageable because components do not get lost between feeders and/or the process is not complicated.

Notation

K	total number of component types
L	the total number of feeder locations
$L(k)$	the set of feeder numbers to which component type k can be allocated
$n(k)$	number of components of type k
$V(k)$	the node set of component type k including the feeder locations (starting nodes) $V(k) = \{1, \ldots, n(k), \ldots, n(k) + L\}$
$\tilde{V}(k)$	the node set excluding the indices of feeder locations $\tilde{V}(k) = \{L + 1, \ldots, n(k) + L\}$
Q	the maximum number of components from one component type that the head can pick up at a time (equivalently, the maximum number of components that can be mounted in one tour)
c_{ij}^k	the cost of traveling (distance) from point i to point j for component type k, $c_{ii}^k = \infty$
d_l	the round-trip distance between the feeder location l and the tool magazine
$m(k)$	the minimum number of tours the arm needs to make for component type k ($m(k)$ is equal to the smallest integer that is greater than or equal to the number of components of type k divided by the head capacity, Q)

Decision variables

$$y_{kl} = \begin{cases} 1 & \text{if component type } k \text{ is assigned to feeder location } l, \\ 0 & \text{otherwise.} \end{cases}$$

$$x_{ij}^k = \begin{cases} 1 & \text{if arc } (i,j) \text{ is traversed for component type } k, \\ 0 & \text{otherwise.} \end{cases}$$

z_{kl} = total cost of assigning component type k to feeder location l.

$$(\text{P1}): Z1^* = \text{Min } Z1 = \sum_{k=1}^{K} \sum_{l \in L(k)} y_{kl}(z_{kl} + d_l) \tag{1}$$

s.t.

$$\sum_{k=1}^{K} y_{kl} \leqslant 1 \quad \forall l, \tag{2}$$

$$\sum_{l \in L(k)} y_{kl} = 1 \quad \forall k, \tag{3}$$

$$\sum_{j=L+1}^{n(k)+L} c_{lj}^k x_{lj}^k + \sum_{j=L+1}^{n(k)+L} c_{lj}^k x_{jl}^k + \sum_{i=L+1}^{n(k)+L-1} \sum_{j=i+1}^{n(k)+L} c_{ij}^k x_{ij}^k = z_{kl} \quad \forall k, \forall l \in L(k) \tag{4}$$

$$\sum_{j=L+1}^{n(k)+L} x_{lj}^k \geqslant m(k) y_{kl} \quad \forall k, \forall l \in L(k), \tag{5}$$

$$\sum_{j=L+1}^{n(k)+L} x_{lj}^k = \sum_{j=L+1}^{n(k)+L} x_{jl}^k \quad \forall k, \forall l \in L(k), \tag{6}$$

$$\sum_{l=1}^{L} x_{lj}^k + \sum_{i=L+1}^{j-1} x_{ij}^k + \sum_{i=j+1}^{n(k)+L} x_{ij}^k + \sum_{i=L+1}^{j-1} x_{ji}^k + \sum_{i=j+1}^{n(k)+L} x_{ji}^k + \sum_{l=1}^{L} x_{jl}^k = 2$$
$$\forall k, \ \forall j = L+1, \ldots, n(k)+L \tag{7}$$

$$\sum_{i=1}^{n(k)+L-1} \sum_{j=i+1}^{n(k)+L} x_{ij}^k = n(k) \quad \forall k \tag{8}$$

$$\sum_{i \in S(k)} \sum_{i<j \in S(k)} x_{ij}^k \leqslant |S(k)| - L_{S(k)}$$
$$\forall k, \forall S(k) \subset \tilde{V}(k) \ L_{S(k)} \geqslant 1, |S(k)| \geqslant 2 \tag{9}$$

$$y_{kl} = (0/1) \quad \forall k, \forall l \in L(k), \tag{10}$$

$$x_{ij}^k = (0/1) \quad \forall k, \forall i < j \in V(k), \tag{11}$$

$$x_{lj}^k = (0/1) \quad \forall k, \forall l \in L(k), \forall j \in \tilde{V}(k), \tag{12}$$

$$z_{kl} \geqslant 0 \quad \forall k, \forall l \in L(k). \tag{13}$$

The above formulation is a combination of assignment-like and vehicle routing-like problems where the decision variables y_{kl} represent the decisions of the assignment problem and variables x_{ij}^k represent the decisions of the vehicle routing problem. Sets (2), (3) and (10) are the constraints of the assignment-like problem. Sets (5)–(9), (11)–(13) represent the constraints of the vehicle routing problem for each component type k. Set (9) is subtour elimination constraints where $L_{S(k)}$ represents an optimal solution of a one-dimensional bin packing problem where bins have length Q and each item to be packed in the bins has a weight of one unit. A detailed description of these constraints can be found in Gavish (1982), Laporte et al. (1985) and Fisher (1994). Set (4) determines the total travel distance (cost) of each component type.

Lemma 2. *Problem* (P1) *is NP-hard.*

Proof. Given the values of y_{kl} in (P1), the problem becomes a vehicle routing problem which is known to be NP-hard and is a restricted version of (P1). Hence, (P1) is NP-hard. □

Since the problem has a special structure, the formulation can be decomposed into two subproblems when constraint set (4) is relaxed. The corresponding Lagrangian variables are denoted by α_{kl} (unrestricted in sign). The resulting Lagrangian relaxation is presented as ($\mathrm{P1^{LR}}$).

$$(\mathrm{P1^{LR}}(\alpha))\colon \ \mathrm{Min}\ Z1^{\mathrm{LR}} = \sum_{k=1}^{K} \sum_{l \in L(k)} y_{kl}(z_{kl} + d_l) + \sum_{l \in L(k)} \sum_{k=1}^{K} \alpha_{kl} \left(- \sum_{j=L+1}^{n(k)+L} c_{lj}^k x_{lj}^k - \sum_{j=L+1}^{n(k)+L} c_{lj}^k x_{jl}^k - \sum_{i=L+1}^{n(k)+L-1} \sum_{j=i+1}^{n(k)+L} c_{ij}^k x_{ij}^k + z_{kl} \right), \tag{14}$$

s.t.

(2), (3), (5), (6), (7), (8), (9),

(10), (11), (12), (13)

where α_{kl} is unrestricted in sign. Recall that in a Lagrangian relaxation the formulation can be represented as $\max_\alpha \{Z1^{\mathrm{LR}}(\alpha)\} \leqslant Z1^*$ subject to suitable sign restrictions on α. In this case, $\boldsymbol{\alpha}$ is unrestricted in sign. α^*, which maximizes the Lagrangian relaxation, $Z1^{\mathrm{LR}}$, gives the tightest lower bound to the original problem (Fisher, 1981; Geoffrion, 1974). Furthermore, (1) is minimized when z_{kl} is equal to the sum of the distances traveled by the head, $\sum_{j=L+1}^{n(k)+L} c_{lj}^k x_{lj}^k + \sum_{j=L+1}^{n(k)+L} c_{lj}^k x_{jl}^k + \sum_{i=L+1}^{n(k)+L-1} \sum_{j=i+1}^{n(k)+L} c_{ij}^k x_{ij}^k$. When this is the case, the value of z_{kl} is equal to the optimal cost of the vehicle routing problem when component type k is assigned to location l, denoted as z_{kl}^{opt}. Constraint set (5) ties all the assignment and touring decisions. If constraint set (5) is enumerated by assigning each y_{kl} value to be equal to 1 and solving for the vehicle routing problem, one can obtain all z_{kl}^{opt} values.

3.1. The integrated algorithm

In this algorithm, we first solve a vehicle routing problem for each component type at every possible feeder location. The feasible solution obtained from the vehicle routing problem is used as the cost of assigning the component type to the particular feeder location in question. By using the feasible solution value as the cost of assigning component types to feeder locations, an assignment problem can be solved. The optimal solution of this assignment problem is a solution to the original problem.

Problem (P1) presents a special form. The variables of the Assignment-like subproblem, y_{kl}, and the variables of the Vehicle Routing-like subproblem, x_{ij}^k, are tied by constraint set (5). When this constraint set is enumerated by assigning each $y_{kl} = 1$ and solving a vehicle routing problem, we obtain the optimal sum of the distances traveled, z_{kl}^{opt}, when component type k is located in feeder location l. This special structure inspired the proposed algorithm that is developed for the original integrated problem (P1).

The integrated algorithm:
Step 1.
for each k {for each component type}
for $l \in L(k)$ {for each possible feeder location}
Find a solution for the vehicle routing problem when component type k is assigned to feeder location l.
$DP_{kl} =$ the cost of the vehicle routing problem
$DP_{kl} = z_{kl} + d_l$
(Note that d_l is a constant and does not depend on the touring decision)
Step 2. Solve the assignment problem by using DP_{kl} as the cost of assigning component type k to feeder location l.

$$\text{Min } \overline{Z1} = \sum_{k=1}^{K} \sum_{l \in L(k)} DP_{kl} y_{kl}, \tag{15}$$

s.t.

$$\sum_{k=1}^{K} y_{kl} \leqslant 1 \quad \forall l, \tag{2}$$

$$\sum_{l \in L(k)} y_{kl} = 1 \quad \forall k, \tag{3}$$

$$y_{kl} = (0/1) \quad \forall k, \forall l \in L(k). \tag{10}$$

Furthermore, constraint set (2) appears as an inequality in the original formulation. In order to make the number of component types equal to the number of feeder locations this constraint set is revised by adding dummy component types with $DP_{kl} = 0$. Now constraint set (2) can be written as an equality constraint. The resulting formulation is the following assignment problem:

$$\text{Min } \overline{Z1} = \sum_{k=1}^{K} \sum_{l \in L(k)} DP_{kl} y_{kl}, \tag{16}$$

s.t.

$$\sum_{k=1}^{K} y_{kl} = 1 \quad \forall l, \tag{2'}$$

$$\sum_{l \in L(k)} y_{kl} = 1 \quad \forall k, \tag{3}$$

$$y_{kl} = (0/1) \quad \forall k, \forall l \in L(k). \tag{10}$$

We can solve this assignment problem by using an efficient assignment code.

The following theorem shows that if the Vehicle Routing Problem is solved optimally and the cost is used as the cost of assigning component type k to feeder location l, DP_{kl}, then the proposed integrated algorithm finds the optimal solution for the original problem.

Theorem 3. *In the case where the feeder locator does not move, assignment problem with cost parameters equal to z_{kl}^{opt} (optimal Vehicle Routing Problem cost when component type k is assigned to the feeder location l) solves the original problem (P1) to optimality.*

Proof. Consider formulation (P1) and relax constraint set (4) multiplying it by vector $\boldsymbol{\alpha}$ ($\boldsymbol{\alpha}$ is unrestricted in sign). The problem is composed of two subproblems: one Assignment-like in variables, y_{kl}, and one Vehicle Routing-like in variables x_{ij}^{k}. The variables of these two subproblems are tied by constraint set (5). When each k and l combination of component types and feeder locations are assigned $y_{kl} = 1$, a Vehicle Routing-like problem can be solved. This provides the optimal cost of the vehicle routing problem, z_{kl}^{opt}, for component type k located in feeder l. When z_{kl}^{opt} is used in the assignment-like problem without constraint set (5), the optimal solution for the original problem (P1) can be obtained. □

The following corollary carries the similar result for the original PCB problem where the feeder moves.

Corollary 4. *In the case where the feeder locator moves, optimal cost of the minimum tour Vehicle Routing Problem for component type k solves the original PCB problem to optimality.*

Proof. The proof follows from Lemma 1. Since the problem can be reduced to independent vehicle routing problems, the optimal value of the PCB problem becomes the sum of the optimal costs of vehicle routing problems solved for each component type k. Therefore, the sum of optimal minimum tour Vehicle Routing Problem for each component type k provides the optimal solution for the PCB problem. □

The Vehicle Routing Problem is an NP-hard problem, and may not be solved optimally for large-scale versions. We now show that if the feasible solution generated for the Vehicle Routing Problem with an error of ε is used as the cost of assigning a component type to a feeder, then the original problem (P1) has an error bound of ε as well. This is important since there are algorithms that have small error bounds. For example, Fisher (1994) empirically presents a heuristic for the Vehicle Routing Problem with an approximate error of 1%. If his algorithm is used to develop the feasible vehicle routing problem cost values, z_{kl}^{f}, with a similar performance then as shown in the following theorem, the percentage error of the integrated algorithm will also be within one percentage of the optimal solution.

Theorem 5. *In the case where the feeder locator does not move, suppose there exists z_{kl}^{f} (feasible solution cost of the vehicle routing problem when component type k is assigned to feeder location l) where $z_{kl}^{f} - z_{kl}^{\mathrm{opt}}/z_{kl}^{\mathrm{opt}} \leqslant \varepsilon$ (z_{kl}^{opt} is the optimal Vehicle Routing Problem cost when component type k is assigned to feeder location l) for every component type k and feeder location l. If these z_{kl}^{f} values are used in the integrated algorithm, then the error for the integrated algorithm for (P1) is less than or equal to ε as well, i.e. $Z1^{f} - Z1^{*}/Z1^{*} \leqslant \varepsilon$ ($Z1^{f}$ is the value of the feasible solution generated by using z_{kl}^{f} values and $Z1^{*}$ is the optimal solution value to the integrated problem).*

Proof. Let us consider the optimal solution for the Assignment Problem:

$$Z1^{*} = \mathrm{Min} \quad \sum_{k=1}^{K} \sum_{l \in L(k)} y_{kl} * z_{kl}^{\mathrm{opt}}$$

s.t.

$$\sum_{k=1}^{K} y_{kl} = 1 \quad \forall l,$$

$$\sum_{l \in L(k)} y_{kl} = 1 \quad \forall k,$$

$$y_{kl} = (0/1) \quad \forall k, \forall l \in L(k).$$

In the optimal solution, there will be only K of y_{kl}'s which are equal to 1. Then, $Z1^{*} = y_{1l}^{*} * z_{1l}^{\mathrm{opt}} + y_{2l}^{*} * z_{2l}^{\mathrm{opt}} + \cdots + y_{Kl}^{*} * z_{Kl}^{\mathrm{opt}}$. Denote the optimal vector of y_{kl}'s with: $y_{kl}^{\mathrm{OPT}} = (y_{1l}^{*}, y_{2l}^{*}, \ldots, y_{Kl}^{*})$. Now, consider the solution for the Assignment Problem by using the heuristic solution of the Vehicle Routing Problem. When the combination of $y_{kl}^{\mathrm{OPT}} = (y_{1l}^{*}, y_{2l}^{*}, \ldots, y_{Kl}^{*})$ assignments (assignment of component types to feeders) is used, it will generate a feasible solution for the integrated algorithm assignment. The cost of this feasible solution, however, will be calculated by using the heuristic costs of the Vehicle Routing Problem. The total assignment cost, $Z^{AP,F} = y_{1l}^{*} * z_{1l}^{\mathrm{f}} + y_{2l}^{*} * z_{2l}^{\mathrm{f}} + \cdots + y_{Kl}^{*} * z_{Kl}^{\mathrm{f}}$, of this combination will be greater than or equal to the minimum cost by using the heuristic solutions of the Vehicle Routing Problem, $Z1^{\mathrm{f}}$.

$$Z1^{\mathrm{f}} = \mathrm{Min} \quad \sum_{k=1}^{K} \sum_{l \in L(k)} y_{kl} * z_{kl}^{\mathrm{f}}$$

s.t.

$$\sum_{k=1}^{K} y_{kl} = 1 \quad \forall l$$

$$\sum_{l \in L(k)} y_{kl} = 1 \quad \forall k$$

$$y_{kl} = (0/1) \quad \forall k, \forall l \in L(k).$$

Calculate the objective function of the Assignment Problem for the combination y_{kl}^{OPT} since it is also a feasible combination for the integrated algorithm. Then,

$$\begin{aligned}
Z^{AP,F} &= y_{1l}^{*} * z_{1l}^{\mathrm{f}} + y_{2l}^{*} * z_{2l}^{\mathrm{f}} + \cdots + y_{Kl}^{*} * z_{Kl}^{\mathrm{f}}, \\
Z^{AP,F} &\leqslant y_{1l}^{*} * \left(z_{1l}^{\mathrm{opt}} + \varepsilon * z_{1l}^{\mathrm{opt}}\right) + y_{2l}^{*} * \left(z_{2l}^{\mathrm{opt}} + \varepsilon * z_{2l}^{\mathrm{opt}}\right) \\
&\quad + \cdots + y_{Kl}^{*} * \left(z_{Kl}^{\mathrm{opt}} + \varepsilon * z_{Kl}^{\mathrm{opt}}\right), \\
Z^{AP,F} &\leqslant \left(y_{1l}^{*} * z_{1l}^{\mathrm{opt}} + y_{2l}^{*} * z_{2l}^{\mathrm{opt}} + \cdots + y_{Kl}^{*} * z_{Kl}^{\mathrm{opt}}\right) \\
&\quad + \varepsilon * \left(y_{1l}^{*} * z_{1l}^{\mathrm{opt}} + y_{2l}^{*} * z_{2l}^{\mathrm{opt}} + \cdots + y_{Kl}^{*} * z_{Kl}^{\mathrm{opt}}\right), \\
Z^{AP,F} &\leqslant Z1^{*} + \varepsilon * Z1^{*}.
\end{aligned}$$

We also know that the optimal solution of the Integrated Algorithm Assignment Problem, $Z1^{\mathrm{f}}$, will be less than or equal to the cost of this combination.

$$\begin{aligned}
Z1^{\mathrm{f}} &\leqslant Z^{AP,F}, \\
Z1^{\mathrm{f}} &\leqslant Z1^{*} + \varepsilon * Z1^{*}, \\
Z1^{\mathrm{f}} &\leqslant Z1^{*} * (1 + \varepsilon), \\
\frac{Z1^{\mathrm{f}}}{Z1^{*}} &\leqslant 1 + \varepsilon, \\
\frac{Z1^{\mathrm{f}} - Z1^{*}}{Z1^{*}} &\leqslant \varepsilon. \qquad \square
\end{aligned}$$

A similar result follows for the original problem where the feeder moves and is shown by the following corollary.

Corollary 6. *In the case where the feeder locator moves, suppose there exists* z_k^{f} *(feasible solution of the minimum tour vehicle routing problem for component type* k*) where* $z_{\mathrm{k}}^{\mathrm{f}} - z_k^{\mathrm{opt}}/z_k^{\mathrm{opt}} \leqslant \varepsilon$ *(*z_k^{opt} *is the optimal minimum tour Vehicle Routing Problem cost for component type* k*) for every component type* k*. If these* z_k^{f} *values are used in the original problem solution, then the error guarantee for this solution is less than or equal to* ε *as well, i.e.* $Z1^{\mathrm{f}} - Z1^{*}/Z1^{*} \leqslant \varepsilon$ *(*$Z1^{\mathrm{f}}$ *is the value of the feasible solution generated by using* z_k^{f} *values and* $Z1^{*}$ *is the optimal solution value to the original problem where the feeder moves).*

Proof. Proof follows from Lemma 1. The optimal cost of the original problem can be written as $Z1^{*} = \sum_{k=1}^{K} z_k^{\mathrm{opt}}$ whereas the cost of the feasible solution is

$$\begin{aligned}
Z1^{\mathrm{f}} \leqslant \sum_{k=1}^{K} z_k^{\mathrm{f}} &= \sum_{k=1}^{K} \left(z_k^{\mathrm{opt}} + \varepsilon * z_k^{\mathrm{opt}}\right) \\
&= \sum_{k=1}^{K} z_k^{\mathrm{opt}} + \sum_{k=1}^{K} \left(\varepsilon * z_k^{\mathrm{opt}}\right) \\
&= \sum_{k=1}^{K} z_k^{\mathrm{opt}} + \varepsilon \sum_{k=1}^{K} z_k^{\mathrm{opt}}.
\end{aligned}$$

Therefore

$$Z1^{\mathrm{f}} - Z1^{*}/Z1^{*} \leqslant \varepsilon \sum_{k=1}^{K} z_k^{\mathrm{opt}} \Big/ \sum_{k=1}^{K} z_k^{\mathrm{opt}} = \varepsilon. \qquad \square$$

Fisher (1994) obtained solutions for the vehicle routing problem with errors around 1%. If we implemented his non-polynomial branch and bound algorithm to develop the feasible solution for the vehicle routing problem for our case and if we could obtain similar errors, then our integrated algorithm would also have errors around 1%. However, if a polynomial algorithm such as the one in Altinkemer and Gavish (1990) is used, then a worst case error guarantee for the integrated algorithm can be obtained as shown in Remark 1. Altinkemer and Gavish (1990) partition the vehicle routing problem with unit weight into Q-clusters where the vehicle visits only Q points in one tour and define the Q-iterated tour partitioning algorithm. The computational complexity of the Q-iterated tour partitioning algorithm is $O(N^3)$.

Remark 1. If Altinkemer and Gavish's (1990) polynomial algorithm (Q-iterated tour partitioning algorithm) is used to develop feasible solutions for the vehicle routing problem, then the worst case error guarantee of the integrated algorithm for the PCB problem is

$$\frac{Z1^{\mathrm{f}} - Z1^{*}}{Z1^{*}} \leqslant \frac{5}{2} - \frac{3}{2Q}$$

where Q is the capacity of the rotary head and $Q \geqslant 3$.

There is a big difference between the empirical and the worst-case-error bounds because in establishing worst-case-error guarantee only simple algorithms could be analyzed. More complex algorithms produce better solutions, however, can not be analyzed in terms of worst-case error behavior.

4. Computational experiments

In this section, we present the computational experiments that show the benefits of using the integrated approach. We start with describing a typical PCB used in these experiments. The PCB is rectangular with dimensions of 500 units of length and 100 units of width. In order to represent the real characteristics of a PCB, some of the components are allocated on a line (vertical, horizontal, diagonal, and circular), some of them are randomly distributed on the free space, and the rest are placed on all the points in a square and in a rectangle. Four representative PCB types are used in these experiments and the number of components for component types are shown in Table 1. It should be noticed that these PCBs are fairly large as the total number of components mounted on the PCB is relatively high.

Table 2 presents the computational results for the proposed integrated approach. The percentage error gap is calculated as the difference between the costs of the integrated algorithm solution and the lower bound divided by the value of the lower bound:

$$\text{Percentage Error Gap} = 100\% * \frac{\overline{Z1} - Z1^{\mathrm{LR}}}{Z1^{\mathrm{LR}}},$$

Table 1
The list and number of the component types for four different PCB types

Component type	No. of components			
	PCB 1	PCB 2	PCB 3	PCB 4
1	20	16	12	8
2	24	12	6	6
3	16	12	8	8
4	16	12	8	8
5	8	8	8	8
6	24	24	16	8
7	24	16	16	16
8	8	8	8	8
Total	140	108	82	70

where $\overline{Z1}$ is the value obtained by using the integrated algorithm and $Z1^{\mathrm{LR}}$ is the lower bound for the PCB problem. The lower bound, $Z1^{\mathrm{LR}}$, is generated by using the approach presented in Altinkemer and Gavish (1991).

Two main results are presented through these computational experiments. The first result shows the performance of the integrated approach. The average percentage error gap ranges between 1.70% (for smaller PCBs) and 6.87% (for larger PCBs). The maximum percentage error gap is recorded as 7.11% whereas the minimum percentage error gap is 1.54%. These results are relatively good given the sizes of the problems considered. Furthermore, these percentage error gaps are the first of its kind and provide a basis for comparison for future algorithms. The second result illustrates our finding in Theorem 5. The percentage error gap of individual vehicle routing problems vary up to 43% whereas the percentage error gaps for the PCB problem are much smaller as presented in Table 2. Furthermore, the feasible solutions and the lower bounds of the vehicle routing problems are generated by using the algorithm in Altinkemer and Gavish (1991). Fisher (1994) presents better empirical results for the vehicle routing problem. We conjecture that if Fisher's (1994) algorithm were used to generate the feasible solutions for the vehicle routing problems with a similar performance, the results for the PCB problem could have been improved even more.

Table 2
The experimental results comparing the integrated formulation with random assignment

	Lower bound	Integrated assignment	%Error gap
PCB1			
Exp.			
1	3250	3465	6.62
2	3241	3456	6.63
3	3053	3268	7.04
4	3091	3306	6.96
5	3024	3239	7.11
Min.			6.62
Avg.			6.87
Max.			7.11
PCB2			
1	3048	3200	4.99
2	2812	2964	5.41
3	2739	2891	5.55
4	2881	3033	5.27
5	2950	3102	5.15
Min.			4.99
Avg.			5.27
Max.			5.55
PCB3			
1	2328	2427	4.25
2	2463	2562	4.02
3	2456	2555	4.03
4	2418	2517	4.09
5	2467	2566	4.01
Min.			4.01
Avg.			4.08
Max.			4.25
PCB4			
1	2520	2561	1.63
2	2668	2709	1.54
3	2365	2406	1.73
4	2263	2304	1.81
5	2306	2347	1.78
Min.			1.54
Avg.			1.70
Max.			1.81

5. Conclusions and future extensions

This paper describes an optimization problem that has significant implications for the productivity of printed circuit board manufacturing using Surface Mount Technology with a rotary head. The problem presents a series of optimization subproblems which need to be addressed simultaneously in order to realize system-wide improvements. Since the overall problem is NP-hard, it is unlikely that optimal solutions can be obtained in polynomial time. Our proposed approach simultaneously accounts for the problems of component assignment to feeders and the sequencing of placement. In addition, it considers minimizing the head movement.

We show that if the optimal vehicle routing problem costs are known, the optimal PCB solution can be obtained by using our integrated algorithm. Since the vehicle routing problem is an NP-hard problem, it may not be easy to obtain optimal vehicle routing problem costs. We show that our integrated algorithm provides a feasible solution with an error gap less than or equal to the *maximum* error gap of the vehicle routing problem costs. If all the error gaps of the vehicle routing problems are equal to ε, then our integrated approach has an ε-error guarantee. Otherwise, the error gap of the integrated algorithm for the PCB problem is determined by the vehicle routing problem that has the largest error gap. One can argue that solving a vehicle routing problem for each possible pair of component types and feeder locations may be a lengthy process. However, this can be justified considering that it is solved only once for a PCB type which is produced in large amounts. The proposed integrated algorithm does not call for a repetitive and extensive effort on the part of the manufacturer.

This paper focuses on one type of machine, a surface mount technology CNC with a rotary head, therefore, the findings may not apply for every CNC. However, it opens future directions to apply optimization methods and algorithms to implement in other types of PCB manufacturing machines. The algorithm proposed in this paper provides a benchmark for future algorithms. Further refinements on this research include two issues: partitioning the components of the same type into smaller groups and the examination of the non-linear time duration when the head moves from one point to another. The first refinement corresponds to relaxing the second assumption and is being studied in Kazaz and Altinkemer (1999). In this work, all components of the same type are assigned to the same feeder location and a linear travel time (in distance) exists between the points. Partitioning combined with a non-linear acceleration/deceleration for the head movement could result in better estimates.

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