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Global Production Planning Under Exchange-Rate Uncertainty

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Motivated by an aggregate production-planning problem in an actual global manufacturing network, we examine the impact of exchange-rate uncertainty on the choice of optimal production policies when the allocation decision can be deferred until the realization of exchange rates. This leads to the formulation of the problem as a two-stage recourse program whose optimal policy structure features two forms of flexibility denoted as operational hedging: (1) *production hedging*, where the firm deliberately produces less than the total demand; and (2) *allocation hedging*, where due to unfavorable exchange rates, some markets are not served despite having unused production. Our characterization of the optimal policy structure leads to an economic valuation of production and allocation hedging. We show that the prevalence of production hedging is moderated by the degree of correlation between exchange rates. A comprehensive examination under the following four generalized settings provides the depth, scope, and relevancy that our proposed operational hedges play to facilitate aggregate planning: (1) multiple periods, (2) demand uncertainty, (3) price setting or monopolistic pricing, and (4) price setting under demand uncertainty. We show that production and allocation hedging are robust for these generalizations and should be integrated into the overall aggregate planning strategy of a global manufacturing firm.

Key words: exchange-rate uncertainty; international operations management; production planning; production hedging; allocation hedging; price setting
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1. Introduction

This study is motivated by a production-planning problem regularly faced by a global electronics manufacturer, and common to many multinational companies. The component manufacturing division of the company periodically receives the projected demand from the company's personal computer (PC) division as well as other original equipment manufacturers (OEMs). These projections are used in the quarterly aggregate plan to determine how much to manufacture of each product in each of its plants to maximize expected profit. Along with setting the production plan, the company allocates budgets and sets transfer prices for sales to all countries. Consequently, the revenue of the manufacturing division is subject to financial risks from currency conversion due to exchange-rate fluctuations.

The conventional approach as detailed in Flaherty's (1996) case, Applichem (A), is to replace the parameters that are influenced by exchange-rate fluctuations by their expected values. A one-period aggregate production-planning problem is then solved in much the same manner as when the problem setting is domestic. Due to production and allocation decisions being made before the realization of exchange rates, this approach has the limitation of foregoing significant operational opportunities to manage the adverse effect of exchange-rate fluctuations. This is because some exchange-rate realizations are known before an allocation decision is made.

In this paper, we develop a comprehensive approach to the production-planning problem that is tailored to respond optimally to the change in the information structure that is a result of deferring or postponing the allocation decision until after the realization of the spot exchange rates. To see how it can improve operational decisions, consider the implication on the allocation for a market that has an adverse fall in its spot exchange rate. When the revenue times the spot exchange rate is sufficiently low that it does not recoup transportation costs, then it might be better to underserve this market, resulting in unused production. This feature, which we term *allocation hedging*, is one of the two operational hedges that we incorporate in the two-stage recourse framework we develop for managing the production-planning function under exchange-rate uncertainty. Because it occurs after the realization of exchange rates, allocation hedging is a reactive strategy.

Our other operational hedge, which we term *production hedging*, is proactive because it is employed before the realization of exchange-rate uncertainty. Effectively, the production planner employs a production hedge if he or she chooses to produce less than the total demand indicated in the aggregate plan. When a production hedge is undertaken, correspondingly the decision maker has determined that there is enough exchange-rate uncertainty that it is economical to consciously choose to not fully serve all markets. As will be seen, these two operational hedging strategies are complementary. Together, they provide insight into the impact of exchange-rate uncertainty on the production-planning function, and lead to an evaluation of the economic cost of fully meeting all demand as dictated by conventional production-planning models.

After reviewing the related literature on production-planning models under exchange-rate uncertainty in §2, the core model and its extensions and generalizations are presented in §3. The objective of the core model is to maximize expected profit by deciding in the first stage of the problem how much to produce, while taking into account the expected contribution, over exchange-rate realizations, from the second (or recourse) stage of the problem. The objective of the second-stage problem is to allocate the production, which is now constrained, to each market.

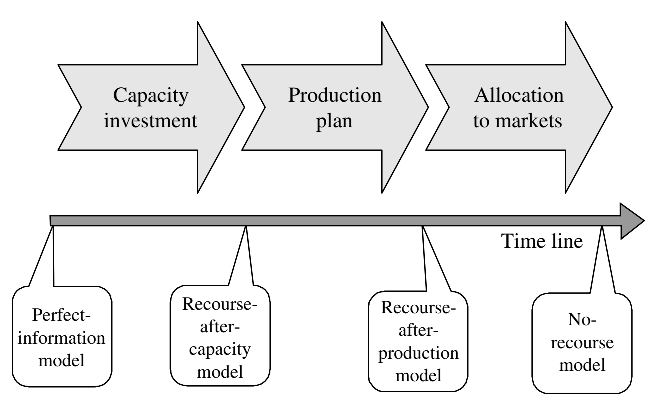
The optimal strategy for this core problem is fully characterized in §3.1. The economic value created by the proposed operational hedges is evaluated in this section. An analysis of correlated exchange rates when both markets are foreign is presented in §3.2. Finally, in §3.3 we show that production and allocation hedging continue to be features of an optimal policy in multiperiod settings. Allocation hedging, however, is more likely to be an optimal choice in a multiperiod model than in a single-period model, because unused products can be sold in future periods. This section not only captures the essential elements of our motivating aggregate production-planning problem, but also lays the foundation for subsequent generalizations of the model by endogenizing prices under demand uncertainty in §§4, 5, and 6.

These generalizations of our modeling framework illustrate the depth, scope, and richness for the aggregate production-planning strategy available in a global manufacturing network. In §4, we allow demand to be uncertain, which makes the resulting structure more complex. In the model of §4.1, demand uncertainty is resolved before the allocation decision, while in the model of §4.2, it is resolved after the allocation decision. In §5, we consider the case where demand is deterministic, but the decision maker is a monopolist who can set the price in each market, thereby incorporating into the core model the option of adjusting prices and, concomitantly, demand. In the model of §5.1, prices and production are determined simultaneously before the realization of exchange rates, while in the model of §5.2 pricing decisions are postponed until the resolution of exchange-rate uncertainty. Finally, in §6, we outline how our modeling framework can be molded to the domain in which the production and pricing decisions must be made knowing that demand is uncertain. In light of these analyses, we conclude that our key innovation, the identification and introduction of two types of operational hedges (i.e., production and allocation hedging), remains an integral and robust part of an optimal global production-planning policy for the generalizations considered in §§4, 5, and 6. Section 7 contains concluding remarks.

2. Literature Review

To put our two uses of operational hedges into perspective, consider the natural sequence of decisions in managing a manufacturing network: (1) invest in capacity, (2) manufacture, and (3) after manufacturing occurs, allocate production to meet demand. Exchange rates are known after either Steps 1, 2, or 3. This leads naturally to thinking of the problem as a stochastic program with recourse. Figure 1 shows a framework to classify modeling approaches for these conditions. For the global electronics manufacturer who motivated this study, exchange rates are realized after Step 2 so that the allocation decisions are made in the recourse stage of the problem. We will refer to this problem as the *recourse-after-production problem*. Obviously, if exchange rates are known prior to all three manufacturing steps, then we can obtain the value of such knowledge (viz. perfect information). Finally, in the conventional domestic analogue

Figure 1 Natural Sequence of Operational Decisions and Models That Have Different Timing of Exchange-Rate Realizations



of our problem, exchange rates are realized after Step 3. Thus, there is no opportunity for recourse, and uncertain exchange rates can be replaced by their expected values. As a general rule, the more decisions are deferred until exchange rates are realized, the higher the expected contribution. The modeling approach developed in this paper is sufficiently general to apply to all four situations outlined above. However, to highlight the motivating application, we focus on the case when exchange rates are realized between Steps 2 and 3.

Earlier models of operational hedging have addressed the long-term, more strategic issues of capacity investment and production switching. Huchzermeier and Cohen (1996), for example, viewed an operational hedge as flexibility to utilize a number of supply contract and production location options in a global supply chain. They employed a recourse-after-capacity model (viz. exchange rates are observed after capacity decisions are made) in selecting the design of the supply chain. Although Huchzermeier and Cohen (1996) did not provide structural results on the nature of the optimal policy, they presented numerical examples that show that operational hedging can reduce the downside risk of exchange rates in the longer term. These authors suggested that financial hedging be used to reduce the variability of the firm's cash flows in the short term. Using the same modeling approach, Huchzermeier (1998) and Lowe et al. (2002) evaluated the option value of operational flexibility in Applichem's network. Kamrad and Siddique (2004) revised the definition of the value of operational flexibility to incorporate the supplier's reactions in the effort to mitigate the risks stemming from the manufacturer's decision to alter order quantities under exchange-rate uncertainty. Rosenfield (1996) provided a complementary approach to that proposed by Huchzermeier and Cohen (1996). His production and capacity decisions were based on minimizing costs influenced by exchange-rate fluctuations. Aspects of Huchzermeier and Cohen's (1996) research may be viewed as extending the Kogut and Kulatilaka (1994) uncapacitated model of costly switching of production from one supplier to another in response to changing macroeconomic data. This switching strategy can also be interpreted as an operational hedge. Using a similar approach, Dasu and Li (1997) determined the production switchover among plants when costs were influenced by exchange rates. In a more recent paper, Kouvelis et al. (2001) prescribed the choice of an ownership structure (e.g., exporting, joint venture, and wholly owned subsidiary) for a set of production facilities under the presence of switchover costs.

The above models complement earlier research on incorporating exchange-rate uncertainty into the strategic decision of capacity investment. Jucker and Carlson (1976) were the first to incorporate exchange-rate uncertainty in the context of an uncapacitated plant-location problem. They considered a model in which there is demand and price uncertainty. Uncertainty in exchange rates was embedded in the uncertainty in prices because exchange rates were assumed to influence prices in local markets. In a series of papers using the notion of an exchange-rate mean-variance trade-off, this model was extended by Hodder and Jucker (1985a, b). The extension of this approach to include limited capacity was considered by Hodder and Dincer (1986). Hodder (1984) simplified the solution technique with a model that introduced a capital market (CAPM) approach with an exchange-rate mean-covariance objective function. These studies permitted other sources of uncertainties such as fluctuations in financial markets to reflect uncertainty. Our paper differs from these studies by providing the firm with the flexibility that it does not have to fulfill the entire market demand, while maximizing its expected profit.

The demand uncertainty generalization of our model resembles that of Kouvelis and Gutierrez (1997), where optimal production quantities and transfer prices within two markets are determined. The demand of each market occurs in nonoverlapping time periods in Kouvelis and Gutierrez (1997), while they occur simultaneously in our model. The impact of exchange-rate fluctuations was also investigated in the context of supplier relationships where order quantities and contract terms were determined based on exchange-rate uncertainty, as in Kouvelis (1999); and on exchange-rate and demand uncertainty as in Scheller-Wolf and Tayur (1999). Overall, our work differs from these studies first by featuring both production and allocation hedging; second, by showing that they hold under more general settings with demand uncertainty and endogenous prices.

3. The Model and Structural Results

This section presents an analysis of operational hedging strategies to manage exchange-rate uncertainty for a multinational company. We first introduce the two-stage stochastic program, then explore its optimal policy structure. One optimal solution suggests that under certain circumstances it is better to produce less than the total demand. As stated earlier, we call this "production hedging." Another key feature of the model is denoted as "allocation hedging," when it is beneficial for the firm not to serve a market completely or partially, notwithstanding unused production. When the impact of correlated exchange rates is investigated, production hedging becomes more beneficial under negatively correlated exchange rates. Finally, it is proven that the production- and allocation-hedging strategies are also relevant in a multiperiod setting.

3.1. The Core Model

While our structural results apply to more complex manufacturing networks, we formulate the core model at just sufficient generality to capture all the nuances explored in the analysis. In the first stage of the model, the decision maker must choose the production quantity X to meet demand of up to d_1 units in Market 1 and d_2 units in Market 2. Because the first stage of our two-stage stochastic program can be viewed as a contract to produce a certain output, the manufacturing costs are paid in advance and can be defined as c per unit in the currency of the home country. Then, exchange rates in Markets 1 and 2 are observed. The exchange rates are modeled as jointly distributed nonnegative random variables, and the case where one of the markets is domestic is also captured. Then, in the second stage of the model, the allocation decision is made, which is now constrained by the first-stage production decision, X. Based on the realized exchange rates, the output from Stage 1 is allocated to markets to maximize expected contribution (revenue less transportation costs) in Stage 2. Using the above sequence, the resulting formulation is a stochastic program with recourse. We use the following notation:

Parameters

j: index representing country of sale (or markets), $j = 1, 2$.

r_j: unit revenue from a sale in market j (in terms of the currency of country j).

c: unit cost of manufacturing in the currency of the home country.

t_j: unit transportation cost between the producing country and market $j = 1, 2$ (in terms of domestic currency).

ε_j: a random variable representing the exchange rate that converts foreign currency $j = 1, 2$ to home country currency ($\varepsilon_j \geq 0$).

(e_1, e_2): a set of random variables representing the realized values of exchange rates ε_1 and ε_2.

$f(e_1, e_2)$: the joint probability density function (pdf) of the realization of exchange rates ε_1 and ε_2.

$f_j(e_j)$: the marginal distribution of the exchange rate ε_j at its realized value e_j, $j = 1, 2$.

ρ: the correlation coefficient between the two exchange rates ε_1 and ε_2.

d_j: the demand in country (market) $j = 1, 2$.

Stage 1 Decision Variable

X: the amount of production at the manufacturing facility.

Stage 2 Decision Variables

X_j: the amount of products shipped for sale to country (market) $j = 1, 2$.

Using this notation, our problem is formulated as a mathematical program that maximizes the expected profit subject to transportation and demand constraints. The first-stage problem can be written as

$$\text{(P1):} \quad \max_{X \geq 0} E[P(X)] = -cX + E[PA(X \mid (e_1, e_2))], \quad (1)$$

where $E[PA(X \mid (e_1, e_2))]$ is the expected revenue obtained from producing X units of products in the first stage and $PA(X \mid (e_1, e_2))$ is the second-stage optimal solution value for given X units of production in Stage 1 and realized exchange rates of (e_1, e_2). $E[\cdot]$ is the expectation operator over $f(e_1, e_2)$.

The second-stage problem, allocation of production to markets, can be written as

$$PA(X \mid (e_1, e_2)) = \max_{(X_1, X_2 \geq 0)} (r_1 e_1 - t_1) X_1 + (r_2 e_2 - t_2) X_2 \quad (2)$$

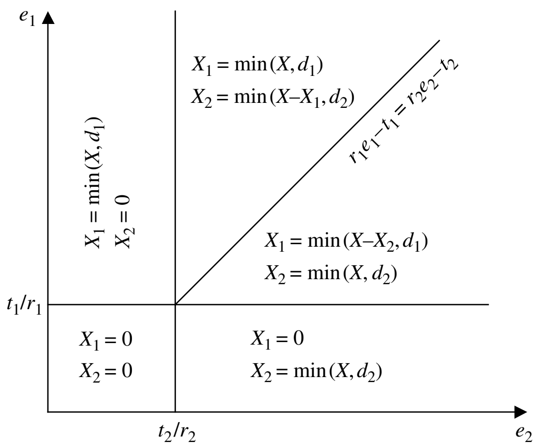
$$\text{s.t.} \quad X_j \leq d_j \quad \forall j = 1, 2, \quad (3)$$

$$X_1 + X_2 \leq X. \quad (4)$$

Compared to traditional aggregate production-planning models, this formulation provides two forms of increased operational flexibility that translate directly into increased expected profits. *Production hedging* is an admissible strategy because the production quantity X in the first stage can be less than the total demand $(d_1 + d_2)$. Under production hedging, at least one of the market constraints in (3) is loose, rather than tight as would be the case if all demand must be met. Analogously, *allocation hedging* is an admissible strategy because constraint (4) can be an inequality. The resulting flexibility can yield optimal policies that employ production or allocation hedging per se, or both simultaneously.

The above model incurs no penalty cost when a market is not entirely served under allocation hedging or when it is partially served if production hedging is employed. The model can be revised to address these concerns in at least two ways. The first is to observe that we can charge an ill-will or penalty cost for each unit that is underserved by incorporating an appropriate penalty cost in the objective function of the second-stage problem $PA(\cdot)$ for lost sales. In fact, it is straightforward to show that after simple algebraic manipulation the resulting problem would have an identical structure to the current formulation, except that each r_j is increased by a unit of ill-will cost so that it can be interpreted as the marginal cost of underage. A second approach is to add a set of constraints to problem $PA(\cdot)$ to ensure a minimum sales level for each market. Once again the ensuing problem would have the identical structure of the current formulation, except now each d_j must be interpreted as demand net of the minimum sales, and each X_j as the incremental allocation that increases sales above the minimum requirement. Therefore, the

Figure 2 Optimal Allocation Decisions in the Second Stage



above formulation is robust in its structural properties because it provides identical results to the models that incorporate either a penalty cost for unsatisfied demand or a constraint that introduces a minimum sales requirement.

The optimal allocation decisions in $PA(X \mid (e_1, e_2))$ can be determined precisely, as depicted in Figure 2. When e_1 and e_2 are sufficiently close to 0, i.e., $e_1 \le t_1/r_1$ and $e_2 \le t_2/r_2$, neither market is served, so that $X_1 = X_2 = 0$. When the value of the Market 1 exchange rate is ex post profitable, i.e., $e_1 > t_1/r_1$; while the Market 2 exchange rate is $e_2 \le t_2/r_2$, Market 1 is served as much as possible so that $X_1 = \min(X, d_1)$ and $X_2 = 0$. Alternatively, when Market 2 is ex post profitable, i.e., $e_2 > t_2/r_2$ while $e_1 \le t_1/r_1$; only Market 2 is served and $X_1 = 0$ and $X_2 = \min(X, d_2)$. Next, we consider the case when both markets are ex post profitable, i.e., $e_1 > t_1/r_1$ and $e_2 > t_2/r_2$. In this situation, when e_2 takes intermediate values, $t_2/r_2 < e_2 < (r_1e_1 - t_1 + t_2)/r_2$, that generate lower ex post returns than that of Market 1, Market 1 receives priority, so $X_1 = \min(X, d_1)$ and $X_2 = \min(X - X_1, d_2)$. Finally, when e_2 is large, $e_2 > (r_1e_1 - t_1 + t_2)/r_2$, Market 2 has priority over Market 1, so that $X_2 = \min(X, d_2)$ and $X_1 = \min(X - X_2, d_1)$.

To develop the optimal policy structure for the first-stage problem, it should be observed that $PA(X \mid (e_1, e_2))$ is the value of the optimal solution to a linear program for a given production quantity of X and the realized values of e_1 and e_2. Hence, $PA(X \mid (e_1, e_2))$ is piecewise linear and concave in X, with break points located at $X = \min(d_1, d_2)$, $X = \max(d_1, d_2)$, and $X = (d_1 + d_2)$. Given the above optimal allocation scheme, we can compute $\lambda(X \mid (e_1, e_2))$, the incremental or marginal value of increasing production by one more unit for given values of X and exchange-rate realizations of e_1 and e_2. Moreover, $\lambda(X \mid (e_1, e_2))$ may also be interpreted as the value of the Lagrangean multiplier associated with constraint (4). Furthermore, for a given X, $\lambda(X \mid (e_1, e_2))$ is increasing in (e_1, e_2). Subscript i is used to represent the different intervals of X values: $i = A$ when $0 < X < \min(d_1, d_2)$; $i = B1$ when $d_2 < X < d_1$ and, $i = B2$ when $d_1 < X < d_2$; $i = C$ when $\max(d_1, d_2) < X < (d_1 + d_2)$; and $i = D$ when $X > (d_1 + d_2)$. Using the values of $\lambda_i(X \mid (e_1, e_2))$ in each interval, the expected value of producing one more unit, defined by $E(\lambda_i(X \mid (e_1, e_2)))$ over $f(e_1, e_2)$, are as follows:

$$E(\lambda_A(X)) = \int_{t_1/r_1}^{\infty} \int_0^{(r_1e_1 - t_1 + t_2)/r_2} (r_1e_1 - t_1) f(e_1, e_2)\, de_2\, de_1 + \int_{t_2/r_2}^{\infty} \int_0^{(r_2e_2 - t_2 + t_1)/r_1} (r_2e_2 - t_2) f(e_1, e_2)\, de_1\, de_2, \quad (5)$$

$$E(\lambda_{B1}(X)) = \int_{t_1/r_1}^{\infty} (r_1e_1 - t_1) f_1(e_1)\, de_1, \quad (6)$$

$$E(\lambda_{B2}(X)) = \int_{t_2/r_2}^{\infty} (r_2e_2 - t_2) f_2(e_2)\, de_2, \quad (7)$$

$$E(\lambda_C(X)) = \int_{t_1/r_1}^{\infty} \int_{t_2/r_2}^{(r_1e_1 - t_1 + t_2)/r_2} (r_2e_2 - t_2) f(e_1, e_2)\, de_2\, de_1 + \int_{t_1/r_1}^{\infty} \int_{(r_1e_1 - t_1 + t_2)/r_2}^{\infty} (r_1e_1 - t_1) f(e_1, e_2)\, de_2\, de_1, \quad (8)$$

$$E(\lambda_D(X)) = 0. \quad (9)$$

Remark 1.

$$E(\lambda_A(X)) > \begin{Bmatrix} E(\lambda_{B1}(X)) \\ E(\lambda_{B2}(X)) \end{Bmatrix} > E(\lambda_C(X)) > E(\lambda_D(X)) = 0.$$

The above remark utilizes $E(\lambda_{B1}(X))$ when $d_1 \ge d_2$, and $E(\lambda_{B2}(X))$ when $d_2 > d_1$. Because these cases are mutually exclusive, it uses either $E(\lambda_{B1}(X))$ or $E(\lambda_{B2}(X))$. This complete ordering by comparing $E(\lambda_i(X))$ against c allows us to state the following theorem.

Theorem 1. *The optimal production quantity X in Stage 1 is either 0, d_1, d_2, or $(d_1 + d_2)$. In an optimal solution*: (a) $X = 0$ *if and only if* $E(\lambda_A) < c$; (b) $X = d_2$ *if and only if* $d_1 > d_2$ *and* $E(\lambda_A) > c > E(\lambda_{B1})$, *and* $X = d_1$ *if and only if* $d_2 > d_1$ *and* $E(\lambda_A) > c > E(\lambda_{B2})$; (c) $X = d_1$ *if and only if* $d_1 > d_2$ *and* $E(\lambda_{B1}) > c > E(\lambda_C)$, *and* $X = d_2$ *if and only if* $d_2 > d_1$ *and* $E(\lambda_{B2}) > c > E(\lambda_C)$; *otherwise*, (d) $X = (d_1 + d_2)$ *if and only if* $E(\lambda_C) > c$.

The above characterization of $E[PA(X)]$ and the corresponding marginal profit perspective is useful in interpreting the optimal production policy. It can be observed that $E(\lambda_A) > c$ eliminates $X = 0$ from optimality, and moves the potentially optimal solution to $X = \min(d_1, d_2)$. When $d_1 > d_2$ and $E(\lambda_{B1}) > c$, the potentially optimal solution moves to $X = \max(d_1, d_2) = d_1$ (alternatively, when $d_2 > d_1$ and $E(\lambda_{B2}) > c$, the potentially optimal solution moves to $X = \max(d_1, d_2) = d_2$), and $X = \min(d_1, d_2)$ is eliminated from consideration. Finally, $E(\lambda_C) > c$ eliminates $X = \max(d_1, d_2)$ from optimality, and moves the potentially optimal solution to $X = (d_1 + d_2)$.

Figure 2 facilitates the computation of $E(\lambda_i)$ and, moreover, leads to a direct computation of $EVAH(X)$, the expected value of allocation hedging for any choice of X. As a consequence of Theorem 1, we need only consider four candidate solutions, $X=0$, $X=d_1$, $X=d_2$, and $X=(d_1+d_2)$. Direct computation shows that these are given by

$$EVAH(X=(d_1+d_2)) = \int_0^{t_1/r_1}(t_1-r_1e_1)d_1f_1(e_1)\,de_1 + \int_0^{t_2/r_2}(t_2-r_2e_2)d_2f_2(e_2)\,de_2,$$

$$\begin{aligned}
&EVAH(X=\max(d_1,d_2)=d_1\ge d_2)\\
&\quad=\int_{\max(0,(t_2-t_1)/r_2)}^{t_2/r_2}\int_{\max(0,(t_1-t_2)/r_1)}^{(r_2e_2-t_2+t_1)/r_1}\big[(t_1-r_1e_1)(d_1-d_2)\\
&\qquad\qquad+(t_2-r_2e_2)d_2\big]f(e_1,e_2)\,de_1\,de_2\\
&\qquad+\int_{\max(0,(t_1-t_2)/r_1)}^{t_1/r_1}\int_{\max(0,(t_2-t_1)/r_2)}^{(r_1e_1-t_1+t_2)/r_2}(t_1-r_1e_1)d_1\\
&\qquad\qquad\cdot f(e_1,e_2)\,de_2\,de_1\\
&\qquad+\int_0^{t_1/r_1}\int_{t_2/r_2}^{\infty}(t_1-r_1e_1)(d_1-d_2)f(e_1,e_2)\,de_2\,de_1,
\end{aligned}$$

$$\begin{aligned}
&EVAH(X=\max(d_1,d_2)=d_2> d_1)\\
&\quad=\int_{\max(0,(t_1-t_2)/r_1)}^{t_1/r_1}\int_{\max(0,(t_2-t_1)/r_2)}^{(r_1e_1-t_1+t_2)/r_2}\big[(t_1-r_1e_1)d_1\\
&\qquad\qquad+(t_2-r_2e_2)(d_2-d_1)\big]f(e_1,e_2)\,de_2\,de_1\\
&\qquad+\int_{\max(0,(t_2-t_1)/r_2)}^{t_2/r_2}\int_{\max(0,(t_1-t_2)/r_1)}^{(r_2e_2-t_2+t_1)/r_1}(t_2-r_2e_2)d_2\\
&\qquad\qquad\cdot f(e_1,e_2)\,de_1\,de_2\\
&\qquad+\int_{t_1/r_1}^{\infty}\int_0^{t_2/r_2}(t_2-r_2e_2)(d_2-d_1)f(e_1,e_2)\,de_2\,de_1,
\end{aligned}$$

$$\begin{aligned}
&EVAH(X=\min(d_1,d_2))\\
&\quad=\int_0^{t_1/r_1}\int_0^{t_2/r_2}\big[(t_1-r_1e_1)\min(d_1,d_2)\\
&\qquad\qquad+(t_2-r_2e_2)\min(d_1,d_2)\big]f(e_1,e_2)\,de_2\,de_1,\\
&\qquad EVAH(X=0)=0.
\end{aligned}$$

It is now possible to infer that when it is optimal to have $X=0$, the expected value of allocation hedging would be zero, and when it is optimal to produce (d_1+d_2), its value is maximum, as there is no need to ever allocate more than this amount. The values for the other cases are intermediate, as captured in the following remark.

Remark 2.

$$\begin{aligned}
EVAH(X=0)&\le EVAH(X=\min(d_1,d_2))\\
&\le EVAH(X=\max(d_1,d_2))\\
&\le EVAH(X=(d_1+d_2)).
\end{aligned}$$

Let X^* be the optimal solution to the recourse problem, and $E[P(X^*)]$ its corresponding expected profit. Then, we can immediately infer that $EVAH(X^*)$ is the portion of $E[P(X^*)]$ that can be attributed directly to allocation hedging in isolation. While the expected value of allocation hedging can be computed directly, it is inappropriate to interpret the residual as the value of production hedging due to the possibility that there can be an interaction between allocation and production hedging when X^* is less than (d_1+d_2). The reason for this is that, for some realizations of e_1 and e_2, both markets are profitable but cannot be fully served. However, we can provide bounds for production hedging relative to a benchmark. Because our key innovation is to introduce allocation and production hedging into the conventional or no-recourse variant of the model, we use the model with no recourse as the appropriate benchmark. By definition, in the no-recourse problem, (d_1+d_2) units are produced and shipped, so its expected value, denoted by $E[P_{nr}]$, is $E[P_{nr}]=(r_1\bar{e}_1-t_1-c)d_1-(r_2\bar{e}_2-t_2-c)d_2$, where $\bar{e}_j$, $j=1,2$, is the mean value of the exchange-rate random variable. It then follows that relative to this benchmark, the value of the recourse problem is $E[P(X^*)]-E[P_{nr}]$, of which $EVAH(X^*)$ is the direct value of allocation hedging. Let $EVPH(X^*)$ be the value of production hedging and $EVPAH(X^*)$ the expected value of the interaction effect. This yields the basic identity

$$\begin{aligned}
&EVPH(X^*)+EVAH(X^*)+EVPAH(X^*)\\
&\quad=E[P(X^*)]-E[P_{nr}].
\end{aligned}$$

This yields an upper bound for $EVPH(X^*)$, because

$$\begin{aligned}
&EVPH(X^*)\\
&\quad=E[P(X^*)]-E[P_{nr}]-EVAH(X^*)-EVPAH(X^*)\\
&\quad\le E[P(X^*)]-E[P_{nr}]-EVAH(X^*).
\end{aligned}$$

It should be observed that when $X=(d_1+d_2)$, there is no production hedging, so that $EVPAH(X^*)$ is zero. This provides the following lower bound:

$$EVPH(X^*)\ge E[P(X^*)]-E[P_{nr}]-EVAH(X=(d_1+d_2)).$$

To complete the development, note that because both markets are fully served in the no-recourse case, each must be ex ante profitable so that $r_j\bar{e}_j-t_j>c$. Direct computation shows that $r_1\bar{e}_1-t_1\le E(\lambda_{B1}(X))$ and $r_2\bar{e}_2-t_2\le E(\lambda_{B2}(X))$ in Problem (P1). This provides the following corollary.

Corollary 1. *When $r_j\bar{e}_j-t_j>c$ is satisfied for each market, $j=1,2$, then the optimal production quantity X in Stage 1 of Problem* (P1) *is either* $\max(d_1,d_2)$ *or* (d_1+d_2).

Hence, we can compute that if $X^* = (d_1 + d_2)$, $E[P(X^*)] - E[P_{nr}] = EVAH(X = (d_1 + d_2))$, leading us to reach the logical conclusion that $EVPH(X^*) = EVPAH(X^*) = 0$. Alternatively, if $X^* = \max(d_1, d_2)$,

$$\begin{aligned} 0 &\le (c - E(\lambda_C))\min(d_1, d_2) \le EVPH(X = \max(d_1, d_2)) \\ &\le \bigg(c - E(\lambda_C) + \int_{t_1/r_1}^{\infty}\int_0^{t_2/r_2}(t_2 - r_2 e_2) f(e_1, e_2)\,de_2\,de_1 \\ &\quad + \int_0^{t_1/r_1}\int_{t_2/r_2}^{\infty}(t_1 - r_1 e_1) f(e_1, e_2)\,de_2\,de_1\bigg)\min(d_1, d_2). \end{aligned}$$

When the optimal policy is $X^* = \max(d_1, d_2)$, the economic value of production hedging is strictly positive because Theorem 1(c) holds, so $E(\lambda_C) < c$, leading to a positive lower bound. Furthermore, the two integral terms of the upper-bound expression are positive in their respective regions making the upper bound positive and greater than the lower bound. These two integral terms of the upper bound can be interpreted as the residual of the interaction effect. Now that we have shown that the optimal policy structure features production and allocation hedging, and that these two operational hedges create value for the firm, we next investigate the impact of the correlation between exchange rates on these operational hedges.

3.2. Impact of Correlated Foreign Exchange Rates

We now focus on examining how the choice between producing the maximum of (d_1, d_2) and $(d_1 + d_2)$ is influenced by the degree of correlation between ε_1 and ε_2. Consider the case where ε_1 and ε_2 are perfectly correlated: Under these circumstances, if it is optimal to serve one market it will be optimal to serve the other market as well. Conversely, if ε_1 and ε_2 are perfectly negatively correlated, and the profit margins are sufficiently small, then it is possible that whenever one market is ex post profitable, the other market tends to be ex post unprofitable. Hence, by strategically giving priority to the more profitable market in the second stage, it is possible to have a higher expected profit by using a production-hedging policy. Using equally attractive ex ante profitable markets, this is formalized as follows.

Theorem 2. *When* $r_1\bar{e}_1 - t_1 = r_2\bar{e}_2 - t_2$, (a) *if it is optimal to produce* $(d_1 + d_2)$ *when* $\rho = -1$, *then it is optimal to produce* $(d_1 + d_2)$ *when* $\rho = +1$; (b) *if it is optimal to produce the maximum of* (d_1, d_2) *when* $\rho = +1$, *then it is optimal to produce the maximum of* (d_1, d_2) *when* $\rho = -1$.

To appreciate Theorem 2, define the function $\delta(\rho)$ as the difference between the expected profits of producing the total demand $(d_1 + d_2)$ and the maximum of (d_1, d_2) for a given ρ:

$$\delta(\rho) = E[P(X = d_1 + d_2 \mid \rho)] - E[P(X = \max(d_1, d_2) \mid \rho)].$$

To decide on the optimal production quantity, it is sufficient to evaluate the sign of $\delta(\rho)$. It follows that when $\delta(\rho)$ is negative the optimal production quantity is the maximum of (d_1, d_2) and the optimal solution corresponds to a production-hedging policy. Conversely, when $\delta(\rho)$ is positive the optimal production quantity is $(d_1 + d_2)$. An important implication of Theorem 2 is that $\delta(\rho = -1) < \delta(\rho = +1)$, suggesting that where $\delta(\rho)$ is monotone and increasing there exists a threshold that would uniquely determine how to choose between production hedging and full production.

Although we are unable to provide a general characterization of $\delta(\rho)$, it can be shown that it is monotone for a restricted class of parameters and density functions. Specifically, we require that $d_1 = d_2 = d$ and $r_1\bar{e}_1 - t_1 = r_2\bar{e}_2 - t_2 > 0$, and the joint density of $f(e_1, e_2)$ is given by

$$\begin{aligned} &f(e_1, e_2 \mid \rho) \\ &= \begin{cases} \dfrac{1}{\pi\sqrt{1-\rho^2}} & \text{if } \left(\dfrac{e_1 - \bar{e}_1}{2\sigma_1}\right)^2 - 2\rho\left(\dfrac{e_1 - \bar{e}_1}{2\sigma_1}\right)\left(\dfrac{e_2 - \bar{e}_2}{2\sigma_2}\right) + \left(\dfrac{e_2 - \bar{e}_2}{2\sigma_2}\right)^2 < 1 - \rho^2, \\ 0 & \text{otherwise} \end{cases} \quad \text{for } -1 < \rho < +1 \\ &= \begin{cases} \dfrac{1}{2\sqrt{2}} & \text{if } \left(\dfrac{e_1 - \bar{e}_1}{2\sigma_1}\right) = -\left(\dfrac{e_2 - \bar{e}_2}{2\sigma_2}\right) \text{ and } -1 \le \left(\dfrac{e_1 - \bar{e}_1}{2\sigma_1}\right) \le +1 \text{ and } -1 \le \left(\dfrac{e_2 - \bar{e}_2}{2\sigma_2}\right) \le +1, \\ 0 & \text{otherwise} \end{cases} \quad \text{for } \rho = -1 \\ &= \begin{cases} \dfrac{1}{2\sqrt{2}} & \text{if } \left(\dfrac{e_1 - \bar{e}_1}{2\sigma_1}\right) = \left(\dfrac{e_2 - \bar{e}_2}{2\sigma_2}\right) \text{ and } -1 \le \left(\dfrac{e_1 - \bar{e}_1}{2\sigma_1}\right) \le +1 \text{ and } -1 \le \left(\dfrac{e_2 - \bar{e}_2}{2\sigma_2}\right) \le +1, \\ 0 & \text{otherwise} \end{cases} \quad \text{for } \rho = +1. \end{aligned} \tag{10}$$

This is a subfamily of densities identified by Van Mieghem (1995), who used this family to provide a rich exploration of the impact of variability in a different context. Specifically, (10) has the attractive features that the marginal distributions are uniform, and, the joint density is uniform on an ellipse. Note that in (10) the density is nonnegative and uniform on an ellipse when $-1 < \rho < +1$, and on a line segment when $\rho = -1$ or $\rho = +1$. Next, we show that $\delta(\rho)$ is monotonically nondecreasing in ρ.

Theorem 3. *If $r_1\bar{e}_1 - t_1 = r_2\bar{e}_2 - t_2$ and $f(e_1, e_2 \mid \rho)$ is given by* (10), *then* (a) *$\delta(\rho)$ is monotonically nondecreasing in ρ;* (b) *$\delta(\rho) = 0$ has a unique solution given by*

$$\rho^* = \frac{1}{2}\left(\frac{r_1\sigma_1}{r_2\sigma_2}\right) + \frac{1}{2}\left(\frac{r_2\sigma_2}{r_1\sigma_1}\right) - \frac{9\pi^2}{32}\frac{(r_1\bar{e}_1 - t_1 - c)}{r_1\sigma_1}\frac{(r_2\bar{e}_2 - t_2 - c)}{r_2\sigma_2}. \quad (11)$$

Proof to Theorems 2 and 3 are provided in the online appendix, available at http://mansci.pubs.informs.org/ecompanion.html.

If $\rho^* > 1$, production hedging is always optimal, and if $\rho^* < -1$, the full production policy is optimal for all values of the correlation coefficient. Theorem 3 complements Theorem 2 by demonstrating that it is indeed plausible that $\delta(\rho)$ is monotone, so that the intuitive property regarding the choice of optimal production policy based on the correlation holds: i.e., production hedging is more likely to be optimal when there is negative or low correlation, and full production of the total demand is more likely when there is high positive correlation. Now that we have established the viability of production and allocation hedging under rather general conditions, in the remainder of the paper we consider the fundamental case where Market 1 is now the domestic market ($e_1 \equiv 1$) and Market 2 is the only foreign market. This simplifies the analysis because we have to consider only the marginal distribution of e_2. More importantly, it allows us to provide sharper results strictly focusing on the impact of exchange-rate uncertainty. We next consider the relevancy of the proposed operational hedging strategies in a multiperiod setting.

3.3. The Multiperiod Extension

In developing the scope of the core model (§3.1), we have considered single-period variants. In this section, we demonstrate that both production hedging and allocation hedging can be optimal in a multiperiod setting as well. We assume that all economic parameters are stationary. An immediate consequence of having multiple periods is that unused inventory from one period is available for use in subsequent periods. We assume that there is a unit charge of h for inventory on hand at the end of any period.

We start with the case $d_1 \geq d_2$ and allow exchange-rate realizations to follow an arbitrary stochastic process. We begin by invoking Theorem 1(c) to conclude that in each period, sales will be either the maximum of (d_1, d_2) or $(d_1 + d_2)$ units. In this case, we would like to start Period 1 with either d_1 or $(d_1 + d_2)$ units. To establish that both production and allocation hedging may occur in this multiperiod setting, it is sufficient to focus on Period 1 with d_1 units; namely, $X = d_1$. Then, the ending inventory is zero for all realizations of e_2, and we start the second period with zero units of inventory. Alternatively, suppose that it is optimal to start Period 1 with $(d_1 + d_2)$ units; in other words, $X = (d_1 + d_2)$. In this case, if the exchange-rate realization of e_2 is favorable we serve both markets and start the second period with no inventory. However, if the exchange-rate realization of e_2 is unfavorable we sell only d_1 units, and start the second period with an inventory of $d_2 < d_1$ units for which we are charged h, $h < c$, per unit, yielding a cost of hd_2. Because we start Period 2 with d_2 units, this case can be interpreted as paying a restocking fee of hd_2 to recover the initial investment of cd_2 through future sales that can be obtained by "selling" d_2 units of inventory to Period 2. The critical consequence of this interpretation is that we can also act as if we are starting the second period with zero units of inventory. Hence, in all cases, by properly accounting for this restocking charge, we can safely assume that the second and subsequent period decisions do not depend on the ending inventory of Period 1. Because this process decouples the future from the decisions of the first period, we can replace the value of the subsequent optimal profit by its expectation.

The case $d_1 < d_2$ is similar; however, we need to impose a restriction on the variability of e_2 over time. This condition is explained later, and assuming that it is satisfied, we can invoke Theorem 1(c) to conclude that the optimal production choices would be either d_2 or $(d_1 + d_2)$. Suppose that $X = d_2$. If the spot rate of e_2 is unfavorable sales are d_1, and if it is favorable sales are d_2, so that ending inventory is either $d_2 - d_1 < d_2$ or 0. Alternatively, $X = (d_1 + d_2)$. If the spot rate of e_2 is unfavorable sales are d_1, and if it is favorable sales are $(d_1 + d_2)$, so that ending inventory is either d_2 or 0. As in all four scenarios ending inventory does not exceed d_2, the minimum purchase quantity from Theorem 1(c), the analysis of the case $d_1 \geq d_2$ can be applied directly to conclude that the decisions regarding future periods can be decoupled from those of Period 1. Consequently, we can directly compare the conditions under which allocation hedging might occur in the single-period setting with that of a multiperiod setting.

In the single-period model, allocation hedging is exercised when the realized exchange rate is so low that the revenue from the foreign market does not justify the unit transportation cost. This corresponds to the following range of exchange-rate values:

$$e_2 < \frac{t_2}{r_2}.$$

In a multiperiod setting, when $(d_1 + d_2)$ units are produced, d_2 units can be kept in inventory for future sale at the unit inventory holding cost of h. However, this also saves the future manufacturing cost of c for each unit held in inventory. Thus, allocation hedging is exercised when the revenue in a foreign market (based on the realized exchange rate) is less than the sum of the unit transportation cost and the unit manufacturing cost less the unit inventory holding cost. The resulting range of exchange-rate values in which allocation hedging is exercised thus corresponds to

$$e_2 < \frac{t_2 + c - h}{r_2}. \tag{12}$$

Because $c > h$ by definition, the expression for allocation hedging in a multiperiod setting corresponds to a larger domain in the probability space than that of the single-period model. Therefore, the probability of allocation hedging in a multiperiod model is higher than that of the single-period model. Moreover, we can conclude that in the multiperiod setting allocation hedging is not predicated on transportation costs being high relative to the sale price. This is formalized as the following theorem.

Theorem 4. *In a multiperiod model when the optimal solution is to produce the total demand, the probability of allocation hedging is higher than that of the single-period model.*

The above theorem shows that the ability to transfer unused inventory to future periods increases the likelihood of allocation hedging. It also reduces the possibility of production hedging in the first period because the presence of future periods lowers the downside cost of allocation hedging. Consequently, producing $(d_1 + d_2)$ in the first period of a multiperiod setting is not as expensive on average.

To complete this argument, the restriction on the stochastic process for e_2 must be developed for $d_1 < d_2$. From Theorem 1(c), it is known that the optimal production quantity reduces to the choice between $\max(d_1, d_2) = d_2$ and $(d_1 + d_2)$ in a single-period setting under the following condition:

$$E(\lambda_{B2}) = \int_{t_2/r_2}^{\infty} (r_2 e_2 - t_2) f_2(e_2)\, de_2 > c \quad \text{for market } j = 2. \tag{13}$$

In a multiperiod setting, however, (13) can be modified to the following stronger requirement as a consequence of (12):

$$\int_{(t_2+c-h)/r_2}^{\infty} (r_2 e_2 - t_2) f_2(e_2)\, de_2 > c. \tag{13a}$$

Note that it must hold with the updated distribution of e_2 for each of the T periods. If an optimal myopic policy is desired, then we can invoke the assumption in Veinott (1965), i.e., all unsold inventory can be returned for a stocking fee of h per unit. This assumption reduces the same requirement to the weaker condition:

$$\int_{(t_2+c-h)/r_2}^{\infty} (r_2 e_2 - t_2) f_2(e_2)\, de_2 > c - (c-h) F_2\left(\frac{t_2 + c - h}{r_2}\right). \tag{13b}$$

Condition (13b) is exactly equal to (13a) except that the threshold c is lowered by the expected value of the cash flows from leftovers obtained through the restocking fee. More importantly, as Condition (13b) is weaker than both (13) and (13a), it is more likely to increase production to the total demand at the beginning of a period in the multiperiod setting. This is formalized in the following theorem.

Theorem 5. *If $d_1 \geq d_2$ or Condition* (13b) *holds when $d_1 < d_2$, then the following are true*: (a) *When it is optimal to produce the maximum of (d_1, d_2) in the first period of a multiperiod model* (*i.e., production hedging is prescribed*), *it is also optimal to produce the maximum of (d_1, d_2) in a single-period model*; (b) *When it is optimal to produce $(d_1 + d_2)$ in a single-period model, it is also optimal to produce $(d_1 + d_2)$ in the first period of the multiperiod model.*

In summary, these two theorems reveal that the optimal production decision is tilted towards producing the total demand, so that the optimal policy tends to include production hedging less frequently in the first period of a multiperiod model. It has been argued earlier that allocation hedging is exercised when the exchange rate is sufficiently low that it does not justify the transportation cost in a single-period model. In a multiperiod model, however, allocation hedging does not depend only on the transportation cost, and is more likely to be exercised so that the net effect is that markets will continue to be underserved. These findings reveal that both production hedging and allocation hedging can still be optimal choices in a multiperiod setting. Therefore, we have shown that the optimal policies that are developed in a single-period model in earlier sections extend to multiperiod environments.

4. The Impact of Demand Uncertainty

In this section, we examine how the core model of §3 changes when demand is uncertain. Now $D_1 \geq 0$ and $D_2 \geq 0$ are modeled as two independent random variables representing demand in Markets 1 and 2 with the realized values, d_1 and d_2, that are distributed with density functions of $g_1(d_1)$ and $g_2(d_2)$ and cumulative density functions of $G_1(\cdot)$ and $G_2(\cdot)$, respectively. As in §3, a production quantity of X is planned before any uncertainty is revealed. However, the model structure is influenced by how demand uncertainty is resolved. If the allocation decisions are made concurrently with the production decision, and exchange rate and demand are observed after allocation decisions, then the resulting problem reduces to two separable newsvendor problems. In this model, X_1 would satisfy $G_1(X_1) = (r_1 - t_1 - c)/r_1$ and for the foreign market X_2 would satisfy $G_2(X_2) = (r_2\bar{e}_2 - t_2 - c)/r_2\bar{e}_2$, where the payoff is adjusted by the expected exchange rate $\bar{e}_2$. However, in our modeling context, exchange-rate uncertainty is resolved after the production decision but before the allocation decision, so that the information structure is different. We first consider the case when, along with exchange-rate uncertainty, demand uncertainty is also resolved between the production and allocation decisions. The resulting second-stage problem is identical to that of the second-stage problem structure of §3.1. We are able to use a Lagrangean approach to facilitate the analysis. In the model of §4.2, only exchange-rate uncertainty is resolved before the allocation decision. The second-stage problem becomes a constrained two-newsvendor problem; this time applying the Lagrangean approach requires additional analysis.

4.1. Demand Is Revealed Between Production and Allocation Decisions

Let d_1, d_2, and e_2 be the realized values of the random variables that are revealed after X, the production quantity, is chosen. Hence, the recourse problem is a deterministic program, and is identical to the second-stage problem of §3.1 for given values of (e_2, d_1, d_2). The key distinction is that because demand was deterministic in §3.1, the expected value of the dual multiplier was precisely one of the five quantities: $E(\lambda_A)$, $E(\lambda_{B1})$, $E(\lambda_{B2})$, $E(\lambda_C)$, or $E(\lambda_D)$. Here, however, for a given X, we can only assign a probability with each of these five expectations of λ. Specifically,

$$\begin{aligned}
\Pr(A) &= \Pr(0 < X < \min(D_1, D_2)) \\
&= \Pr(D_1 > X \text{ and } D_2 > X), \\
\Pr(B1) &= \Pr(D_2 < X < D_1) = \Pr(D_1 > X \text{ and } D_2 < X), \\
\Pr(B2) &= \Pr(D_1 < X < D_2) = \Pr(D_1 < X \text{ and } D_2 > X), \\
\Pr(C) &= \Pr(\max(D_1, D_2) < X < D_1 + D_2) \\
&= \Pr(D_1 < X \text{ and } D_2 < X \text{ and } D_1 + D_2 > X), \\
\Pr(D) &= \Pr(D_1 + D_2 > X).
\end{aligned}$$

Note that the second and third probabilities represent the two subevents in which X is between the minimum and maximum of D_1 and D_2. Recognizing that these probabilities depend on X and remembering that $E(\lambda_D) = 0$ yields

$$\begin{aligned}
E(\lambda(X)) &= E(\lambda_A)\Pr(A) + E(\lambda_{B1})\Pr(B1) \\
&\quad + E(\lambda_{B2})\Pr(B2) + E(\lambda_C)\Pr(C).
\end{aligned}$$

Because $E(\lambda(X))$ is nonincreasing in X, and X may take any nonnegative value, it follows that the unique optimal solution satisfies

$$E(\lambda(X)) = c.$$

The case in which demand uncertainty is resolved prior to the allocation decision inherits the structural properties of §3. In the second stage, the allocation decision is made under realized demand values. Due to the fact that the allocation scheme of §3.1 is employed, allocation hedging is part of the optimal policy structure for this generalization. However, it is more difficult to define production hedging because demand, and therefore X, can take a continuum of values. Nevertheless, insight into the optimal production policy can be provided by considering each market in isolation. Let X_d^* be the solution to the problem where Market 1 is served independently of Market 2, and let X_f^* be the solution to the problem where Market 2 is served in isolation. Each of these quantities entails solving newsvendorlike problems. Then, direct computation shows that the optimal production quantity would be at least the maximum of X_d^* and X_f^*. This is formalized as follows.

Theorem 6. *The optimal production quantity X is greater than or equal to* $\max(X_d^*, X_f^*)$.

The implication of Theorem 6 is that, in contrast to Theorem 1(b), $X = \min(X_d^*, X_f^*)$ can never be optimal. However, $X = 0$ still remains to be an admissible strategy in the event that $X_d^* = X_f^* = 0$. When the optimal production choice is such that $X < (X_d^* + X_f^*)$, the optimal solution mimics production hedging. Therefore, production hedging is also an integral part of the optimal policy for this generalization. As in §3, production and allocation hedging are in the optimal policy set when demand is revealed before the allocation decision; as we show next, a similar conclusion is reached for the case when demand is observed after the allocation decision.

4.2. Demand Is Revealed After Allocation Decisions

Here, we consider the case with the following sequence of events. First, the production quantity X is chosen, then e_2 is observed, and the allocation decisions X_1 and X_2 are made before the values of demand in Markets 1 and 2, d_1 and d_2, are revealed. Hence, each market in the second stage represents a newsvendor problem that must be served from a jointly limited supply of X. Let λ represent the Lagrange multiplier for the supply constraint (4). Hence, the marginal benefit of selling a unit in Market 1 is $r_1 - t_1 - \lambda$ and in Market 2 is $r_2 e_2 - t_2 - \lambda$. There are three potentially optimal policies. If only Market 1 is served, the newsvendor fractile is

$$G_1(X_1) = \frac{r_1 - t_1 - \lambda}{r_1} \quad \text{or} \tag{14}$$

$$\lambda = r_1(1 - G_1(X_1)) - t_1 \equiv \lambda_1(X_1) \geq 0. \tag{15}$$

If only Market 2 is served, the newsvendor fractile is

$$G_2(X_2) = \frac{r_2 e_2 - t_2 - \lambda}{r_2 e_2} \quad \text{or} \tag{16}$$

$$\lambda = r_2 e_2(1 - G_2(X_2)) - t_2 \equiv \lambda_2(X_2). \tag{17}$$

If both markets are served, then $X_1 + X_2 = X$, so

$$G_1^{-1}\left(\frac{r_1 - t_1 - \lambda}{r_1}\right) + G_2^{-1}\left(\frac{r_2 e_2 - t_2 - \lambda}{r_2 e_2}\right) = X. \tag{18}$$

It is easy to see from (14) and (16) that X_1 and X_2 decrease with increasing values of λ. Similarly, it should be observed that the left-hand side of (18) decreases with increasing λ. Therefore, if X increases, λ must decrease to satisfy the equality in (18). Finally, it should be observed that in all three potentially optimal policies of the second stage, λ is decreasing in X. Moreover, it is continuous in X for a given e_2; therefore, its expectation is continuous and decreasing in X. Once again, the unique optimal value of X satisfies $E(\lambda(X)) = c$.

Having established that this Lagrangean approach can be used to find the optimal production quantity X, we precisely study how the optimal allocation is determined. First, consider the case $r_2 e_2 - t_2 \leq 0$, so that $X_2 = 0$. Because λ must be nonnegative, it follows from (14) that

$$X_1 < G_1^{-1}\left(\frac{r_1 - t_1}{r_1}\right) = X_1^{\max}.$$

Hence, $X_1 = X$ when $X < X_1^{\max}$ as depicted in region A1, and $X_1 = X_1^{\max}$ when $X \geq X_1^{\max}$ as depicted in region A2 in Figure 3. Thus, allocation hedging is exercised in region A2. Now consider the more complex case when $r_2 e_2 - t_2 > 0$. Then, $X_1 = X$ and $\lambda = \lambda_1(X_1)$, and because $X_2 = 0$, $\lambda_1(X_1) = r_1(1 - G_1(X_1)) - t_1 > r_2 e_2 - t_2$. This can be rewritten as

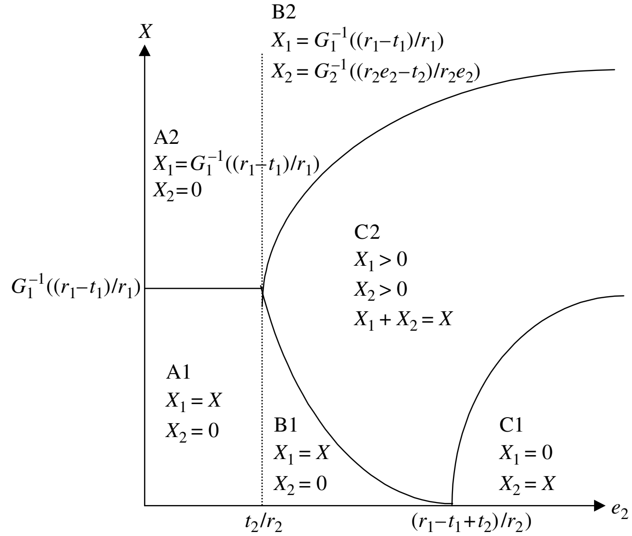
$$\frac{t_2}{r_2} < e_2 < \frac{r_1 - t_1 + t_2 - r_1 G_1(X)}{r_2} < \frac{r_1 - t_1 + t_2}{r_2} \tag{19}$$

and

$$G_1(X) < \frac{r_1 - t_1 - (r_2 e_2 - t_2)}{r_1}, \quad \text{or}$$
$$X < G_1^{-1}\left(\frac{r_1 - t_1 - (r_2 e_2 - t_2)}{r_1}\right). \tag{20}$$

The region specified by (19) is depicted as $B1$ in Figure 3. X reaches zero when $r_1 - t_1 = r_2 e_2 - t_2$ and approaches $X_1^{\max}$ as $r_2 e_2$ approaches t_2.

Figure 3 **Optimal Allocation Policies in the Second Stage When Demand Is Revealed After Allocation Decisions**



Now we consider the event when only Market 2 is served. Then, $X_2 = X$, and because $X_1 = 0$, $\lambda = \lambda_2(X)$, so $\lambda_2(X) = r_2 e_2(1 - G_2(X)) - t_2 > r_1 - t_1$. This can be rewritten as

$$e_2 > \frac{r_1 - t_1 + t_2 + r_2 e_2 G_2(X)}{r_2} > \frac{r_1 - t_1 + t_2}{r_2} \quad \text{and} \tag{21}$$

$$G_2(X) < \frac{r_2 e_2 - t_2 - r_1 + t_1}{r_2 e_2}. \tag{22}$$

This region is depicted as $C1$ in Figure 3. X reaches zero when $r_1 - t_1 = r_2 e_2 - t_2$, and because $G_2(X)$ approaches 1 as e_2 approaches infinity, X approaches infinity.

Now consider the final case where both markets are served so that the allocations satisfy $X_1 + X_2 = X$. If X is sufficiently small, the entire production is allocated as depicted in region C2. However, if X is sufficiently high for a given e_2, then $\lambda = 0$, yielding

$$X_{1\&2}^{\max}(e_2) = X_1^{\max} + G_2^{-1}\left(\frac{r_2 e_2 - t_2}{r_2 e_2}\right) > X_1^{\max}. \tag{23}$$

Hence, if $X > X_{1\&2}^{\max}(e_2)$ some production is not allocated as depicted in region B2. The expression $X_{1\&2}^{\max}(e_2)$ is analogous to the sum of $(X_d^* + X_f^*)$ of §4.1; however, it depends on e_2 because the allocation decision is made after the exchange rate is revealed. Therefore, the value of $X_{1\&2}^{\max}(e_2)$ can be higher or lower than $(X_d^* + X_f^*)$.

The optimal allocation scheme, as shown in Figure 3, suggests that allocation to the foreign market (X_2) increases with increasing values of e_2, and allocation to the domestic market (X_1) decreases in e_2. Therefore, it can be proven that, in an optimal response, the expected sales in the foreign market is positively correlated with the exchange rate, and the expected sales in the domestic market is negatively correlated with e_2.

The allocation decisions of the models under demand uncertainty provide similar structural properties with that of deterministic demand. When demand is revealed after the production but before the allocation decision, the second-stage model inherits the entire allocation scheme of the model in §3.1. When demand is revealed after the allocation decision, the allocation scheme is qualitatively similar to the results presented in §3. We conclude that, as in the core model of §3, allocation hedging is an integral part of an optimal policy as shown in region A2 of Figure 3. However, as demand becomes less clearly defined, interpreting when the optimal choice is a production hedge gets muddled. However, the choice of $X < X_1^{\max}$ mimics production hedging in the sense that with probability one the entire production is used and there is some probability that a beneficial market will not be served as depicted in regions B1 and C1 of Figure 3. Next, we show that the case of endogenous prices reveals qualitatively similar results.

5. Impact of Price Setting

In the models of §§3 and 4, demand and prices are exogenously determined as they are in conventional aggregate planning models. In other words, the global manufacturer is a price taker and has inelastic demand. In this section, we establish the policy structure for the global manufacturer who is a price setter or a monopolist who faces a downward-sloping demand curve in each of its markets. We restrict the analysis of this section to the case of deterministic demand, the case when demand is considered in §6. In addition to the production and allocation decisions, two additional quantities must be determined: the demand values d_1 and d_2 and the corresponding prices, $p_1(d_1)$ and $p_2(d_2)$, respectively. We make the standard assumption that the revenue function $p_j(d_j)d_j$ is unimodal; consequently, when it is positive the marginal revenue function, $MR_j(d_j)$, which is defined as the derivative of $p_j(d_j)d_j$, is decreasing in its argument in both markets, $j = 1, 2$. It also follows that once marginal revenue becomes negative it remains negative.

The optimal policy structure depends crucially on the timing of the pricing decision. For example, in the scenario when production, allocation, and pricing decisions are made at the start of the planning period, the second-stage problem becomes degenerate. In this case, the standard monopoly pricing rule that marginal revenue equals marginal cost applies with the proviso that the marginal revenue is multiplied by the expected value of the exchange-rate variable to adjust for uncertainty. However, the optimal policy deviates from this basic pricing rule under the two more complex scenarios in which the allocation decision occurs after the exchange rate is observed. In the model developed in §5.1, prices and production decisions are made simultaneously before the realization of exchange-rate uncertainty. In this case, the single-period core model (of §3.1) is a submodel, so that its entire policy structure is inherited by this more general problem. In §5.2, we consider another model in which pricing decisions are postponed until the realization of the exchange rate so that its effect can be passed through to the consumers.

5.1. Price Setting in the First Stage

We begin our analysis with a model that features the pricing and production decisions in the first stage. First-stage decision variables d_1 and d_2 represent the demand generated by the choice of prices $p_1(d_1)$ and $p_2(d_2)$ in each market, respectively, and the decision variable X represents the production quantity. After realizing the value of the exchange rate (corresponding to the second stage of our model), the manufacturer decides on the allocation of available products to markets (denoted by X_1 and X_2). The resulting first-stage problem can be expressed as follows:

$$\text{(P2):} \quad \max_{(d_1, d_2, X \geq 0)} E[P(d_1, d_2, X)] = -cX + E[PB(d_1, d_2, X \mid e_2)],$$

where $E[P(d_1, d_2, X)]$ is the expected revenue obtained in the first stage when prices are set as $p_1(d_1)$ and $p_2(d_2)$, and X units are produced. Similarly, the second-stage objective function can be written as follows:

$$PB(d_1, d_2, X \mid e_2): \quad \max_{(X_1, X_2 \geq 0)} \; (p_1(d_1) - t_1)X_1 + (p_2(d_2)e_2 - t_2)X_2 \quad \text{s.t. } (3), (4).$$

Note that $PB(d_1, d_2, X \mid e_2)$ is precisely the same problem as defined in §3.1, except e_1 is identically equal to one. Hence, Theorem 1 applies, and because d_1 and d_2 are endogenous, the optimal policy can be restricted to one of the following six choices: (1) $X = d_1 + d_2$, (2) $X = d_1 < d_2$, (3) $X = d_1 \geq d_2$, (4) $X = d_2 \leq d_1$, (5) $X = d_2 > d_1$, and (6) $X = 0$. The first policy may be interpreted as a full production policy because it is designed to meet the total demand, while the next four policies are variants of production hedging. The final policy is the null case of no market entry. Because demand is set endogenously, the two production-hedging policies—(2) and (4)—that represent producing the minimum of the two endogenous demands, can be eliminated by a direct argument.

This is because, without reducing potential unit sales, the price can be raised in the market with higher demand to increase expected revenue and therefore profit. Because $X = d_1 = d_2 = 0$ is a feasible solution with zero profit, policy (6) may be viewed as a special case of policy (3). Hence, analogous to Theorems 1 and 6, we get Theorem 7.

Theorem 7. *In an optimal strategy, the production quantity is either $X = d_1 \geq d_2$, or $X = d_2 > d_1$, or $X = (d_1 + d_2)$, where d_1 and d_2 are determined by optimally choosing p_1 and p_2 for each policy type.*

As in Theorem 6, Theorem 7 states that the optimal production quantity is at least the maximum of the two market demand values. Like Theorem 6, and in contrast to Theorem 1(b), $X = \min(d_1, d_2)$ can never be optimal in Problem (P2). Therefore, the manufacturer now has to compute three potentially optimal policies when demand is endogenous and choose the one with the highest expected profit. Two of the three candidate solutions are production-hedging policies, while the remaining one is the full production policy. In contrast to Theorem 6, the full production policy is well defined in Theorem 7. Hence, we have shown that, as in the case when demand is exogenous, production hedging and allocation hedging are integral parts of the optimal policy. As we show next, when pricing decisions can be postponed until after the allocation decision, the optimal problem structure provides a perspective analogous to that of §4.2.

5.2. Postponing Pricing Decisions

Here, we examine a model in which prices and allocation decisions are made after the realization of the exchange rate. As in the previous section, we ship only the amount we wish to sell in each market. The crucial difference is that now, in the second stage, the amount shipped is precisely the demand, so that the market-clearing price can be inferred, making it a direct function of the allocation to that market, i.e., $p_1 = p_1(d_1 = X_1)$ and $p_2 = p_2(d_2 = X_2)$. Therefore, the first-stage objective function can be expressed as

$$\text{(P3):}\quad \max_{(X\geq 0)} E[P(X)] = -cX + E[PC(X \mid e_2)],$$

where $E[P(X)]$ is the expected revenue obtained in the first stage when X units are produced, $E[PC(X \mid e_2)]$ is the expected second-stage return function (over the pdf of exchange-rate variable e_2), and cX is the manufacturing cost of X units. The second-stage objective function is

$$PC(X \mid e_2):\quad \max_{(X_1, X_2\geq 0)} (p_1(X_1) - t_1)X_1 + (p_2(X_2)e_2 - t_2)X_2 \quad \text{s.t. (4).}$$

Note that if the capacity constraint (4) was not binding, the optimal solution to $PC(X \mid e_2)$ would merely be the solution to two independent pricing problems, each of which would be solved by setting marginal revenue equal to marginal cost. We can essentially obtain this solution by dualizing constraint (4). Let $\lambda(e_2, X)$ represent the value of the dualized constraint, i.e., the marginal contribution from one more unit of first-stage production. Then, the Lagrangean

$$L(e_2, X, \lambda) = \max_{(X_1, X_2\geq 0)} (p_1(X_1) - t_1)X_1 + (p_2(X_2)e_2 - t_2)X_2 - \lambda(X_1 + X_2 - X). \tag{24}$$

Its solution is

$$MR_1(d_1) = t_1 + \lambda,$$
$$MR_2(d_2) = \frac{t_2 + \lambda}{e_2}.$$

When $X_1 + X_2 = X$,

$$MR_1^{-1}(t_1 + \lambda) + MR_2^{-1}\left(\frac{t_2 + \lambda}{e_2}\right) = X. \tag{25}$$

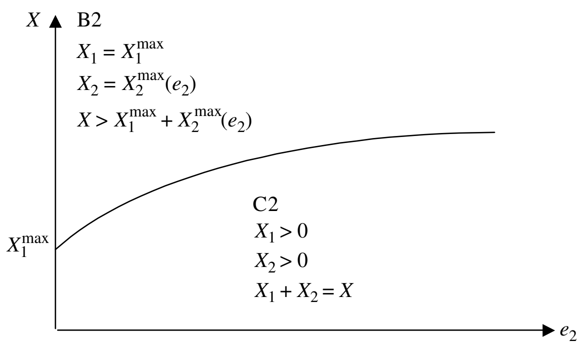
It is important to point out that (25) has essentially the same functional form as (18) of §4.2. To make this connection transparent, let $\bar{G}(\cdot) = 1 - G(\cdot)$. Then, (18) can be rewritten as

$$\bar{G}_1^{-1}\left(\frac{t_1 + \lambda}{r_1}\right) + \bar{G}_2^{-1}\left(\frac{t_2 + \lambda}{r_2 e_2}\right) = X.$$

Hence, the left-hand side of (25) has the same functional properties in terms of λ as does (18). In particular, the allocation to the foreign market (X_2) increases with increasing values of e_2, and the allocation to the domestic market (X_1) decreases in e_2. It now follows that the expected demand in the foreign market increases in e_2, and therefore the price in the foreign market decreases with increasing values of e_2. Conversely, the expected demand decreases in the domestic market with increasing values of e_2, and therefore the price in the domestic market increases with e_2. Thus, it can be concluded that the price in the foreign market is negatively correlated with the exchange rate, while the price in the domestic market is positively correlated with e_2. Consequently, ignoring the boundary conditions such as price or demand equal to zero, it follows that $\lambda(e_2, X)$ is decreasing in X, and therefore, $E(\lambda(X))$ is also decreasing in X. Because $E(\lambda(X))$ is the expected value of one more unit of X, the first-stage problem is the unique solution to $E(\lambda(X)) = c$, which exists if $E(\lambda(X = 0)) > c$.

To complete the characterization of the optimal solution to $PC(\cdot)$, as in §4.2, suppose that for a given realization e_2, X is just large enough that λ becomes zero. Refer to this value as $X^{\min}(e_2)$, which is merely the value of the left-hand side of Equation (25). The first term of $X^{\min}(e_2)$ is independent of e_2, while

Figure 4 **Optimal Allocation Policies in the Second Stage When Demand Curves Are Convex**



the second term increases from zero as e_2 increases. Hence, if X is greater than $X^{min}(e_2)$, both markets are fully served in the sense that the total allocation is less than the available production. Otherwise, both markets are profitable and X is fully allocated between them. If the demand curve for each market is convex so that $MR_j(d_j = 0)$ is unbounded, then each market receives a positive allocation, as depicted in Figure 4.

To reconcile the seemingly qualitative different policy representations of Figures 3 and 4, consider the implications of (25) when the demand curves are concave. For example, if the demand curves are linear, then $MR_1^{-1}(t_1) = a_1 - t_1$, with $p_1(0) = a_1$ and $p_2(0) = a_2$. In Figure 3, replacing X_1^{max} by $MR_1^{-1}(t_1) = a_1 - t_1$, r_1 by $p_1(0) = a_1$, and r_2 by $p_2(0) = a_2$ yields the solution for the case of linear demand. Now note that if the demand curves are convex, then $p_j(0)$ approaches infinity; hence, the thresholds equivalent to t_2/r_2 and $(r_1 - t_1 + t_2)/r_2$ each approach zero. Consequently, regions A1, A2, B1, and C1 in Figure 3 disappear, yielding Figure 4. Hence, we have shown in this section that this market-clearing price case is merely a modest generalization of the demand uncertainty case solved in §4.2.

To summarize, the analysis of a price-setting global manufacturer provides structural properties similar to those of §§3 and 4. When prices are set along with the production quantity in the first stage, as in §4.1, the second-stage model inherits the allocation decisions of the model in §3.1 entirely, yielding the same allocation scheme for a given set of prices. As shown in Theorem 7, two of the three potentially optimal solutions recommend production hedging; thus, both production and allocation hedging are integral parts of the optimal policy structure. When pricing decisions are postponed until the allocation decision, the structural properties are similar to the ones developed in §4.2. Once again, optimal production quantity choices such as $X < MR_1^{-1}(t_1)$ mimic production hedging in the sense that the entire production is used, and there is some probability that a beneficial market will not be served. In this model, allocation hedging is also part of the optimal policy when demand curves are concave. However, when demand curves are convex, there is no possibility of allocation hedging because the impact of transportation costs can be mitigated by setting sufficiently high prices in the second stage. We now have fully demonstrated the impact of endogenous pricing under deterministic demand on the optimal production and allocation decisions. As we show next, when prices are endogenous and demand is uncertain, the resulting modeling variants inherit much of the problem structure of the models of §§3, 4, and 5.

6. The Combined Effect of Demand Uncertainty and Price Setting

Starting with the core model in §3, we considered the case where demand was prespecified and deterministic, and sequentially examined the impact of demand uncertainty and monopolistic pricing. In this section, we consider the global production-planning problem in which the production and pricing decisions must be made simultaneously under demand uncertainty. For this analysis of the combined effect of demand uncertainty and price setting, the benchmark case would be the problem in which all uncertainty is resolved after all production, pricing, and allocation decisions are made. This scenario would result in modeling each market as a newsvendor with pricing, but with the revenue for the foreign market adjusted by the expected value of the exchange rate. However, as in earlier sections, we are interested in problem variants in which exchange-rate uncertainty is resolved between the production and allocation stages of the problem. This results in four variants that are described below, along with their solution approach.

As in earlier variants, the decision maker must choose X, the production quantity, in the first stage of the program and choose the market allocations X_1 and X_2 in the second stage. Moreover, the decision maker can influence price, and therefore demand, by appropriately managing price effects. Adapting the notation from §4, we let D_j be the random demand for market j, $\bar{D}_j$ be the mean demand for market j, and $p_j(D_j)$ as the corresponding price in market $j = 1, 2$. As reported by Petruzzi and Dada (1999), the random demand is typically represented by either the additive form $D_j = \bar{D}_j + \zeta_j$, or the multiplicative form $D_j = \bar{D}_j \zeta_j$, where ζ_j is the uncertain shock term. In these cases, to characterize the distribution of D_j, it is sufficient to characterize the distribution of ζ_j. To preserve the interpretation of $\bar{D}_j$ as the mean demand, we assume that ζ_j has mean zero in the additive case and has mean 1 for the multiplicative case.

In variant 6.1, as in §4.1, all uncertainty is resolved between the production and allocation stages. Moreover, as in §5.1, the mean demand or prices are chosen at the production stage. Consequently, as in the models of §§4.1 and 5.1, the second-stage problem is merely the problem of §3.1. As a result, allocation hedging is a part of the optimal policy. As in §§4.1 and 5.1, some choices of optimal production quantity of this variant resemble production hedging.

In the next three variants, as in §4.2, exchange-rate uncertainty is resolved between the production and allocation stages, but demand uncertainty is resolved after the allocation decision has been made. In variant 6.2.1, as in the model of §5.1, the mean demand or prices are chosen at the production stage. Hence, the second-stage problem, as in §4.2, is a constrained newsvendor problem. It too is amenable to a dual approach easily yielding $E(\lambda(X))$. Once again, allocation hedging is an integral part of the optimal policy, and there are choices of the optimal production quantity that mimic production hedging.

In variant 6.2.2, as in the model of §5.2, mean demand or prices are set at the allocation stage. For this subproblem, a dual approach separates the problem into two independent problems, one for each market. Because demand uncertainty has not been resolved, as in the benchmark case, it results in two newsvendor models with pricing. However, here the expected profit from the foreign market is adjusted by the actual spot rate e_2. Such newsvendor problems can be solved tractably under mild regularity conditions similar to those in Petruzzi and Dada (1999). As in §5.2, production hedging is part of the optimal policy, and allocation hedging depends on the type of demand curves.

The final variant 6.2.3 differs from variant 6.2.2 in that prices are not set until after the allocation decision. This is facilitated by positing that prices are such that markets clear—in other words, prices adjust so that demand is exactly equal to the allocation to the market. While this assumption is equivalent to that in §5.2 when demands are deterministic, there is a subtle distinction. Under market clearance, randomness in demand leads to randomness in the market-clearing price. We explain this key distinction for the case of multiplicative demand: $D_j = X_j = \overline{D}_j\zeta_j$. Hence, $\overline{D}_j = X_j/\zeta_j$ so that the market-clearing price must be the random variable $p_j(X_j/\zeta_j)$. Therefore, given e_j, the expected contribution from market j equals $(E(p_j(X_j/\zeta_j))e_j - t_j)X_j$. Hence, this problem is equivalent to a deterministic demand problem of §5.2 that has been adjusted for risk. For example, when demand is isoelastic so that $p_j(X_j/\zeta_j) = \overline{D}_j^{-\beta}$, we have

$$E\left(p_j\left(\frac{X_j}{\zeta_j}\right)\right) = E(\zeta_j^{\beta})X_j^{-\beta}.$$

When demand is elastic so that $0 < \beta < 1$, p_j is concave in ζ_j, so that it follows from Jensen's inequality that the expected price, and therefore the expected marginal revenue, is less than in the corresponding deterministic demand case with ζ_j replaced by its expected value. It then follows that for this demand specification the decision maker would produce less than in the corresponding deterministic case. In general, however, the production choice can be higher or lower depending on the interaction between the demand specification and the distribution of ζ_j. However, in all cases, the problem reduces to a risk-adjusted version of the deterministic problem of §5.2.

7. Conclusions and Managerial Insights

Motivated by a periodic aggregate production-planning problem routinely faced by a global manufacturer, we have examined the impact of exchange-rate uncertainty on the optimal production-planning and allocation decisions. In §3, we developed the core model for the case of two markets, and identified two operational hedges, production and allocation hedging, that are beneficial in the aggregate planning strategy of a global firm. We then performed an extensive analysis to show our conclusion, that production hedging and allocation hedging are features of a robust optimal policy under more generalized settings. In §§4, 5, and 6, we demonstrated that our modeling approach is broad in scope because it applies to the cases when the production and allocation decisions must take into account the effects of endogenizing pricing under demand uncertainty. In these model extensions, the core model plays a crucial role in demonstrating that production hedging and allocation hedging provide flexibility to enhance operational decision making. In general, except for a subset of models in §5.2, allocation hedging is an integral feature of the optimal policy. Also, production hedging, while harder to define when demand is not known in the production stage, also remains a persistent feature of the optimal policy.

Our proposed operational hedging strategies apply to more complex global manufacturing networks that include multiple markets and multiple plants. In Kazaz (1997), the generalized version of Problem (P1) with two plants, one associated with a domestic market and the other with a foreign market, is considered. In this setting, as in our problem, production and allocation hedging are integral parts of the optimal policy. Moreover, using data from our motivating example, the following are illustrated in Kazaz (1997): (1) how the same modeling framework can be used to evaluate the expected profit for different production plans, and (2) for the given data set, the optimal policy is to employ production hedging in the corresponding setting with two plants and three markets.

The above results lead to a rich modeling framework that show operations managers how to make trade-offs in their production planning and allocation decisions from an enterprise perspective. Our analysis in §5 suggests that prices are negatively correlated with the exchange rate in foreign markets and positively correlated with the exchange rate in the domestic market. The implications of the relationship between prices and exchange rates can be beneficial when the marketing function is engaged in the exploration of production and sales across global markets. In sum, we show: (1) how a rich modeling framework (with the series of problem variants) can be developed for the important problem of production planning in global manufacturing networks, and (2) the robustness of production and allocation hedging as aggregate planning strategies.

The allocation-hedging strategy has also been recently employed by Ding et al. (2004) and Ding and Kouvelis (2001). Their scenario setting is similar to §4.1, where the allocation decision is made after both the exchange rate and demand values are realized (their localization cost is mathematically equivalent to our transportation cost). In these two related papers, the above authors develop a comprehensive approach to examining the interplay between operational hedging strategies and financial markets. Their results complement and enrich our framework and approach by integrating the risk attitude of the firm and hedging strategies in aggregate production planning.

An online appendix to this paper is available at http://mansci.pubs.informs.org/ecompanion.html.

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Appendix

Proof of Theorem 1. Using the allocation scheme presented in Figure 2, we can write the objective function for different values of X. Because the objective function of the stochastic program with recourse is concave and piecewise linear in X, the optimal solution can only occur at break points: 0, $\min(d_1, d_2)$, $\max(d_1, d_2)$, (d_1+d_2).

(a) When $0 \le X < \min(d_1, d_2)$, the objective function is

$$E[P(0 \le X < \min(d_1, d_2))] = -cX + \int_{t_1/r_1}^{\infty}\int_0^{(r_1e_1-t_1+t_2)/r_2}(r_1e_1-t_1)Xf(e_1,e_2)\,de_2\,de_1 + \int_{t_2/r_2}^{\infty}\int_0^{(r_2e_2-t_2+t_1)/r_1}(r_2e_2-t_2)Xf(e_1,e_2)\,de_1\,de_2.$$

Because the second-order derivative with respect to X is zero, the first-order derivative is sufficient to determine the optimal behavior of X:

$$\frac{\partial E[P(0 \le X < \min(d_1, d_2))]}{\partial X} = -c + \int_{t_1/r_1}^{\infty}\int_0^{(r_1e_1-t_1+t_2)/r_2}(r_1e_1-t_1)f(e_1,e_2)\,de_2\,de_1 + \int_{t_2/r_2}^{\infty}\int_0^{(r_2e_2-t_2+t_1)/r_1}(r_2e_2-t_2)f(e_1,e_2)\,de_1\,de_2 = -c + E(\lambda_A(X)).$$

Therefore, if $E(\lambda_A(X)) < c$, the optimal value of X is zero.

(b) Suppose that $E(\lambda_A(X)) > c$. Then, the optimal value is at least $\min(d_1, d_2)$. In the region when $\min(d_1, d_2) \le X < \max(d_1, d_2)$, we study the objective function under two cases depending on the relative values of d_1 and d_2. First, let us consider the case $d_1 \le d_2$, then $\min(d_1, d_2) = d_1$ and $\max(d_1, d_2) = d_2$, and the objective function is

$$\begin{aligned} E[P(d_1 \le X < d_2)] &= -cX + \int_0^{t_1/r_1}\int_{t_2/r_2}^{\infty}(r_2e_2-t_2)Xf(e_1,e_2)\,de_2\,de_1 + \int_{t_1/r_1}^{\infty}\int_0^{t_2/r_2}(r_1e_1-t_1)d_1f(e_1,e_2)\,de_2\,de_1 \\ &\quad + \int_{t_1/r_1}^{\infty}\int_{t_2/r_2}^{(r_1e_1-t_1+t_2)/r_2}[(r_1e_1-t_1)d_1+(r_2e_2-t_2)(X-d_1)] \cdot f(e_1,e_2)\,de_2\,de_1 \\ &\quad + \int_{t_1/r_1}^{\infty}\int_{(r_1e_1-t_1+t_2)/r_2}^{\infty}(r_2e_2-t_2)Xf(e_1,e_2)\,de_2\,de_1. \end{aligned}$$

Because the second-order derivative with respect to X is zero, the first-order derivative is sufficient to determine the optimal behavior of X:

$$\frac{\partial E[P(d_1 \le X < d_2)]}{\partial X} = -c + \int_{t_2/r_2}^{\infty}(r_2e_2-t_2)f_2(e_2)\,de_2 = -c + E(\lambda_{B2}(X)).$$

Therefore, if $E(\lambda_{B2}(X)) < c$, the optimal value of X is $\min(d_1, d_2) = d_1$. Now consider the case $d_1 > d_2$. Then, $\min(d_1, d_2) = d_2$ and $\max(d_1, d_2) = d_1$, and the objective function is

$$\begin{aligned} E[P(d_2 \le X < d_1)] &= -cX + \int_0^{t_1/r_1}\int_{t_2/r_2}^{\infty}(r_2e_2-t_2)d_2f(e_1,e_2)\,de_2\,de_1 + \int_{t_1/r_1}^{\infty}\int_0^{t_2/r_2}(r_1e_1-t_1)Xf(e_1,e_2)\,de_2\,de_1 \\ &\quad + \int_{t_1/r_1}^{\infty}\int_{t_2/r_2}^{\frac{r_1e_1-t_1+t_2}{r_2}}[(r_1e_1-t_1)X]f(e_1,e_2)\,de_2\,de_1 \\ &\quad + \int_{t_1/r_1}^{\infty}\int_{(r_1e_1-t_1+t_2)/r_2}^{\infty}[(r_2e_2-t_2)d_2+(r_1e_1-t_1)(X-d_2)] \cdot f(e_1,e_2)\,de_2\,de_1. \end{aligned}$$

Because the second-order derivative with respect to X is zero, the first-order derivative is sufficient to determine the optimal behavior of X:

$$\frac{\partial E[P(d_2 \le X < d_1)]}{\partial X} = -c + \int_{t_1/r_1}^{\infty} (r_1 e_1 - t_1) f_1(e_1)\, de_1 = -c + E(\lambda_{B1}(X)).$$

Therefore, if $E(\lambda_{B1}(X)) < c$, the optimal value of X is $\min(d_1, d_2) = d_2$.

(c) If $E(\lambda_{B1}(X)) > c$ when $d_1 > d_2$, or $E(\lambda_{B2}(X)) > c$ when $d_1 \le d_2$, the optimal value of X is greater than $\min(d_1, d_2)$ and is at least $\max(d_1, d_2)$. The objective function when $\max(d_1, d_2) \le X < (d_1 + d_2)$ is

$$\begin{aligned} &E[P(\max(d_1, d_2) \le X < (d_1 + d_2))] \\ &\quad = -cX + \int_0^{t_1/r_1} \int_{t_2/r_2}^{\infty} (r_2 e_2 - t_2) d_2 f(e_1, e_2)\, de_2\, de_1 \\ &\qquad + \int_{t_1/r_1}^{\infty} \int_0^{t_2/r_2} (r_1 e_1 - t_1) d_1 f(e_1, e_2)\, de_2\, de_1 \\ &\qquad + \int_{t_1/r_1}^{\infty} \int_{t_2/r_2}^{(r_1 e_1 - t_1 + t_2)/r_2} [(r_1 e_1 - t_1) d_1 + (r_2 e_2 - t_2)(X - d_1)] \cdot f(e_1, e_2)\, de_2\, de_1 \\ &\qquad + \int_{t_1/r_1}^{\infty} \int_{(r_1 e_1 - t_1 + t_2)/r_2}^{\infty} [(r_2 e_2 - t_2) d_2 + (r_1 e_1 - t_1)(X - d_2)] \cdot f(e_1, e_2)\, de_2\, de_1. \end{aligned}$$

Because the second-order derivative with respect to X is zero, the first-order derivative is sufficient to determine the optimal behavior of X:

$$\begin{aligned} &\frac{\partial E[P(\max(d_1, d_2) \le X < (d_1 + d_2))]}{\partial X} \\ &\quad = -c + \int_{t_1/r_1}^{\infty} \int_{t_2/r_2}^{(r_1 e_1 - t_1 + t_2)/r_2} (r_2 e_2 - t_2) f(e_1, e_2)\, de_2\, de_1 \\ &\qquad + \int_{t_1/r_1}^{\infty} \int_{(r_1 e_1 - t_1 + t_2)/r_2}^{\infty} (r_1 e_1 - t_1) f(e_1, e_2)\, de_2\, de_1 \\ &\quad = -c + E(\lambda_C(X)). \end{aligned}$$

Therefore, if $E(\lambda_C(X)) < c$, the optimal value of X is $\max(d_1, d_2)$.

(d) When $E(\lambda_C(X)) > c$, the optimal value of X is $(d_1 + d_2)$.

Proof of Corollary 1. It should be noted that $r_1 \bar{e}_1 - t_1 \le E(\lambda_{B1}(X))$ and $r_2 \bar{e}_2 - t_2 \le E(\lambda_{B2}(X))$. When $r_j \bar{e}_j - t_j > c$ for $j = 1, 2$, both $E(\lambda_{B1}(X))$ and $E(\lambda_{B2}(X))$ are greater than c, and the conditions necessary for Theorem 1(c) are satisfied. Therefore, the optimal value of X is at least $\max(d_1, d_2)$.

Proofs of Theorems 2 and 3. The proofs are available in the online appendix.

Proof of Theorem 4. In a single-period model, allocation hedging is exercised when the realized exchange rate is unfavorable so that the revenue from a foreign market does not justify the unit transportation cost. This corresponds to the following range of exchange-rate values: $e_2 < t_2/r_2$. In a multiperiod setting, when the total demand is produced, d_2 units can be kept in inventory for future sale at the unit inventory holding cost of h. However, this also saves the future manufacturing cost of c for each unit kept in inventory. Thus, allocation hedging is exercised when the revenue in a market (based on the realized exchange rate) is less than the sum of unit transportation cost and unit manufacturing cost less unit inventory holding cost. The resulting range of exchange-rate values that allocation hedging is exercised corresponds to: $e_2 < (t_2 + c - h)/r_2$. Because $c > h$ by definition, the expression for allocation hedging in a multiperiod setting corresponds to a bigger region than that of the single-period model. Therefore, the probability of allocation hedging in a multiperiod model is higher than that of the single-period model.

Proof of Theorem 5. We first establish the expected profit function for the potentially optimal decisions of a single-period model. From the analysis of §3.1, we know that the expected profit from producing the total demand of $(d_1 + d_2)$ is

$$E[P(X = (d_1 + d_2))] = (r_1 - t_1 - c) d_1 + \left(\int_{t_2/r_2}^{\infty} (r_2 e_2 - t_2) f_2(e_2)\, de_2 - c \right) d_2.$$

It should be noted here that allocation hedging occurs when $e_2 < t_2/r_2$. The expected profit from a potentially optimal production-hedging policy of producing $\max(d_1, d_2)$ changes with the relative values of d_1 and d_2. We show the proof for the case of $d_1 \ge d_2$:

$$\begin{aligned} &E[P(X = \max(d_1, d_2) = d_1 \ge d_2)] \\ &\quad = -c d_1 + \int_0^{(r_1 - t_1 + t_2)/r_2} (r_1 - t_1) d_1 f_2(e_2)\, de_2 \\ &\qquad + \int_{(r_1 - t_1 + t_2)/r_2}^{\infty} [(r_2 e_2 - t_2) d_2 + (r_1 - t_1)(d_1 - d_2)] f_2(e_2)\, de_2. \end{aligned}$$

The difference between the expected profits of the two potentially optimal decisions, producing $(d_1 + d_2)$ and $\max(d_1, d_2) = d_1 \ge d_2$, can be expressed with

$$E[P(X = (d_1 + d_2))] - E[P(X = \max(d_1, d_2) = d_1 \ge d_2)].$$

Later, we compare the value of this difference with a similar expression developed for the multiperiod model. For the multiperiod setting, we consider the T-period problem. We assume that all parameters are the same in each period. While not enforcing any assumptions regarding the distribution of exchange rates in future periods, we use the same exchange-rate distribution in the first period for comparison purposes. We define the value obtained from the decisions made in the first period with $M_1(X)$. The production decisions of $(d_1 + d_2)$ and $\max(d_1, d_2) = d_1 \ge d_2$ in the first period can be shown as follows:

$$\begin{aligned} &M_1(X = (d_1 + d_2)) \\ &\quad = (r_1 - t_1 - c) d_1 + \left(\int_0^{(t_2 + c - h)/r_2} (c - h) f_2(e_2)\, de_2 + \int_{(t_2 + c - h)/r_2}^{\infty} (r_2 e_2 - t_2) f_2(e_2)\, de_2 - c \right) d_2 \\ &\quad = E[P(X = (d_1 + d_2))] + \left(\int_0^{t_2/r_2} (c - h) f_2(e_2)\, de_2 + \int_{t_2/r_2}^{(t_2 + c - h)/r_2} [(t_2 + c - h) - r_2 e_2] f_2(e_2)\, de_2 \right) d_2 \\ &\quad > E[P(X = (d_1 + d_2))]. \end{aligned}$$

Because $c > h$ by definition and $(t_2 + c - h) - r_2 e_2$ is always positive for all values of $e_2 < (t_2 + c - h)/r_2$, the two integral terms are positive. Therefore, when a total of (d_1+d_2) is produced, the value obtained in the first period of a multiperiod model is higher than that of the single-period model:

$$\begin{aligned}M_1(X=\max(d_1,d_2)=d_1\ge d_2)\\&=-cd_1+\int_0^{(r_1-t_1+t_2)/r_2}(r_1-t_1)d_1f_2(e_2)\,de_2\\&\quad+\int_{(r_1-t_1+t_2)/r_2}^{\infty}[(r_2e_2-t_2)d_2+(r_1-t_1)(d_1-d_2)]f_2(e_2)\,de_2\\&=E[P(X=\max(d_1,d_2)=d_1\ge d_2)].\end{aligned}$$

When only $\max(d_1,d_2)=d_1\ge d_2$ units are produced in the first period of a multiperiod model, the value obtained is equal to that of a single-period model. We next define $H_2(e_2)$ as the optimal profit that can be obtained from Period 2 onwards given that the exchange rate in Period 1 is e_2. The expectation of $H_2(e_2)$ over exchange-rate values of e_2 is denoted with $E[H_2(e_2)]$. Similarly, $H_1(X_1)$ represents the expected profit obtained from Period 1 onwards when X units are produced in Period 1. We express it as follows:

$$\begin{aligned}H_1(X=(d_1+d_2))\\&=-c(d_1+d_2)\\&\quad+\int_0^{(t_2+c-h)/r_2}\big[(r_1-t_1)d+(c-h)d_2+H_2(e_2)\big]f_2(e_2)\,de_2\\&\quad+\int_{(t_2+c-h)/r_2}^{\infty}[(r_1-t_1)d_1+(r_2e_2-t_2)d_2+H_2(e_2)]f_2(e_2)\,de_2\\&=M_1(X=(d_1+d_2))+E[H_2(e_2)],\\H_1\big(X=\max(d_1,d_2)=d_1\big)\\&=M_1\big(X=\max(d_1,d_2)=d_1\big)+E[H_2(e_2)].\end{aligned}$$

The difference of expected profits from producing (d_1+d_2) and $\max(d_1,d_2)=d_1\ge d_2$ in the first period can be expressed as follows:

$$\begin{aligned}&H_1(X=(d_1+d_2))-H_1(X=\max(d_1,d_2)=d_1\ge d_2)\\&\quad=M_1(X=(d_1+d_2))-M_1(X=\max(d_1,d_2)=d_1\ge d_2)\\&\quad=E[P(X=(d_1+d_2))]-E[P(X=\max(d_1,d_2)=d_1\ge d_2)]\\&\qquad+\bigg(\int_0^{t_2/r_2}(c-h)f_2(e_2)\,de_2\\&\qquad\quad+\int_{t_2/r_2}^{(t_2+c-h)/r_2}[(t_2+c-h)-r_2e_2]f_2(e_2)\,de_2\bigg)d_2\\&\quad>E[P(X=(d_1+d_2))]-E[P(X=\max(d_1,d_2)=d_1\ge d_2)].\end{aligned}$$

The above expression states that the difference in the optimal values from producing (d_1+d_2) and $\max(d_1,d_2)=d_1\ge d_2$ is higher in a multiperiod model than a single-period model. For part (a) of the theorem, when

$$E[P(X=(d_1+d_2))]>E[P(X=\max(d_1,d_2)=d_1\ge d_2)],$$

the above expression states that

$$H_1(X=(d_1+d_2))>H_1(X=\max(d_1,d_2)=d_1\ge d_2).$$

For part (b), when

$$H_1(X=(d_1+d_2))<H_1(X=\max(d_1,d_2)=d_1\ge d_2),$$

the above expression provides the result that

$$E[P(X=(d_1+d_2))]<E[P(X=\max(d_1,d_2)=d_1\ge d_2)].$$

Therefore, the difference of expected profits from producing (d_1+d_2) and $\max(d_1,d_2)=d_1\ge d_2$ in the first period increases in a multiperiod model. As a result, it is more likely to produce (d_1+d_2) in a multiperiod model than a single-period model. The same approach can be used to prove the theorem when $d_1<d_2$.

Proof of Theorem 6. The incremental benefit from producing one more unit can be expressed with $E(\lambda(X))$, where

$$\begin{aligned}E(\lambda(X))\\&=\int_0^{(r_1-t_1+t_2)/r_2}(r_1-t_1)\Pr(D_1>X)f_2(e_2)\,de_2\\&\quad+\int_{(r_1-t_1+t_2)/r_2}^{\infty}(r_2e_2-t_2)\Pr(D_2>X)f_2(e_2)\,de_2\\&\quad+\int_{t_2/r_2}^{(r_1-t_1+t_2)/r_2}(r_2e_2-t_2)\Pr(D_1\le X<D_1+D_2)f_2(e_2)\,de_2\\&\quad+\int_{(r_1-t_1+t_2)/r_2}^{\infty}(r_1-t_1)\Pr(D_2\le X<D_1+D_2)f_2(e_2)\,de_2\end{aligned}$$

in the problem variant explained in §4.1. However, let us consider the variant of the problem where there is only Market 1 (domestic) and no other (foreign) markets. The optimal production quantity, denoted by X_d^*, satisfies $E_1(\lambda(X_d^*))=(r_1-t_1)\Pr(D_1>X_d^*)=c$. Similarly, let us now consider the problem variant where there is a foreign market (Market 2) and no other markets. The optimal production quantity, denoted by X_f^*, satisfies

$$E_2\big(\lambda\big(X_f^*\big)\big)=\int_{t_2/r_2}^{\infty}(r_2e_2-t_2)\Pr\big(D_2>X_f^*\big)f_2(e_2)\,de_2=c.$$

Adding and subtracting the term

$$\int_{(r_1-t_1+t_2)/r_2}^{\infty}(r_1-t_1)\Pr(D_1>X)f_2(e_2)\,de_2$$

to the above $E(\lambda(X))$ expression provides

$$\begin{aligned}E(\lambda(X))\\&=E_1(\lambda(X))\\&\quad+\int_{(r_1-t_1+t_2)/r_2}^{\infty}(r_1-t_1)\Pr\left(\begin{Bmatrix}D_1\\D_2\end{Bmatrix}\le X<D_1+D_2\right)f_2(e_2)\,de_2\\&\quad+\int_{t_2/r_2}^{(r_1-t_1+t_2)/r_2}(r_2e_2-t_2)\Pr(D_1\le X<D_1+D_2)f_2(e_2)\,de_2\\&\quad+\int_{(r_1-t_1+t_2)/r_2}^{\infty}(r_2e_2-t_2)\Pr(D_1\le X<D_2<D_1+D_2)f_2(e_2)\,de_2\\&\quad+\int_{(r_1-t_1+t_2)/r_2}^{\infty}((r_2e_2-t_2)-(r_1-t_1))\Pr\left(\begin{Bmatrix}D_1\\D_2\end{Bmatrix}>X\right)f_2(e_2)\,de_2.\end{aligned}$$

Evaluating its value at $X=X_d^*$, it can be observed that $E(\lambda(X_d^*))\ge E_1(\lambda(X_d^*))=c$ because all four integral terms are positive in their respective exchange-rate regions. Therefore, the optimal production quantity is greater than or equal to X_d^*. Similarly, when we add and subtract the term

$$\int_{t_2/r_2}^{(r_1-t_1+t_2)/r_2}(r_2e_2-t_2)\Pr(D_2>X)f_2(e_2)\,de_2$$

to $E(\lambda(X))$, we obtain

$$
\begin{aligned}
&E(\lambda(X))\\
&=E_2(\lambda(X))+\int_{(r_1-t_1+t_2)/r_2}^{\infty}(r_1-t_1)\Pr(D_2\le X<D_1)f_2(e_2)\,de_2\\
&\quad+\int_{t_2/r_2}^{(r_1-t_1+t_2)/r_2}(r_1-t_1)\Pr(D_2\le X<D_1<D_1+D_2)f_2(e_2)\,de_2\\
&\quad+\int_{t_2/r_2}^{(r_1-t_1+t_2)/r_2}(r_2e_2-t_2)\Pr\left(\left\{\begin{matrix}D_1\\D_2\end{matrix}\right\}\le X<D_1+D_2\right)f_2(e_2)\,de_2\\
&\quad+\int_{t_2/r_2}^{(r_1-t_1+t_2)/r_2}((r_1-t_1)-(r_2e_2-t_2))\Pr\left(\left\{\begin{matrix}D_1\\D_2\end{matrix}\right\}>X\right)f_2(e_2)\,de_2\\
&\quad+\int_{0}^{t_2/r_2}(r_1-t_1)\Pr(D_1>X)f_2(e_2)\,de_2.
\end{aligned}
$$

Evaluating its value at $X=X_f^*$, it can be observed that $E(\lambda(X_f^*))\ge E_2(\lambda(X_f^*))=c$ because all five terms are positive in their respective exchange-rate regions. Therefore, the optimal production quantity is greater than or equal to X_f^*. As a result, the optimal production quantity is greater than or equal to both X_d^* and X_f^*, and thus it is greater than or equal to the maximum of X_d^* and X_f^*.

Proof of Theorem 7. Consider Strategy (2) $X=d_1<d_2$, thus $p_1>p_2$. The corresponding expected profit for this policy is

$$
\begin{aligned}
&E[P(p_1,p_2,X=d_1)\mid p_1>p_2]\\
&\quad=-cd_1(p_1)+\int_0^{((p_1-t_1+t_2)/p_2)}(p_1-t_1)d_1(p_1)f_2(e_2)\,de_2\\
&\qquad+\int_{((p_1-t_1+t_2)/p_2)}^{\infty}(p_2e_2-t_2)d_1(p_1)f_2(e_2)\,de_2.
\end{aligned}
$$

We show that when $X=d_1<d_2$, the first-order derivative of the objective function with respect to p_2 is positive, indicating that p_2 should be increased as much as possible. This results in the violation of d_1 being the minimum demand amount. The first-order derivative is

$$
\frac{\partial E[P(p_1,p_2,X)]}{\partial p_2}=\int_{((p_1-t_1+t_2)/p_2)}^{\infty}e_2d_1(p_1)f_2(e_2)\,de_2>0.
$$

Thus, p_2 can no longer be less than p_1, and this solution cannot be optimal. Now consider Strategy (4) $X=d_2\le d_1$ and $p_1\le p_2$. The corresponding expected profit is

$$
\begin{aligned}
&E[P(p_1,p_2,X=d_2)\mid p_1\le p_2]\\
&\quad=-cd_2(p_2)+\int_0^{((p_1-t_1+t_2)/p_2)}(p_1-t_1)d_2(p_2)f_2(e_2)\,de_2\\
&\qquad+\int_{((p_1-t_1+t_2)/p_2)}^{\infty}(p_2e_2-t_2)d_2(p_2)f_2(e_2)\,de_2.
\end{aligned}
$$

The first-order derivative with respect to p_1 is

$$
\frac{\partial E[P(p_1,p_2,X)]}{\partial p_1}=\int_0^{((p_1-t_1+t_2)/p_2)}d_2(p_2)f_2(e_2)\,de_2>0.
$$

This suggests that p_1 needs to be as large as possible and can no longer be less than p_2. Therefore, because Strategies (2) and (4) cannot be optimal, the optimal production quantity can never be equal to the minimum of the two demand values. Considering the remaining strategies and using the optimal choices of p_1 and p_2, an optimal production policy can be either $X=\max(d_1,d_2)$ or $X=(d_1+d_2)$.

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