OPERATIONS RESEARCH
Articles in Advance, pp. 1–5
ISSN 0030-364X (print) | ISSN 1526-5463 (online)
http://dx.doi.org/10.1287/opre.2015.1366

Technical Note—Price-Setting Newsvendor Problems with Uncertain Supply and Risk Aversion

Burak Kazaz
Whitman School of Management, Syracuse University, Syracuse, New York 13244, bkazaz@syr.edu
Scott Webster
W. P. Carey School of Business, Arizona State University, Tempe, Arizona 85287, scott.webster@asu.edu

The price-setting newsvendor problem, which models the economic trade-offs associated with uncertain demand of a perishable product, is fundamental to supply chain analysis. However, in settings such as agriculture, there is significant economic risk associated with supply uncertainty. We analyze how risk aversion and the source of uncertainty—demand and/or supply—affect tractability and optimal decisions. We find that concavity of the objective function is preserved under the introduction of risk aversion if the source of uncertainty is demand, but it is not necessarily preserved if the source of uncertainty is supply. We identify a structural difference that explains this result, and show that this difference can lead to opposing directional effects of risk aversion on optimal decisions.

Subject classifications: inventory; production; marketing; pricing; supply uncertainty; risk aversion.
Area of review: Operations and Supply Chains.
History: Received January 2013; revisions received April 2014, November 2014, February 2015; accepted February 2015. Published online in *Articles in Advance*.

1. Introduction

The classic price-setting newsvendor problem (PSNP) is to determine the price and quantity of a perishable product assuming uncertain demand, deterministic supply, and a risk-neutral decision maker. We study a generalization of the classic PSNP that includes uncertain supply and a risk-averse decision maker. Our motivation comes from an agricultural setting, and in particular, a winemaker who leases vineyard space and sells direct-to-consumer through wine-club memberships. The leasing of farm space is common among many agribusinesses, including winemaking. In comparison with selling wine through distributors, the direct-to-consumer channel loses the benefit of pricing flexibility in response to the realized crop yield but offers less volatile demand in the marketplace. Smith-Madrone Winery, for example, sells exclusively through its wine club and tasting room and does not change its wine price according to crop yield (Heron 2010). This is equivalent to fixing the selling price of the wine before the growing season begins, at the time the lease quantity is determined, yielding a PSNP with uncertain supply. Compared to large diversified firms such as Constellation Brands that owns and distributes wines under many different brands, smaller wineries such as Smith-Madrone have a relatively narrow product range and limited financial resources and are therefore more sensitive to risk in their decision making.

This note is most closely related to two papers on the PSNP with demand uncertainty. Kocabıyıkoğlu and Popescu (2011) consider a risk-neutral newsvendor. They introduce an elasticity measure and use this measure to identify conditions that assure the optimal price and quantity can be obtained from a solution to the first-order condition. The nature of the price-demand function and the probability distribution determine whether or not the conditions are satisfied. Eeckhoudt et al. (1995) study the impact of risk aversion on the newsvendor's quantity decision. They assume that price is exogenous and show that a risk-averse newsvendor will order less than a risk-neutral newsvendor.

We analyze the impact of supply uncertainty and risk aversion on the classic PSNP. We generalize the elasticity measure of Kocabıyıkoğlu and Popescu (2011) to accommodate uncertain supply, and we show that the tractability conditions they identify continue to hold under a risk-neutral objective when supply, as well as demand, is stochastic. We investigate conditions for tractability when a newsvendor is risk averse. Toward this end, we consider the question of whether a concave expected profit function is assured to remain concave under a risk-averse utility function. We prove that the answer is "yes" when the source of uncertainty is demand and "no" when the source of uncertainty is supply. We show that the underlying cause of this result also drives opposing directional effects of risk aversion on univariate optimal price and quantity decisions. In particular, for a given price, a risk-averse newsvendor will order less than a risk-neutral newsvendor when demand is uncertain, whereas the opposite is generally the case when supply is uncertain. Alternatively, for a given quantity, a risk-averse newsvendor will price higher than a risk-neutral decision maker when supply is uncertain, whereas the opposite is generally the case when demand is uncertain.

2. Model

We consider the price-setting newsvendor problem with a risk-averse decision maker and either supply or demand uncertainty. We model the newsvendor's risk aversion through concave utility function $U(x)$, $U'(x) > 0$, and $U''(x) \leqslant 0$ for all x. If $U'(x) = k$ for all x, then the newsvendor is risk neutral and expected utility is maximized by maximizing expected profit. Otherwise, the newsvendor is risk averse.

The firm's decision variables are quantity q and price p, and we capture randomness in supply and demand through random variables Y and Z, respectively. We use lower case y and z to denote realizations of these random variables. Random yield and random demand functions are $q \times Y$ and $d(p, Z)$. The finite support of Y is denoted $[y_L, y_H]$ with $y_L \geqslant 0$, and the mean of Y is normalized to 1. We assume that the demand function $d(p, z)$ is strictly decreasing in price p and is strictly increasing in realization z of Z, which has finite support $[z_L, z_H]$. We write the inverse of $d(p, z)$ with respect to z as $z(p, d)$, i.e., $d(p, z(p, d)) = d$ and $z(p, d(p, z)) = z$. As in Kocabıyıkoğlu and Popescu (2011), we assume that the revenue function $pd(p, z)$ is strictly concave in price for any realization z and that $d(p, z)$ is twice differentiable in p and z. The pdf and cdf of Y and Z are $\phi_i(\cdot)$ and $\Phi_i(\cdot)$ for $i \in \{Y, Z\}$, respectively. If supply is deterministic, then the yield function is simply q; if demand is deterministic, then we write the demand function as $d(p)$.

The end-of-season salvage value per unit is s. The purchase cost per unit is c. We define price and cost as net-of-salvage-value. Accordingly, the random profit under uncertain supply and deterministic demand is $\tilde{S}(p, q, Y) = p\min\{d(p), qY\} - cq + sq(Y - 1)$.[1] The expected utility is $S(p, q) = E[U(\tilde{S}(p, q, Y))]$, and the newsvendor's problem is

$$P_S\colon \max_{(p,\,q)\in \mathbf{X}} S(p, q)$$

where $\mathbf{X}$ denotes a convex set of viable (p, q) values. The corresponding functions and problem for the model with deterministic supply and uncertain demand are $\tilde{D}(p, q, Z) = p\min\{d(p, Z), q\} - cq$, $D(p, q) = E[U(\tilde{D}(p, q, Z))]$, and

$$P_D\colon \max_{(p,\,q)\in \mathbf{X}} D(p, q).$$

3. Sufficient Conditions for Concavity

A general expression for expected lost sales is $E[(d(p, Z) - qY)^+]$, which reduces to $E[(d(p) - qY)^+]$ for P_S and to $E[(d(p, Z) - q)^+]$ for P_D. We identify sufficient conditions for joint concavity of the expected utility function that use the price elasticity of marginal lost sales:

$$\varepsilon(p, q) = \frac{-p\big(\partial^2 E\big[(d(p, Z) - qY)^+\big]/\partial p \partial q\big)}{\big(\partial E\big[(d(p, Z) - qY)^+\big]/\partial q\big)}. \tag{1}$$

This is a general function that applies to both P_S and P_D by substituting in the appropriate deterministic terms. This function was originally introduced by Kocabıyıkoğlu and Popescu (2011) for a risk-neutral analysis of P_D, though with a slightly different interpretation. In particular, they define the price elasticity of the lost-sales rate. For P_D, the lost-sales rate is $P(d(p, Z) > q) = 1 - \Phi_Z(z(p, q))$. Note that the lost-sales rate is, excepting the sign, the marginal lost sales, i.e.,

$$\partial E\big[(d(p, Z) - q)^+\big]/\partial q = -(1 - \Phi_Z(z(p, q))).$$

Thus, for P_D, the price elasticity of marginal lost sales given in (1) reduces to the price elasticity of the lost-sales rate. However, for P_S, the price elasticity of marginal lost sales and the price elasticity of the lost-sales rate are different functions.

Before introducing the results in this section, we require additional notation. For P_S, we define two measures of marginal utility,

$$u(p, q) = U'(\tilde{S}(p, q, d(p)/q)) \quad \text{and}$$

$$u^+(p, q) = \int_{y_L}^{d(p)/q} U'(\tilde{S}(p, q, y)) \frac{y\phi_Y(y)}{\int_{y_L}^{d(p)/q} t\phi_Y(t)\, dt}\, dy.$$

The first function, $u(p, q)$, is the marginal utility when demand is equal to supply. Because $y\phi_Y(y)/\int_{y_L}^{d(p)/q} t \cdot \phi_Y(t)\, dt$ is a valid density function defined over $[y_L, d(p)/q]$ (i.e., nonnegative and integrates to 1), $u^+(p, q)$ is a measure of marginal utility when demand is more than supply. Thus, from the concavity of U, it follows that $u(p, q)/u^+(p, q) \leqslant 1$, with equality if the decision maker is risk neutral.

The random yield function has a more restrictive structure than the random demand function (i.e., supply has a multiplicative form). Consequently, one might anticipate that the tighter structural limitations on the yield function translate into simpler or less restrictive conditions for assurance of concave $S(p, q)$ than for concave $D(p, q)$. We find the opposite.

Proposition 1. (a) *If*

$$\varepsilon(p, q)u(p, q)/u^+(p, q) \geqslant \frac{1}{2} \quad \textit{for all } (p, q) \in \mathbf{X}, \tag{2}$$

then $S(p, q)$ is concave. (b) *If $E[\tilde{D}(p, q, Z)]$ is concave, then $D(p, q)$ is concave.* (c) *If*

$$\varepsilon(p, q) \geqslant \frac{1}{2} \quad \textit{for all } (p, q) \in \mathbf{X}, \tag{3}$$

then $D(p, q)$ is concave, $E[\tilde{S}(p, q, Y)]$ is concave, but $S(p, q)$ is not necessarily concave.

Kocabıyıkoğlu and Popescu (2011) show that (3) assures that $E[\tilde{D}(p, q, Z)]$ is concave. Proposition 1(c) shows that (3) is also sufficient for concave $D(p, q)$. Indeed, (3) is a fundamental indicator of tractability because it assures that the expected profit function is concave even when both supply and demand uncertainty are present (see Corollary 1 in the online appendix (all proofs are available as supplemental material at http://dx.doi.org/10.1287/opre.2015.1366). However, (3) is not sufficient to assure concavity when supply is uncertain and the newsvendor is risk averse. The more restrictive condition (2) assures concave $S(p, q)$; this condition also assures concave expected utility when both supply and demand uncertainty are present (see Proposition 4 in the online appendix).

Why do we see this difference in sufficient conditions for concavity? The reason stems from the polar-opposite character of the realized profit functions for these two problems. Specifically, in the neighborhood of (p, q), $\tilde{S}(p, q, Y)$ is not concave at low-profit realizations (i.e., $Y < d(p)/q$) and concave at high-profit realizations (i.e., $Y > d(p)/q$), whereas the opposite is true for $\tilde{D}(p, q, Z)$, i.e., $\tilde{D}(p, q, Z)$ is concave at low-profit realizations (i.e., $Z < z(p, q)$) and not concave at high-profit realizations (i.e., $Z > z(p, q)$).[2] With the introduction of risk aversion, this structural difference is problematic for P_S because the character of the function at low-profit realizations is amplified relative to the character of the function at high-profit realizations because of the diminishing marginal utility of profit. This intuition is generalized in Lemma 2 of the online appendix where we show that the expected utility of a multivariate function is concave if (i) the expected value of the function is concave and, informally speaking, (ii) any nonconcave realizations of the function correspond to high-value realizations of the function. In Section 4, we will see that this structural difference between P_S and P_D explains why the introduction of risk aversion can lead to opposite directional effects on optimal decisions.

4. Effect of Risk Aversion on Optimal Decisions

We compare optimal decisions of risk-neutral and risk-averse newsvendors. We assume a strictly concave utility function; otherwise the inequalities in our results below would not be strict. We assume the risk-neutral and risk-averse newsvendor objective functions have a unique stationary point that is a global maximum (e.g., conditions of Proposition 1 hold). For both P_S and P_D, we let $p^o(q)$ denote the optimal price for a given quantity, and we let $q^o(p)$ denote the optimal quantity for a given price. We use superscript * to denote optimal values for risk-neutral models. We use (p^*, q^*) and (p^o, q^o) to denote the joint optimal price and quantity for the risk-neutral and risk-averse models, respectively.

Eeckhoudt et al. (1995) consider P_D and show that a risk-averse newsvendor will order less than a risk-neutral newsvendor for a given price (stated as Proposition 2(c) below). Proposition 2 expands on this result and highlights how the source of uncertainty affects the impact of risk aversion on optimal univariate decisions.

Proposition 2. *For P_S:*

(a) *If $[y_L, y_H] \subset [(c+s)/(p+s), (c+s)/s]$, then $q^o(p) > q^*(p)$.*

(b) *$p^o(q) > p^*(q)$.*

For P_D:

(c) *$q^o(p) < q^*(p)$.*

(d) *If $d(p, z) = d(p)z$, then $p^o(q) < p^*(q)$.*

(e) *If $d(p, z) = d(p) + z$, then $p^o(q) < p^*(q)$.*

When the conditions of Proposition 2 hold, we see that the directional impact of risk aversion on optimal univariate decisions is affected by the source of uncertainty. If the source of uncertainty is supply, then optimal univariate decisions increase. If the source of uncertainty is demand, then optimal univariate decisions decrease. As discussed above, the introduction of risk aversion amplifies the concern for low profits, which are associated with low realizations of Y in the case of random supply and low realizations of Z in the case of random demand. Low values of Y translate into low sales and low profit because of insufficient supply, and the risk-averse decision maker increases the quantity (for fixed price) or increases the price (for fixed quantity) to protect against this risk. In contrast, low values of Z translate into low sales and low profit because of excess supply, and the risk-averse decision maker lowers the quantity (for fixed price) or lowers the price (for fixed quantity) to protect against this risk.

Proposition 2(a) specifies a sufficient condition for the quantity inequality for P_S: the support of Y lies within the ratio of cost-to-price and the ratio of cost-to-salvage value (recall that p and c are defined as net of salvage value s). If the support extends beyond this range, then the realized profit per unit and the realized cost per unit (i.e., purchase cost less salvage revenue) are negative at realizations of Y outside of this range (i.e., margin is negative at low realizations of supply, and cost is negative at high realizations of supply). Relative to the risk-neutral newsvendor, the risk-averse newsvendor is more concerned about the possibility of negative margins and is less concerned about the possibility of negative costs. Both of these differences put downward pressure on the order quantity, potentially to the point where the risk-averse quantity is lower than the risk-neutral quantity (see Example 1 in the online appendix).

Propositions 2(d) and 2(e) state that multiplicative random demand or additive random demand is sufficient for the price inequality to hold for P_D. Interestingly, this price relationship does not hold under a combined multiplicative-additive form of demand, e.g., $d(p, z) = \alpha(p)z + \beta(p)$ (see Example 2 in the online appendix). The reason is that the conclusions in Propositions 2(d) and 2(e) derive from distinct underlying sufficient conditions on the revenue function $pd(p, z)$, and both of these conditions break down under a multiplicative-additive form of random demand (see Lemma 4 and the proof of Proposition 2 in the online appendix).

We next consider the impact of the introduction of risk aversion on the joint optimal price and quantity. When the conditions of Proposition 2 hold, there is a force that puts a pressure on price and quantity to move in the same direction when the other decision is held fixed, either up or down, depending on the source of uncertainty. However, there may be a countervailing force that puts pressure on decisions to move in opposite directions. This force stems from the inverse relationship in the price-demand function (i.e., $d_p(p,z) < 0$), e.g., an increase in price exerts downward pressure on quantity, and vice versa. Thus, in general, the impact of the introduction of risk aversion on joint optimal decisions can go either way (see Example 3 in the online appendix). However, the impact of risk aversion on the joint optimal decisions is determinate if the optimal risk-neutral price and quantity are increasing in their respective arguments, or equivalently, if the expected profit function is supermodular (see Theorem 2.8.2 in Topkis 1998). (Kocabıyıkoğlu and Popescu 2011 show, in Proposition 1, that expected profit under P_D is supermodular if and only if $\varepsilon(p,q) \leqslant 1$; it is straightforward to verify that this result continues to hold under P_S.) As an example, consider P_S at the risk-neutral optimal price and quantity given that the conditions in Proposition 2 hold. The optimal response to the introduction of risk aversion is an increase in price (for fixed quantity) and an increase in quantity (for fixed price). In a risk-neutral model, the optimal response to an increase in price (quantity) is an increase in quantity (price). This alignment of directional forces assures that the risk-averse price and quantity are larger than the risk-neutral price and quantity.

Proposition 3.

(a) *For* P_S: *If* $E[\tilde{S}(p,q,Y)]$ *and* $S(p,q)$ *each have a unique global maximum,* $[y_L, y_H] \subset [(c+s)/(p^*+s), (c+s)/s]$, $q^*(p)$ *is nondecreasing for all* $(p, q^*(p)) \in \mathbf{X}$, *and* $p^*(q)$ *is nondecreasing for all* $(p^*(q), q) \in \mathbf{X}$, *then* $p^o > p^*$ *and* $q^o > q^*$.

(b) *For* P_D: *If* $E[\tilde{D}(p,q,Z)]$ *and* $D(p,q)$ *each have a unique global maximum, either* $d(p,z) = d(p)z$ *or* $d(p,z) = d(p) + z$, $q^*(p)$ *is nondecreasing for all* $(p, q^*(p)) \in \mathbf{X}$, *and* $p^*(q)$ *is nondecreasing for all* $(p^*(q), q) \in \mathbf{X}$, *then* $p^o < p^*$ *and* $q^o < q^*$.

5. Summary

In this note we extend technical results on the tractability of the classic PSNP in Kocabıyıkoğlu and Popescu (2011) to problems with supply uncertainty and risk aversion. In addition, we analyze the impact of risk aversion on optimal decisions.

We show how demand uncertainty and supply uncertainty are structurally different, and that this difference affects some results but not others. In particular, conditions on the price-elasticity-of-marginal-lost-sales function that assure concavity of the risk-neutral PSNP objective function are robust to the introduction of supply uncertainty. However, there are two areas where results are affected by the source of uncertainty. First, if only demand is uncertain, then previously identified tractability conditions for the risk-neutral PSNP extend to the risk-averse newsvendor (see (3)), whereas this is the not case if supply uncertainty is introduced. In particular, the problem with supply uncertainty exhibits a polar-opposite single-switch property than the problem with demand uncertainty, and this difference inhibits tractability under a risk-averse objective. As a consequence, the problem requires a more restrictive condition for assurance of tractability (see (2)). We note that our conditions for tractability are general in the sense that they extend to the problem with both supply and demand uncertainty, i.e., (3) is sufficient for concave expected profit and (2) is sufficient for concave expected utility. Second, the two sources of uncertainty tend to drive opposing directional effects of risk aversion on optimal decisions. Our results lend insight into how risk mitigation tactics differ between traditional settings in the literature where the main source of uncertainty is demand and settings where the main source of uncertainty is supply.

Supplemental Material

Supplemental material to this paper is available at http://dx.doi.org/10.1287/opre.2015.1366.

Acknowledgments

The authors are grateful to the anonymous associate editor and the three reviewers whose feedback improved the paper significantly. The paper has benefited from participants' feedback through presentations at Pennsylvania State University, Rutgers University, and Indiana University. This study was partially supported by the Robert H. Brethen Operations Management Institute and the H. H. Franklin Center for Supply Chain Management at Syracuse University.

Endnotes

1. For example, given price p' and cost c', the net price and cost are $p = p' - s$ and $c = c' - s > 0$, and

$$\begin{aligned}\tilde{S} &= p' \min\{d(p), qY\} - c'q + s(qY - \min\{d(p), qY\}) \\ &= p\min\{d(p), qY\} - cq + s(Y-1)q.\end{aligned}$$

2. At a more detailed level, P_S and P_D exhibit opposing *single-switch* properties, which are defined and discussed in §A.2 of the online appendix.

References

Eeckhoudt L, Gollier C, Schlesinger H (1995) The risk-averse (and prudent) newsboy. *Management Sci.* 41(5):786–794.

Heron K (2010) Try the red: Napa learns to sell. *The New York Times* (February 16) D1.

Kocabıyıkoğlu A, Popescu I (2011) The newsvendor with pricing: A stochastic elasticity perspective. *Oper. Res.* 59(2):301–312.

Topkis DM (1998) *Supermodularity and Complementarity* (Princeton University Press, Princeton, NJ).

Burak Kazaz is the Stephen Becker Professor, the Laura J. and L. Douglas Meredith Professor for Teaching Excellence, executive director of the H. H. Franklin Center for Supply Chain Management, and associate professor of supply chain management at the Whitman School of Management, Syracuse University. His research focuses on supply chain finance and risk mitigation in agricultural, manufacturing, and retail supply chains.

Scott Webster is the Bob Herberger Arizona Heritage Chair in Supply Chain Management at the W. P. Carey School of Business, Arizona State University. His research focuses on managing risk and uncertainty in supply chains, most recently in agriculture, manufacturing, and retail settings.