Online Supplement

The Impact of Yield-Dependent Trading Costs on Pricing and Production Planning under Supply Uncertainty

Appendix A

Proof of Proposition 1. a) Expected profit is concave in Q because the revenue function is concave, i.e.,

$$\partial^2 E\left[\Pi\left(Q\right)\right]/\partial Q^2 = \int_{u_l}^{u_h} [2p'(Qu)u^2 + p''(Qu)u^3]g\left(u\right)du < 0$$

and thus the first-order condition

$$\partial E\left[\Pi\left(Q\right)\right]/\partial Q = -\left(c_l + c_p\bar{u}\right) + \int_{u_l}^{u_h} [p(Qu)u + p'(Qu)Qu^2]g\left(u\right)du = 0$$

provides (3).

b) From the first-order condition, we have $\int_{u_l}^{u_h} p(Q^*u)ug\left(u\right)du = \left(c_l + c_p\bar{u}\right) - \int_{u_l}^{u_h} p'(Q^*u)Q^*u^2 g\left(u\right)du$. Substituting this expression in (2) provides (4). From the fact that $p(Qu)Qu$ is concave in u, it follows from Jensen's inequality that

$$\Psi(Q) - E\left[\Pi\left(Q\right)\right] = \int_{u_l}^{u_h} [p\left(Q\bar{u}\right) - p(Qu)]Qug\left(u\right)du = p\left(Q\bar{u}\right)Q\bar{u} - E\left[p(Q\tilde{u})Q\tilde{u}\right] > 0,$$

and thus $\Psi(Q^0) - E[\Pi(Q^*)] \geq \Psi(Q^*) - E[\Pi(Q^*)] > 0$.

c) Note that if $f(Q, u) = p(Qu)u + p'(Qu)Qu^2$ is concave in u, then, due to Jensen's inequality, we have

$$E\left[f\left(Q^*,\tilde{u}\right)\right] = c_l + c_p\bar{u} = f\left(Q^0,\bar{u}\right) > E\left[f\left(Q^0,\tilde{u}\right)\right],$$

and from $\frac{\partial f\left(Q,u\right)}{\partial Q} = 2p'\left(Qu\right)u^2 + p''\left(Qu\right)Qu^3 < 0$, it follows that $Q^* < Q^0$. From

$$\begin{aligned}\frac{\partial^2 f\left(Q,u\right)}{\partial u^2} &= 4p'\left(Qu\right)Q + 5p''\left(Qu\right)Q^2u + p'''\left(Qu\right)Q^3u^2 \\ &= Q[4p'\left(Qu\right) + 5p''\left(Qu\right)Qu + p'''\left(Qu\right)Q^2u^2] \\ &= 2Q[2p'\left(Qu\right) + p''\left(Qu\right)Qu] + Q^2u[3p''\left(Qu\right) + p'''\left(Qu\right)Qu]\end{aligned}$$

wherein $2p'\left(Qu\right) + p''\left(Qu\right)Qu < 0$, and $3p''\left(Qu\right) + p'''\left(Qu\right)Qu\, p''(Qu) \leq 0$ implies $\frac{\partial^2 f\left(Q,u\right)}{\partial u^2} < 0$ (i.e., $f(Q, u)$ is concave in u) and thus $Q^* < Q^0$. ❒

The derivations using linear demand:

The following analysis shows the optimal amount of farm space to be leased and the optimal profit of the firm under deterministic and stochastic supply using linear demand, i.e., $d(p) = a - bp$.

Remark A1. *a) The optimal amount of farm space to be leased under deterministic supply is* $Q^0 = \frac{\left[a - b\left(\frac{c_l}{\bar{u}} + c_p\right)\right]}{2\bar{u}}$ *; b) The optimal deterministic profit is* $\Psi\left(Q^0\right) = \frac{1}{4b}\left[a - b\left(\frac{c_l}{\bar{u}} + c_p\right)\right]^2$.

Proof of Remark A1: The deterministic objective function

$\Psi(Q) = -c_l Q + \left(\frac{a - Q\bar{u}}{b} - c_p\right)Q\bar{u} = \frac{1}{b}\left(a - b\left(\frac{c_l}{\bar{u}} + c_p\right) - Q\bar{u}\right)Q\bar{u}$ is concave in Q because

$\frac{\partial \Psi(Q)}{\partial Q} = \frac{1}{b}\left(a - b\left(\frac{c_l}{\bar{u}} + c_p\right) - 2Q\bar{u}\right)\bar{u}$ and $\frac{\partial^2 \Psi(Q)}{\partial Q^2} = -\frac{2}{b}(\bar{u})^2 \le 0$. The first-order condition provides the deterministic optimal amount of farm space to be leased: $Q^0 = \frac{\left[a - b\left(\frac{c_l}{\bar{u}} + c_p\right)\right]}{2\bar{u}}$. Substituting the deterministic optimal amount of farm space to be leased back into the objective function leads to

$\Psi\left(Q^0\right) = \frac{1}{4b}\left[a - b\left(\frac{c_l}{\bar{u}} + c_p\right)\right]^2$. ❐

Proposition A1. *Under stochastic supply: a) The first-stage objective function is concave in Q, and the optimal amount of farm space to be leased is* $Q^* = \frac{\left[a - b\left(\frac{c_l}{\bar{u}} + c_p\right)\right]\bar{u}}{2\left(\bar{u}^2 + \sigma^2\right)}$ *; b) The optimal amount of farm space to be leased is less than that of the deterministic supply, i.e.,* $Q^* < Q^0$ *; c) The optimal profit is* $E\left[\Pi\left(Q^*\right)\right] = \frac{\left[a - b\left(\frac{c_l}{\bar{u}} + c_p\right)\right]^2 \bar{u}^2}{4b\left(\bar{u}^2 + \sigma^2\right)}$*, and is less than its deterministic equivalent; d) The optimal amount of farm space to be leased and the optimal profit are both decreasing in the variance of supply uncertainty.*

Proof of Proposition A1. a)

$$E\left[\Pi(Q)\right] = -\left(c_l + c_p\bar{u}\right)Q + \int_{u_l}^{u_h} p(Qu)Qug(u)\,du = \frac{\left[a - b\left(\frac{c_l}{\bar{u}} + c_p\right)\right]Q\bar{u} - Q^2\left(\bar{u}^2 + \sigma^2\right)}{b},$$

$$\frac{\partial E\left[\Pi(Q)\right]}{\partial Q}=-\left(c_l+c_p\bar{u}\right)+\int_{u_l}^{u_h}\left(\frac{a-2Qu}{b}\right)ug(u)\,du=\frac{\left[a-b\left(\frac{c_l}{\bar{u}}+c_p\right)\right]\bar{u}-2Q\left(\bar{u}^2+\sigma^2\right)}{b},$$

$$\frac{\partial^2 E\left[\Pi(Q)\right]}{\partial Q^2}=-\frac{2}{b}\int_{u_l}^{u_h}u^2g(u)\,du=-\frac{2}{b}\left(\bar{u}^2+\sigma^2\right)\leq 0.$$

Therefore, the first-order condition, when equated to zero, provides the optimal amount of farm space to be leased:

$$Q^*=\frac{\left[a-b\left(\frac{c_l}{\bar{u}}+c_p\right)\right]\bar{u}}{2\left(\bar{u}^2+\sigma^2\right)}.$$

b) Observe that the above optimal amount of farm space to be leased can also be expressed as:

$$Q^*=\frac{\left[a-b\left(\frac{c_l}{\bar{u}}+c_p\right)\right]}{2\left(\bar{u}+\frac{\sigma^2}{\bar{u}}\right)}<Q^0=\frac{\left[a-b\left(\frac{c_l}{\bar{u}}+c_p\right)\right]}{2\bar{u}}.$$

c) Substituting Q^* back into the objective function provides $E\left[\Pi\left(Q^*\right)\right]=\frac{\left[a-b\left(\frac{c_l}{\bar{u}}+c_p\right)\right]^2\bar{u}^2}{4b\left(\bar{u}^2+\sigma^2\right)}$.

Moreover, $E\left[\Pi\left(Q^*\right)\right]=\frac{\left[a-b\left(\frac{c_l}{\bar{u}}+c_p\right)\right]^2\bar{u}^2}{4b\left(\bar{u}^2+\sigma^2\right)}=\frac{\left[a-b\left(\frac{c_l}{\bar{u}}+c_p\right)\right]^2}{4b\left(1+cv^2\right)}=\frac{\Psi\left(Q^0\right)}{\left(1+cv^2\right)}<\Psi\left(Q^0\right)$.

d) Because $\frac{\partial Q^*}{\partial\sigma^2}=-\frac{\left[a-b\left(\frac{c_l}{\bar{u}}+c_p\right)\right]\bar{u}}{2\left(\bar{u}^2+\sigma^2\right)^2}\leq 0$ and $\frac{\partial E\left[\Pi\left(Q^*\right)\right]}{\partial\sigma^2}=-\frac{\left[a-b\left(\frac{c_l}{\bar{u}}+c_p\right)\right]^2\bar{u}^2}{4b\left(\bar{u}^2+\sigma^2\right)^2}\leq 0$, the optimal amount of farm space to be leased and the optimal profit are monotonically decreasing in the variance term of supply uncertainty. ❐

Denoting the coefficient of variation in supply uncertainty as $cv=\sigma/\bar{u}$, the optimal amount of farm space to be leased can also be expressed as follows:

$$Q^*=\frac{\left[a-b\left(\frac{c_l}{\bar{u}}+c_p\right)\right]\bar{u}}{2\left(\bar{u}^2+\sigma^2\right)}=\frac{\left[a-b\left(\frac{c_l}{\bar{u}}+c_p\right)\right]}{2\bar{u}\left(1+cv^2\right)}=\frac{Q^0}{1+cv^2}.$$

It can be easily seen from the last expression that the optimal amount of farm space to be leased is decreasing in the coefficient of variation. Similarly, the optimal value of the objective function is decreasing in the coefficient of variation,

$$E\left[\Pi\left(Q^*\right)\right]=\frac{\left[a-b\left(\frac{c_l}{\bar{u}}+c_p\right)\right]^2\bar{u}^2}{4b\left(\bar{u}^2+\sigma^2\right)}=\left[Q^*\bar{u}\right]^2\left(\frac{1+cv^2}{b}\right)=\frac{\Psi\left(Q^0\right)}{\left(1+cv^2\right)}<\Psi\left(Q^0\right),$$

and is less than its deterministic equivalent.

Proof of Proposition 2. a) Expected profit is concave in q_i because the demand function is concave, i.e.,

$$\partial^2\pi\left(q_i\left|Q,u\right.\right)/\partial q_i^2=2p'(q_i)+p''(q_i)q_i<0$$

and thus the first-order condition

$$\partial\pi\left(q_i\left|Q,u\right.\right)/\partial q_i=p\left(q_i\right)-c_p-s\left(u\right)+p'\left(q_i\right)q_i=0$$

provides $TS(u)$.

b) To characterize $TS(u)$, we need to solve

$$\max_{p,q_i}\pi(p,q_i)=\max_{p,q_i}\left\{\left[p-c_p-s(u)\right]\min\left\{d(p),q_i\right\}+s(u)Qu\right\}.$$

For any viable $p \geq c_p + s(u)$, it is clear that $\pi(p, q_i)$ is maximized at $q_i = d(p)$, and thus the problem reduces to a single variable optimization problem $\max_p \pi(p)$. Expected profit $\pi(p)$ is concave in p, i.e.,

$$\pi''(p)\ =\ 2d'(p)+\left[p-c_p-s(u)\right]d''(p)\leq 0$$

and thus for any given yield u the optimal price p^* uniquely satisfies

$$\pi'(p^*)=d(p^*)+\left[p^*-c_p-s(u)\right]d'(p^*)=0,$$ or equivalently,

$$p^*\left(1-\frac{d(p^*)}{-p^*d'(p^*)}\right)=c_p+s(u)$$

and $TS(u) = d(p^*)$. The optimal price is a function of the open market price s, which in turn is a decreasing function of yield u. Recall that $d'(p) < 0$, $s'(u) < 0$, and note that $\frac{dp^*}{ds} > 0$ (i.e., optimal price is increasing in s). Thus,

$$TS'(u)=\ d'\left(p^*\right)\times\frac{dp^*}{ds}\times\frac{ds}{du}>0.\ \square$$

Proposition A2. *For a given realized supply of Qu and the revenue of $s(u)$, the optimal selling price and the production quantities that maximize $\pi\left(p,q_i,q_s\left|Q,u\right.\right)$ in the second stage are*

$$p^* = \begin{cases} p(Qu) & \text{if } Qu \le TS(u) \\ p(TS(u)) & \text{if } Qu \ge TS(u) \end{cases} \quad \textit{and} \quad (q_i^*, q_s^*) = \begin{cases} (Qu, 0) & \text{if } Qu \le TS(u) \\ (TS(u), Qu - TS(u)) & \text{if } Qu \ge TS(u) \end{cases} \tag{10}$$

and the optimal second-stage profit is

$$\pi\left(p^*, q_i^*, q_s^* \middle| Q, u\right) = \left[p\left(\min\{TS(u), Qu\}\right) - c_p - s(u)\right]\min\{TS(u), Qu\} + s(u)Qu .$$

Proof of Proposition A2. The second-stage objective function can be expressed as

$$\max_{\substack{p, q_i \ge 0 \\ q_i \le Qu}} \pi\left(p, q_i \middle| Q, u\right) = \max_{\substack{p, q_i \ge 0 \\ q_i \le Qu}} \left\{p \min\{d(p), q_i\} - c_p q_i + s(u)(Qu - q_i)\right\}.$$

For any viable $p \ge c_p + s(u)$, it is clear that $\pi(p, q_i)$ is maximized at $q_i = d(p)$, and thus the problem reduces to a single variable optimization problem

$$\max_{\substack{p \ge 0 \\ d(p) \le Qu}} \left\{\left[p - c_p - s(u)\right] d(p) + s(u)Qu\right\},$$

which is concave in p. The first-order condition provides the optimal price expression in the event there is sufficient supply; otherwise price is set to clear available supply:

$$p^* = \begin{cases} p(Qu) \text{ when } u \le u_s(Q) \\ p(TS(u)) \text{ when } u > u_s(Q) \end{cases} ; \; q_i^* = \begin{cases} Qu \text{ when } u \le u_s(Q) \\ TS(u) \text{ when } u > u_s(Q) \end{cases}.$$

where

$$u_s(Q) = \begin{cases} u_l, \text{ if } Qu > TS(u) \text{ for all } u \in [u_l, u_h] \\ u_h, \text{ if } Qu < TS(u) \text{ for all } u \in [u_l, u_h] \\ TS(u)/Q \text{ otherwise} \end{cases}$$

Since $q_s = Qu - q_i$, the amount sold in the open market is

$$q_s^* = \begin{cases} 0 \text{ when } u \le u_s(Q) \\ Qu - TS(u) \text{ when } u > u_s(Q) \end{cases}. \; ❑$$

Proof of Proposition 3. The expected profit is

$$\pi(p, q_b) = \left[p - c_p - b(u)\right]\min\{d(p), q_b\}.$$

The proof follows the proof of Proposition 2 except that $b(u)$ replaces $s(u)$. We omit the details. ❑

Proposition A3. *For a given realized yield of Qu and the unit purchasing cost of* $b(u)$*, the optimal selling price and the production quantities that maximize* $\pi\left(p, q_i, q_b \middle| Q, u\right)$ *in the second stage are*

$$p^* = \begin{cases} p(TB(u)) & \text{if } Qu \le TB(u) \\ p(Qu) & \text{if } Qu \ge TB(u) \end{cases} \quad \textit{and} \quad (q_i^*, q_b^*) = \begin{cases} (Qu, TB(u) - Qu) & \text{if } Qu \le TB(u) \\ (Qu, 0) & \text{if } Qu \ge TB(u) \end{cases}. \tag{11}$$

and the optimal second-stage profit is

$$\pi\left(p^*,q_i^*,q_b^*\middle|Q,u\right) = \left[p\left(\max\left\{TB(u),Qu\right\}\right)-c_p-b(u)\right]\max\left\{TB(u),Qu\right\}+b(u)Qu .$$

Proof of Proposition A3. The proof follow the proof of Proposition A2 except that *b*(*u*) replaces *s*(*u*). We omit the details. ❑

Proof of Proposition 4. Because *b*(*u*) > *s*(*u*) for all *u*, *TS*(*u*) > *TB*(*u*) for all *u*. The rest of the proof follows from expressions (10) and (11). ❑

Proof of Proposition 5. Profit given realization *u* is

$$\Pi(Q,u)=\begin{cases}\Pi_B(Q,u)=\left[p\left(TB(u)\right)-c_p-b(u)\right]TB(u)+b(u)Qu-c_lQ,\ Q\le TB(u)/u\\ \Pi_N(Q,u)=\left[p(Qu)-c_p\right]Qu-c_lQ,\ Q\in\left[TB(u)/u,TS(u)/u\right]\\ \Pi_S(Q,u)=\left[p\left(TS(u)\right)-c_p-s(u)\right]TS(u)+s(u)Qu-c_lQ,\ Q\ge TS(u)/u\end{cases}$$

From Proposition 3, it is known that at *Qu* = *TB*(*u*), the optimal price satisfies

$$p'\left(TB(u)\right)TB(u)+p\left(TB(u)\right)-c_p=b(u) .$$

From Proposition 2, it is known that at *Qu* = *TS*(*u*), the optimal price satisfies

$$p'\left(TS(u)\right)TS(u)+p\left(TS(u)\right)-c_p=s(u) .$$

Recall that second-stage revenue function is concave, i.e., $2p'(d)+p''(d)d<0$. Thus

$$\frac{\partial\Pi(Q,u)}{\partial Q}=\begin{cases}\dfrac{\partial\Pi_B(Q,u)}{\partial Q}=b(u)u-c_l,\ Q\le TB(u)/u\\ \dfrac{\partial\Pi_N(Q,u)}{\partial Q}=\left[p'(Qu)Qu+p(Qu)-c_p\right]u-c_l,\ Q\in\left[TB(u)/u,TS(u)/u\right]\\ \dfrac{\partial\Pi_S(Q,u)}{\partial Q}=s(u)u-c_l,\ Q\ge TS(u)/u\end{cases}$$

$$Q=TB(u)/u:\ \frac{\partial\Pi_N(Q,u)}{\partial Q}=\left[p'\left(TB(u)\right)TB(u)+p\left(TB(u)\right)-c_p\right]u-c_l=\frac{\partial\Pi_B(Q,u)}{\partial Q}$$

$$Q=TS(u)/u:\ \frac{\partial\Pi_N(Q,u)}{\partial Q}=\left[p'\left(TS(u)\right)TS(u)+p\left(TS(u)\right)-c_p\right]u-c_l=\frac{\partial\Pi_S(Q,u)}{\partial Q}$$

$$\frac{\partial^2\Pi(Q,u)}{\partial Q^2}=\begin{cases}\dfrac{\partial^2\Pi_B(Q,u)}{\partial Q^2}=0,\ Q\le TB(u)/u\\ \dfrac{\partial^2\Pi_N(Q,u)}{\partial Q^2}=\left[2p'(Qu)+p''(Qu)Qu\right]u^2<0,\ Q\in\left[TB(u)/u,TS(u)/u\right]\\ \dfrac{\partial^2\Pi_S(Q,u)}{\partial Q^2}=0,\ Q\ge TS(u)/u\end{cases}$$

$$\le 0$$

We see that $\Pi(Q,u)$ is a continuously differentiable function that is concave in Q for any realization $u \in [u_l, u_h]$. Therefore $E[\Pi(Q)] = \int_{u_l}^{u_h} \Pi(Q,u)\, g(u)\, du$ is concave. ❑

Proof of Proposition 6. Note that $E[\Pi_{NT}^*]$ is unaffected by changes in Δ. Let Q_{NT}^* denote the optimal lease quantity without the option of trading in the open market and let $Q^*(\Delta)$ denote the optimal lease quantity with the option of trading in the open market. Finally, we define

$$V_T(\Delta, Q, Q_{NT}) = E[\Pi(Q, \Delta)] - E[\Pi_{NT}(Q_{NT})],$$

which is the gain from the trading option as a function of Δ and the lease quantities with (Q) and without (Q_{NT}) the trading option. Recall that $\hat{b}(u) = b(u) + \Delta/2$ and $\hat{s}(u) = s(u) - \Delta/2$. Accordingly, the value of trading in the open market can be expressed as:

$$\begin{aligned}
V_T^*(\Delta) &= V_T(\Delta, Q^*(\Delta), Q_{NT}^*) \\
&= \int_{N(Q^*(\Delta))} \Big(\big[p(Q^*(\Delta)u) - c_p\big]Q^*(\Delta)u - \big[p(Q_{NT}^*u) - c_p\big]Q_{NT}^*u\Big) g(u)\,du + \\
&\quad \int_{B(Q^*(\Delta))} \Big(\big[p(TB(u)) - c_p\big]TB(u) - \big[p(Q_{NT}^*u) - c_p\big]Q_{NT}^*u\Big) g(u)\,du + \\
&\quad \int_{S(Q^*(\Delta))} \Big(\big[p(TS(u)) - c_p\big]TS(u) - \big[p(Q_{NT}^*u) - c_p\big]Q_{NT}^*u\Big) g(u)\,du - \\
&\quad \int_{B(Q^*(\Delta))} \Big[\big(b(u) + \Delta/2\big)\big(TB(u) - Q^*(\Delta)u\big)\Big] g(u)\,du + \\
&\quad \int_{S(Q^*(\Delta))} \Big[\big(s(u) - \Delta/2\big)\big(Q^*(\Delta)u - TS(u)\big)\Big] g(u)\,du .
\end{aligned}$$

Note that for any $u \in B(Q^*(\Delta))$, we have $TB(u) > Q^*(\Delta)u$, and that for any $u \in S(Q^*(\Delta))$, we have $TS(u) < Q^*(\Delta)u$. Therefore, for any $x \in (0, \Delta)$,

$$\begin{aligned}
V_T^*(\Delta - x) - V_T^*(\Delta) &= V_T(\Delta - x, Q^*(\Delta - x), Q_{NT}^*) - V_T(\Delta, Q^*(\Delta), Q_{NT}^*) \\
&\geq V_T(\Delta - x, Q^*(\Delta), Q_{NT}^*) - V_T(\Delta, Q^*(\Delta), Q_{NT}^*) \\
&= \int_{B(Q^*(\Delta))} \Big[(x/2)\big(TB(u) - Q^*(\Delta)u\big)\Big] g(u)\,du + \\
&\quad \int_{S(Q^*(\Delta))} \Big[(x/2)\big(Q^*(\Delta)u - TS\big)\Big] g(u)\,du \\
&\geq 0.
\end{aligned}$$

Thus, $V_T^*(\Delta)$ is decreasing in Δ. ❑

Proof of Proposition 7. Profit given realization u is

$$\Pi\left(Q,Q_f,u\right)=\begin{cases}\Pi_B\left(Q,Q_f,u\right), & Qu+Q_f\le TB(u)\\ \Pi_N\left(Q,Q_f,u\right), & Qu+Q_f\in\left[TB(u),TS(u)\right]\\ \Pi_S\left(Q,Q_f,u\right), & Qu+Q_f\ge TS(u)\end{cases}$$

where

$$\Pi_B\left(Q,Q_f,u\right)=\left[p\left(TB(u)\right)-c_p-b(u)\right]TB(u)+b(u)\left[Qu+Q_f\right]-c_lQ-c_fQ_f$$

$$\Pi_N\left(Q,Q_f,u\right)=\left[p\left(Qu+Q_f\right)-c_p\right]\left[Qu+Q_f\right]-c_lQ-c_fQ_f$$

$$\Pi_S\left(Q,Q_f,u\right)=\left[p\left(TS(u)\right)-c_p-s(u)\right]TS(u)+s(u)\left[Qu+Q_f\right]-c_lQ-c_fQ_f$$

From Proposition 3, it is known that at $Qu + Q_f = TB(u)$, the optimal price satisfies

$$p'\left(TB(u)\right)TB(u)+p\left(TB(u)\right)-c_p=b(u).$$

From Proposition 2, it is known that at $Qu + Q_f = TS(u)$, the optimal price satisfies

$$p'\left(TS(u)\right)TS(u)+p\left(TS(u)\right)-c_p=s(u).$$

Recall that second-stage revenue function is concave, i.e., $2p'(d) + p''(d)d < 0$. Thus

$$\frac{\partial\Pi\left(Q,Q_f,u\right)}{\partial Q}=\begin{cases}\dfrac{\partial\Pi_B\left(Q,Q_f,u\right)}{\partial Q}=b(u)u-c_l, & Qu+Q_f\le TB(u)\\ \dfrac{\partial\Pi_N\left(Q,Q_f,u\right)}{\partial Q}=\begin{bmatrix}p'\left(Qu+Q_f\right)\left[Qu+Q_f\right]+\\ p\left(Qu+Q_f\right)-c_p\end{bmatrix}u-c_l, & Qu+Q_f\in\left[TB(u),TS(u)\right]\\ \dfrac{\partial\Pi_S\left(Q,Q_f,u\right)}{\partial Q}=s(u)u-c_l, & Qu+Q_f\ge TS(u)\end{cases}$$

$$\frac{\partial\Pi\left(Q,Q_f,u\right)}{\partial Q_f}=\begin{cases}\dfrac{\partial\Pi_B\left(Q,Q_f,u\right)}{\partial Q_f}=b(u)-c_f, & Qu+Q_f\le TB(u)\\ \dfrac{\partial\Pi_N\left(Q,Q_f,u\right)}{\partial Q_f}=\begin{bmatrix}p'\left(Qu+Q_f\right)\left[Qu+Q_f\right]+\\ p\left(Qu+Q_f\right)-c_p\end{bmatrix}-c_f, & Qu+Q_f\in\left[TB(u),TS(u)\right]\\ \dfrac{\partial\Pi_S\left(Q,Q_f,u\right)}{\partial Q_f}=s(u)-c_f, & Qu+Q_f\ge TS(u)\end{cases}$$

$Qu + Q_f = TB(u)$: $$\frac{\partial\Pi_N\left(Q,Q_f,u\right)}{\partial Q}=\left[p'\left(TB(u)\right)TB(u)+p\left(TB(u)\right)-c_p\right]u-c_l=\frac{\partial\Pi_B\left(Q,Q_f,u\right)}{\partial Q}$$

$$\frac{\partial\Pi_N\left(Q,Q_f,u\right)}{\partial Q_f}=p'\left(TB(u)\right)TB(u)+p\left(TB(u)\right)-c_p-c_l=\frac{\partial\Pi_B\left(Q,Q_f,u\right)}{\partial Q_f}$$

$Qu + Q_f = TS(u)$: $$\frac{\partial\Pi_N\left(Q,Q_f,u\right)}{\partial Q}=\left[p'\left(TS(u)\right)TS(u)+p\left(TS(u)\right)-c_p\right]u-c_l=\frac{\partial\Pi_B\left(Q,Q_f,u\right)}{\partial Q}$$

$$\frac{\partial \Pi_N\left(Q,Q_f,u\right)}{\partial Q_f} = p'\left(TS\left(u\right)\right)TS\left(u\right) + p\left(TS\left(u\right)\right) - c_p - c_l = \frac{\partial \Pi_B\left(Q,Q_f,u\right)}{\partial Q_f}$$

$$\frac{\partial^2 \Pi_B\left(Q,Q_f,u\right)}{\partial Q^2} = \frac{\partial^2 \Pi_B\left(Q,Q_f,u\right)}{\partial {Q_f}^2} = \frac{\partial^2 \Pi_B\left(Q,Q_f,u\right)}{\partial Q \partial Q_f} = 0$$

$$\frac{\partial^2 \Pi_S\left(Q,Q_f,u\right)}{\partial Q^2} = \frac{\partial^2 \Pi_S\left(Q,Q_f,u\right)}{\partial {Q_f}^2} = \frac{\partial^2 \Pi_S\left(Q,Q_f,u\right)}{\partial Q \partial Q_f} = 0$$

$$\frac{\partial^2 \Pi_N\left(Q,Q_f,u\right)}{\partial Q^2} = \left[p''\left(Qu + Q_f\right)\left[Qu + Q_f\right] + 2p'\left(Qu + Q_f\right)\right]u^2 < 0$$

$$\frac{\partial^2 \Pi_N\left(Q,Q_f,u\right)}{\partial {Q_f}^2} = p''\left(Qu + Q_f\right)\left[Qu + Q_f\right] + 2p'\left(Qu + Q_f\right) < 0$$

$$\frac{\partial^2 \Pi_N\left(Q,Q_f,u\right)}{\partial Q \partial Q_f} = \left[p''\left(Qu + Q_f\right)\left[Qu + Q_f\right] + 2p'\left(Qu + Q_f\right)\right]u < 0$$

and the determinant of the Hessian is

$$\Delta\left(u\right) = \begin{cases} \left(\frac{\partial^2 \Pi_B\left(Q,Q_f,u\right)}{\partial Q^2}\right)\left(\frac{\partial^2 \Pi_B\left(Q,Q_f,u\right)}{\partial {Q_f}^2}\right) - \left(\frac{\partial^2 \Pi_B\left(Q,Q_f,u\right)}{\partial Q \partial Q_f}\right)^2 = 0 \\ \left(\frac{\partial^2 \Pi_N\left(Q,Q_f,u\right)}{\partial Q^2}\right)\left(\frac{\partial^2 \Pi_N\left(Q,Q_f,u\right)}{\partial {Q_f}^2}\right) - \left(\frac{\partial^2 \Pi_N\left(Q,Q_f,u\right)}{\partial Q \partial Q_f}\right)^2 = 0 \\ \left(\frac{\partial^2 \Pi_S\left(Q,Q_f,u\right)}{\partial Q^2}\right)\left(\frac{\partial^2 \Pi_S\left(Q,Q_f,u\right)}{\partial {Q_f}^2}\right) - \left(\frac{\partial^2 \Pi_S\left(Q,Q_f,u\right)}{\partial Q \partial Q_f}\right)^2 = 0 \end{cases}$$

We see that $\Pi\left(Q,Q_f,u\right)$ is a continuously differentiable function that is concave in (Q, Q_f) for any realization $u \in [u_l, u_h]$. Therefore $E[\Pi\left(Q,Q_f\right)] = \int_{u_l}^{u_h} \Pi\left(Q,Q_f,u\right) g\left(u\right) du$ is concave. ❑

Proof of Proposition 8. Recall that $E\left[\Pi\left(Q,Q_f\right)\right]$ = optimal expected profit given lease quantity Q and futures quantity Q_f. We define

$q_b(Q, Q_f, u)$ = open market purchase quantity function

$q_s(x, u)$ = open market sell quantity function where x denotes the total available supply of fruit, e.g., $x = Qu + Q_f + q_b(Q, Q_f, u)$

$E\left[\Pi\left(Q,Q_f,q_b\left(Q,Q_f,\tilde{u}\right),q_s\left(Q\tilde{u}+Q_f+q_b\left(Q,Q_f,\tilde{u}\right),\tilde{u}\right)\right)\right]$ = expected profit given lease quantity Q, futures quantity Q_f, open market buying function $q_b(Q, Q_f, u)$, open market selling function $q_s(x, u)$, and the optimal price and production quantity

Applying the above notation, we have

$$
\begin{aligned}
E\left[\Pi\left(Q,Q_f\right)\right] &= \max_{q_b\left(Q,Q_f,u\right)\ge 0,\, q_s(x,u)\ge 0}\left\{E\left[\Pi\left(Q,Q_f,q_b\left(Q,Q_f,\tilde{u}\right),q_s\left(Q\tilde{u}+Q_f+q_b\left(Q,Q_f,\tilde{u}\right),\tilde{u}\right)\right)\right]\right\} \\
&= \max_{q_b\left(Q,0,u\right)\ge Q_f,\, q_s(x,u)\ge 0}\left\{E\left[\Pi\left(Q,0,q_b\left(Q,0,\tilde{u}\right),q_s\left(Q\tilde{u}+q_b\left(Q,0,\tilde{u}\right),\tilde{u}\right)\right)\right]\right\}
\end{aligned}
$$

(because buying futures at unit cost $c_f = E\left[b\left(\tilde{u}\right)\right]$ is equivalent to introducing a lower bound constraint on the number of units to purchase in the open market)

$$
\le \max_{q_b\left(Q,0,u\right)\ge 0,\, q_s(x,u)\ge 0}\left\{E\left[\Pi\left(Q,0,q_b\left(Q,0,\tilde{u}\right),q_s\left(Q\tilde{u}+q_b\left(Q,0,\tilde{u}\right),\tilde{u}\right)\right)\right]\right\} \;\; \forall Q_f \ge 0
$$

(because the above problem is a relaxation of the previous problem)

$$
= E\left[\Pi\left(Q,0\right)\right]. \;\square
$$

Proof of Proposition 9. From the proof of Proposition 7 we know that $\Pi(Q, Q_f, u)$ is concave in (Q, Q_f) for any realization $u \in [u_l, u_h]$. We will show that $U(\Pi(Q, Q_f, u))$ is concave in (Q, Q_f) for any realization $u \in [u_l, u_h]$, from which it follows that $E\left[U\left(\Pi\left(Q,Q_f\right)\right)\right]$ is concave.

To simplify the presentation, let $Q_1 = Q$ and $Q_2 = Q_f$. Let A be the Hessian of $\Pi\left(Q_1,Q_2,u\right)$ and let B be the Hessian of $U\left(\Pi\left(Q_1,Q_2,u\right)\right)$, i.e.,

$$
a_{ij} = \frac{\partial^2 \Pi\left(Q_1,Q_2,u\right)}{\partial Q_i \partial Q_j}
$$

$$
\begin{aligned}
b_{ij} &= \frac{\partial^2 U\left(\Pi\left(Q_1,Q_2,u\right)\right)}{\partial Q_i \partial Q_j} \\
&= U'\left(\Pi\left(Q_1,Q_2,u\right)\right)\frac{\partial^2 \Pi\left(Q_1,Q_2,u\right)}{\partial Q_i \partial Q_j} + U''\left(\Pi\left(Q_1,Q_2,u\right)\right)\left(\frac{\partial \Pi\left(Q_1,Q_2,u\right)}{\partial Q_i}\right)\left(\frac{\partial \Pi\left(Q_1,Q_2,u\right)}{\partial Q_j}\right).
\end{aligned}
$$

Since $\Pi\left(Q_1,Q_2,u\right)$ is concave in (Q_1, Q_2), the quadratic form of A is nonpositive, i.e.,

$$
\mathbf{y}^{\mathrm{T}}A\mathbf{y} = y_1^2\frac{\partial^2 \Pi\left(Q_1,Q_2,u\right)}{\partial Q_1^2} + 2y_1y_2\frac{\partial^2 \Pi\left(Q_1,Q_2,u\right)}{\partial Q_1 \partial Q_2} + y_2^2\frac{\partial^2 \Pi\left(Q_1,Q_2,u\right)}{\partial Q_2^2} \le 0 \text{ for any } (y_1, y_2) \qquad (12)
$$

The quadratic form of B is

$$\mathbf{y}^{\mathrm{T}}B\mathbf{y} = y_1^{\,2}\frac{\partial^2 U\left(\Pi\left(Q_1,Q_2,u\right)\right)}{\partial Q_1^{\,2}} + 2y_1y_2\frac{\partial^2 U\left(\Pi\left(Q_1,Q_2,u\right)\right)}{\partial Q_1\partial Q_2} + y_2^{\,2}\frac{\partial^2 U\left(\Pi\left(Q_1,Q_2,u\right)\right)}{\partial Q_2^{\,2}}$$

$$= U'\left(\Pi\left(Q_1,Q_2,u\right)\right)\left[y_1^{\,2}\frac{\partial^2 \Pi\left(Q_1,Q_2,u\right)}{\partial Q_1^{\,2}} + 2y_1y_2\frac{\partial^2 \Pi\left(Q_1,Q_2,u\right)}{\partial Q_1\partial Q_2} + y_2^{\,2}\frac{\partial^2 \Pi\left(Q_1,Q_2,u\right)}{\partial Q_2^{\,2}}\right] +$$

$$U''\left(\Pi\left(Q_1,Q_2,u\right)\right)\left[\begin{array}{l}\left(y_1\frac{\partial \Pi\left(Q_1,Q_2,u\right)}{\partial Q_1}\right)^2 + \left(y_2\frac{\partial \Pi\left(Q_1,Q_2,u\right)}{\partial Q_2}\right)^2 + \\ +2y_1y_2\left(\frac{\partial \Pi\left(Q_1,Q_2,u\right)}{\partial Q_1}\right)\left(\frac{\partial \Pi\left(Q_1,Q_2,u\right)}{\partial Q_2}\right)\end{array}\right]$$

$$= U'\left(\Pi\left(Q_1,Q_2,u\right)\right)\mathbf{y}^T A\mathbf{y} + U''\left(\Pi\left(Q_1,Q_2,u\right)\right)\left(\sum_{i=1}^{2} y_i\frac{\partial \Pi\left(Q_1,Q_2,u\right)}{\partial Q_i}\right)^2$$

$$\leq 0 \qquad \text{(due to (12), } U'(x) > 0,\ U''(x) \leq 0\text{)}$$

and thus $U\left(\Pi\left(Q,Q_f,u\right)\right)$ is concave and $E\left[U\left(\Pi\left(Q,Q_f\right)\right)\right]$ is concave. ❑

Proof of Proposition 10. The proof follows the spirit of the proof of Proposition 8 in which the elimination of fruit futures is equivalent to relaxing a constraint: if open market buying cost is static (i.e., $b(u) = b$ for all u), then for any Q, the expected utility associated with the purchase of Q_f futures at price $c_f = b = E\left[b(\tilde{u})\right]$ is equal to the expected utility with no purchased futures and a constraint that at least Q_f units must be purchased in the open market after yield is realized, and relaxing this constraint cannot decrease expected utility. ❑

Appendix B

Let $\tilde{b}(u)$ and $\tilde{s}(u)$ denote random buying and selling price in the open market with $b(u) = E\left[\tilde{b}(u)\right]$ and $s(u) = E\left[\tilde{s}(u)\right]$. Recall that $b(u)$ and $s(u)$ differ by the market spread $\delta(u)$. Let $\tilde{v}(u) = \tilde{b}(u) - \delta(u)/2 = \tilde{s}(u) + \delta(u)/2$, i.e., $\tilde{v}(u)$ is the market center line. The joint pdf of $\tilde{u}$ and $\tilde{v}$ is $\phi(u, v)$, and thus $g(u) = \int_0^\infty \phi(u,v)\,dv$. Let $\Omega_v(u)$ denote the support of $\tilde{v}(u)$ and let Ω denote the support of $(\tilde{u},\tilde{v})$.

We use the subscript 2 for a model with less than perfect correlation between the market center line and the realized yield (correlation is less than perfect if and only if $Var\left[\tilde{v}(u)\right] > 0$ for some $u \in [u_l, u_h]$). For a given realization v, there is an optimal buy threshold denoted $TB_2(v)$, and there is an optimal sell threshold denoted $TS_2(v)$, i.e., purchase up to $TB_2(v)$ if $Qu < TB_2(v)$ and sell down to $TS_2(v)$ if $Qu > TS_2(v)$. The second-stage optimal buy, no trade, and sell regions are denoted $B_2(Q)$, $N_2(Q)$, and $S_2(Q)$, i.e.,

$$B_2(Q) = \{(u, v) : Qu < TB_2(v),\ (u, v) \in \Omega\}$$

$N_2(Q) = \{(u, v) : Qu \in [TB_2(v), TS_2(v)], (u, v) \in \Omega\}$

$S_2(Q) = \{(u, v) : Qu > TS_2(v), (u, v) \in \Omega\}$.

The first-stage expected profit given optimal second-stage decisions is

$$E[\Pi_2(Q)] = -c_l Q + \begin{cases} \iint\limits_{B_2(Q)} \left[\left[p(TB_2(v)) - c_p - v - \delta(u)/2\right] TB_2(v) + (v + \delta(u)/2)Qu\right] \phi(u,v)\, dudv + \\ \iint\limits_{N_2(Q)} \left[p(Qu) - c_p\right] Qu\phi(u,v)\, dudv + \\ \iint\limits_{S_2(Q)} \left[\left[p(TS_2(v)) - c_p - v + \delta(u)/2\right] TS_2(v) + (v - \delta(u)/2)Qu\right] \phi(u,v)\, dudv \end{cases}$$

The first-stage expected profit given optimal second stages decisions and $Var\left[\tilde{v}(u)\right] = 0$ for all $u \in [u_l, u_h]$) (i.e., perfect correlation) is denoted $E[\Pi_1(Q)]$.

Proposition B1. $\max\limits_{Q \geq 0}\left\{E\left[\Pi_1(Q)\right]\right\} \leq \max\limits_{Q \geq 0}\left\{E\left[\Pi_2(Q)\right]\right\}$.

Proof of Proposition B1. Let Q_1 denoted the optimal lease quantity under perfect correlation, i.e., $Q_1 = \arg\max\limits_{Q \geq 0}\left\{E\left[\Pi_1(Q)\right]\right\}$. Note that $b(u) = E\left[\tilde{v}(u)\right] + \delta(u)/2$ and $s(u) = E\left[\tilde{v}(u)\right] - \delta(u)/2$. Therefore

$$E[\Pi_1(Q_1)] = -c_l Q_1 + \begin{cases} \int_{B(Q_1)} \left[\left[p(TB(u)) - c_p - b(u)\right] TB(u) + b(u) Q_1 u\right] g(u)\, du + \\ \int_{N(Q_1)} \left[\left(p(Q_1 u) - c_p\right) Q_1 u\right] g(u)\, du + \\ \int_{S(Q_1)} \left[\left[p(TS(u)) - c_p - s(u)\right] TS(u) + s(u) Q_1 u\right] g(u)\, du \end{cases}$$

$$= -c_l Q_1 + \begin{cases} \int\limits_{B(Q_1)} \left[\left[p(TB(u)) - c_p - E\left[\tilde{v}(u)\right] - \frac{\delta(u)}{2}\right] TB(u) + \left(E\left[\tilde{v}(u)\right] + \frac{\delta(u)}{2}\right) Q_1 u\right] g(u)\, du + \\ \int\limits_{N(Q_1)} \left[(p(Q_1 u) - c_p) Q_1 u\right] g(u)\, du + \\ \int\limits_{S(Q_1)} \left[\left[p(TS(u)) - c_p - E\left[\tilde{v}(u)\right] + \frac{\delta(u)}{2}\right] TS(u) + \left(E\left[\tilde{v}(u)\right] + \frac{\delta(u)}{2}\right) Q_1 u\right] g(u)\, du \end{cases}$$

$$= -c_l Q_1 + \begin{cases} \iint\limits_{B_3(Q_1)} \left[\left[p(TB(u)) - c_p - v - \delta(u)/2\right] TB_2(v) + (v + \delta(u)/2)Q_1 u\right] \phi(u,v)\, dvdu + \\ \iint\limits_{N_3(Q_1)} \left[p(Q_1 u) - c_p\right] Q_1 u\phi(u,v)\, dvdu + \\ \iint\limits_{S_3(Q_1)} \left[\left[p(TS(u)) - c_p - v + \delta(u)/2\right] TS_2(v) + (v - \delta(u)/2)Q_1 u\right] \phi(u,v)\, dvdu \end{cases}$$

$$
\leq \max_{Q\geq 0}\left\{-c_l Q + \left\{\begin{array}{l}\iint\limits_{B_2(Q)} \left[\left[p\left(TB_2(v)\right)-c_p - v - \delta\left(u\right)/2\right]TB_2(v) + (v+\delta\left(u\right)/2)Qu\right]\phi\left(u,v\right)dvdu + \\ \iint\limits_{N_2(Q)} \left[p\left(Qu\right)-c_p\right]Qu\phi\left(u,v\right)dvdu + \\ \iint\limits_{S_2(Q)} \left[\left[p\left(TS_2(v)\right)-c_p - v + \delta\left(u\right)/2\right]TS_2(v) + (v-\delta\left(u\right)/2)Qu\right]\phi\left(u,v\right)dvdu\end{array}\right.\right\}
$$

$$
= \max_{Q\geq 0}\left\{E\left[\Pi_2\left(Q\right)\right]\right\}
$$

where

$B_3(Q_1) = \{(u, v) : u \in B_1(Q_1), v \in \Omega_v(u)\}$

$N_3(Q_1) = \{(u, v) : u \in N_1(Q_1), v \in \Omega_v(u)\}$

$S_3(Q_1) = \{(u, v) : u \in N_1(Q_1), v \in \Omega_v(u)\}$. ❐