Global Sourcing under Exchange-Rate Uncertainty

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We study a firm's global sourcing decisions under exchange-rate and demand uncertainty. The firm initially reserves capacity from one domestic and one international supplier in the presence of exchange-rate and demand uncertainty. After observing exchange rates, the firm determines the amount of capacity to utilize for manufacturing under demand uncertainty.

The paper makes four contributions. First, we identify the set of optimal sourcing policies (one onshore, two offshore, and two dual sourcing policies), and the conditions that lead to each policy. Two dual sourcing policies emerge: The first one is a defensive policy where the firm rations limited capacity in order to minimize the negative consequences of exchange-rate fluctuations, while the second one is an opportunistic policy and features excess capacity investment in order to benefit from currency fluctuations. Moreover, our analysis shows how the optimal sourcing policy changes with increasing degrees of exchange-rate volatility. Second, we find that lower capacity and manufacturing costs are neither necessary nor sufficient to reserve capacity at a supplier. Third, we show that risk aversion reduces the likelihood of single sourcing (specifically offshore sourcing) and increases the likelihood of dual sourcing. Fourth, we show that financial hedging can eliminate the negative consequences of risk aversion, and make our policy findings more pronounced as they continue to hold under risk aversion and financial hedging.

Keywords: *exchange-rate uncertainty*, *dual sourcing, onshore sourcing, offshore sourcing.*

1. Introduction

Multinational firms have dramatically increased their sourcing from abroad in recent decades. Sourcing from other countries provides multinational firms with the opportunity to access low-cost sources, but it can also bring out additional challenges associated with currency fluctuations. When acquisition and operational costs are denominated in foreign currencies, fluctuations in exchange rates can make a source significantly less costly or more expensive. This paper explores the impact of exchange-rate uncertainty on the sourcing decisions of multinational firms. Recent literature has identified various reasons for multinational firms to engage in dual sourcing, including unequal and positive lead times under demand uncertainty, or when the delivery reliability is random. Our work adds to this list by investigating the impact of exchange-rate uncertainty on sourcing decisions. It identifies the type of single and dual sourcing policies that emerge as a consequence of exchange-rate uncertainty in the context of capacity reservation contracts. It shows how uncertainties in exchange-rate and demand, and risk aversion influence the firm's optimal sourcing decisions.

Our work is motivated by global sourcing practices of a furniture company in the United States that specializes in school and library furniture. Selling in a domestic market, this company outsources most of its products from either a domestic or an international supplier. In this industry, the firm faces three seasons of demand (corresponding to each school semester). The company prepares for each selling season by initially reserving capacity with its suppliers well in advance. Depending on the product, the international supplier may be located in Europe or in Asia. After capacity reservation contracts are signed, the firm observes the realization of the random exchange rate, then determines how much to order from each supplier to prepare for the selling season. Fluctuating Euro and appreciating Chinese Yuan in recent years motivated the managers of this firm to revisit their sourcing policies.

Our model applies to a variety of manufacturing settings with long lead times where one firm outsources its production activities to contract manufacturers serving as suppliers. In addition to the furniture industry that motivated our problem, capacity reservation contracts are extensively used in other industries such as telecommunication, electronics and semi-conductor equipment manufacturing (Cohen et al. 2003, Erkoç and Wu 2005, Özer and Wei 2006, Peng et al. 2012, Cachon and Lariviere 2001). Long lead times and the fact that custom-designed products may not possibly be procured from a spot market, force the buying firms (e.g., original equipment manufacturers) to decide on production quantities well in advance of the selling season. Capacity reservation provides a guaranteed amount of capacity for the buyer and also allows the supplier to more efficiently plan its production and capacity expansion when necessary. Our model helps such buying firms in determining the most effective use of the domestic and foreign suppliers, and when to engage in dual sourcing.

Our analysis considers sourcing agreements made with four types of costs. The firm initially pays a per-unit capacity reservation cost to the supplier in order to reserve capacity for production in the future. Later when the season approaches, there is another per unit operational cost paid to the supplier for production. The production cost at the foreign supplier incurs in the foreign currency. In addition to the production cost, the buying firm incurs a transportation cost which is considered to be inclusive of duties and other localization costs. Using the common theme of free-on-board shipments in global logistics, we consider the case that the buying firm pays for the transportation cost from the supplier, and therefore, our model features a transportation cost in the domestic currency of the buying firm. The fourth cost term involves unused capacity; the buying firm pays a penalty cost for the reserved but not utilized capacity, and in the case of the foreign supplier, this payment occurs in the foreign currency. Thus, the total landed cost is the sum of capacity reservation, production, transportation (inclusive of duties and localization costs), and if any, the unused capacity penalty costs. It is noteworthy that transportation costs made in foreign currency and/or penalty cost from unused capacity in the domestic currency do not alter the structural properties in our model, and thus, our results apply to these other cost settings.

The paper makes four contributions. First, we show that the set of potentially optimal decisions includes five distinct policies with one onshore sourcing, two offshore sourcing and two dual sourcing policies. The two dual sourcing policies are characteristically different. In the first dual sourcing policy, the firm takes a conservative action that mitigates the negative consequences of currency fluctuations by splitting a constant total capacity between the domestic and foreign suppliers. In the second dual sourcing policy, the firm reserves extra capacity in order to benefit from exchange-rate fluctuations. We also show how the firm switches from one optimal sourcing policy to another with increasing degrees of exchange-rate volatility. Second, our analysis shows that exchange rate uncertainty can drive a firm's sourcing decision. In particular, we find that a firm may source only from the high-cost foreign supplier under exchange-rate uncertainty, and this can be optimal even if the expected cost of sourcing is higher than the selling price. This finding complements earlier literature that has characterized the role of cost and lead time in sourcing decisions. Third, the introduction of risk aversion makes dual sourcing a more desirable policy structure. It reduces the likelihood of single sourcing by reducing the likelihood of featuring an offshoring policy; these policies switch to dual sourcing when risk aversion is introduced into the model. Fourth, financial hedging helps the firm to mitigate the negative consequences of risk aversion, and more importantly, it enables the firm to replicate the expected profit from each policy identified in the risk-neutral setting. Therefore, we conclude that our results are robust as the policy findings continue to hold under risk aversion and financial hedging.

The remainder of the paper is structured as follows. Section 2 describes the most related literature to our paper. Section 3 presents the model, Section 4 analyzes it and discusses the results. Section 5 examines the impact of increasing exchange-rate uncertainty. Section 6 introduces risk aversion and Section 7 analyzes the influence of financial hedging. Section 8 presents numerical illustrations from the furniture maker that motivated our study. Section 9 provides concluding remarks. All the proofs are relegated to the online appendix.

2. Literature Review

This paper studies global sourcing policies in the presence of uncertain exchange rate and demand. There is vast literature that investigates different aspects of sourcing decisions with an emphasis on dual sourcing. Most recently, Jain et al. (2014) establish the potential benefit of dual sourcing by empirically showing that switching from single to dual sourcing policy reduces the inventory investment by almost 11%. One stream of research shows that the asymmetric lead time among suppliers is one of the drivers of dual sourcing. Fuduka (1964) is one of the earliest studies that shows a dual-base-stock policy is optimal when there is only one review period difference between the lead times of the two sources. Whittemore and Saunders (1977), Moinzadeh and Nahmias (1988), Moinzadeh and Schmidt (1991), Tagaras and Vlachos (2001), and Veeraraghavan and Scheller-Wolf (2008) extend this stream of literature by optimizing and/or evaluating the performance of given dual sourcing policies in the presence of lead time differences. Allon and Van Mieghem (2010) examine the cost-responsiveness trade-off when splitting the supply base between a low-cost offshore and a responsive near-shore supplier. Wu and Zhang (2014) show that when two sources are equally costly, sourcing from the long-lead time supplier may still be optimal under Cournot competition. However, their study does not allow for dual sourcing. Our work differs from this stream of literature focusing on sourcing policies under lead-time differences in several ways. While these studies assume deterministic exchange rate, our paper does not feature asymmetric or stochastic lead-times. Our work complements this literature by showing that exchange-rate uncertainty and the recourse flexibility (stemming from capacity reservation contracts) can lead to dual sourcing among optimal policies.

Another stream of literature explores the impact of reliability on sourcing decisions. Yano and Lee (1995) present an extensive review of the early dual sourcing literature addressing the reliability aspect of sourcing in terms of both supply uncertainty and lead-time uncertainty. More recent studies include Tomlin and Wang (2005), Dada et al. (2007), Burke et al. (2009) and Kouvelis and Lee (2013). Particularly, Dada et al. (2007) investigate the cost-reliability trade-off in choosing the portfolio of suppliers with random capacity. Their main finding is consistent with Hill's (2000) strategic note and conclude that cost is an order qualifier. Burke et al. (2009) reach the same conclusion in a similar setting. They point out that supplier's cost is the key criterion and thus the lowest-cost supplier always receives some order quantity share. Our results substantially differ from theirs as we show that, in the presence of cost uncertainty, the most-expensive source may receive even the entire order.

Our focus of study departs from these streams of literature as we analyze the impact of cost uncertainty caused by exchange-rate uncertainty on sourcing policies. From this perspective, our work is also related to the global supply chain literature. Within this body of research, Kogut and Kulatilaka (1994) investigate the benefits of the flexibility to shift production between geographically-dispersed facilities based on exchange-rate fluctuations. Huchzermeier and Cohen (1996) use a global supply chain model in order to analyze the value of operational hedging including holding excess capacity and production switching option. Kazaz et al. (2005), Ding et al. (2007), and Dong et al. (2010) focus on the impact of exchange-rate uncertainty in the revenues generated from sales in multiple markets. These earlier publications feature a recourse flexibility that benefits the firm in mitigating the uncertainty in distribution operations to foreign markets. In their model, exchange-rate and demand uncertainty are revealed at the same time, and the firm makes efficient production and (responsive) pricing decisions in the second stage. Our paper departs from these publications in three ways. First, our paper focuses on the impact of exchange-rate uncertainty in sourcing decisions, and thus, the recourse flexibility stemming from the capacity reservation contracts provide the ability to mitigate the uncertainty in procurement costs. Second, our model features a single market in order to determine the drivers for dual sourcing. Dual sourcing becomes more prevalent in the presence of multiple markets. Third, while these earlier publications do not explore the cost implications of various sourcing policies, our study examines the impact of sourcing costs in depth. Gurnani and Tang (1999) study the ordering decisions of a retailer under demand and cost uncertainty where the retailer has two instants to order from a manufacturer prior to the selling season. The unit cost at the second instant is uncertain and the retailer has to evaluate the trade-off between a more accurate forecast and a potential higher unit cost at the second instant. They show that regardless of the value of information, the retailer never utilizes the cost-certain option (first instant) when its associated unit cost is higher than the expected unit cost of the cost-uncertain option (second instant) (propositions 3.1. and 3.2.). Our problem setting, however, features dual sourcing (sourcing from both cost-certain and cost-uncertain suppliers) even if the cost of sourcing from the domestic (cost-certain) supplier is higher than that of the foreign (cost-uncertain) supplier. Chen et al. (2015) as well explore the implications of uncertainty in operational costs for global supply chains. While they study the optimal inventory policy in a periodic-review inventory system, our paper examines the optimal sourcing policy in a single-period setting.

The major driver of global sourcing practice is well acknowledged to be seeking for cost reduction. As a consequence, the common assumption in this stream of literature is that the offshore supplier is the low-cost source. Lu and Van Mieghem (2009) develop a transportation cost threshold to determine centralization (equivalent to single sourcing) versus decentralization (resembling dual sourcing) of manufacturing common components. Our work complements their study by examining the impact of exchange-rate uncertainty. Li and Wang (2010) and Chen et al. (2014), for instance, examine the trade-offs between the expensive domestic sourcing and low-cost offshore sourcing under exchange-rate risk. The models in these two papers feature a different setup as demand uncertainty is realized at the same time as exchange-rate. The firm, in our model, continues to make its second-stage decisions under demand uncertainty. As a consequence of the difference in their modeling approach, these papers do not develop dual sourcing policies with the characterization of rationing capacity between the two suppliers in order to mitigate exchange-rate risk and/or investing in excess capacity in order to benefit from currency fluctuations. Shunko et al. (2014) study the role of transfer pricing and sourcing strategies in achieving low tax rates and low production costs, respectively. While they investigate how transfer pricing decisions impact sourcing decisions of a local manager and, thus, a multinational corporation's profits, the main focus of our paper is the impact of exchange-rate fluctuations on sourcing policies of a multinational corporation. Feng and Lu (2012) investigates strategic perils of low-cost outsourcing and find that low-cost outsourcing may result in a win-lose outcome under competition. However, only single sourcing policies can be adopted in their model. Kouvelis (1998) justifies the use of an expensive supplier when the firm does not want to incur a switching cost. Our study, on the other hand, shows that the expensive foreign supplier might be the only source utilized in the absence of switchover costs. Fox et al. (2006) and Zhang et al. (2012) focus on a different aspect of global sourcing costs by distinguishing between fixed costs and variable costs of sourcing in an optimal inventory control study. Zhang et al. (2012) introduce an order size constraint which leads to dual sourcing becoming a potentially optimal policy. In the absence of this constraint their model does not feature dual sourcing as the optimal solution. Contrary to the common belief that offshore sourcing is utilized because of low sourcing costs associated with foreign suppliers, we show that lower cost is neither a necessary nor a sufficient condition for optimality of offshore sourcing under exchange-rate uncertainty.

Finally, our results provide insights into the ongoing debate on whether offshore sourcing is still economically viable (Ferreira and Prokopets, 2009; Ellram et al. 2013) even after the recent growing costs in emerging markets.

3. The Model

The model considers a firm that sells a product in its home country at price p, and outsources its manufacturing to two suppliers, one in the home country (denoted H) and the other in a foreign country (denoted F). The random exchange rate is denoted $\tilde{e}$, its realization is denoted e with a probability density function (pdf) $g(e)$, a cumulative distribution function (cdf) $G(e)$ on a support $[e_l, e_h]$ with a mean $\bar{e}$. Random demand is denoted $\tilde{x}$, its realization is denoted x with a pdf $f(x)$ and cdf $F(x)$ on a support $[x_l, x_h]$ where $x_h > x_l > 0$ with a mean of D.

Figure 1 illustrates the setting and notation, and Figure 2 describes the sequence of decisions. In the first stage, the firm determines the amount of capacity to reserve from the two suppliers, denoted Q_H and Q_F, in the presence of exchange-rate and demand uncertainty. A per-unit capacity reservation cost, denoted k_i where $i = H, F$, is incurred in order to reserve capacity at each supplier in advance.

The first-stage objective function maximizes the expected profit $E\left[\Pi\left(Q_H, Q_F\right)\right]$ and is as follows:

$$\max_{Q_H, Q_F \geq 0} E\left[\Pi\left(Q_H, Q_F\right)\right] = -k_H Q_H - k_F Q_F + \int_{e_l}^{e_h} \pi^*\left(Q_H, Q_F, e\right) g\left(e\right) de \tag{1}$$

where $\pi^*\left(Q_H, Q_F, e\right)$ is the optimal second-stage expected profit over random demand for a given set of capacity reservation decisions Q_H and Q_F and realized exchange rate e.

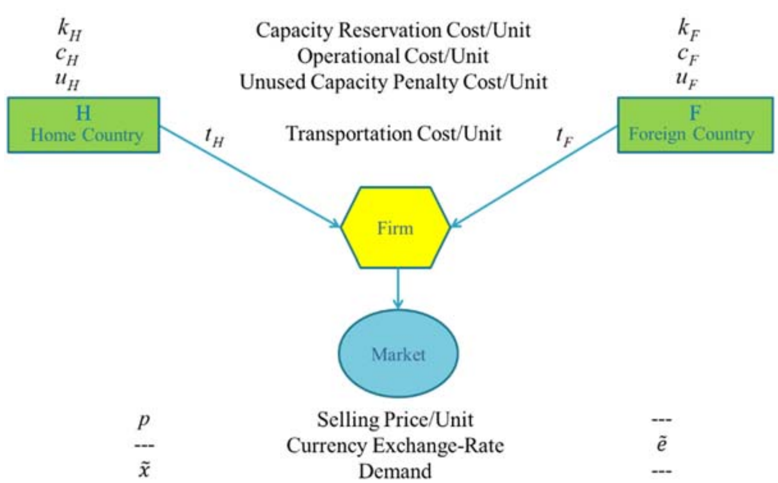


Figure 1. Two suppliers and one market network.

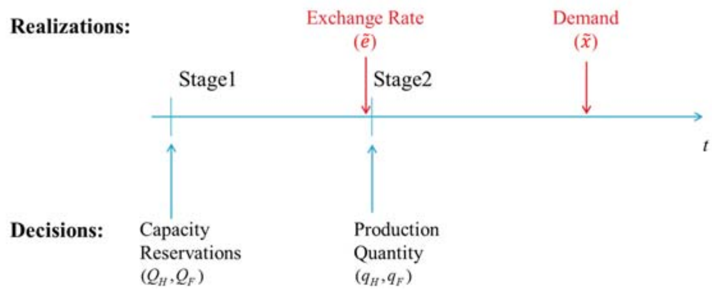


Figure 2. The natural sequence of events for a firm reserving production capacity from two suppliers.

After observing the realized value of the exchange rate, correspondingly in Stage 2, the firm determines how much to order from each supplier in the presence of demand uncertainty, denoted q_i, subject to the constraint of first-stage capacity reservation decisions, i.e., $q_i \le Q_i$ where $i = H, F$. In Stage 2, the firm incurs three different kinds of costs. First, the firm pays an operational cost associated with production processing, denoted c_i where $i = H, F$, denominated in the local currency. Thus, c_F is paid in the foreign currency for each unit produced by the foreign supplier, and therefore, it can be seen as a random cost as its value changes with exchange-rate fluctuations. Second, the firm incurs a transshipment cost that is inclusive of duties and localization costs in the home country currency denoted t_i where $i = H, F$. Third, the firm is penalized by the supplier to pay an additional fee for unused capacity (reserved from Stage 1, but not utilized in Stage 2) paid in the supplier's currency denoted u_i where $i = H, F$. Like the processing cost c_F, the penalty cost for unused capacity u_F can also be perceived as a random cost as its value changes with exchange-rate fluctuations. The second-stage objective function maximizes the expected profit over random demand, denoted $E\left[\pi_2\left(q_H, q_F, x \mid Q_H, Q_F, e\right)\right]$, for a given set of first-stage capacity reservation decisions (Q_H, Q_F) and realized exchange rate e:

$$\begin{aligned}\pi^*\left(Q_H, Q_F, e\right) &= \max_{q_H, q_F \ge 0} E\left[\pi_2\left(q_H, q_F, x \mid Q_H, Q_F, e\right)\right] \\ &= -c_H q_H - c_F e q_F - t_H q_H - t_F q_F - u_H\left(Q_H - q_H\right)^+ - u_F e\left(Q_F - q_F\right)^+ \quad (2) \\ &\quad + \int_{x_l}^{x_h} p \min\left\{x, q_H + q_F\right\} f\left(x\right) dx\end{aligned}$$

$$\text{s.t. } q_i \le Q_i \text{ for } i = H, F \quad (3)$$

where constraint (3) ensures that the production order quantities do not exceed the first-stage capacity reservation decisions.

We note that the above formulation can be transformed into an equivalent formulation where the first-stage objective function is expressed as:

$$\max_{Q_H, Q_F \geq 0} E\left[\Pi\left(Q_H, Q_F\right)\right] = -(k_H + u_H)Q_H - (k_F + u_F\bar{e})Q_F + \int_{e_l}^{e_h} \pi^*\left(Q_H, Q_F, e\right) g\left(e\right) de \tag{4}$$

and the second-stage objective function is written as:

$$\pi^*\left(Q_H, Q_F, e\right) = \max_{q_H, q_F \geq 0} E\left[\pi_2\left(q_H, q_F, x \mid Q_H, Q_F, e\right)\right]$$
$$= -(c_H - u_H + t_H)q_H - \left((c_F - u_F)e + t_F\right)q_F + \int_{x_l}^{x_h} p \min\left\{x, q_H + q_F\right\} f\left(x\right) dx \tag{5}$$

$$\text{s.t. } q_i \leq Q_i \text{ for } i = H, F \tag{6}$$

This formulation can be interpreted as follows: The buying firm pays the penalty fee for the unused capacity upfront (at the same time with reserving capacity) and receives a credit for each unit of utilized capacity when ordering production. It is also beneficial to state that if the unit penalty cost from unused capacity is paid in the buying firm's domestic currency, then $u_F e$ is replaced with u_F in (5) and $u_F\bar{e}$ with u_F in (4); the change does not cause any changes in the structural properties of the problem.

We introduce additional notation in order to simplify expressions: m_H is the margin in the second stage when the product is sourced from the domestic supplier, i.e., $m_H = p - c_H + u_H - t_H$, and $\tilde{m}_F$ and m_F describe the random and realized second-stage margin if the product is sourced from the foreign supplier, i.e., $\tilde{m}_F = p - c_F\tilde{e} + u_F\tilde{e} - t_F$ and $m_F = p - c_F e + u_F e - t_F$, respectively. We denote the first stage profit margins of the product sourced from the home market and foreign market as M_H and M_F, respectively, and assume that sourcing from both suppliers is economically viable (i.e., to avoid the trivial case of not reserving capacity):

(A1) $M_H = p - c_H - t_H - k_H > 0$ and $M_F = E\left[\left(p - c_F\tilde{e} + u_F\tilde{e} - t_F\right)^+\right] - k_F - u_F\bar{e} = E\left[\tilde{m}_F^{\ +}\right] - k_F - u_F\bar{e} > 0$

The superscript "+" indicates the maximum of zero and the value of the term, and we use "increase", "decrease", "concave" and "convex" in their weak sense throughout the manuscript. Note that assumption (A1) is not a restrictive assumption for the foreign supplier as its value is less than the expected margin (when the firm reserves and utilizes its entire capacity) $M_F \geq p - c_F\bar{e} - t_F - k_F$.

Our model provides the firm with the flexibility to alter its production orders based on the realized value of the exchange rate as long as the firm has reserved capacity. The order allocation flexibility enables the firm to utilize the lower cost supplier. Such variations in the optimal second-stage decisions influence the first-stage capacity reservation decisions. Thus, both first-stage capacity reservation and second-stage production decisions are affected by exchange-rate uncertainty.

The optimal second-stage production decisions can be classified in three regions of exchange-rate realizations. In the first region the realized exchange rate is so low ($e_l \leq e \leq \tau_1 = (c_H - u_H + t_H - t_F)/(c_F - u_F)$) that sourcing from the foreign supplier (i.e., offshore sourcing) is less costly than sourcing from the home supplier (i.e., onshore sourcing). In other words, for these realized values of exchange rates, offshore sourcing is more desirable than onshore sourcing (i.e., $m_F \geq m_H > 0$). In the second region, the realized exchange rate is higher but not sufficiently high to cause offshore sourcing to be eliminated from consideration. Specifically, in this region we have $\tau_1 \leq e \leq \tau_2 = (p - t_F)/(c_F - u_F)$ corresponding to exchange rate realizations where onshore sourcing is more profitable than offshore sourcing but offshore sourcing is still profitable, i.e., $m_H > m_F \geq 0$. In the third and final region, the realized value of the exchange rate is so high that offshore sourcing is no more a viable alternative, i.e., $m_F < 0$. In this region, the firm does not order the product from the foreign supplier even if it has already reserved capacity in the first stage.

The threshold point τ_1 can be lower or higher than the mean of the exchange rate $\bar{e}$ depending on the relative magnitude of the cost terms. Throughout the manuscript, we do not impose any assumptions regarding their relative magnitudes. Figure 3 illustrates the three regions for an example where τ_1 and τ_2 are located within the support of $\tilde{e}$, i.e., $e_l < (c_H - u_H + t_H - t_F)/(c_F - u_F)$ and $e_h > (p - t_F)/(c_F - u_F)$.

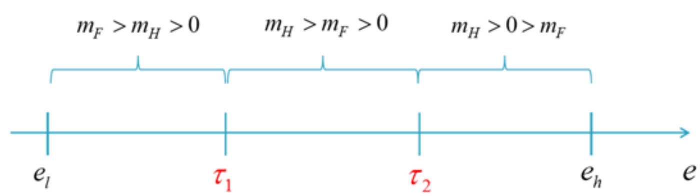


Figure 3. Exchange-rate realization in the second stage.

4. Analysis

4.1. Demand Uncertainty

We begin our analysis by focusing on the influence of demand uncertainty. We consider the special case when the exchange-rate random variable is replaced by its deterministic equivalent, its mean $\bar{e}$. When uncertainty is only associated with demand, the problem becomes a single-stage Newsvendor Problem with two suppliers. We define the total unit sourcing cost as the sum of unit capacity reservation, production, transportation, duties and localization costs: $c_H^T = k_H + c_H + t_H$ and $c_F^T = k_F + c_F\,\bar{e} + t_F$. It is easy to verify that the firm would choose the supplier with the lowest total unit sourcing cost in this special case, leading to a single-sourcing option.

4.2. Demand and Exchange-Rate Uncertainty

The second-stage problem conforms to the standard newsvendor structure with the first-stage capacity constraints. With no capacity constraints in the second stage, the optimal order quantities from each supplier can be determined easily by solving two independent Newsvendor Problems.

4.2.1. Onshore Sourcing

If the firm restricts its sourcing activities to an onshore supplier, the problem becomes a single-stage Newsvendor Problem and exchange-rate uncertainty becomes irrelevant. In Stage 2, if the firm ignores the first-stage capacity reservation contract, it would order $q_H^0 = F^{-1}((p - c_H + u_H - t_H)/p)$ units of products from the onshore source. The optimal amount of capacity reserved in Stage 1 is:

$$Q_H^0 = F^{-1}((p - k_H - c_H - t_H)/p) < q_H^0. \tag{7}$$

In other words, when the domestic supplier is the only alternative, the firm utilizes the reserved capacity in its entirety in the second stage.

4.2.2. Offshore Sourcing

If the firm utilizes only the offshore source, then its second-stage production amount is limited by the amount of capacity reserved in stage 1 (denoted Q_F) as well as the amount established by the Newsvendor fractile (denoted $q_F^0(e)$). In the absence of a limitation caused by capacity reservation, the firm would prefer to produce $q_F^0(e) = F^{-1}((p - c_F e + u_F e - t_F)^+/p)$. We describe the exchange rate threshold value where the firm's first-stage production amount equals the desired level of second-stage production amount as $\tau(Q_F) = (p[1 - F(Q_F)] - t_F)/(c_F - u_F)$. If the realized exchange rate is less than $\tau(Q_F)$, the firm utilizes the entire capacity reserved from Stage 1 despite the preference to produce more than reserved. If the realized exchange rate is greater than $\tau(Q_F)$, however, the firm does not make use of the entire capacity reserved in Stage 1 and produces less than Q_F. We describe the exchange-rate threshold that equates the second-stage margin to zero as τ_2; when the realized exchange rate is greater than τ_2, the firm does not produce in Stage 2 because of the guaranteed loss from the negative margin. Thus, the second-stage order quantity is:

$$q_F^*(e) = \begin{cases} Q_F & \text{if } e_l \le e \le \tau\left(Q_F\right) \\ q_F^0\left(e\right) & \text{if } \tau\left(Q_F\right) < e \le \tau_2 \\ 0 & \text{if } \tau_2 < e \le e_h \end{cases}. \tag{8}$$

The optimal capacity amount to be reserved from the offshore supplier is denoted Q_F^0. There is no closed-form solution to Q_F^0, however, it has a unique solution that satisfies the following first-order condition (FOC):

$$-\left(k_F + u_F \overline{e}\right) + \left(c_F - u_F\right) \int_{e_l}^{\tau(Q_F^0)} \left(\tau(Q_F^0) - e\right) g\left(e\right) de = 0\,. \tag{9}$$

4.2.3. Global Sourcing

We next analyze the optimal capacity reservation decisions for the general problem described in (4) – (6). In Stage 2, if the realized exchange rate is low, below the threshold τ_1 that equates the second-stage returns from the domestic and foreign suppliers, then the firm prioritizes sourcing from the foreign supplier. If the realized exchange rate is high and above the threshold value τ_2 that equates the second-stage returns from the foreign source to zero, then the foreign source is not utilized at all. The following proposition provides the optimal second-stage production decisions at each realization of exchange rate.

Proposition 1. *The optimal second-stage production decisions are*:

$$\left(q_H^*(e), q_F^*(e)\right) = \begin{cases} \left(\min\left\{Q_H, \left(q_H^0 - Q_F\right)^+\right\}, \min\left\{Q_F, q_F^0(e)\right\}\right) & \text{if } e_l \le e < \tau_1 \\ \left(\min\left\{Q_H, q_H^0\right\}, \min\left\{Q_F, \left(q_F^0(e) - Q_H\right)^+\right\}\right) & \text{if } \tau_1 \le e < \tau_2 \\ \left(\min\left\{Q_H, q_H^0\right\}, 0\right) & \text{if } \tau_2 \le e \le e_h \end{cases}. \tag{10}$$

In the remainder of the paper, we suppress the exchange rate parameter in the optimal second-stage production functions unless necessary for clarity.

We next establish that the objective function is jointly concave in its decision variables.

Proposition 2. *The objective function in* (4) *is jointly concave in Q_H and Q_F.*

From Proposition 1, it can be seen that the optimal amount of capacity reserved from the domestic supplier cannot exceed q_H^0. The optimal capacity decisions in the first stage can be classified into the following three sets: Region $R1 = \{Q_H, Q_F \mid Q_H + Q_F \le q_H^0\}$, region $R2 = \{Q_H, Q_F \mid Q_H \le q_H^0, Q_F \le q_H^0$ and $Q_H + Q_F > q_H^0\}$, and region $R3 = \{Q_H, Q_F \mid Q_H \le q_H^0$ and $Q_F > q_H^0\}$. These three regions are illustrated in Figure 4.

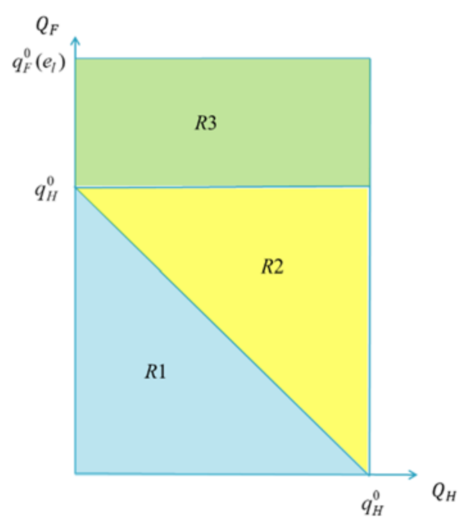


Figure 4. Optimal regions for the general case of the problem.

We can determine the optimal capacity decisions in each of the three regions depicted in Figure 4. From Proposition 2, we know that the problem is jointly concave in Q_H and Q_F. Therefore, we can identify optimal decisions in each region through the FOC. The optimal capacity reservation decisions in region $R1$ satisfies the following system of equations:

$$\begin{cases} \displaystyle\int_{\tau(Q_H)}^{e_h} \left(e-\tau(Q_H)\right) g(e)\,de = \frac{\left(k_F + c_F\overline{e} + t_F\right) - \left(k_H + c_H + t_H\right)}{c_F - u_F} \\ \displaystyle\int_{e_l}^{\tau(Q_H+Q_F)} \left(\tau(Q_H+Q_F) - e\right) g(e)\,de = \frac{k_F + u_F\overline{e}}{c_F - u_F} \end{cases}. \tag{11}$$

Defining the values of $\tau(Q_H)$ and $\tau(Q_H + Q_F)$ that solve the system of equations (11) as τ_H^* and τ_{HF}^*, respectively, we can express the optimal capacity choices as follows:

$$\begin{cases} Q_H^* = F^{-1}\left(\dfrac{p-(c_F-u_F)\tau_H^* - t_F}{p}\right) \\ Q_F^* = F^{-1}\left(\dfrac{p-(c_F-u_F)\tau_{HF}^* - t_F}{p}\right) - F^{-1}\left(\dfrac{p-(c_F-u_F)\tau_H^* - t_F}{p}\right) \end{cases}. \tag{12}$$

The optimal solution is never located in region $R2$; this is formalized in the following proposition. Considering the fact that it is never optimal to reserve more capacity than q_H^0 at the home country, this proposition guarantees that the capacity to be reserved from the foreign supplier is either greater than q_H^0, or it is sufficiently low that the total amount of capacity to be reserved is not more than q_H^0.

Proposition 3. *The optimal solution does not lie in region R2.*

The optimal capacity reservation decisions in region $R3$ satisfies the following system of equations:

$$\begin{cases} \displaystyle\int_{\tau(Q_H)}^{e_h} \left(e-\tau(Q_H)\right) g(e)\,de = \frac{E\left[\left(m_H - \tilde{m}_F\right)^+\right] - k_H - u_H}{c_F - u_F} \\ \displaystyle\int_{e_l}^{\tau(Q_F)} \left(\tau(Q_F) - e\right) g(e)\,de = \frac{k_F + u_F\overline{e}}{c_F - u_F} \end{cases}. \tag{13}$$

Describing the optimal value of $\tau(Q_F)$ as τ_F^*, we can express the optimal capacity choices as follows:

$$\begin{cases} Q_H^* = F^{-1}\left(\dfrac{p-(c_F-u_F)\tau_H^* - t_F}{p}\right) \\ Q_F^* = F^{-1}\left(\dfrac{p-(c_F-u_F)\tau_F^* - t_F}{p}\right) \end{cases}. \tag{14}$$

4.3. Optimal Sourcing Policies

We next show that there are five potentially optimal policies: one onshore sourcing, two offshore sourcing, and two dual sourcing policies. We describe them as follows:

(1) Policy H: Onshore sourcing with $Q_H^* = Q_H^0$ and $Q_F^* = 0$,

(2) Policy F_L: Offshore sourcing with a smaller capacity reservation $Q_F^* = Q_F^0 \leq q_H^0$ and $Q_H^* = 0$,

(3) Policy F_H: Offshore sourcing with a higher capacity reservation $Q_F^* = Q_F^0 > q_H^0$ and $Q_H^* = 0$,

(4) Policy D_R: Dual sourcing featuring a rationing perspective with $Q_H^* + Q_F^* = Q_F^0$,

(5) Policy D_E: Dual sourcing featuring excess capacity with $Q_F^* = Q_F^0$ and $Q_H^* < Q_H^0$.

Policy H is the onshore policy where the firm reserves capacity only at the domestic supplier. The optimal amount of capacity to reserve is equal to $Q_H^* = Q_H^0$ where Q_H^0 is determined through (7). The next two policies, F_L and F_H, are offshore policies where the optimal capacity reservation decisions are $Q_F^* = Q_F^0$ where Q_F^0 is determined through (9). Recall that Q_F^0 can be less than or greater than q_H^0. We denote the offshore sourcing policy that leads to limited capacity investment $Q_F^* = Q_F^0 \leq q_H^0$ as F_L, and the offshore sourcing policy with a higher capacity commitment $Q_F^* = Q_F^0 > q_H^0$ as F_H.

The set of potentially optimal policies features two dual sourcing policies that differ characteristically. Policy D_R is an intermediate solution in region $R1$ and the optimal amount of total capacity reserved can be obtained through the set of equations in (12), i.e., $Q_H^* + Q_F^* = Q_F^0$. Depending on the cost parameters and the distribution of the random exchange rate, the total capacity is rationed between the two sources. This is a conservative and a defensive policy where the firm mitigates the negative consequences of currency fluctuations by distributing the total capacity investment between the two sources. The second dual sourcing policy is denoted D_E and it is aggressive and opportunistic as it features excess capacity in order to enjoy the benefits of fluctuating exchange rates. The optimal capacity decisions are obtained through the set of equations in (14). Under D_E, the firm commits to the level of capacity investment that it would have invested in the offshore sourcing policy F_H (i.e., $Q_F^* = Q_F^0$) and it reserves an additional amount of capacity from the domestic source. However, this amount is strictly less than the ideal amount it would have reserved under the onshore sourcing policy, i.e., $Q_H^* < Q_H^0$. Figure 5 illustrates the five potentially optimal policies.

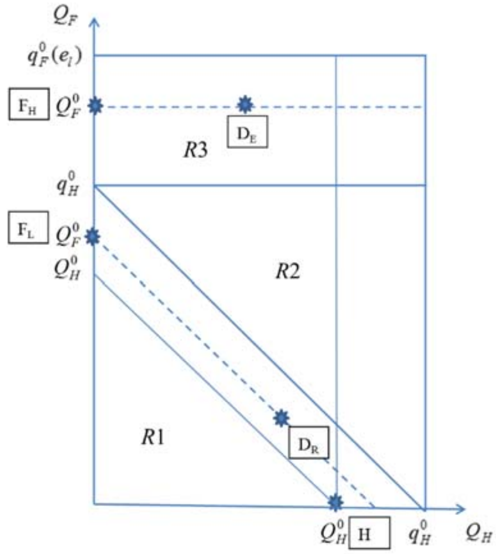


Figure 5. Set of all possible optimal solutions.

The above set of potentially optimal policies can be obtained by reviewing four optimality conditions. These four conditions provide the necessary and sufficient conditions for each policy to be the optimal decision. These four optimality conditions are:

(OC1): $E\left[\left(\tilde{m}_F^{\;+} - M_H\right)^+\right] - k_F - u_F\bar{e} > 0$,

(OC2): $E\left[\left(\tilde{m}_F^{\;+} - m_H\right)^+\right] - k_F - u_F\bar{e} > 0$,

(OC3): $m_H - k_H - u_H - \left(E\left[\tilde{m}_F^{\;+}\right] - k_F - u_F\bar{e}\right) > 0$,

(OC4): $E\left[\left(m_H - \tilde{m}_F^{\;+}\right)^+\right] - k_H - u_H > 0$.

Proposition 4 shows how the five potentially optimal policies are obtained through the above four optimality conditions.

Proposition 4.

(a) *Policy* H *is optimal iff* (OC1) *does not hold;*

(b) *Policy* $\mathrm{F_L}$ *is optimal iff* (OC2) *and* (OC3) *do not hold;*

(c) *Policy* $\mathrm{F_H}$ *is optimal iff* (OC2) *holds and* (OC4) *does not hold;*

(d) *Policy* $\mathrm{D_R}$ *is optimal iff* (OC1) *and* (OC3) *hold and* (OC2) *does not hold;*

(e) *Policy* $\mathrm{D_E}$ *is optimal iff* (OC2) *and* (OC4) *hold.*

Table 1 presents the necessary and sufficient conditions for each policy to be the optimal solution for the problem in (4) – (6). Dual sourcing is the prevailing policy under certain conditions. The following proposition shows that a quick comparison between the optimal offshore capacity with the desired level of second-stage order quantity from the domestic source reveals which one of these two dual sourcing policies can be featured in the optimal solution. We also observe that Q_H^0 establishes a minimum total capacity reservation amount for the global sourcing problem (see Lemma A5 in the appendix).

Proposition 5. (a) *If $Q_F^0 < q_H^0$, then policy* D_E *cannot be optimal, leaving policy* D_R *as the only viable dual sourcing policy;* (b) *If $Q_F^0 > q_H^0$, then policy* D_R *cannot be optimal, leaving policy* D_E *as the only viable dual sourcing policy.*

Dual sourcing policies D_R and D_E exhibit completely different characteristics. We next provide a discussion of these two policies.

Optimal Sourcing Policy / Optimality Condition	Onshore Sourcing	Offshore Sourcing		Dual Sourcing	
	H	F_L	F_H	D_R	D_E
(OC1)	×			✓	
(OC2)		×	✓	×	✓
(OC3)		×		✓	
(OC4)			×		✓

Table 1. Necessary and sufficient conditions for the first-stage optimal decisions. A check mark ("✓") indicates that the corresponding inequality (i.e., optimality condition) holds when a particular sourcing policy is optimal, and a cross mark ("×") indicates that the opposite inequality holds.

4.3.1. Rationing Dual Sourcing with Policy D_R

In policy D_R, the firm reserves capacity from both sources where the sum of these reserved capacities equals the amount of capacity it would have reserved from the offshore source, i.e., $Q_H^* + Q_F^* = Q_F^0$. The firm's allocation of capacity can be perceived as rationing capacity in order to mitigate cost uncertainty stemming from currency fluctuations. In this policy, the firm does not necessarily benefit much from currency swings. Thus, it is a defensive and a conservative policy that can be perceived as mitigating the negative consequences of currency fluctuations. Under D_R, the firm always utilizes the home supplier to its maximum, i.e., $q_H^* = Q_H^*$ at every realization of the random exchange rate. However, it utilizes the foreign supplier up to its limit only when the realized exchange rate is desirable, i.e., $e \leq \tau_1$.

According to policy D_R, the firm diversifies its supply base between a cost-uncertain and a cost-certain supplier in order to mitigate the negative consequences of exchange-rate uncertainty. However, it cannot capitalize completely in the event that exchange rate makes the foreign supplier an economically desirable source; this can be seen from $q_F^* = Q_F^0 - Q_H^* < Q_F^0$ when $e_l \leq e \leq \tau_1$.

4.3.2. Excess Dual Sourcing with Policy $\mathrm{D_E}$

Two observations can be made regarding policy $\mathrm{D_E}$. First, the firm considers the foreign supplier as its primary source and reserves the exact amount of capacity it would have reserved under the offshore sourcing policies, i.e., $Q_F^* = Q_F^0$. Second, the firm reserves additional capacity from the domestic source. However, this amount is strictly less than what it would have reserved under the onshore sourcing policy, i.e., $Q_H^* < Q_H^0$. The domestic supplier appears to serve as a backup source in this policy. The amount reserved at the domestic source Q_H^* is utilized only when the realized exchange rate makes the foreign source an expensive supplier. Similarly, the foreign source is not always utilized at its maximum reserved capacity. By reserving a total capacity that exceeds the optimal amount that would be reserved from the offshore source, the firm always ends up wasting some reserved capacity, but in turn, takes advantage of the swings in the exchange rate. Thus, additional capacity reserved at the domestic source leads to an opportunistic behavior and provides the flexibility to enjoy the benefits of cost fluctuations.

5. Impact of Exchange-Rate Uncertainty

In this section, we compare the optimal sourcing decisions under exchange-rate and demand uncertainty with those obtained under deterministic exchange rate and stochastic demand by replacing the random exchange rate with its deterministic equivalent. The comparison provides insights regarding the impact of exchange-rate uncertainty on capacity reservation decisions. It is shown earlier that, under deterministic exchange-rate in our model, the firm engages only in single-sourcing utilizing either the onshore source or the offshore source depending on the lower total cost of sourcing. We examine the capacity choices under the cases with one source featuring the lower total sourcing cost.

Case 1: Lower expected cost at the foreign supplier: $k_F + c_F\bar{e} + t_F < k_H + c_H + t_H$. When the foreign supplier has the lower expected total sourcing cost, the firm always chooses offshore sourcing under deterministic exchange rate. However, this is not necessarily the case if the exchange rate is uncertain.

Proposition 6. *When the foreign supplier has the lower expected total unit sourcing cost, the firm utilizes either an offshore sourcing policy* ($\mathrm{F_L}$ *or* $\mathrm{F_H}$) *or the dual sourcing policy* $\mathrm{D_E}$ *under exchange-rate and demand uncertainty.*

The above proposition implies that lower expected cost of sourcing is not a sufficient condition for offshore sourcing. More specifically, when the exchange-rate uncertainty is taken into account, it may be optimal for the firm to utilize dual sourcing, rather than offshore sourcing, under specific conditions even if the expected total unit sourcing cost is lower for the foreign supplier.

Case 2: Lower cost at the domestic supplier: $k_F + c_F\bar{e} + t_F \geq k_H + c_H + t_H$. When sourcing from the domestic supplier is less costly, the firm always chooses onshore sourcing under deterministic exchange rate. The next proposition indicates that the offshore sourcing policy can be optimal despite featuring a more expensive foreign supplier.

Proposition 7. *When the domestic supplier has the lower total unit sourcing cost, offshore sourcing policies* (F_L *or* F_H) *can be optimal under exchange-rate and demand uncertainty.*

Proposition 7 shows that it is not necessary to have the lowest total sourcing cost in order to reserve capacity at a single source. It shows that lower sourcing cost from the domestic supplier does not eliminate the possibility of offshore sourcing. Alternatively said, offshore sourcing does not need to feature the lower expected sourcing cost to be the optimal policy. This result contrasts the common rationale behind offshore sourcing practices that often justify working with foreign sources because of the lower cost feature. In our finding, however, foreign source is utilized only when the exchange rate is lower, and thus, the effective cost of utilizing the foreign source is lower than its expected sourcing cost.

Table 2 illustrates how introducing exchange-rate uncertainty can significantly influence the optimal sourcing policy when compared to the case where uncertainty is associated only with demand. In other words, ignoring the exchange-rate uncertainty may result in decisions that are far from optimal.

<table>
<tr><th>Cost Structure</th><th>Demand Uncertainty</th><th colspan="3">Optimality Conditions</th><th>Exchange-Rate and Demand Uncertainty</th></tr>
<tr><td rowspan="3">Case 1
$k_F + c_F\,\bar{e} + t_F <$
$k_H + c_H + t_H$</td><td rowspan="3">Offshore Sourcing</td><td rowspan="2" colspan="2">OC2</td><td>OC4</td><td>Excess Dual Sourcing (D_E)</td></tr>
<tr><td>$\overline{OC4}$</td><td>Offshore Sourcing (F_H)</td></tr>
<tr><td colspan="3">$\overline{OC2}$</td><td>Offshore Sourcing (F_L)</td></tr>
<tr><td rowspan="5">Case 2
$k_F + c_F\,\bar{e} + t_F \geq$
$k_H + c_H + t_H$</td><td rowspan="5">Onshore Sourcing</td><td rowspan="2">OC2</td><td colspan="2">OC4</td><td>Excess Dual Sourcing (D_E)</td></tr>
<tr><td colspan="2">$\overline{OC4}$</td><td>Offshore Sourcing (F_H)</td></tr>
<tr><td rowspan="3">$\overline{OC2}$</td><td rowspan="2">OC1</td><td>OC3</td><td>Rationing Dual Sourcing (D_R)</td></tr>
<tr><td>$\overline{OC3}$</td><td>Offshore Sourcing (F_L)</td></tr>
<tr><td colspan="2">$\overline{OC1}$</td><td>Onshore Sourcing (H)</td></tr>
</table>

Table 2. The impact of exchange-rate uncertainty on sourcing decisions. An overscore implies the reversed condition.

Through propositions 6 and 7, we conclude that lower unit costs are neither necessary nor sufficient to reserve capacity at a supplier. We next show that exchange-rate uncertainty can be influential in offshore sourcing even if the foreign source has a negative expected profit margin in the second stage.

Proposition 8. *The optimal sourcing policy may be dual sourcing* ($\mathrm{D_R}$ *or* $\mathrm{D_E}$) *or offshore sourcing* ($\mathrm{F_L}$ *or* $\mathrm{F_H}$) *when* $p - (c_F - u_F)\bar{e} - t_F < 0$.

The consequence of the above proposition is that the firm can choose offshore sourcing even if it is expected to lose money in the second stage with $p - (c_F - u_F)\bar{e} - t_F < 0$. Note that this is a stronger condition than negative expected margin in the first stage with $p - c_F\bar{e} - t_F - k_F < 0$. Under deterministic exchange rate, the firm may not even consider the foreign supplier when the second-stage return is negative. Thus, exchange-rate uncertainty creates the opportunity for the firm to reserve capacity at a supplier even with negative expected margin in the second-stage. Proposition 8 implies that the firm may benefit from giving up a deterministic positive profit margin from sourcing through the domestic supplier, and instead engage with a single foreign source even if the expected unit cost from the offshore source (in stage 2) is higher than the unit price. In this case, postponing the sourcing decision until the revelation of the exchange rate provides the benefit of potentially high profit margin caused by low exchange-rate realizations. It also comes at the expense of incurring a potential loss (the sum of capacity reservation cost and penalty cost of unused capacity) at high realizations of exchange rate. This flexibility can increase the desirability of the foreign supplier so much that the firm prefers to utilize offshore sourcing without a domestic supplier even if the foreign supplier has an expected unit cost in Stage 2 higher than the market price.

We next investigate the impact of exchange-rate volatility on the optimal sourcing policy. The next proposition shows that the optimal domestic capacity does not behave monotonically in exchange-rate volatility as it shows both an increasing and decreasing behavior.

Proposition 9. *The optimal foreign capacity* Q_F^* *always increases in exchange-rate volatility. But the optimal domestic capacity* Q_H^* *may increase or decrease.*

One would intuit that higher degrees of exchange-rate volatility create the incentive to invest in additional flexibility through domestic capacity. However, the next proposition establishes the condition under which the firm reduces its domestic capacity investment under a uniform exchange-rate distribution.

Proposition 10. *When the exchange rate is uniformly distributed, under policy* $\mathrm{D_E}$*, the optimal domestic capacity* Q_H^* *decreases in exchange-rate volatility iff* $k_H + c_H + t_H < (c_F - u_F)\bar{e} + t_F$.

The condition in Proposition 10 seems counter-intuitive at first sight as it suggests that when total unit cost of sourcing from the domestic source is lower than the expected cost of sourcing from the foreign source, the domestic capacity decreases in exchange-rate uncertainty. It is worth noting that this proposition does not imply that reducing the cost of onshore sourcing leads to lower optimal capacity at the domestic source. In fact, when $k_H + c_H + t_H < (c_F - u_F)\bar{e} + t_F$, the firm reserves higher levels of domestic capacity at any degree of exchange-rate volatility compared to the opposite case (i.e., $k_H + c_H + t_H > (c_F - u_F)\bar{e} + t_F$). The condition in Proposition 10 requires that the cost of sourcing from the domestic source is low, and thus, the firm has already reserved a sufficiently high level of domestic capacity under policy $\mathrm{D_E}$.

Consequently, the domestic capacity becomes a substitute for the foreign capacity which increases in exchange-rate volatility (according to Proposition 9). As a result, the firm reduces its high capacity commitment to the domestic source in order to capitalize more on the prospects of sourcing from the foreign supplier.

The following proposition sheds light on how the optimal sourcing policy evolves with exchange-rate volatility.

Proposition 11. *As the exchange-rate volatility increases*:

(a) *if* $k_F + c_F\bar{e} + t_F < k_H + c_H + t_H$, *then the optimal sourcing policy changes according to the following path* (*or a continuous portion thereof*): $F_L \rightarrow F_H \rightarrow D_E$;

(b) *if* $k_F + c_F\bar{e} + t_F \geq k_H + c_H + t_H$, *then the optimal sourcing policy changes according to the following paths* (*or a continuous portion thereof*): $H \rightarrow D_R \rightarrow F_L \rightarrow F_H \rightarrow D_E$, *or* $H \rightarrow D_R \rightarrow D_E$.

Proposition 11 establishes the optimal policy paths that the firm follows as exchange-rate volatility increases. Specifically, when the foreign supplier is associated with lower expected unit sourcing cost, the firm either keeps offshore sourcing or switches to policy D_E utilizing excess capacity. On the other hand, when the unit cost of sourcing from the domestic supplier is lower, the firm switches from onshore sourcing to the dual sourcing policy exhibiting rationing behavior (D_R). With increasing exchange-rate volatility the firm either directly switches to the excess dual sourcing policy (D_E) or first adopts offshore sourcing policies before implementing the excess dual sourcing policy. We observe that as the degree of exchange-rate variation increases, the foreign supplier becomes a more desirable supplier. This is because there are higher chances of a large savings due to low realization of the exchange rate while the possible loss due to the appreciation of the exchange rate—which is as much as the capacity reservation cost—remains unchanged. Consequently, high volatility of exchange rate results in choosing the foreign supplier as the primary source (corresponding to policies F_L, F_H, and D_E).

6. Risk Aversion

This section presents the influence of risk aversion on the part of the buying firm. We utilize the value-at-risk (VaR) measure to limit the risk associated with the realized profits in the global sourcing problem under exchange-rate and demand uncertainty. VaR is the most widely employed risk measure in practice and is the prevailing risk approach in the Basel II and III Accords specifying the banking laws and regulations issued by the Basel Committee on Banking Supervision (2013). There are two parameters that describe the firm's risk preferences in VaR: $\beta \geq 0$ represents the loss (value at risk) that the firm is willing to tolerate at probability α, where $0 \leq \alpha \leq 1$. For a given α, if VaR is more than the tolerable loss β, then first-stage decisions (Q_H, Q_F) correspond to an infeasible solution. We incorporate the firm's VaR concern into the model in (1) – (3) by supplementing the first-stage problem with the following probability constraint:

$$P_{(\tilde{e},\tilde{x})}\left[\Pi\left(Q_H, Q_F\right) < -\beta\right] \leq \alpha \tag{15}$$

where

$$\Pi\left(Q_H, Q_F\right) = -k_H Q_H - k_F Q_F - \left(c_H + t_H\right) q_H^*\left(\tilde{e}\right) - \left(c_F \tilde{e} + t_F\right) q_F^*\left(\tilde{e}\right)$$
$$- u_H\left(Q_H - q_H^*\left(\tilde{e}\right)\right)^+ - u_F \tilde{e}\left(Q_F - q_F^*\left(\tilde{e}\right)\right)^+ + p \min\left\{\tilde{x}, q_H^*\left(\tilde{e}\right) + q_F^*\left(\tilde{e}\right)\right\}$$

is the random profit from the optimal second-stage decisions and first-stage capacity reservation decisions (Q_H, Q_F) and $P_{(\tilde{e},\tilde{x})}[\cdot]$ represents the probability over the exchange-rate and demand random variables. Constraint (15) states that the probability that the realized loss exceeds β should be less than or equal to the firm's tolerable loss probability α.

Before proceeding with the analysis of risk aversion, it is important to make several observations. In the absence of exchange-rate uncertainty, the introduction of risk aversion through a VaR constraint as in (15) does not lead to dual sourcing. It is already pointed out earlier that when random exchange rate is replaced with its certainty equivalent $\bar{e}$ in the risk-neutral setting, the firm works with only one supplier and reserves capacity at the lower cost source. When the optimal capacity reserved at the low-cost source (let us denote it with Q^N) violates the VaR constraint due to the stochastic demand, then the firm would reduce its initial capacity reservation to satisfy the constraint at the tolerated loss. Specifically, let x_α denote the value of the demand random variable that corresponds to α probability in its cdf. It is sufficient to check the value of the realized profit at x_α from reserving Q^N units of capacity at the low cost supplier j: If $p\, x_\alpha - (k_j + c_j + t_j)Q^N < -\beta$, then the firm reduces its initial capacity investment from Q^N to $Q^A = (p\, x_\alpha - \beta)/(k_j + c_j + t_j)$ where Q^A describes the amount of capacity reserved in Stage 1 due to risk aversion. Thus, in the absence of exchange-rate uncertainty, the firm reduces its initial capacity reservation commitment as a result of risk aversion but does not switch to dual sourcing.

We next examine the impact of risk aversion in the presence of exchange-rate uncertainty. Let e_α denote the exchange rate realization at fractile $1 - \alpha$ (i.e., $[1 - G(e_\alpha)] = \alpha$). Because demand uncertainty in isolation does not lead to any policy change in our model, but rather a reduction in reserved capacity, we focus on problem settings where exchange-rate is a source of uncertainty in violating the VaR requirement with $e_\alpha > \tau_2$. Exchange-rate realizations greater than τ_2 are most detrimental to the firm because it would waste the entire capacity reserved at the foreign source due to the fact that $q_F^*(e) = 0$ for $e \geq \tau_2$.

In the presence of exchange-rate uncertainty, incorporating risk aversion encourages the firm to engage in dual sourcing. This can be seen when the optimal policy in the risk-neutral setting is an offshore sourcing policy as in the case of policies F_H and F_L. When risk aversion is included and the VaR constraint is violated, the firm can decrease the level of capacity investment in the foreign source in order to comply with the VaR constraint in (15). Let us define Q_F^A as the level of capacity reserved at the foreign source that yields realized profit equal to $-\beta$ at exchange-rate realization e_α, i.e.,

$$Q_F^A = \beta / \left(k_F + u_F e_\alpha\right). \tag{16}$$

We use the following two conditions in Proposition 12, which characterizes when the firm switches from single sourcing at the foreign source (i.e., offshore sourcing policies F_H and F_L) to dual sourcing:

$$\text{(RA1): } M_H\left[\left(\frac{c_F - u_F}{k_F + u_F\bar{e}}\right)\int_{e_l}^{\tau(Q_F^A)} G(e)\,de - 1\right] > k_H + u_H - E\left[\left(m_H - \tilde{m}_F^{\,+}\right)^+\right],$$

$$\text{(RA2): } M_H\left[\left(\frac{c_F - u_F}{k_F + u_F\bar{e}}\right)\int_{e_l}^{\tau(Q_F^A)} G(e)\,de - 1\right] > E\left[\tilde{m}_F^{\,+}\right] - M_H - \left(c_F - u_F\right)\int_{e_l}^{\tau(Q_F^A)} G(e)\,de.$$

Proposition 12. *Suppose the risk-neutral optimal solution violates the VaR constraint, and* $e_\alpha \geq \tau_2$, $u_F > 0$.

(a) *When the optimal sourcing policy in the risk-neutral setting is* F_H *and* $Q_F^A > q_H^0$, *the firm switches to dual sourcing under risk aversion if* RA1 *holds;*

(b) *When the optimal sourcing policy in the risk-neutral setting is either* F_L *or* F_H *with* $Q_F^A \leq q_H^0$, *the firm switches to dual sourcing under risk aversion if* RA2 *holds.*

From Proposition 12, we see conditions that cause the firm to switch from offshore sourcing policies that are optimal in a risk-neutral setting to dual sourcing. Condition RA1 is a stronger condition because the right hand side (RHS) of RA1 is greater than that of RA2 when $Q_F^A > q_H^0$. In this case, when RA1 holds, condition RA2 also holds. Thus, RA1 can be perceived as a sufficient condition that, when an offshore sourcing policy is optimal in the risk-neutral setting, then the firm switches to dual sourcing as a consequence of risk aversion. It can also be shown that when dual sourcing policies are optimal in the risk-neutral setting, they continue to be optimal under risk aversion in our model.

In conclusion, our analysis shows that the introduction of risk aversion through a VaR constraint leads to a higher likelihood of dual sourcing.

7. Financial Hedging

We next examine the impact of introducing the flexibility to purchase financial hedging instruments on the firm's risk concern and expected profit. We consider the case when the firm obtains a certain number of currency futures contracts, denoted H, in stage 1 along with its capacity reservation decisions (Q_H, Q_F). Each unit of financial hedging contract has a unit cost of $h(e_s)$ (also referred to as the premium) and a strike (or, exercise) price of e_s. We assume that the financial institution sells the hedging instrument at cost, i.e.,

$$h\left(e_s\right) = \int_{e_s}^{e_h}\left(e - e_s\right) f\left(e\right) de - \int_{e_l}^{e_s}\left(e_s - e\right) f\left(e\right) de. \tag{17}$$

In Stage 1, the firm now determines the optimal values of (Q_H, Q_F, H) in order to maximize the expected profit subject to the same VaR requirement:

$$\max_{Q_H,Q_F,H\geq 0} E\left[\Pi\left(Q_H,Q_F,H\right)\right] = -k_H Q_H - k_F Q_F - h\left(e_s\right)H + \int_{e_l}^{e_h} \pi^*\left(Q_H,Q_F,H,e\right)g\left(e\right)de\,. \tag{18}$$

In Stage 2, all financial hedging contracts purchased in Stage 1 are exercised. Subject to the same capacity reservation constraint in (3), the second-stage objective function in (2) is then revised as follows:

$$\begin{aligned}\pi^*\left(Q_H,Q_F,H,e\right) &= \max_{q_H,q_F\geq 0} E\left[\pi_2\left(q_H,q_F,x\,|\,Q_H,Q_F,H,e\right)\right]\\ &= -c_H q_H - c_F e q_F - t_H q_H - t_F q_F - u_H\left(Q_H-q_H\right)^+ - u_F e\left(Q_F-q_F\right)^+\\ &\quad +\left(e-e_s\right)H + \int_{x_l}^{x_h} p\min\left\{x,q_H+q_F\right\}f\left(x\right)dx\end{aligned} \tag{19}$$

The objective function in (19) adds the term $(e - e_s)H$ into (2). The advantage of employing financial hedging in Stage 1 enables the firm to eliminate the negative consequences of the VaR constraint in (15), which can now be expressed as follows:

$$P_{(\tilde{e},\tilde{x})}\left[\left\{\begin{array}{l}-k_H Q_H - k_F Q_F - h\left(e_s\right)H - \left(c_H+t_H\right)q_H^*\left(e\right) - \left(c_F e + t_F\right)q_F^*\left(e\right)\\ -u_H\left(Q_H - q_H^*\left(e\right)\right)^+ - u_F e\left(Q_F - q_F^*\left(e\right)\right)^+ + \left(e-e_s\right)H + p\min\left\{x, q_H^*\left(e\right)+q_F^*\left(e\right)\right\}\end{array}\right\} < -\beta\right] \leq \alpha\,. \tag{20}$$

The firm's tolerated loss probability α is not exceeded when the firm is ensured to have realized profits greater than or equal to $-\beta$ when the exchange-rate random variable takes values in the range $e_l \leq e \leq e_\alpha$. For any given (Q_H, Q_F), if the VaR constraint in (15) is violated, then the firm determines the number of hedging contracts that would satisfy the same risk concern expressed in (20) even at the lowest second-stage revenues occurring at $x = x_l$. This is formalized in the next proposition.

Proposition 13. *If the first-stage decisions* (Q_H, Q_F) *does not satisfy the VaR constraint in* (15) *then*

(a) *the firm can purchase the following number of financial hedging contracts with a strike price* $e_s \leq e_\alpha$ *and premium* $h(e_s)$ *as defined in* (17)

$$H^*\left(e_s\right) = \frac{\left\{\begin{array}{c}k_H Q_H + k_F Q_F + \left(c_H+t_H\right)q_H^*\left(e_\alpha\right) + \left(c_F e_\alpha + t_F\right)q_F^*\left(e_\alpha\right) - p\min\left\{x_l, q_H^*\left(e_\alpha\right)+q_F^*\left(e_\alpha\right)\right\} - \beta\\ +u_H\left(Q_H - q_H^*\left(e_\alpha\right)\right)^+ + u_F e_\alpha\left(Q_F - q_F^*\left(e_\alpha\right)\right)^+\end{array}\right\}}{\left(e_\alpha - e_s - h\left(e_s\right)\right)} \tag{21}$$

and satisfy (20)*; and*

(b) *the expected profit* $E[\Pi(Q_H, Q_F, H^*(e_s))]$ *under risk aversion and financial hedging is equivalent to* $E[\Pi(Q_H, Q_F, 0)]$ *of the risk-neutral setting in the absence of the VaR constraint* (15).

Proposition 13(a) shows that financial hedging enables the firm to satisfy the VaR constraint. The number of hedging contracts specified in (21) accounts for the losses that can incur at all of the undesirable exchange-rate realizations in the range of $e_\alpha \leq e \leq e_h$ corresponding to α percent in the cdf. Thus, the number of financial hedging contracts in (21) guarantees that the firm's losses exceeding β is less than or equal to the firm's tolerated loss probability α.

Proposition 13(b) shows that financial hedging is also beneficial in terms of protecting the expected profit. For any capacity reservation decisions (Q_H, Q_F) in stage 1, financial hedging enables the firm to obtain the same expected profit it earned in the risk-neutral setting without having to sacrifice initial capacity reservation in order to satisfy the VaR constraint.

The consequence of Proposition 13(b) is that the firm's set of potential optimal policies under risk aversion and financial hedging is identical to the set of policies developed in Section 4 for the risk-neutral setting. Thus, we conclude that financial hedging not only eliminates the negative consequences of risk aversion, but also makes our five potentially optimal policies to hold under more general settings.

In sum, we find that our insights into the role of exchange rate uncertainty on optimal sourcing decisions are robust. While the introduction of risk aversion through a VaR constraint can make dual sourcing more likely, our earlier conclusions are unaffected by the use of financial hedging to mitigate risk.

8. Numerical Illustration

In this section, we provide numerical illustrations from the operating environment of the furniture manufacturer that motivated our study. For this furniture company specializing in school and library furniture, there are three selling seasons, each representing a four-month time window (following the traditional school semesters). The furniture maker develops a forecast for its products for each selling season (for four months). The firm makes capacity reservation decisions one selling season in advance. Book carts, one of the firm's best known products, are sourced from small suppliers that charge a unit capacity reservation cost ranging from 1% to 5% of the selling price. The firm has other products where the unit capacity reservation cost is between 5% and 10% of the selling price (e.g., office desks and chairs). We present the results associated with surprising insights, and ignore the expected results associated with product with higher unit capacity reservation costs.

The capacity reservation payment is made at the spot exchange rate one season before the selling season approaches (equivalent to four months). Thus, it can be converted to domestic currency using the spot exchange rate at the time of the initial payment. Four months later, corresponding to the beginning of the selling season, the furniture maker specifies the exact amount of products to be manufactured at each supplier. It is important to note that the firm continues to operate under demand uncertainty during the selling season; schools and libraries continue to place orders during the selling season.

We next describe the data used to represent exchange-rate fluctuations. Our analysis uses data on the daily Euro-Dollar exchange rate from the beginning of 2010 through the end of 2012 (3-year period), corresponding to the planning period for the furniture maker that motivated our problem. We first analyze the rate of change in the exchange rate in four months. We accomplish this goal by examining each daily exchange rate (or, spot rate), denoted s_t, and comparing it with the daily exchange rate of four months later, denoted s_{t+120}, within our three-year data set. Specifically, for each day over the 3-year period, we calculate the proportion of the exchange-rate four months into the future relative to the current exchange rate, i.e., $e_t = s_{t+120}/s_t$. The comparison of the daily exchange rate with its counterpart in four months results in an empirical distribution which we use in our analysis. Figure 6(a) provides the histogram of the empirical distribution representing the fluctuations in the Euro-Dollar exchange rate used in our numerical illustrations. Figure 6(b) shows the frequency distribution of the change in the value of the exchange rate in four months. As can be seen from Figure 6(b), there would not be a well-fitting statistical distribution to represent the change in exchange rates in four months. Therefore, we use the entire data set of the changes during the period of 2010 – 2012 as our distribution in the analysis.

In order to provide a meaningful and comparative analysis without revealing costs and prices, we normalize the selling price to \$100, and scale the capacity reservation and transportation costs accordingly. Figure 7 demonstrates the optimal decisions with changing values of the second-stage operational costs, c_H and c_F, as described in horizontal and vertical axes, respectively. The unit capacity reservation costs, k_H and k_F, are smallest in Figure 7(a), at 1% of the selling price, and their values increase in figures 7(b) and 7(c) up to 5% of the selling price. The same policy indicators, {H, F_L, F_H, D_R, D_E}, are used to designate the five potentially optimal policies identified in Section 4.

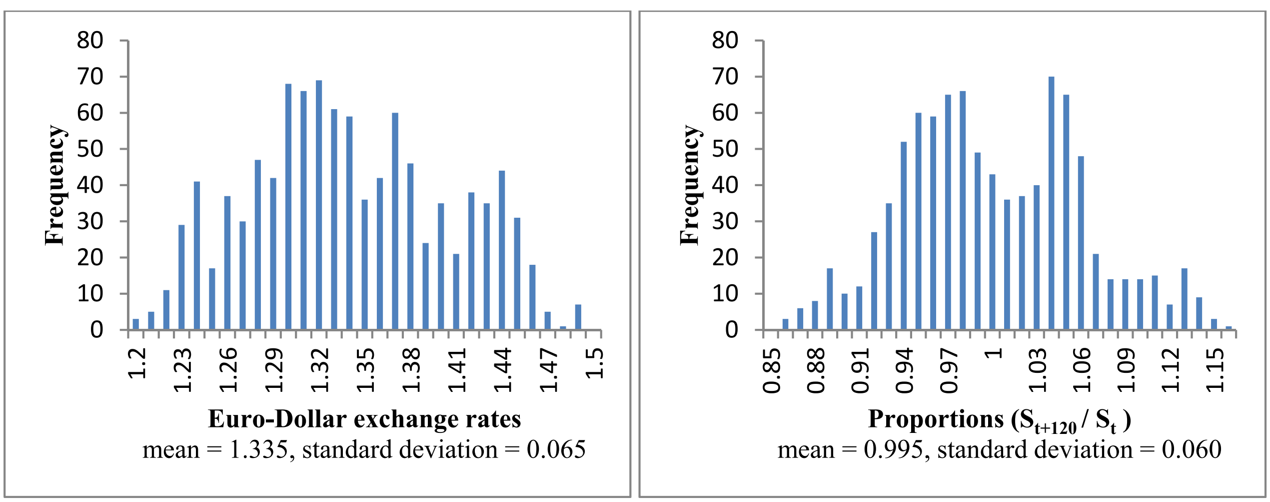
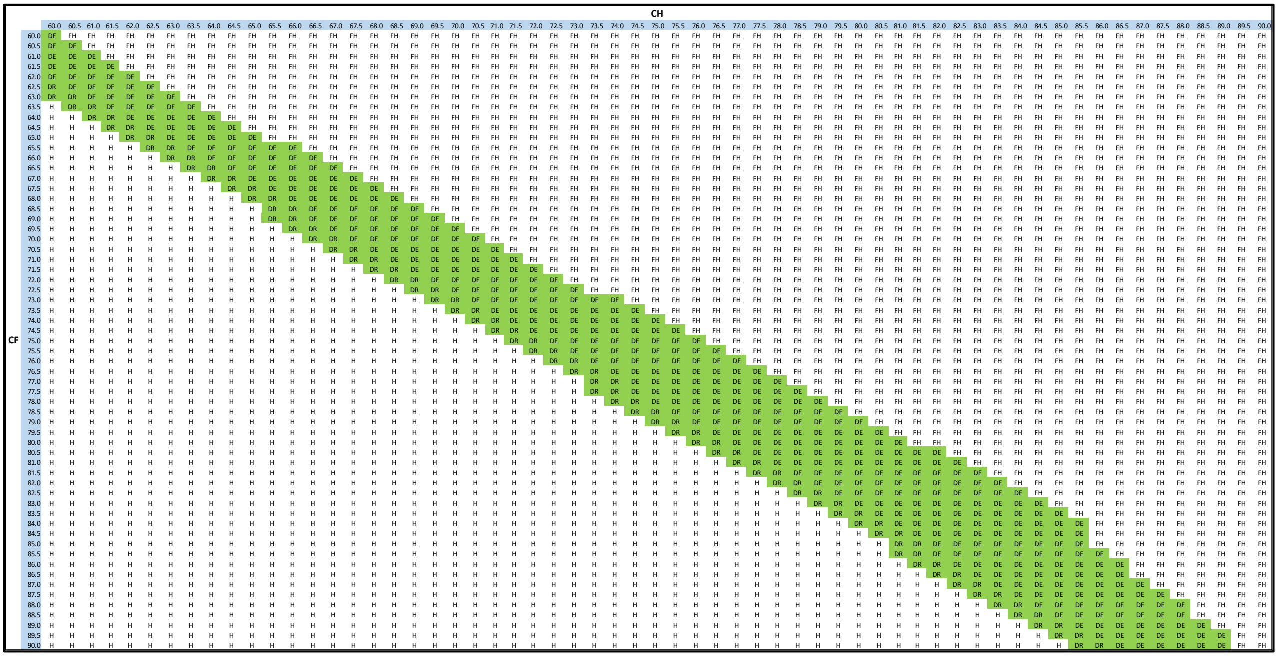
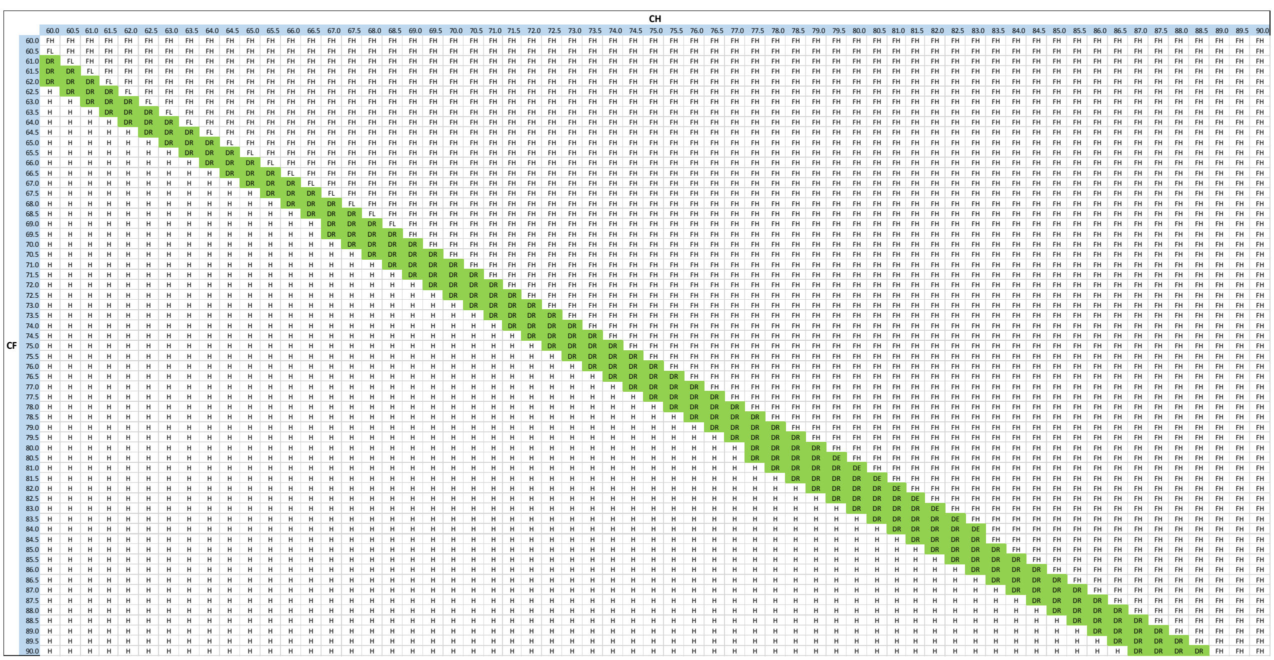


Figure 6. The Euro-Dollar exchange rate between 2010 – 2012. (a) Histogram of the actual values representing the variation in the Euro-Dollar exchange. (b) Frequency distribution of the proportions representing the change in the value of the exchange rate in four months.



(a) $k_H = \$1$, $k_F = \$1$



(b) $k_H = \$2$, $k_F = \$2$

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		60.0	60.5	61.0	61.5	62.0	62.5	63.0	63.5	64.0	64.5	65.0	65.5	66.0	66.5	67.0	67.5	68.0	68.5	69.0	69.5	70.0	70.5	71.0	71.5	72.0	72.5	73.0	73.5	74.0	74.5	75.0	75.5	76.0	76.5	77.0	77.5	78.0	78.5	79.0	79.5	80.0	80.5	81.0	81.5	82.0	82.5	83.0	83.5	84.0	84.5	85.0	85.5	86.0	86.5	87.0	87.5	88.0	88.5	89.0	89.5	90.0
	60.0	FL	FL	FL	FL	FL	FL	FL	FL	FH	FH	FH	FH	FH	FH	FH	FH	FH	FH	FH	FH	FH	FH	FH	FH	FH	FH	FH	FH	FH	FH	FH	FH	FH	FH	FH	FH	FH	FH	FH	FH	FH	FH	FH	FH	FH	FH	FH	FH	FH	FH	FH	FH	FH	FH	FH	FH	FH	FH	FH	FH	---
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	79.5	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	DR	FL	FL	FL	FL	FL	FL	FL	FL	FL	FH	FH	FH	FH	FH	FH	FH	FH	FH	FH	FH	FH	FH	---
	80.0	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	DR	FL	FL	FL	FL	FL	FL	FL	FL	FL	FH	FH	FH	FH	FH	FH	FH	FH	FH	FH	FH	FH	---
	80.5	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	DR	FL	FL	FL	FL	FL	FL	FL	FL	FL	FH	FH	FH	FH	FH	FH	FH	FH	FH	FH	FH	---
	81.0	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	DR	FL	FL	FL	FL	FL	FL	FL	FL	FL	FH	FH	FH	FH	FH	FH	FH	FH	FH	FH	---
	81.5	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	DR	FL	FL	FL	FL	FL	FL	FL	FL	FL	FH	FH	FH	FH	FH	FH	FH	FH	FH	---
	82.0	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	DR	DR	FL	FL	FL	FL	FL	FL	FL	FL	FH	FH	FH	FH	FH	FH	FH	FH	FH	---
	82.5	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	DR	DR	FL	FL	FL	FL	FL	FL	FL	FL	FH	FH	FH	FH	FH	FH	FH	FH	---
	83.0	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	DR	DR	FL	FL	FL	FL	FL	FL	FL	FL	FH	FH	FH	FH	FH	FH	FH	---
	83.5	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	DR	DR	FL	FL	FL	FL	FL	FL	FL	FL	FH	FH	FH	FH	FH	FH	---
	84.0	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	DR	DR	FL	FL	FL	FL	FL	FL	FL	FL	FH	FH	FH	FH	FH	---
	84.5	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	DR	FL	FL	FL	FL	FL	FL	FL	FL	FL	FH	FH	FH	FH	---
	85.0	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	DR	FL	FL	FL	FL	FL	FL	FL	FL	FL	FH	FH	FH	---
	85.5	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	DR	FL	FL	FL	FL	FL	FL	FL	FL	FL	FH	FH	---
	86.0	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	DR	FL	FL	FL	FL	FL	FL	FL	FL	FL	FH	---
	86.5	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	DR	FL	FL	FL	FL	FL	FL	FL	FL	FL	---
	87.0	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	DR	FL	FL	FL	FL	FL	FL	FL	FL	---
	87.5	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	DR	FL	FL	FL	FL	FL	FL	FL	---
	88.0	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	DR	FL	FL	FL	FL	FL	FL	---
	88.5	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	DR	FL	FL	FL	FL	FL	---
	89.0	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	DR	FL	FL	FL	FL	---
	89.5	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	DR	FL	FL	FL	---
	90.0	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	DR	FL	FL	---

(c) k_H = \$5, k_F = \$5

Figure 7. Optimal sourcing policies with changing second-stage operational costs (c_H, c_F) between \$60 and \$90, with varying unit capacity reservation costs (k_H, k_F), with unit transportation costs t_H = \$2, t_F = \$4, and zero penalty cost of unused capacity ($u_H = u_F = 0$). The "---" sign means the cost terms do not satisfy assumption A1. The values of k_F and c_F are equivalent to their corresponding dollar values at time 0.

Several observations can be made from the comparison of figures 7(a) – (c). First, it is easy to see that when the operational cost of the home (foreign) source is significantly less expensive than that of the foreign (domestic) source, then the optimal policy is a single source policy H (F).

Second, dual sourcing policies are optimal in a region where the second-stage operational costs are relatively similar. This region of dual sourcing (see the diagonal axis) is larger when the unit reservation costs, k_H and k_F, are relatively small. The region of dual sourcing shrinks with increasing values of k_H and k_F (as can be seen from the comparison of figures 7(a) – (c)). This result indicates that when the unit capacity reservation costs are small in comparison to selling prices, the firm can enjoy the benefits from the fluctuations in exchange rates by adjusting the production levels in Stage 2. Thus, dual sourcing policies D_R and D_E are more desirable. On the other hand, if the unit capacity reservation costs are higher in value relative to the selling price, then the firm is less likely to commit to two suppliers upfront. Thus, single source policies H, F_L and F_H are more desirable. In terms of the products offered by the furniture company that motivated our study, dual sourcing is more desirable for products such as book carts (with lower capacity reservation costs relative to the selling price), and single sourcing is the preferred sourcing policy for office furniture (with relatively higher unit capacity reservation costs).

Third, the region for dual sourcing shows an increasing and a decreasing behavior with increasing values of the second-stage production costs (c_H and c_F). There are two opposing drivers for this phenomenon. First, higher operational costs in Stage 2 diminish the relative magnitude of the unit capacity reservation costs of Stage 1. Therefore, the firm perceives reserving capacity to be less expensive in sourcing decisions; this leads to a wider range of second stage operational costs that feature dual sourcing policies as the optimal choice. On the other hand, higher operational costs reduce the value that can be gained from dual sourcing policies. Specifically, as the operational costs increase, sourcing from both suppliers become less profitable, and thus the potential gain from dual sourcing policies declines with higher second-stage operational costs. As a consequence, the region of dual sourcing shrinks as the operational costs continue to increase in relative to the selling price.

Fourth, policy D_R is more desirable than policy D_E at higher values of k_H and k_F as in Figure 7(c). This is because there is not a sufficient degree of exchange-rate volatility to benefit from having excess capacity in place. Similarly, policy D_E is more often the desired dual sourcing policy (over policy D_R) in the bandwidth representing dual sourcing preferences at lower values of k_H and k_F as in Figure 7(a). With lower capacity reservation costs (relative to the selling price), the firm has a higher degree of benefit from exchange rate fluctuations under policy D_E in Stage 2, without having to make a significant payout in Stage 1. While not presented in these numerical illustrations, the range of dual sourcing increases with a higher degree of variation in exchange rates. The Euro-Dollar exchange rate is perceived to be the most stable exchange rate by industry professionals, and our data represents a conservative impact of exchange rate variations in capacity reservation decisions. Higher variation in exchange rates would be observed in other currency conversions. A higher degree of variation in the exchange rate increases the benefits from recourse, and specifically increases the value gained from adjusting production levels based on the realized values of exchange rates. Thus, a higher degree of exchange-rate variability makes dual sourcing a more desirable alternative over single sourcing policies.

9. Conclusions and Managerial Insights

This paper examines the impact of exchange-rate uncertainty on capacity reservation decisions for a global firm. We develop an analytical model for a firm that sources from two suppliers, one domestic and one foreign, and sells in a single market. While demand uncertainty in isolation (i.e., ignoring the impact of exchange-rate uncertainty) does not lead to dual sourcing in our model, exchange-rate uncertainty creates the incentive for the firm to engage in dual sourcing.

The paper makes four contributions. First, we identify the set of potentially optimal policies and the conditions that lead to these policies. Five potentially optimal sourcing policies emerge: One onshore sourcing, two kinds of offshore souring policies, and two characteristically different dual sourcing policies. One dual sourcing policy commits to a total capacity amount that would equal the amount it would have reserved under the offshore sourcing policy, but it rations this total capacity investment between the domestic and foreign suppliers according to the exchange-rate uncertainty. This dual-sourcing policy can be perceived as a defensive and a conservative approach as it is motivated to negate the detrimental consequences of an appreciating exchange rate making the foreign supplier an expensive source. The same policy foregoes the benefits of a lower cost foreign supplier under realizations of exchange-rate devaluation. The second dual sourcing policy features excess capacity. The same amount of capacity is reserved at the foreign supplier as with the offshore sourcing policy, and there is additional capacity reserved at the domestic supplier. However, the amount of capacity reserved at the domestic supplier is less than the amount that would have been reserved under the onshore sourcing policy. Thus, the domestic supplier capacity is perceived as a backup capacity, and is intended to be utilized in order to benefit from exchange-rate fluctuations. We show that these five policies can be located by checking four optimality conditions. The conditions clarify how the firm switches its optimal policy choice from one sourcing policy to another with increasing degrees of exchange-rate volatility.

Second, our analysis shows that a lower sourcing cost is neither a necessity nor a sufficient condition to reserve capacity at a supplier. Under exchange-rate uncertainty, lower unit sourcing cost does not necessarily qualify a supplier as a potential source. It can be optimal to source only from a foreign supplier that has a higher expected sourcing cost. This finding goes against the common practice of low-cost sourcing and suggests that managers should think more deeply about their sourcing policies under exchange-rate uncertainty. Moreover, our results show that the firm may reserve capacity only at the foreign supplier even if the expected operational cost (inclusive of production, transportation, duty, and localization expenses) is higher than the selling price. This result makes our finding associated with reserving capacity only at the more expensive supplier even more pronounced as it exemplifies the opportunity gains from the currency fluctuations.

Third, we show that risk aversion makes dual sourcing more desirable. In particular, when the firm's optimal policy is offshoring in the risk-neutral setting, the introduction of risk aversion with a VaR requirement can trigger a policy switch to dual sourcing. Thus, under risk aversion, dual sourcing becomes more pronounced over single sourcing.

Fourth, financial hedging can help eliminate the negative consequences of risk aversion and enable the firm to replicate the expected profit of the risk-neutral policies. Thus, the firm obtains the same set of five potentially optimal global sourcing policies in the presence of financial hedging. As a consequence, our policy findings are robust as they continue to hold under risk aversion and financial hedging.

In addition to the above four contributions, our study provides insights into the impact of exchange-rate volatility on capacity reservation decisions. While the reserved capacity of the foreign supplier increases with volatility, capacity reservation decisions at the domestic supplier can exhibit both an increasing and decreasing behavior. The conditions for the increasing and decreasing behavior are described under increasing exchange-rate uncertainty. Greater degrees of exchange-rate volatility do not always increase domestic capacity and do not regularly lead to higher flexibility. The firm may prefer to give up some of its allocation flexibility under policy D_E, for example, by reducing its domestic capacity and capitalizing more on the foreign capacity.

We have stated earlier that our work is motivated from the challenges faced by a specialty furniture maker. Our paper demonstrates numerically how our model applies to the products of this firm. The firm's prior sourcing decisions have ignored exchange-rate uncertainty, and have relied on utilizing a single supplier which is selected based on the lower cost. Incorporating exchange-rate uncertainty, our analysis shows that, when unit capacity reservation costs are smaller relative to the selling price as in the case of book carts, the firm prefers dual sourcing policies. When the unit capacity reservation cost is higher in comparison to the selling price as in the case of office desks and chairs, then a single-sourcing policy is desired. The firm now engages in dual sourcing for some of its products. We also report that both the rationing dual sourcing policy and dual sourcing policy with excess capacity are currently being utilized among the firm's product portfolio.

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Appendix

Proposition A1. *The first-stage expected profit function for offshore sourcing is concave in Q_F*; (b) $Q_F^0 > 0$.

Proof. Available upon request.

Lemma A1. *In a Capacitated Newsvendor problem, the optimal solution is min* (q^*, Q) *where q^* is the optimal quantity for the typical newsvendor problem* (*without capacity constraints*) *and Q is the capacity level.*

Lemma A2. *For a newsvendor who is initially granted an initial inventory of q_I, it is optimal to order* $(q^* - q_I)^+$ *where q^* is the optimal quantity for the typical newsvendor problem* (*no initial inventory*).

Proof of Proposition 1. When there are two suppliers each associated with a different set of second-stage costs, and given capacity level (i.e., Q_H and Q_F), the firm prioritizes sourcing from the less costly supplier. Therefore, by Lemma A1, $q_H^* = \min\{Q_H, q_H^0\}$ when $m_H \geq m_F$ and $q_F^* = \min\{Q_F, q_F^0(e)\}$ when $m_F > m_H$.

If the desired level of second-stage production is not completed then the firm continues to source from the expensive supplier. We present the proof for q_H^*. The proof for q_F^* is analogous.

When $m_F > m_H$, if the foreign supplier is in short of capacity (i.e., $\min\{Q_F, q_F^0(e)\} = Q_F$)), we know from Lemma A2 that the domestic source can still be utilized as long as $q_H^0 > Q_F = q_F^*$. In this case, the optimal production quantity would be $(q_H^0 - Q_F)^+$ if there were no capacity constraints. It follows that $q_H^* = \min\{Q_H, (q_H^0 - Q_F)^+\}$ when Q_H restricts the production quantity. ❑

Proposition A2. *The first-stage expected profit function in region $R1$ is jointly concave in* (Q_H, Q_F).

Proof. The profit function in region $R1$ can be expressed as follows:

$$
\begin{aligned}
E\left[\Pi\left(Q_H, Q_F \middle| R1\right)\right] &= -\left(k_H + u_H\right)Q_H - \left(k_F + u_F\bar{e}\right)Q_F \\
&+\int_{e_l}^{\tau_1}\left[pD - \left(c_H - u_H + t_H\right)Q_H - \left(c_F e - u_F e + t_F\right)Q_F - \int_{Q_H+Q_F}^{\infty} p\left(x - Q_H - Q_F\right)f\left(x\right)dx\right]g\left(e\right)de \\
&+\int_{\tau_1}^{\tau(Q_H+Q_F)}\left[pD - \left(c_H - u_H + t_H\right)Q_H - \left(c_F e - u_F e + t_F\right)Q_F - \int_{Q_H+Q_F}^{\infty} p\left(x - Q_H - Q_F\right)f\left(x\right)dx\right]g\left(e\right)de \qquad (22)\\
&+\int_{\tau(Q_H+Q_F)}^{\tau(Q_H)}\left[pD - \left(c_H - u_H + t_H\right)Q_H - \left(c_F e - u_F e + t_F\right)\left(q_F^0(e) - Q_H\right) - \int_{q_F^0(e)}^{\infty} p\left(x - q_F^0(e)\right)f\left(x\right)dx\right]g\left(e\right)de \\
&+\int_{\tau(Q_H)}^{e_h}\left[pD - \left(c_H - u_H + t_H\right)Q_H - \int_{Q_H}^{\infty} p\left(x - Q_H\right)f\left(x\right)dx\right]g\left(e\right)de
\end{aligned}
$$

$$
\frac{\partial E\left[\Pi\left(Q_H, Q_F \middle| R1\right)\right]}{\partial Q_F} = -\left(k_F + u_F\bar{e}\right) + \left(c_F - u_F\right)\int_{e_l}^{\tau(Q_H+Q_F)}\left(\tau(Q_H+Q_F) - e\right)g\left(e\right)de
$$

$$
\begin{aligned}
\frac{\partial E\left[\Pi\left(Q_H, Q_F \middle| R1\right)\right]}{\partial Q_H} &= -\left(k_H + c_H + t_H\right) + \left(c_F - u_F\right)\bar{e} + t_F - \left(c_F - u_F\right)\int_{\tau(Q_H)}^{e_h}\left(e - \tau(Q_H)\right)g\left(e\right)de \\
&+\left(c_F - u_F\right)\int_{e_l}^{\tau(Q_H+Q_F)}\left(\tau(Q_H+Q_F) - e\right)g\left(e\right)de
\end{aligned}
$$

$$
\frac{\partial^2 E\left[\Pi\left(Q_H, Q_F \middle| R1\right)\right]}{\partial Q_F^2} = \frac{\partial^2 E\left[\Pi\left(Q_H, Q_F \middle| R1\right)\right]}{\partial Q_H \partial Q_F} = -pf\left(Q_H + Q_F\right)G\left(\tau(Q_H + Q_F)\right) < 0,
$$

$$
\frac{\partial^2 E\left[\Pi\left(Q_H, Q_F \middle| R1\right)\right]}{\partial Q_H^2} = -pf\left(Q_H + Q_F\right)G\left(\tau(Q_H + Q_F)\right) - pf\left(Q_H\right)\left[1 - G\left(\tau(Q_H)\right)\right] < 0,
$$

$$\frac{\partial^2 E\left[\Pi\left(Q_H,Q_F\,|R1\right)\right]}{\partial Q_H^2}=\frac{\partial^2 E\left[\Pi\left(Q_H,Q_F\,|R1\right)\right]}{\partial Q_F^2}-pf\left(Q_H\right)\left[1-G\left(\tau(Q_H)\right)\right]<\frac{\partial^2 E\left[\Pi\left(Q_H,Q_F\,|R1\right)\right]}{\partial Q_F^2}\;.$$

The determinant is positive, and the objective function jointly concave in Q_H and Q_F in region $R1$ because

$$\frac{\partial^2 E\left[\Pi\left(Q_H,Q_F\,|R1\right)\right]}{\partial Q_H^2}\times\frac{\partial^2 E\left[\Pi\left(Q_H,Q_F\,|R1\right)\right]}{\partial Q_F^2}>\left(\frac{\partial^2 E\left[\Pi\left(Q_H,Q_F\,|R1\right)\right]}{\partial Q_H\partial Q_F}\right)^2=\left(\frac{\partial^2 E\left[\Pi\left(Q_H,Q_F\,|R1\right)\right]}{\partial Q_H^2}\right)^2.\ \square$$

Proposition A3. *The first-stage expected profit function in region R2 is jointly concave in* (Q_H, Q_F).

Proof. The objective function in region $R2$ can be expressed as follows:

$$\begin{aligned}E\left[\Pi\left(Q_H,Q_F\,|R2\right)\right]&=-\left(k_H+u_H\right)Q_H-\left(k_F+u_F\bar{e}\right)Q_F\\&+\int_{e_l}^{\tau_1}\left[pD-\left(c_H-u_H+t_H\right)\left(q_H^0-Q_F\right)-\left(c_Fe-u_Fe+t_F\right)Q_F-\int_{q_H^0}^{\infty}p\left(x-q_H^0\right)f\left(x\right)dx\right]g\left(e\right)de\\&+\int_{\tau_1}^{\tau(Q_H)}\left[pD-\left(c_H-u_H+t_H\right)Q_H-\left(c_Fe-u_Fe+t_F\right)\left(q_F^0(e)-Q_H\right)-\int_{q_F^0(e)}^{\infty}p\left(x-q_F^0(e)\right)f\left(x\right)dx\right]g\left(e\right)de\\&+\int_{\tau(Q_H)}^{e_h}\left[pD-\left(c_H-u_H+t_H\right)Q_H-\int_{Q_H}^{\infty}p\left(x-Q_H\right)f\left(x\right)dx\right]g\left(e\right)de\end{aligned}\qquad(23)$$

$$\begin{aligned}\frac{\partial E\left[\Pi\left(Q_H,Q_F\,|R2\right)\right]}{\partial Q_H}&=-\left(k_H+u_H\right)+\int_{\tau_1}^{\tau(Q_H)}\left[-\left(c_H-u_H+t_H\right)+\left(c_Fe-u_Fe+t_F\right)\right]g\left(e\right)de\\&+\int_{\tau(Q_H)}^{e_h}\left[-\left(c_H-u_H+t_H\right)+\left(c_F-u_F\right)\tau\left(Q_H\right)+t_F\right]g\left(e\right)de\end{aligned}$$

$$\frac{\partial^2 E\left[\Pi\left(Q_H,Q_F\,|R2\right)\right]}{\partial Q_H^2}=\int_{\tau(Q_H)}^{e_h}\left(c_F-u_F\right)\left[\frac{-pf\left(Q_H\right)}{\left(c_F-u_F\right)}\right]g\left(e\right)de=-pf\left(Q_H\right)\left[1-G\left(\tau(Q_H)\right)\right]<0.$$

Therefore, the objective function is concave in Q_H.

We next show that the objective function is linear in Q_F.

$$\frac{\partial E\left[\Pi\left(Q_H,Q_F\,|R2\right)\right]}{\partial Q_F}=-\left(k_F+u_F\bar{e}\right)+E\left[\left(\tilde{m}_F^{\,+}-m_H\right)^+\right]\qquad(24)$$

Equation (24) is constant and independent of Q_F and Q_H implying that

$$\frac{\partial^2 E\left[\Pi\left(Q_H,Q_F\,|R2\right)\right]}{\partial Q_F^2}=\frac{\partial^2 E\left[\Pi\left(Q_H,Q_F\,|R2\right)\right]}{\partial Q_H\partial Q_F}=0.$$

The Hessian for the objective function in $R2$ is 0, and the objective function is jointly concave in Q_H and Q_F in region $R2$.❑

Proposition A4. *The first-stage expected profit function in region R3 is jointly concave in* (Q_H, Q_F).

Proof. The objective function in region $R3$ can be expressed as follows:

$$\begin{aligned}E\left[\Pi\left(Q_H,Q_F\,|R3\right)\right]&=-\left(k_H+u_H\right)Q_H-\left(k_F+u_F\bar{e}\right)Q_F\\&+\int_{e_l}^{\tau(Q_F)}\left[pD-\left(c_Fe-u_Fe+t_F\right)Q_F-\int_{Q_F}^{\infty}p\left(x-Q_F\right)f\left(x\right)dx\right]g\left(e\right)de\end{aligned}$$

$$+\int_{\tau(Q_F)}^{\tau_1}\left[pD-\left(c_F e-u_F e+t_F\right)q_F^o(e)-\int_{q_F^0(e)}^{\infty}p\left(x-q_F^0(e)\right)f\left(x\right)dx\right]g\left(e\right)de$$

$$+\int_{\tau_1}^{\tau(Q_H)}\left[pD-\left(c_H-u_H+t_H\right)Q_H-\left(c_F e-u_F e+t_F\right)\left(q_F^0(e)-Q_H\right)-\int_{q_F^0(e)}^{\infty}p\left(x-q_F^0(e)\right)f\left(x\right)dx\right]g\left(e\right)de. \quad (25)$$

$$+\int_{\tau(Q_H)}^{e_h}\left[pD-\left(c_H-u_H+t_H\right)Q_H-\int_{Q_H}^{\infty}p\left(x-Q_H\right)f\left(x\right)dx\right]g\left(e\right)de$$

$$\frac{\partial E\left[\Pi\left(Q_H,Q_F\left|R3\right.\right)\right]}{\partial Q_F}=-\left(k_F+u_F\bar{e}\right)+\left(c_F-u_F\right)\int_{e_l}^{\tau(Q_F)}\left(\tau(Q_F)-e\right)g\left(e\right)de$$

$$\frac{\partial E\left[\Pi\left(Q_H,Q_F\left|R3\right.\right)\right]}{\partial Q_H}=-\left(k_H+u_H\right)-\left(c_F-u_F\right)\int_{\tau(Q_H)}^{e_h}\left(e-\tau(Q_H)\right)g\left(e\right)de+E\left[\left(m_H-\tilde{m}_F\right)^+\right]$$

$$\frac{\partial^2 E\left[\Pi\left(Q_H,Q_F\left|R3\right.\right)\right]}{\partial Q_H^2}=\left(c_F-u_F\right)\int_{\tau(Q_H)}^{e_h}\frac{-pf\left(Q_H\right)}{\left(c_F-u_F\right)}g\left(e\right)de=-pf\left(Q_H\right)\left[1-G\left(\tau(Q_H)\right)\right]<0$$

$$\frac{\partial^2 E\left[\Pi\left(Q_H,Q_F\left|R3\right.\right)\right]}{\partial Q_F^2}=\left(c_F-u_F\right)\int_{e_l}^{\tau(Q_F)}\frac{-pf\left(Q_F\right)}{\left(c_F-u_F\right)}g\left(e\right)de=-pf\left(Q_F\right)\left[1-G\left(\tau(Q_F)\right)\right]<0$$

$$\frac{\partial^2 E\left[\Pi\left(Q_H,Q_F\left|R3\right.\right)\right]}{\partial Q_H\partial Q_F}=0\ .$$

The determinant of the Hessian is positive, leading to joint concavity in region *R*3. ❑

Lemma A3. *The objective function in* (4) *is continuous.*

Proof. Available upon request.

Lemma A4. *The profit function in* (4) *is differentiable.*

Proof. Available upon request.

Lemma A5. *Onshore sourcing* (H) *is the optimal policy if and only if* $Q_F^0 < Q_H^0$.

Proof. From Proposition A2, we observe that in *R*1 the shadow price of Q_H monotonically decreases in Q_H as long as the total capacity is fixed. Note that this shadow price is positive at solution ($Q_H = Q_F^0$, $Q_F = 0$) due to concavity of the profit function in Q_H. Moreover, the shadow price of Q_F in *R*1 remains the same as long as the total capacity is unchanged. Therefore, the solution ($Q_H = Q_F^0$, $Q_F = 0$) dominates all solutions along the line $Q_H + Q_F = Q_F^0$. However, this dominant solution corresponds to onshore sourcing which itself is dominated by the optimal onshore sourcing policy H ($Q_H^* = Q_H^0$, $Q_F^* = Q_F^0$).

Consequently, below the line $Q_H + Q_F = Q_H^0$, the optimal solution is the onshore sourcing policy H. ❑

Proof of Proposition 2. From differentiability (lemmas A3 and A4) and piecewise concavity of the objective function (propositions A2, A3, and A4), it follows that the objective function is jointly concave in Q_H and Q_F everywhere.❑

Proof of Proposition 3. The first derivative of the objective function in *R*2 with respect to Q_F is given in (24) as follows:

$$\frac{\partial E\left[\Pi\left(Q_H,Q_F\left|R2\right.\right)\right]}{\partial Q_F}=-\left(k_F+u_F\bar{e}\right)+E\left[\left(\tilde{m}_F^{\,+}-m_H\right)^+\right].$$

The above derivative is constant and is independent of Q_F and Q_H. Therefore, the objective function in region *R*2 is linear in Q_F, which implies that there is no interior solution in this region. ❑

Proof of Proposition 4. We have already shown that the objective function in (4) is jointly concave in Q_H and Q_F, and that there is no interior solution in region $R2$ where the shadow price of Q_F is constant (independent of Q_H and Q_F). Therefore, the sign of this shadow price in equation (24) leads us to the region where the optimal solution is located. If it is positive (negative), the optimal solution lies in region $R3$ (region $R1$) and that the optimal solution is an interior solution if the solution to system of equations in (13) (system of equations in (11)) exists. Suppose the shadow price in (24) is positive, i.e.,

$$E\left[\left(\tilde{m}_F^{\ +} - m_H\right)^+\right] - k_F - u_F\bar{e} > 0 \tag{26}$$

The proof for parts (c) and (e) of the proposition requires analyzing region $R3$.

Region $R3$: In this region, it can observed from (13) that the second equation (first-order condition for Q_F) is a rearrangement of equation (9), and results in the same optimal solution Q_F^0.

In order for Q_H^* to be positive, we must have $\frac{\partial E\left[\Pi\left(Q_H, Q_F \mid R3\right)\right]}{\partial Q_H}\Big|_{Q_H=0}$ as positive. It is shown in the proof of Proposition A4 that this derivative does not depend on the value of Q_F. Hence,

$$\frac{\partial E\left[\Pi\left(Q_H, Q_F \mid R3\right)\right]}{\partial Q_H}\Big|_{Q_H=0} = -\left(k_H + u_H\right) + \int_{\tau_2}^{e_h} m_H\, g(e)\, de + \int_{\tau_1}^{\tau_2} \left(m_H - m_F\right) g(e)\, de$$

must be positive, implying that

$$\frac{\partial E\left[\Pi\left(Q_H, Q_F \mid R3\right)\right]}{\partial Q_H}\Big|_{Q_H=0} = E\left[\left(m_H - \tilde{m}_F^{\ +}\right)^+\right] - k_H - u_H > 0\,. \tag{27}$$

Therefore, $E\left[\left(m_H - \tilde{m}_F^{\ +}\right)^+\right] - k_H - u_H$ must be positive for D_E to be optimal. Otherwise, the first-order derivative with respect to Q_H is negative and $Q_H^* = 0$, which results in the offshore sourcing policy F_H.

On the other hand, defining $\tau' = \tau\,(Q_H + Q_F = Q_H^0) = (c_H + t_H + k_H - t_F\,)/(c_F - u_F)$

$$\frac{\partial E\left[\Pi\left(Q_H, Q_F\right)\right]}{\partial Q_H}\Big|_{Q_H=Q_H^0} = E\left[\left(\tilde{m}_F^{\ +} - m_H\right)^+\right] - E\left[\left(\tilde{m}_F^{\ +} - M_H\right)^+\right] < 0. \tag{28}$$

As a result, Q_H^* is lower than Q_H^0 in this region.

For parts (a), (b) and (d) of this proposition, we investigate Region $R1$.

Note that OC2 is identical to inequality (26). Therefore, if OC2 does not hold (i.e., the first-order derivative of the objective function in (23) with respect to Q_F in region $R2$ is negative), the optimal solution must lie in region $R1$.

Region $R1$: In this region we can observe from the system of equations in (11) that the solution to the second equation (corresponding to the first-order condition for Q_F), i.e., τ_{HF}^*, is identical to the solution in equation (9), i.e., τ_F^*. Therefore, an interior optimal solution must satisfy

$$Q_H^* + Q_F^* = Q_F^0 \tag{29}$$

From (29), we have $\frac{\partial E\left[\Pi\left(Q_H, Q_F \mid R1\right)\right]}{\partial Q_F}\Big|_{\left(Q_H+Q_F=Q_F^0\right)} = 0$ at the optimal solution. In order for Q_H^* to be positive, we need $\frac{\partial E\left[\Pi\left(Q_H, Q_F \mid R1\right)\right]}{\partial Q_H}\Big|_{Q_H=0,\,Q_F=Q_F^0}$ to be positive. We have

$$\frac{\partial E\left[\Pi\left(Q_H, Q_F \mid R1\right)\right]}{\partial Q_H}\Big|_{Q_H=0,\,Q_F=Q_F^0} = \left(\frac{\partial E\left[\Pi\left(Q_H, Q_F \mid R1\right)\right]}{\partial Q_H} - \frac{\partial E\left[\Pi\left(Q_H, Q_F \mid R1\right)\right]}{\partial Q_F}\right)\Big|_{Q_H=0,\,Q_F=Q_F^0}$$

$$\frac{\partial E\left[\Pi\left(Q_H, Q_F \middle| R1\right)\right]}{\partial Q_H}\bigg|_{Q_H=0, Q_F=Q_F^0} = m_H - k_H - u_H - \left(E\left[\tilde{m}_F^{\ +}\right] - k_F - u_F \bar{e}\right). \tag{30}$$

Thus, $m_H - k_H - u_H - \left(E\left[\tilde{m}_F^{\ +}\right] - k_F - u_F \bar{e}\right)$ must be positive for D_R to be optimal. Otherwise, $Q_H^* = 0$ corresponding to the offshore sourcing policy F_L.

On the other hand, from Lemma A5, in order for Q_F^* to be positive, $Q_F^0 > Q_H^0$ must hold. Equivalently, the derivative of the objective function in (22) with respect to Q_F along the line $Q_H + Q_F = Q_H^0$ (i.e., $\frac{\partial E\left[\Pi\left(Q_H, Q_F \middle| R1\right)\right]}{\partial Q_F}\Big|_{Q_H+Q_F=Q_H^0}$) must be positive. We have

$$\frac{\partial E\left[\Pi\left(Q_H, Q_F \middle| R1\right)\right]}{\partial Q_F}\bigg|_{Q_H+Q_F=Q_H^0} = E\left[\left(\tilde{m}_F^{\ +} - M_H\right)^+\right] - k_F - u_F \bar{e}\,. \tag{31}$$

Hence, $E\left[\left(\tilde{m}_F^{\ +} - M_H\right)^+\right] - k_F - u_F \bar{e}$ must also be positive for D_R to be optimal. Otherwise, $Q_F^* = 0$ corresponding to the onshore sourcing policy H.

Furthermore, as $\frac{\partial E\left[\Pi\left(Q_H, Q_F\right)\right]}{\partial Q_H}\Big|_{Q_H=Q_H^0, Q_F=0} = 0$ by definition, and $\frac{\partial^2 E\left[\Pi\left(Q_H, Q_F\right)\right]}{\partial Q_H \partial Q_F} < 0$, we have $\frac{\partial E\left[\Pi\left(Q_H, Q_F\right)\right]}{\partial Q_H}\Big|_{Q_H=Q_H^0, Q_F>0} < 0$, which implies that Q_H^* must be lower than Q_H^0 in region $R1$.

<u>The Special Case of Low Volatility in Exchange Rate and/or High Profit Margin:</u>

This is the special case where the volatility in exchange rate is so low, or the profit margin is so high, that $m_F > 0$ for all exchange-rate realizations in the second stage (i.e., $\tau_2 > e_h$ in Figure 3). First, the set of potentially optimal solutions remains the same. This is because the expected profit function in the first stage does not depend on τ_2 in any of the regions. Consequently, the system of equations in (11) and (13) remain the same, which results in the same set of potentially optimal solutions. Second, the optimality conditions also remain the same. The reason is as follows: First, inequlity (26) is independent of τ_2. Second, in the derivations of equations (27), (28), (30) and (31), τ_2 is replaced with e_h, which causes $\tilde{m}_F^+$ to be replaced with $\tilde{m}_F$. Therefore, if m_F is always positive, then we can simplify all of the optimality conditions by replacing $\tilde{m}_F^+$ with $\tilde{m}_F$. However, keeping $\tilde{m}_F^+$ leads to the general conditions.

Therefore, the set of potentially optimal solutions and the optimality conditions are robust to the magnitude of exchange-rate volatility and to the variation in profit margin. ❑

Proof of Proposition 5. Note that the first-order derivative of the objective function in an offshore sourcing policy with respect to Q_F evaluated at q_H^0 is equal to the derivative of the global sourcing objective function with respect to Q_F evaluated at the same point. i.e.,

$$\frac{\partial E\left[\Pi\left(Q_H = 0, Q_F\right)\right]}{\partial Q_F}\bigg|_{Q_F=q_H^0} = \frac{\partial E\left[\Pi\left(Q_H, Q_F\right)\right]}{\partial Q_F}\bigg|_{Q_F=q_H^0} = E\left[\left(\tilde{m}_F^{\ +} - m_H\right)^+\right] - k_F - u_F \bar{e}$$

Hence, $Q_F^0 > q_H^0$ is equivalent to the optimal global sourcing policy being in region $R3$, and $Q_F^0 < q_H^0$ is equivalent to the optimal global sourcing policy being in region $R1$. Moreover, for part (b), $Q_F^0 > Q_H^0$ ensures that the optimal policy is not H. ❑

Proof of Proposition 6. If $k_F + c_F\,\bar{e} + t_F < k_H + c_H + t_H$, then $p - c_F\,\bar{e} - t_F - k_F > p - c_H - t_H - k_H$, which means $E\left[\tilde{m}_F\right] - k_F - u_F \bar{e} > m_H - k_H - u_H$. This has two implications: First, the opposite of OC3 holds (i.e., $E\left[\tilde{m}_F^{\ +}\right] - k_F - u_F \bar{e} > m_H - k_H - u_H$), which in turn implies D_R is never optimal in this case. Second, OC1 holds because

$$E\left[\left(\tilde{m}_F{}^{+}-M_H\right)^{+}\right]\geq E\left[\tilde{m}_F{}^{+}-M_H\right]=E\left[\tilde{m}_F{}^{+}\right]-\left(m_H-k_H-u_H\right)>E\left[\tilde{m}_F\right]-\left(m_H-k_H-u_H\right)>k_F+u_F\bar{e}\ .$$

Therefore from Table 1, if OC2 does not hold, then F_L is the optimal policy. Otherwise, if OC2 holds but OC4 does not hold, then F_H is the optimal policy. Finally if both OC2 and OC4 hold, then D_E is the optimal policy. ❑

Proof of Proposition 7. The inequality $k_F + c_F\,\bar{e} + t_F \geq k_H + c_H + t_H$ by itself does not imply that the optimality conditions hold or not. If OC2 holds and OC4 does not hold, the optimal policy is F_H. Moreover, if OC2 and OC3 do not hold and OC1 holds, the optimal policy is F_L. ❑

Proof of Proposition 8. The inequality $p - (c_F - u_F)\,\bar{e} - t_F < 0$ does not eliminate the possibility of the optimality conditions associated with dual sourcing and offshore sourcing to hold. Thus, they may still be optimal sourcing policies. ❑

Proof of Proposition 9. In region $R3$, the shadow price of Q_F is

$$\lambda_{Q_F}^{R3}=\frac{\partial E\left[\Pi\left(Q_H,Q_F\,|R3\right)\right]}{\partial Q_F}=-\left(k_F+u_F\bar{e}\right)+\left(c_F-u_F\right)\int_{e_l}^{e_h}\left(\tau(Q_F)-e\right)^{+}g\left(e\right)de$$

Let us define $n(e) = (\tau(Q_F) - e)^{+}$. Then, the shadow price can be expressed as $-\left(k_F+u_F\bar{e}\right)+\left(c_F-u_F\right)E\left[n\left(\tilde{e}\right)\right]$. Note that $n(e)$ is piecewise linear and convex in e. Therefore, by definition of second-degree stochastic dominance, $E\left[n\left(\ddot{e}\right)\right]\geq E\left[n\left(\dot{e}\right)\right]$ if $\ddot{e}$ is a mean-preserving spread of $\dot{e}$. Consequently, the higher exchange-rate volatility, the higher the shadow price, and the higher the value of Q_F^* in this region.

In region $R1$, the shadow price of Q_F is

$$\lambda_{Q_F}^{R1}=\frac{\partial E\left[\Pi\left(Q_H,Q_F\,|R1\right)\right]}{\partial Q_F}=-\left(k_F+u_F\bar{e}\right)+\left(c_F-u_F\right)\int_{e_l}^{e_h}\left(\tau(Q_H+Q_F)-e\right)^{+}g\left(e\right)de$$

By similar argument Q_F^* increases in exchange-rate volatility in this region as well.

Note that in region $R2$, the shadow price is constant, and its sign, positive or negative, determines that the optimal solution is whether in region $R3$ or region $R1$, respectively. This shadow price is

$$E\left[\left(\tilde{m}_F{}^{+}-m_H\right)^{+}\right]-k_F-u_F\bar{e}=E\left[\left(c_H-u_H+t_H-c_Fe+u_Fe-t_F\right)^{+}\right]-k_F-u_F\bar{e}$$

where $(c_H - u_H + t_H - c_F\,e + u_F\,e - t_F)^{+}$ is convex in e. Therefore, this shadow price increases in exchange-rate volatility, which implies that as volatility increases the location of the optimal solution may switch from region $R1$ to region $R3$ but not in an opposite way. Since Q_F^* is always higher in region $R3$ than in region $R1$, it follows that Q_F^* always increases in exchange-rate volatility.

For Q_H^*, the shadow price in region $R3$ is

$$\lambda_{Q_H}^{R3}=\frac{\partial E\left[\Pi\left(Q_H,Q_F\,|R3\right)\right]}{\partial Q_H}=-\left(k_H+c_H+t_H\right)+\left(c_F-u_F\right)\bar{e}+t_F+\left(c_F-u_F\right)E\left[n\left(\tilde{e}\right)\right]$$

where $n(e)=[(\tau_l - e)^{+} - (e - \tau(Q_H))^{+}]$ is piecewise linear but neither convex nor concave in e, which leads to an inconclusive result regarding the behavior of the domestic capacity in exchange-rate volatility. ❑

Proof of Proposition 10. From the first equation of the system of equations in (13) for uniformly distributed exchange rate ($\tilde{e}\sim U\left[\bar{e}-d,\bar{e}+d\right]$), we have

$$\left(c_F-u_F\right)\int_{\tau(Q_H)}^{\bar{e}+d}\left(e-\tau(Q_H)\right)\frac{1}{2d}de-\int_{\tau_1}^{\bar{e}+d}\left(\left(c_Fe-u_Fe+t_F\right)-\left(c_H-u_H+t_H\right)\right)\frac{1}{2d}de+k_H+u_H=0$$

$$\frac{\left(c_F-u_F\right)}{4d}\left(\bar{e}+d-\tau(Q_H)\right)^2-\frac{\left[\left(c_F-u_F\right)\left(\bar{e}+d\right)+t_F-\left(c_H-u_H+t_H\right)\right]^2}{4d\left(c_F-u_F\right)}+k_H+u_H=0$$

$$\left(\bar{e}+d-\tau(Q_H)\right)^2=\frac{\left[\left(c_F-u_F\right)\left(\bar{e}+d\right)+t_F-\left(c_H-u_H+t_H\right)\right]^2-4d\left(c_F-u_F\right)\left(k_H+u_H\right)}{\left(c_F-u_F\right)^2}.$$

The feasible solution to this quadratic equation is

$$\tau_H^*=\bar{e}+d-\frac{1}{\left(c_F-u_F\right)}\sqrt{\left[\left(c_F-u_F\right)\left(\bar{e}+d\right)+t_F-\left(c_H-u_H+t_H\right)\right]^2-4d\left(c_F-u_F\right)\left(k_H+u_H\right)}$$

The square root term is guaranteed to be positive by OC4 so that it holds under policy D_E. It follows that

$$\frac{\partial\tau_H^*}{\partial d}=1-\frac{\left[\left(c_F-u_F\right)\left(\bar{e}+d\right)+t_F-\left(c_H-u_H+t_H\right)\right]-2\left(k_H+u_H\right)}{\sqrt{\left[\left(c_F-u_F\right)\left(\bar{e}+d\right)+t_F-\left(c_H-u_H+t_H\right)\right]^2-4d\left(c_F-u_F\right)\left(k_H+u_H\right)}}.$$

The root of the above partial derivative occurs when

$$\begin{aligned}&\left[\left(c_F-u_F\right)\left(\bar{e}+d\right)+t_F-\left(c_H-u_H+t_H\right)\right]^2-4d\left(c_F-u_F\right)\left(k_H+u_H\right)\\&-\left(\left[\left(c_F-u_F\right)\left(\bar{e}+d\right)+t_F-\left(c_H-u_H+t_H\right)\right]-2\left(k_H+u_H\right)\right)^2=0\end{aligned}$$

which reduces to $(c_F - u_F)\,\bar{e} + t_F - (k_H + c_H + t_H) = 0$. As a result, when $k_H + c_H + t_H < (c_F - u_F)\,\bar{e} + t_F$, we have $\frac{\partial\tau_H^*}{\partial d}>0$ which by definition ($\tau_H^* = \tau(Q_H^*) = (p\,[(1 - F(Q_H^*)] - t_F)/(c_F - u_F)$) implies $\frac{\partial Q_H^*}{\partial d}<0$. ❑

Proof of Proposition 11. In case 1, by Proposition 6, the potentially optimal policies are F_L, F_H, and D_E. From Proposition 9, we know that the left-hand side (LHS) of OC2 and Q_F^* increase in exchange-rate volatility. As a result, the optimal policy can switch from F_L in region $R1$ to F_H in region $R3$, and not in the opposite way. Depending on whether OC4 holds in region $R3$, the optimal policy may change to D_E.

In case 2, all policies can potentially be optimal. At sufficiently low degrees of exchange-rate volatility, the firm adopts policy H due to the lower cost of onshore sourcing. Note that under policy H, when OC1 does not hold, OC2 does not hold either. Moreover, if OC1 does not hold, OC3 holds because

$$E\left[\tilde{m}_F^{\,+}\right]-k_F-u_F\bar{e}-\left(m_H-k_H-u_H\right)=E\left[\tilde{m}_F^{\,+}-M_H\right]-k_F-u_F\bar{e}<E\left[\tilde{m}_F^{\,+}-M_H\right]^+-k_F-u_F\bar{e}<0. \quad (32)$$

Higher levels of exchange-rate volatility causes the LHS of OC1 to increase (by similar argument as presented in proof of proposition 9) until OC1 holds. Observe that due to inequality (32), at this switching point, OC3 still holds. This is where D_R becomes the optimal policy. As exchange-rate volatility increases, the left-hand side of OC2 increases and the left-hand side of OC3 decreases. The latter is because m_F^+ is convex in e, thus $E\left[\tilde{m}_F^{\,+}\right]-k_F-u_F\bar{e}-\left(m_H-k_H-u_H\right)$ increases in exchange-rate volatility by a similar argument as mentioned in proof of Proposition 9. Depending on whether OC3 does not hold or OC2 holds first, the optimal policy path is different. If OC3 does not hold first, F_L becomes the optimal policy first and similar to case 1, the next optimal policies are F_H (when OC2 holds) and D_E if OC4 holds as well. Otherwise, if OC2 holds before OC3 is reversed, the optimal solution moves to $R3$. But this also causes OC4 to hold because the LHS of OC4 is the summation of the LHS of OC2 and that of OC3. Therefore, if both OC2 and OC3 hold, OC4 holds as well. That implies that the optimal solution may directly switch from D_R to D_E.❑

Proof of Proposition 12. (a) For $e_\alpha \geq \tau_2$, we have $q_F^*(e_\alpha) = 0$. By supposition, we have

$$P_{(\tilde{e},\tilde{x})}\left[\Pi\left(0,Q_F^*\right)<-\beta\right]>\alpha.$$

Furthermore,

$$\Pi\left(0,Q_F^A\,|\,e_\alpha\right)=-\beta,\ \Pi\left(0,Q_F^A\,|\,e\right)<-\beta\ \text{ for all } e>e_\alpha,\ \Pi\left(0,Q_F^A\,|\,e\right)>-\beta\ \text{ for all } e<e_\alpha,$$

and thus, $P_{(\tilde{e},\tilde{x})}\left[\Pi\left(0,Q_F^A\right)<-\beta\right]=\alpha$.

Define

$$Q_F^A\left(Q_H\right)\equiv\left[\left(p-k_H-c_H-t_H\right)Q_H+\beta\right]/\left(k_F+u_Fe_\alpha\right)=\left[M_HQ_H+\beta\right]/\left(k_F+u_Fe_\alpha\right) \quad (33)$$

and note that $P_{(\tilde{e},\tilde{x})}\left[\Pi\left(Q_H, Q_F^A\left(Q_H\right)\right) < -\beta\right] = \alpha$ for all $0 \le Q_H \le x_L$.

Part (*a*). If $Q_F^A = Q_F^A(0) > q_H^0$, we can substitute the capacity investment in (33) into the first-stage objective function associated with *R*3 and take its first-order derivative with respect to Q_H. The total derivative of that function with respect to Q_H can be expressed as:

$$\frac{dE\left[\Pi\left(Q_H, Q_F^A\left(Q_H\right)\middle|R3\right)\right]}{dQ_H} = \frac{\partial E\Pi\left(Q_H, Q_F \middle| R3\right)}{\partial Q_F}\Big|_{Q_F^A(Q_H)} \cdot \frac{dQ_F^A\left(Q_H\right)}{dQ_H} + \frac{\partial E\Pi\left(Q_H, Q_F\middle|R3\right)}{\partial Q_H}\Big|_{Q_F^A(Q_H)}$$

From Proposition A4,

$$\frac{\partial E\left[\Pi\left(Q_H, Q_F\middle|R3\right)\right]}{\partial Q_F} = -\left(k_F + u_F\overline{e}\right) + \left(c_F - u_F\right)\int_{e_l}^{\tau(Q_F)}\left(\tau(Q_F) - e\right)g\left(e\right)de \text{, and,}$$

$$\frac{\partial E\left[\Pi\left(Q_H, Q_F\middle|R3\right)\right]}{\partial Q_H} = -\left(k_H + u_H\right) - \left(c_F - u_F\right)\int_{\tau(Q_H)}^{e_h}\left(e - \tau(Q_H)\right)g\left(e\right)de + E\left[\left(m_H - \tilde{m}_F\right)^+\right].$$

Thus,

$$\frac{dE\left[\Pi\left(Q_H, Q_F^A\left(Q_H\right)\middle|R3\right)\right]}{dQ_H}\Big|_{Q_H=0} = \frac{M_H}{k_F + u_F e_\alpha}\left[-\left(k_F + u_F\overline{e}\right) + \left(c_F - u_F\right)\int_{e_l}^{\tau\left(Q_F^A(Q_H)\right)}\left(\tau\left(Q_F^A\left(Q_H\right)\right) - e\right)g(e)de\right]$$

$$-\left(k_H + u_H\right) - \left(c_F - u_F\right)\int_{\tau_2}^{e_h}\left(e - \tau_2\right)g\left(e\right)de + E\left[\left(m_H - \tilde{m}_F\right)^+\right]$$

Because the last two terms simplify to $E\left[\left(m_H - \tilde{m}_F{}^+\right)^+\right]$, we can rewrite the derivative as

$$\frac{dE\left[\Pi\left(Q_H, Q_F^A\left(Q_H\right)\middle|R3\right)\right]}{dQ_H}\Big|_{Q_H=0} = \frac{M_H}{k_F + u_F e_\alpha}\begin{bmatrix}-\left(k_F + u_F\overline{e}\right) \\ +\left(c_F - u_F\right)\int_{e_l}^{\tau\left(Q_F^A(Q_H)\right)}\left(\tau\left(Q_F^A\left(Q_H\right)\right) - e\right)g(e)de\end{bmatrix} + E\left[\left(m_H - \tilde{m}_F{}^+\right)^+\right] - k_H - u_H$$

Note that the first term is positive since $Q_F{}^A < Q_F{}^0$, and the last three terms form the LHS of OC4 is negative because of the definition of policy F_H. Because $\int_{e_l}^{\tau}\left(\tau - e\right)g\left(e\right)de = \int_{e_l}^{\tau}G\left(e\right)de$, the above condition is equivalent to RA1. Therefore, when $Q_F{}^A > q_H{}^0$, if RA1 holds, the firm satisfies the VaR constraint and increases expected profit by increasing Q_H from 0 to any $Q_H \in (0, x_L]$ while simultaneously increasing Q_F from $Q_F^A(0)$ to $Q_F^A(Q_H)$, and thus engages in dual sourcing under risk aversion.

Part (b) If $Q_F^A = Q_F^A(0) \le q_H{}^0$, regardless of whether F_H or F_L is the optimal policy in the risk-neutral setting, we can substitute the capacity investment in (33) into the first-stage objective function associated with *R*1 and take its first-order derivative with respect to Q_H.

$$\frac{dE\left[\Pi\left(Q_H, Q_F^A\left(Q_H\right)\middle|R1\right)\right]}{dQ_H} = \frac{\partial E\Pi\left(Q_H, Q_F\middle|R1\right)}{\partial Q_F}\Big|_{Q_F^A(Q_H)} \cdot \frac{dQ_F^A\left(Q_H\right)}{dQ_H} + \frac{\partial E\Pi\left(Q_H, Q_F\middle|R1\right)}{\partial Q_H}\Big|_{Q_F^A(Q_H)}$$

From Proposition A2,

$$\frac{\partial E\left[\Pi\left(Q_H, Q_F\middle|R1\right)\right]}{\partial Q_F} = -\left(k_F + u_F\overline{e}\right) + \left(c_F - u_F\right)\int_{e_l}^{\tau(Q_H+Q_F)}\left(t(Q_H + Q_F) - e\right)g\left(e\right)de \text{, and,}$$

$$\frac{\partial E\left[\Pi\left(Q_H,Q_F\left|R1\right.\right)\right]}{\partial Q_H}=-\left(k_H+c_H+t_H\right)+\left(c_F-u_F\right)\overline{e}+t_F-\left(c_F-u_F\right)\int_{\tau(Q_H)}^{e_h}\left(e-\tau(Q_H)\right)g\left(e\right)de$$

$$+\left(c_F-u_F\right)\int_{e_l}^{\tau(Q_H+Q_F)}\left(\tau(Q_H+Q_F)-e\right)g\left(e\right)de$$

Thus,

$$\frac{dE\left[\Pi\left(Q_H,Q_F^A\left(Q_H\right)\left|R1\right.\right)\right]}{dQ_H}\Big|_{Q_H=0}=\frac{M_H}{k_F+u_F\overline{e}}\left[-\left(k_F+u_F\overline{e}\right)+\left(c_F-u_F\right)\int_{e_l}^{\tau\left(Q_F^A\left(Q_H\right)\right)}\left(\tau\left(Q_F^A\left(Q_H\right)\right)-e\right)g(e)\left(e\right)de\right]$$

$$+M_H-E\left[\tilde{m}_F^{\ +}\right]+\left(c_F-u_F\right)\int_{e_l}^{\tau\left(Q_F^A\left(Q_H\right)\right)}\left[\tau\left(Q_F^A\left(Q_H\right)\right)-e\right]g\left(e\right)de$$

Therefore, when $Q_F^A \le q_H^0$ and dual sourcing is the optimal sourcing policy under risk aversion if

$$\frac{M_H}{k_F+u_F\overline{e}}\left[-\left(k_F+u_F\overline{e}\right)+\left(c_F-u_F\right)\int_{e_l}^{\tau\left(Q_F^A\left(Q_H,e_\alpha,\beta\right)\right)}\left(\tau\left(Q_F^A\left(Q_H,e_\alpha,\beta\right)\right)-e\right)g(e)\left(e\right)de\right],$$

$$+M_H-E\left[\tilde{m}_F^{\ +}\right]+\left(c_F-u_F\right)\int_{e_l}^{\tau\left(Q_F^A\left(Q_H,e_\alpha,\beta\right)\right)}\left[\tau\left(Q_F^A\left(Q_H,e_\alpha,\beta\right)\right)-e\right]g\left(e\right)de>0$$

Due to submodularity of the expected profit function, LHS of RA2 is larger than that of RA1. Thus, under risk aversion, it is more likely for dual sourcing to be adopted when $Q_F^A \le q_H^0$ than when $Q_F^A > q_H^0$. ❑

Proof of Proposition 13. a) Recall that e_α denotes the exchange rate realization at fractile $1-\alpha$ (i.e., $[1-G(e_\alpha)]=\alpha$). The realized profit is greater than or equal to $-\beta$ when the exchange-rate random variable takes values in the range $e_l \le e \le e_\alpha$, and the probability of loss greater than or equal to β is less than or equal to α. In (20), we can equate the profit at $e = e_\alpha$ inside the probability expression to $-\beta$ in order to determine the number of hedging contracts necessary to warrant profitability that satisfies the VaR requirement:

$$\left\{\begin{matrix}-k_HQ_H-k_FQ_F-h\left(e_s\right)H-\left(c_H+t_H\right)q_H^*\left(e_\alpha\right)-\left(c_Fe_\alpha+t_F\right)q_F^*\left(e_\alpha\right)\\ -u_H\left(Q_H-q_H^*\left(e_\alpha\right)\right)^+-u_Fe_\alpha\left(Q_F-q_F^*\left(e_\alpha\right)\right)^++\left(e_\alpha-e_s\right)H+p\min\left\{x,q_H^*\left(e_\alpha\right)+q_F^*\left(e_\alpha\right)\right\}\end{matrix}\right\}=-\beta\,.$$

Solving for H at the lowest demand realization $x = x_l$ provides the minimum (optimal) number of hedging contracts satisfying VaR at $e = e_\alpha$:

$$H^*\left(e_s\right)=\frac{\left\{\begin{matrix}k_HQ_H+k_FQ_F+\left(c_H+t_H\right)q_H^*\left(e_\alpha\right)+\left(c_Fe_\alpha+t_F\right)q_F^*\left(e_\alpha\right)\\ +u_H\left(Q_H-q_H^*\left(e_\alpha\right)\right)^++u_Fe_\alpha\left(Q_F-q_F^*\left(e_\alpha\right)\right)^+-p\min\left\{x_l,q_H^*\left(e_\alpha\right)+q_F^*\left(e_\alpha\right)\right\}-\beta\end{matrix}\right\}}{\left(e_\alpha-e_s-h\left(e_s\right)\right)}.$$

b) Taking the expectation of the term $(e - e_s)H$ in (19) over all exchange-rate realizations provides the expected return of $H\left(\int_{e_l}^{e_h}\left(e-e_s\right)f\left(e\right)de\right)$ in stage 2, which is equal to the first-stage payment of

$h\left(e_s\right)H=H\left(\int_{e_s}^{e_h}\left(e-e_s\right)f\left(e\right)de-\int_{e_l}^{e_s}\left(e_s-e\right)f\left(e\right)de\right)=H\left(\int_{e_l}^{e_h}\left(e-e_s\right)f\left(e\right)de\right)$ for H units of hedging contracts in (18). Thus, the expected profit for any given (Q_H, Q_F) decision made in stage 1 in the risk-neutral setting can be replicated through purchasing $H^*(e_s)$ units of contracts in (21) under risk aversion to satisfy VaR. ❑