Risk Mitigation of Production Hedging

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This paper examines how a firm can mitigate global economic risk through production hedging, defined as producing less than the total demand. We investigate a firm's production planning, pricing, and financial hedging decisions under exchange-rate and demand uncertainty with the objective of maximizing expected profit while complying with a value-at-risk constraint that limits the firm's losses in amount and probability.

The paper makes three contributions. First, we show that production hedging, when compared to matching demand with production, can substantially reduce risk both from value-at-risk and conditional-value-at-risk perspectives while increasing expected profit. Our second contribution relates to the optimal pricing decisions. When a firm has pricing flexibility, it is commonly expected that the optimal price would increase under production hedging. Our paper, however, shows that production hedging causes the firm to decrease the optimal price below the riskless price in order to benefit from exchange-rate fluctuations. The pressure from risk aversion on the optimal price decision is not one directional, and can lead to both an increase and a decrease in price. Third, our work examines the interactions between financial hedging and production hedging. It identifies when financial hedging serves as a complement, and when as a substitute, to production hedging. Our work shows that financial hedging cannot always eliminate production hedging from being an optimal solution.

Keywords: *exchange-rate risk*, *value at risk*, *production hedging*, *financial hedging, pricing*

1. Introduction

This paper examines a global firm's manufacturing, pricing, and financial hedging decisions under exchange-rate and demand risk. It shows that production hedging, defined as producing less than the total demand, is an effective policy mechanism to mitigate global economic risk. In recent years, the fluctuations in exchange rates have increased due to global economic crises (e.g., the US credit crisis, Japanese tsunami disaster, the ongoing Greek debt crisis, etc.), resulting in a greater amount of uncertainty for global businesses. On October 24, 2011, for example, the US Dollar to the Japanese Yen exchange rate reached its lowest value since World War II, trading at 75.71 Yen/USD. For Japanese firms that manufacture in Japan and sell in the US, the US dollar's depreciation against the Japanese Yen means it is virtually impossible to remain profitable. Fluctuations in exchange rates have profound effects on the bottom-line profits, making it necessary to provide multinational corporations with prescriptions for ways to cope with the increased amount of global economic uncertainty. This paper responds to this need by investigating the influence of exchange-rate and demand risks on a global manufacturer's production planning, pricing and financial hedging decisions.

We consider a firm that manufactures a product in one country, and sells it in two countries: one domestic and one international market where the revenues, in terms of domestic currency, fluctuate with exchange rates. The firm makes three decisions in the presence of exchange-rate and demand uncertainty: (1) manufacturing quantity, (2) selling prices of the product in both markets, and (3) the number of financial hedging instruments to purchase. Upon the realization of the random exchange rate, the firm makes two decisions under demand uncertainty: (1) the allocation of production to two markets, and (2) whether to exercise the purchased financial hedging instruments.

Our paper finds relevance and motivation from the practices observed at The Gap Inc. The firm gathers its suppliers in its New York office for the Buyers Week. During this week, the Gap Inc. buyers select which garments to order eight to nine months before the season begins. The company reports that an increasing number of garment suppliers prefer to be paid in the foreign currency, and the firm views managing the currency risk can be a cumbersome activity/task in ensuring profitability. During the same Buyers Week, The Gap Inc. determines (1) the order quantity, (2) the retail selling price of the apparel, and (3) the level of financial hedging to engage in. Once the apparel is produced and sent to the company's worldwide distribution center in London, the firm determines how to allocate it to various markets based on the observed fluctuations in exchange rates. This is also the time that the firm can exercise its financial hedging contracts. One might wonder why the firm determines the garment's selling price this early in the planning cycle, specifically in the presence of exchange-rate uncertainty, and not after the product is shipped to the distribution center. The company executives explain the early pricing decision by citing two specific reasons: (1) The selling price enables the firm to determine its margins at the time the regional buyers make their inventory commitment (without a price, these buyers would not have a margin to determine order quantities); (2) the firm sets equal prices in all markets and complies with the anti-dumping laws described in Article VI.1a in the General Agreement on Tariffs and Trade (GATT). We provide Article VI.1 of the GATT agreement below.

Article VI: Anti-dumping and Countervailing Duties

1. The contracting parties recognize that dumping, by which products of one country are introduced into the commerce of another country at less than the normal value of the products, is to be condemned if it causes or threatens material injury to an established industry in the territory of a contracting party or materially retards the establishment of a domestic industry. For the purposes of this Article, a product is to be considered as being introduced into the commerce of an importing country at less than its normal value, if the price of the product exported from one country to another

(a) is less than the comparable price, in the ordinary course of trade, for the like product when destined for consumption in the exporting country, or,

(b) in the absence of such domestic price, is less than either (i) the highest comparable price for the like product for export to any third country in the ordinary course of trade, or (ii) the cost of production of the product in the country of origin plus a reasonable addition for selling cost and profit.

Our model complies with this legal requirement which dictates that the firm has to set equal prices in each market at the time prices are determined. It is important to note that our main findings do not rely on the assumption that the firm sets equal prices; we show in Appendix B.1 that our results continue to hold when the firm sets unequal prices. Equally, our main findings are not an outcome of the pricing decision made in the presence of exchange rate uncertainty; Appendix B.5 provides derivations where our results continue to hold when the firm can postpone its pricing decisions until after exchange rates are realized.

In our model, the firm considers a value-at-risk (VaR) measure that finds wide acceptance among practicing risk managers. VaR is the preferred approach in the Basel II and III Accords, which specifies the banking laws and regulations issued by the Basel Committee on Banking Supervision (2013). European banks (e.g., Citibank) are now requiring firms to comply with similar VaR requirements in their financing requests, and thus, VaR is becoming the widely-used risk measure for non-financial institutions as well. According to VaR, the firm's losses are limited in amount and probability, i.e., the realized losses after observing exchange-rate and demand random variables cannot exceed a certain amount with a limited probability. While we employ VaR in order to show the effectiveness of production hedging in mitigating the firm's risk exposure, we also demonstrate its benefits according to conditional-value-at-risk (CVaR); production hedging's risk mitigation benefits extend to risk measures associated with CVaR.

Our paper makes three contributions. First, we show that production hedging is an effective risk-mitigation approach. The reason that production hedging becomes an effective risk-mitigation policy is twofold. First, by manufacturing a quantity less than the total demand, the firm creates the flexibility to sell a bigger portion of its limited supply in the foreign market when the exchange rate appreciates, and in the domestic market when the exchange rate depreciates. This allocation flexibility does not exist when the firm is required to match the demand through its manufacturing decisions. Second, the domestic market provides a downside protection under production hedging without having to lose on the upside potential. By manufacturing a smaller quantity than the total demand, when the exchange rate is low, the firm reduces its losses by limiting its product allocation to the foreign market and increasing its production allocation to the domestic market. Thus, under production hedging, the domestic market resembles the benefits of a currency option commonly seen in financial hedging.

Our second contribution relates to the firm's optimal price choice under exchange-rate uncertainty and risk aversion. Let us begin our discussion with the risk-neutral setting and ignore the impact of a value-at-risk measure. Recall that production hedging advocates manufacturing less than the total demand. Thus, one would intuit that the firm would increase its selling price when it follows production hedging. On the contrary to this intuition, we show that production hedging causes the firm to reduce its selling price below the riskless price. The reduced price inflates demand in markets, and allocation flexibility creates higher benefits under a fluctuating exchange rate. This result also departs from the findings presented for the

Price-Setting Newsvendor Problem (PSNP) where the firm determines price and quantity simultaneously, but under demand uncertainty rather than exchange-rate uncertainty. Petruzzi and Dada (1999) report that selling price increases beyond the riskless price when demand uncertainty is described with a multiplicative error term, similar to the definition of the exchange rate random variable in our problem. However, our work shows that exchange-rate uncertainty causes the firm to reduce its selling price below the riskless price. Next, we examine the influence of risk aversion on the optimal price decision. The impact of risk aversion on the optimal price is not one directional. One would intuit that incorporating a VaR constraint would cause the firm to increase its selling price and decrease its manufacturing quantity in order to reduce the risk exposure. While this is true in some settings, our work shows that incorporating risk aversion with a VaR constraint can also cause the price to further decrease. The manufacturing quantity in this case can increase (due to reduced price) or decrease (due to the pressure from risk aversion). Our pricing results under risk aversion also depart from those reported for PSNP. Agrawal and Seshadri (2000) show that the optimal price increases when compared to the risk-neutral optimal price under demand risk described with a multiplicative random error term. We also use a multiplicative random error term to describe exchange-rate uncertainty, however, we show that the optimal price can exhibit both an increasing and a decreasing behavior under exchange-rate risk when compared with the risk-neutral optimal and riskless prices.

Our third contribution relates to the interactions between production and financial hedging. In our analysis, we provide an answer to the common question: Can financial hedging eliminate the need for production hedging? The common notion is that financial hedging is a substitute to production hedging. Our paper, however, shows that financial hedging can play both a substitute and complementary role to production hedging. When the unit manufacturing cost is low and the exchange rate does not exhibit a great degree of uncertainty, the firm might prefer to manufacture the total demand. The VaR constraint, however, might force the firm to switch to a production hedging policy. Under this scenario, financial hedging can help the firm by eliminating the need to switch to production hedging; thus, financial hedging serves as a substitute to production hedging. However, when the unit manufacturing cost is high and/or the exchange rate shows sufficient volatility, then the firm's preference can be production hedging even in the absence of a VaR constraint. In this case, financial hedging helps the firm improve the expected profit, however, it cannot eliminate production hedging from being an optimal policy. Thus, financial hedging and production hedging complement each other under this scenario.

The paper is organized as follows. Section 2 provides a literature review and Section 3 introduces the general model. Section 4 examines the impact of exchange-rate uncertainty along with the VaR requirement on manufacturing quantities. Section 5 presents the impact of pricing and Section 6 provides the impact of financial hedging. Section 7 shows that our results continue to hold under demand uncertainty. Section 8 presents our conclusions and managerial insights. All derivations and examples are provided in an online supplement: Proofs are presented in Appendix A; Appendix B provides additional analysis demonstrating that our main insight continue to hold under various modeling extensions with examples demonstrating the optimal pricing decisions under risk aversion; and, Appendix C has the details of the solution approach developed under demand uncertainty.

2. Literature Review

The majority of the global supply chain literature examining the impact of global economic uncertainty has been investigated under flexibility accrued through excess capacity in multi-national production facilities. Kogut and Kulatilaka (1994) demonstrate that having a supply network located in different countries is equivalent to owning an option, where the value is dependent upon the volatility of the exchange rate. Huchzermeier and Cohen (1996) develop operational hedging strategies via the configuration of the supply chain network. Through the Harvard Business School case of Applichem (Flaherty, 1985), Lowe et al. (2002) evaluate the value of having excess capacity. Rosenfield (1996) shows that excess capacity can reduce costs compared to a single plant. Dasu and Li (1997) concentrate on the optimal production allocation policy to minimize the production cost assuming a stochastic exchange rate and switching costs. Li et al. (2001) complement Dasu and Li (1997) by having the same objective of pursuing the optimal operating policies, but with different assumptions in the model which are: (1) stochastic demand and processing times, (2) make-to-stock environment. Kouvelis et al. (2001) focus on the choice of ownership strategies among exporting, joint venture, and wholly owned production facilities for the foreign production facility. Wang (2012) extends this work by determining whether to enter a new market with an export strategy or foreign direct investment. Our work differs from this literature as we do not advocate excess capacity. Rather, our study focuses on the operational strategy termed as "production hedging" where the firm manufactures less than its global demand. Moreover, our paper complements these publications by featuring price-setting flexibility in the presence of exchange-rate and demand uncertainty.

Contrary to the majority of the operational hedging related literature, Kazaz et al. (2005) is the first to define production hedging as producing less than the total demand. Their study shows that, under production hedging, the expected profit can be greater than a traditional policy that fulfills the global demand. Their work does not examine risk aversion or financial hedging, and does not offer any insights for the firm's pricing choices. Our work differs from their study in three ways: (1) We focus on the risk mitigation aspect of production hedging by examining a risk-averse firm; (2) it develops a comprehensive structural analysis under the flexibility of price setting; and (3) our model features financial hedging.

Price setting within global supply chains is only examined in the form of postponed prices; specifically, the firm determines prices after exchange-rate uncertainty is revealed. Kouvelis and Gutierrez (1997) focus on the production quantity and the coordination between decentralized production facilities under demand and exchange rate uncertainty at a multinational firm that sells its "style goods" in two non-overlapping selling seasons, and determine a transfer price from one market to the other after observing the exchange rate. Dong et al. (2010) consider the optimal supply chain configuration using a responsive pricing mechanism under stochastic demand and exchange rates. Our work complements these studies by examining pricing decisions, along with production and financial hedging, in the presence of exchange-rate uncertainty.

Our analysis builds on a model presented in Park et al. (2016) who provide an alternative explanation for why a firm may set price below cost in a foreign market. Our model extends their model in two ways: (1) we provide a comprehensive analysis of risk aversion through a VaR constraint and demonstrate that production hedging leads to smaller risk both from VaR and CVaR perspectives; (2) we incorporate financial hedging into the analysis. In addition, their work seeks to understand price-below-cost behavior whereas our work focuses the role of production hedging in risk mitigation.

There is a growing literature that examine the interface of operational and financial hedging decisions under risk. Kim et al. (2006) define operational hedging as firms having foreign sales; they show that firms with foreign sales also engage in financial hedging. Our definition of production hedging is different than the operational hedging definition of Kim et al. (2006). For example, a firm might have foreign sales and manufacture a quantity equaling the total demand. According to Kim et al. (2006), this firm is engaging in operational hedging, however, it is not utilizing production hedging according to our description. Ding et al. (2007) incorporate a risk-averse perspective by using a mean-variance objective function to determine the optimal operational and financial hedging decisions in the presence of demand and exchange rate uncertainty. Other studies that utilize both operational and financial instruments to hedge against exchange rate uncertainty include Mello et al. (1995) and Chowdhry and Howe (1999). Chod et al. (2010) show that financial hedging complements capacity investment in product flexibility, but works as a substitute to postponement. Li and Wang (2010) examine the outsourcing decision and capacity planning under exchange-rate uncertainty. Chen et al. (2014) investigate a firm's capacity investments in multiple sources to serve a single market, and build an analysis of financial and operational hedging decisions under exchange-rate risk. Our work, however, considers multiple markets to generate revenues through a single manufacturing facility, thus, the network infrastructure is the opposite of their model. Zhu and Kapuscinski (2015) also examine capacity decisions under exchange rate and demand uncertainty in a two-stage stochastic program. After uncertainty are revealed, the firm makes efficient pricing, production, allocation to markets, and financial hedging decisions in stage 2. Our model and conclusions are distinctly different than those presented in Zhu and Kapuscinski (2015): (1) They use market-clearing prices, and therefore, do not provide a characterization of optimal price decisions. Our model determines the optimal prices in the presence of uncertainty, and we characterize the optimal price choices. (2) When the production amount decreases, the selling price increases in Zhu and Kapuscinski (2015), whereas the optimal selling price decreases under production hedging in our model. (3) They find that financial hedging is complementary to their definition of operational hedging only when the domestic market share is greater than the foreign market. In our model, we show the opposite where financial hedging only plays a complementary role to production hedging when the foreign market is greater than the domestic market. The above-mentioned publications restrict the firm to the traditional practice of matching demand with production, and often lead to satisfying the global demand; moreover, they ignore pricing decisions. Operational hedging in our model, however, consists of both quantity and price decisions, and we show their interactions with financial hedging decisions. Our work complements this literature in three ways: (1) We show that production hedging is an effective risk mitigation approach that can serve as a substitute to financial hedging; (2) we show how the optimal price is influenced by production hedging and exchange-rate risk; and, (3) we present a comprehensive analysis of the structural properties for the price-setting behavior at the interface of operational and financial hedging, and offer new managerial insights.

3. The Model

This section introduces the general model where the firm determines the optimal choices for the manufacturing quantity x, selling price p, and the number of financial hedging contracts h in the presence of exchange-rate and demand uncertainty subject to a VaR constraint. The firm pays a manufacturing cost c for each unit manufactured, and an acquisition cost (premium) c_h for each unit of financial hedging contract. The selling price of the product is denoted p_i, where $i = H$ corresponds to the home market price denominated in the home country currency and $i = F$ represents the foreign market price denominated in the foreign currency.

The random exchange rate is represented by $\tilde{e}$, where e is the realization, $f(e)$ is the probability density function (pdf) defined on a support $[e_l, e_h]$ with a mean $\bar{e} = E[\tilde{e}]$ where $e_h > e_l > 0$. We make no assumptions regarding the distribution of $f(e)$, except that we scale it such that $\bar{e} = 1$ without loss of generality.

The demand random error term in each market is represented by $\tilde{z}_i$, its realization is denoted z_i following a pdf $g_i(z_i)$ and a cumulative distribution function (cdf) $G_i(z_i)$ on a support $[z_{il}, z_{ih}]$ for $i = H, F$. We express random demand as $d_i\left(p_i, \tilde{z}_i\right)$ and its realization with $d_i(p_i, z_i)$. We assume that $d_i(p_i, z_i)$ is decreasing in p_i and increasing in z_i, and that revenue $p_i d_i(p_i, z_i)$ is concave in p_i, i.e., $2\, d_{ip}(p_i, z_i) + p_i d_{ipp}(p_i, z_i) \leq 0$ where $d_{ip}(p_i, z_i)$ and $d_{ipp}(p_i, z_i)$ represent the first- and second-order derivatives of the demand function d_i with respect to price for $i = H, F$. We restrict the selling price in each market to be $c \leq p_i \leq p^{\max}$ where $p^{\max}$ is the minimum price that equates $d_i(p^{\max}, z_{il}) = 0$; thus, realized demand is non-negative at each feasible price level. We express the maximum and minimum demand as $d_x = \max\{d_H(p_H, z_H), d_F(p_F, z_F)\}$ and $d_m = \min\{d_H(p_H, z_H), d_F(p_F, z_F)\}$, respectively.

We begin our analysis with a firm that complies with the anti-dumping law, specifically Article VI.1.a of the 1994 GATT agreement. According to this law, the selling price in the foreign market, when converted to the home-country currency by the mean exchange rate $\bar{e} = 1$, should equal the same return from the home market, i.e., $p_F\bar{e} = p_H$. As the random exchange rate fluctuates, the actual return from a sale in the foreign market differs from that of the home market. This is legally allowed because the selling price is determined in the presence of exchange-rate uncertainty, and the firm's pricing decision is not subject to dumping, even if the firm does not adjust its foreign selling price instantaneously with exchange-rate fluctuations.

It is important to note that our main results are not an artifact of the anti-dumping law; as shown in Appendix B.1, our main results and the ensuing managerial insights continue to hold when $p_H \neq p_F\bar{e}$. The only consequence of the anti-dumping law is that the firm establishes a single selling price p that applies in both markets, i.e., $p_H = p_F\bar{e} = p$.

We examine a two-stage stochastic program with recourse. In the first stage, the firm determines the optimal values of manufacturing quantity x, selling price p, and financial hedging contracts h in the presence of exchange-rate and demand uncertainty in order to maximize the expected profit, denoted $E\left[\Pi\left(x, p, h\right)\right]$ under a VaR constraint. We utilize the VaR measure to limit the risk associated with the realized returns from sales in two markets after exchange rate is observed. In VaR, two parameters describe the firm's risk preference: β represents the loss (value at risk) that the firm is willing to tolerate at probability α, where $0 \leq \alpha \leq 1$. For a given α, if the realized profit (VaR) is more than the tolerable loss β, then the first-stage decision triplet (x, p, h) is an infeasible solution since the risk exceeds the tolerable probability. We can then express the first-stage formulation as follows:

$$\max_{(x,p,h)\geq 0} E\left[\Pi\left(x, p, h\right)\right] = -cx - c_h h + \int_{e_l}^{e_h} \pi^*\left(x, p, h, e\right) f\left(e\right) de \tag{1}$$

$$\text{subject to } P_{(\tilde{e},\tilde{z}_H,\tilde{z}_F)}\left[-cx - c_h h + \pi_2\left(y_H^*, y_F^*, s^*, z_H, z_F \,\middle|\, x, p, h, e\right) < -\beta\right] \leq \alpha \tag{2}$$

where $\pi^*\left(x, p, h, e\right)$ in (1) is the optimal second-stage return from the first-stage decisions (x, p, h) and exchange rate realization e, $P_{(\tilde{e},\tilde{z}_H,\tilde{z}_F)}\left[\cdot\right]$ in (2) represents the probability over the three random variables $\left(\tilde{e}, \tilde{z}_H, \tilde{z}_F\right)$, and $\pi_2\left(y_H^*, y_F^*, s^*, z_H, z_F \,\middle|\, x, p, h, e\right)$ describes the realized second-stage profit from the optimal second-stage decisions (y_H^*, y_F^*, s^*) at realized demand values of z_H and z_F for a given set of first-stage decisions (x, p, h) and exchange-rate e.

Given the realized exchange rate e and the first-stage production planning decisions (x, p, h), at the beginning of Stage 2, the firm determines (1) the allocation quantities to home and foreign markets, defined as y_H and y_F, respectively, where $y_H + y_F \le x$, and (2) the amount of financial hedging contracts to be exercised, denoted s, where $s \le h$. In this formulation, futures contracts can be examined by enforcing $s = h$ in stage 2; $s \le h$ enables the firm to exercise some of the hedging commitments from stage 1 in stage 2 as is the case in option contracts. Each unit of hedging contract that gets exercised in Stage 2 pays $(e_s - e)$; thus, the firm is protected for the cases when the realized exchange rate goes below the contracted exercise price (also known as the strike price) of e_s. The second-stage objective function maximizes the expected revenue in the presence of demand uncertainty, defined as $E\left[\pi_2\left(y_H, y_F, s, \tilde{z}_H, \tilde{z}_F \mid x, p, h, e\right)\right]$:

$$\pi^*\left(x, p, h, e\right) = \max_{\substack{(y_H, y_F, s) \ge 0 \\ y_H + y_F \le x \\ s \le h}} E\left[\pi_2\left(y_H, y_F, s, \tilde{z}_H, \tilde{z}_F \mid x, p, h, e\right)\right] = \begin{bmatrix} p\int \min\left\{y_H, d_H\left(p, z_H\right)\right\} g_H\left(z_H\right) dz_H \\ +pe\int \min\left\{y_F, d_F\left(p, z_F\right)\right\} g_F\left(z_F\right) dz_F \\ +\left(e_s - e\right) s \end{bmatrix}. \quad (3)$$

Stage 2 profit expression in (3) ignores transportation, duties, and other localization costs. We define the selling price in each market as net revenue, corresponding to price minus the sum of transportation, duties, and localization costs, and describe the demand function in terms of this net revenue. Because these changes do not alter the structural properties of our problem, we proceed with the present formulation.

In order to highlight the impact of exchange-rate risk, we begin our analysis with a simplified version of the model in (1) – (3) where financial hedging is ignored ($h = s = 0$), price is exogenous, and demand is deterministic. We incorporate endogenous pricing in Section 5, financial hedging in Section 6, and demand uncertainty in Section 7. Table 1 provides a comprehensive description of the portfolio of models developed in our analyses.

		Section 4	Section 5			Section 6	Section 7
			5.1	5.2	5.3		
Demand	Stochastic						✓
	Deterministic	✓	✓	✓	✓	✓	
Exchange Rate	Stochastic	✓		✓	✓	✓	✓
	Deterministic		✓				
Price Setting	Y		✓	✓	✓	✓	✓
	N	✓					
Financial Hedging	Y					✓	✓
	N	✓	✓	✓	✓		
Risk Aversion	Y	✓			✓	✓	✓
	N		✓	✓			

Table 1. Summary of models and their features that will be used in the analyses.

4. The Impact of Exchange-Rate Risk

This section presents the impact of exchange-rate risk on the optimal production decisions. This is accomplished by analyzing a variation of the model in (1) – (3) where the selling price is exogenous and the firm's financial hedging activity is not activated (by setting $h = s = 0$). We replace the demand random error term with its certainty equivalent, and define the deterministic demand as $d_i = d_i\left(p, \bar{z}_i\right)$ for $i = H, F$. This variation of the model enables us to derive the structural properties of the problem. It also serves as the foundation of the main conclusions. We then show that the basic structure and the main findings continue to hold under the settings where price-setting and financial hedging are included in the model (in Sections 5 and 6, respectively), and when demand uncertainty is incorporated into the analysis (in Section 7).

The second-stage problem in (3) can be rewritten as:

$$\pi^*\left(x, p, h, e, \bar{z}_H, \bar{z}_F\right) = \max_{\substack{(y_H, y_F) \geq 0 \\ y_H + y_F \leq x \\ s=h=0}} \pi_2\left(y_H, y_F, s \middle| x, h, p, e\right) = p\left[\min\left\{y_H, d_H\right\} + e\min\left\{y_F, d_F\right\}\right], \tag{4}$$

and the optimal allocation decisions can be expressed as:

$$\left(y_H^*, y_F^*\right) = \begin{cases} \left(\min\left\{d_H, x\right\}, \min\left\{d_F, \left(x - d_H\right)^+\right\}\right) & \text{if } e < 1 \\ \left(\min\left\{d_H, \left(x - d_F\right)^+\right\}, \min\left\{d_F, x\right\}\right) & \text{if } e \geq 1 \end{cases}. \tag{5}$$

Expressions (4) and (5) imply that if the realized exchange rate is below its mean (i.e., $e < 1$), then the firm prioritizes its allocation of products to the home market because the domestic revenue per unit is greater than the revenue per unit from the foreign market. If there are any leftovers, they can be sold in the foreign market after the home market demand is completely satisfied. If $e \geq 1$, however, the firm prioritizes the foreign market in its allocation decisions because the revenue from the foreign market is greater than the domestic revenue. Any leftovers after satisfying the foreign market demand are sold in the domestic market.

Using the above observation associated with (4) and (5), we introduce θ as a risk measure describing the expected loss from selling the product in the less desirable market (as opposed to the desirable market) due to the volatility in the exchange rate:

$$\theta = \int_{e_l}^{1} f\left(e\right)de - \int_{e_l}^{1} ef\left(e\right)de = \int_{1}^{e_h} ef\left(e\right)de - \int_{1}^{e_h} f\left(e\right)de. \tag{6}$$

The value of θ is computed by taking the difference between the cdf and the partial expectation until the switching point $\bar{e} = 1$ for the allocation preference in Stage 2. Its value is proportional to the standard deviation for common distributions such as the Normal, Uniform, and Exponential Distributions, and thus, increases with higher degrees of exchange-rate uncertainty. The risk measure θ also describes the upside potential that can be gained from selling in the foreign market. Thus, θ is a measure of: (1) exchange-rate volatility, (2) downside risk protection associated with producing less than global demand, and (3) upside profit potential of the foreign market over the home market when the exchange rate is high.

4.1 Optimal Manufacturing Quantity

This section develops the optimal production policies for the problem presented in (1), (2), and (4) where the firm determines the manufacturing quantity x (for a given p and when $h = s = 0$) in the presence of exchange-rate uncertainty subject to the VaR constraint. After realizing the exchange rate, the firm sets allocation quantities y_H and y_F subject to $y_H + y_F \leq x$ under deterministic demand values d_H and d_F. As will be shown later, the optimal manufacturing quantity is positive and Proposition 1 establishes the following four potentially optimal policies:

1. Total demand (TD) policy: $x = d_H + d_F$
2. Production hedging above maximum demand (PHA) policy: $d_x < x < d_H + d_F$
3. Production hedging at maximum demand (PHX) policy: $x = d_x$
4. Production hedging below maximum demand (PHB) policy: $d_m < x < d_x$

The TD policy is where the firm satisfies the demand in both markets; this is the policy recommendation in traditional aggregate production plans where the firm is forced to satisfy the global demand. Policies PHA, PHX, and PHB are production hedging policies where the firm deliberately produces a smaller amount than the global demand. Policies PHA and PHB emerge when policies TD and PHX, respectively, do not satisfy the VaR constraint in (2). Expected profit from each of these four policies is as follows:

$$E[\Pi^{TD}(X = d_H + d_F)] = (p - c)d_H + (p - c)d_F \quad (7)$$

$$E[\Pi^{PHA}(d_x < x < d_H + d_F)] = (p - c)x + p(d_H + d_F - x)\theta \quad (8)$$

$$E[\Pi^{PHX}(x = d_x)] = (p - c)d_x + pd_m\theta \quad (9)$$

$$E[\Pi^{PHB}(d_m < x < d_x)] = (p - c)x + pd_m\theta \quad (10)$$

The following proposition prescribes the optimal manufacturing quantities, and the conditions that lead to these potentially optimal policies under the VaR constraint. We let e_α denote the realized value of exchange rate corresponding to α probability, i.e., $P[\tilde{e} \leq e_\alpha] = \alpha$, and consider the reasonable cases when $e_\alpha < 1$ in our analysis.

Proposition 1. *For a given price p,* (a) *the objective function in* (1) *is piecewise linear in x;* (b) *there are four potentially optimal production policies, and the corresponding optimality conditions are described in Table 2:*

Policy	Optimality Condition	Optimal Production Quantity, x^*
PHB	$c \le p < \min\{\max\{c, (cd_F - \beta)/(d_H(1 - e_\alpha) + d_F e_\alpha)\}, c/(1 - \theta)\}$	$x^{PHB} = \dfrac{(1 - e_\alpha) d_H + \beta/p}{c/p - e_\alpha}$
PHX	$\min\{\max\{c, (cd_F - \beta)/(d_H(1 - e_\alpha) + d_F e_\alpha)\}, c/(1 - \theta)\} \le p \le c/(1 - \theta)$	$x^{PHX} = d_x$
PHA	$c/(1 - \theta) < p < \max\{((d_H + d_F)c - \beta)/(d_H + d_F e_\alpha), c/(1 - \theta)\}$	$x^{PHA} = \dfrac{(1 - e_\alpha) d_H + \beta/p}{c/p - e_\alpha}$
TD	$\max\{((d_H + d_F)c - \beta)/(d_H + d_F e_\alpha), c/(1 - \theta)\} \le p$	$x^{TD} = d_H + d_F$

Table 2. Optimal production quantity expressions for alternative policies with exogenous price.

(c) *policy* PHX *satisfies the risk constraint* (2) *and dominates policy* PHB *when* $d_H \ge d_F$ *for* $c \le p < c/(1-\theta)$.

Table 2 in Proposition 1 indicates that the exogenous selling price p has to be greater than $c/(1 - \theta)$ and $((d_H + d_F)c - \beta)/(d_H + d_F e_\alpha)$ in order for the firm to follow policy TD and manufacture the total demand. In the absence of a VaR constraint, the value of $c/(1 - \theta)$ serves as a threshold for the production hedging policy PHX to be optimal. A risk-neutral firm would manufacture the total demand when its price exceeds $c/(1 - \theta)$. Observe that the second threshold $((d_H + d_F)c - \beta)/(d_H + d_F e_\alpha)$ can be greater in value than $c/(1 - \theta)$; thus it is possible to have $c/(1 - \theta) < p < ((d_H + d_F)c - \beta)/(d_H + d_F e_\alpha)$. In this case, while the risk-neutral firm's preference would be to follow policy TD, risk aversion causes to firm to switch to production hedging. Thus, unit manufacturing cost (relative to the selling price) and the degree of exchange-rate uncertainty are not sufficient to make the firm switch to production hedging – risk aversion can trigger the switch from policy TD to production hedging policy PHA.

4.2 Risk Mitigation Aspect of Production Hedging

Proposition 1(c) shows that policy PHX is risk-free and dominates the expected profit of PHB at every price value in the interval of $c \le p \le c/(1-\theta)$ when $d_H \ge d_F$. This is because policy PHX is always risk-free when the firm has a larger home country market than its foreign market (i.e., $d_H \ge d_F$) due to the fact that the firm can sell all manufactured goods in the home market whenever the exchange rate takes on an unfavorable realization. Thus, a large domestic market serves as a significant downside risk protection under policy PHX. The consequence of this observation is that policy PHB only exists when the firm has a larger foreign market; alternatively said, PHB does not evolve as a viable policy when the home market is larger. It is important to note that having a larger domestic market is not sufficient to eliminate risk and satisfy (2). When the firm follows the TD policy and manufactures the total demand, the firm can violate the VaR constraint even if it has a larger domestic market, and thus, TD cannot dominate policy PHA when the domestic market is larger.

Is production hedging better at mitigating risk than the traditional TD policy? We first examine the realized profits under various production hedging policies and compare it with that of the TD policy at lower realizations of exchange rate. Proposition A1 in the online appendix shows that, when compared with the

TD policy, production hedging policies generate higher profits at lower realizations of stochastic exchange rate. Thus, from a realized profit perspective, production hedging policies can be more effective than the TD policy in mitigating the downside risk of the stochastic exchange rate.

We next examine the probability of realized profits being less than the tolerable loss β under various policies, and the expected profit when the exchange rate depreciates below the threshold value e_α representing the VaR probability. The next proposition formalizes that production hedging policies are less risky when compared to the traditional TD policy.

Proposition 2. (a) *For a given loss* β *where* $\beta \geq 0$, $P[\Pi(x) < -\beta] \leq P[\Pi^{TD}(x^{TD}) < -\beta]$ *for any* $x \in [0, d_H + d_F)$; (b) *For a given* α, *suppose that the VaR under* TD *corresponds to a loss, i.e.,* $\Pi(x^{TD} \mid e = e_\alpha) \leq 0$. *Then,* $E[\Pi(x \mid \tilde{e} \leq e_\alpha)] \geq E[\Pi(x^{TD} \mid \tilde{e} \leq e_\alpha)]$ *for any* $x \in [0, d_H + d_F)$.

The consequence of Proposition 2(a) is that, as the firm commits to full production $x = d_H + d_F$, it increases its likelihood of violating the VaR constraint. Thus, it can be concluded from Proposition 2(a) is that production hedging policies lead to a lower VaR than the traditional TD policy. While our analysis involves primarily the VaR measure in defining the risk perspective of the firm, similar observations can be made if we were to analyze the problem from a conditional value-at-risk (CVaR) measure. Proposition 2(b) sheds light into CVaR at probability α denoted as CVaR_α where $\text{CVaR}_\alpha = -E[\Pi(x \mid \tilde{e} \leq e_\alpha)]$. Proposition A1 in the online appendix shows that the realized profit is the lowest under the TD policy for exchange rate values in the range of $e_l \leq e \leq e_\alpha \leq c/p$. As a consequence, when $e_\alpha \leq c/p$, CVaR_α is maximized when the firm manufactures the total demand. Thus, even if the firm were to employ a constraint on maximum allowable CVaR instead of VaR, our insights will not change. Example B1 of Appendix B demonstrates an incident where production hedging policies reduce the firm's risk exposure and policy TD creates the highest risk exposure. Thus, we conclude that production hedging, defined as producing less than the total demand, can substantially reduce risk while maximizing expected profit.

Proposition 2 has demonstrated the risk mitigation benefits of production hedging policies. The following proposition shows that production hedging becomes more desirable with increasing exchange-rate uncertainty, risk aversion, and manufacturing cost (equivalently, with smaller profit margin).

Proposition 3. *Production hedging policy becomes more likely to be optimal* (*and TD becomes less likely to be optimal*) *with* (a) *increasing risk aversion, corresponding to decreasing values of* α *and/or* β; (b) *increasing exchange-rate uncertainty* θ; *and* (c) *increasing unit manufacturing cost* c.

We have established the risk mitigation benefits of production hedging under exogenous price. We next examine how the firm would adjust its pricing decision when it possesses the price-setting flexibility.

5. The Impact of Pricing-Setting Flexibility

This section presents the impact of incorporating price-setting into the production planning problem presented in Section 4 under exchange-rate risk. Because the firm manufactures a smaller amount of products according to a production hedging policy, one would intuit that a price-setting firm would increase its selling price under production hedging. However, the analysis in this section shows that a price-setting firm enjoys the benefits from the recourse flexibility when it reduces price and increases demand.

The firm now determines the production amount and selling price (x, p) in Stage 1 under exchange-rate uncertainty. We modify the model in (1) – (3) as follows:

$$\max_{(x,p)\geq 0} E\left[\Pi\left(x,p\right)\right] = -cx + \int_{e_l}^{e_h} \pi^*\left(x,p,h=0,e\right) f\left(e\right) de \tag{11}$$

$$\text{subject to } P_{(\tilde{e})}\left[-cx - c_h h + \pi_2\left(y_H^*, y_F^*, s=0, \bar{z}_H, \bar{z}_F \,\middle|\, x,p,h=0,e\right) < -\beta\right] \leq \alpha \tag{12}$$

where $\pi^*(x, p, h = 0, e)$ is the optimal second-stage objective function for a given set of first-stage decisions (x, p) and exchange-rate realization e. Demand in each market is now price-sensitive and is expressed as $d_i(p) = d_i(p, \bar{z}_i)$ for $i = H, F$ where demand is decreasing in price. In Stage 2, the firm sets allocation quantities y_H and y_F subject to $y_H + y_F \leq x$ under deterministic but price-sensitive demand values of $d_H(p)$ and $d_F(p)$:

$$\pi^*\left(x,p,h=0,e\right) = \max_{\substack{(y_H,y_F,s)\geq 0\\ y_H+y_F\leq x\\ s=h=0}} E\left[\pi_2\left(y_H,y_F,s,\bar{z}_H,\bar{z}_F \,\middle|\, x,p,h,e\right)\right] = p\begin{bmatrix}\min\left\{y_H, d_H\left(p\right)\right\}\\ +e\min\left\{y_F, d_F\left(p\right)\right\}\end{bmatrix}. \tag{13}$$

In Stage 2, the firm's allocation preference remains the same as in Section 3 where the firm prioritizes its allocation of products to the home market if the realized exchange rate is below its mean (i.e., $e \leq 1$), and prioritizes the foreign market otherwise (i.e., $e > 1$).

In order to provide insight into the impact of exchange-rate uncertainty and risk aversion, we first establish two benchmarks, and compare them with the firm's optimal decisions.

5.1 Riskless Price

The first benchmark is established by solving the joint price and quantity decisions under deterministic exchange rate and in the absence of risk aversion. We replace the random exchange rate variable in (11) with its expectation, and ignore the VaR constraint (12). We denote the decision variables (x^0, p^0), where p^0 represents the riskless price. In this setting, the firm would manufacture a quantity that is equal to the total demand, i.e., $x^0 = d_H(p^0) + d_F(p^0)$. Substituting x^0 into (11), the riskless price can be established as:

$$p^0 = c + [d_H(p^0) + d_F(p^0)]/[-(d_{Hp}(p^0) + d_{Fp}(p^0))]. \tag{14}$$

It should be observed that the riskless price in (14) is greater than the unit manufacturing cost, i.e., $p^0 > c$.

5.2 Price under Uncertainty

The second benchmark is established by solving the joint price and quantity decisions under exchange-rate uncertainty while ignoring risk aversion and the VaR constraint in (12). The first-stage decisions (x, p) are made in the presence of exchange-rate uncertainty as in (11). For a given set of (x, p), the firm determines the best allocation decisions (y_H, y_F) in stage 2 as in (13). Like in Section 4, we restrict the selling price to be greater than or equal to the unit manufacturing cost, i.e., $p \geq c$, in order to have a meaningful comparison. We first build an analysis in order to answer the following research question: If the firm has the ability to set prices in the presence of exchange-rate uncertainty, would it set a higher or lower price when compared to the riskless price in (14)? Production hedging recommends manufacturing a smaller amount than the total demand in order to mitigate the exchange-rate risk. One would intuit that, the firm would prefer to increase its selling price beyond the riskless price as a consequence of the smaller manufacturing quantity under production hedging. Our analysis, however, shows that the firm reduces its selling price in order to enjoy the benefits from the allocation flexibility based on realized exchange rates in stage 2.

We denote the optimal manufacturing quantity and price decisions in the presence of exchange-rate uncertainty as (x^u, p^u). We describe the price-elasticity of the total demand function evaluated at price p as $\varepsilon^{TD}(p) = -p(d_{Hp} + d_{Fp})/(d_H + d_F)$. The next proposition shows that only two policies, TD and PHX, emerge as potentially optimal policies, and it identifies the optimal price and quantity decisions under exchange-rate uncertainty.

Proposition 4. *When* (12) *is ignored,*

(a) *The problem under exchange-rate uncertainty is bimodal, and policies* PHX *and* TD *are the only two potentially optimal policies;* (b) *Expected profit functions under policies* PHX *and* TD *are concave in p, and the optimal price and production quantities under these policies are as follows:*

Policy	Optimal Price, p^u	Optimal Production Quantity, x^u
PHX	$p^{PHX} = -\dfrac{d_x - cd_{x_p} + d_m\theta}{d_{x_p} + d_{m_p}\theta}$	$x^{PHX} = d_x^{PH} = -p\left[d_{x_p} + d_{m_p}\theta\right] + cd_{x_p} - d_m\theta$
TD	$p^{TD} = p^0$	$x^{TD} = d_H^{TD} + d_F^{TD} = -(p-c)\left(d_{H_p} + d_{F_p}\right)$

Table 3. Optimal price and quantity expressions for alternative policies.

(c) *The following inequalities on optimal prices are not possible simultaneously:* $p^{TD} < c/(1-\theta)$ *and* $p^{PHX} > c/(1-\theta)$; (d) *Production hedging policy* PHX *is optimal when* $\varepsilon^{TD}(c/(1-\theta)) > 1/\theta$; (e) *The optimal manufacturing quantity and price decisions are* $(x^u, p^u) = (x^{PHX}, p^{PHX})$ *when* $p^{TD} < c/(1-\theta)$ *or when* $\{p^{TD} \geq c/(1-\theta) \cup p^{PHX} < c/(1-\theta) \cup E[\Pi(x^{TD}, p^{TD})] < E[\Pi(x^{PHX}, p^{PHX})]\}$; (f) *The optimal manufacturing quantity and price decisions are* $(x^u, p^u) = (x^{TD}, p^{TD})$ *when* $p^{PHX} \geq c/(1-\theta)$ *or when* $\{p^{TD} \geq c/(1-\theta) \cup p^{PHX} < c/(1-\theta) \cup$

$E[\Pi(x^{TD}, p^{TD})] \geq E[\Pi(x^{PHX}, p^{PHX})]\}$; (g) *The optimal price under exchange-rate uncertainty is less than or equal to the riskless price, i.e.,* $p^u \leq p^0$.

Proposition 4 provides two important findings associated with the optimal price decision. First, the optimal price under the TD policy is equivalent to the riskless price established in (14), i.e., $p^{TD} = p^0$. Thus, when the firm manufactures to match demand, it would not be able to take advantage of the exchange-rate fluctuations in stage 2. Second, the optimal price under uncertainty is less than or equal to the riskless price: $p^u \leq p^0$. Thus, incorporating exchange-rate uncertainty into the problem can only cause the firm to decrease its price below the riskless price in the absence of a VaR constraint. The firm's decision to reduce the selling price under exchange-rate uncertainty can be explained as follows: The firm is willing to forgo some of its profit margin by decreasing the selling price because the reduction in price inflates the level of demand in each region, and the firm enjoys the financial benefits from adjusting the allocation decisions in stage 2 based on exchange-rate fluctuations. The benefit from allocation flexibility in stage 2 does not materialize when the firm follows the TD policy. Proposition 4(d) establishes a sufficient condition using the price-elasticity of demand, $\varepsilon^{TD}(c/(1-\theta)) > 1/\theta$, that ensures the optimality of production hedging and warrants the reduction in the optimal selling price.

The result associated with reducing the selling price under exchange-rate uncertainty differs from similar results established under demand uncertainty. Recall that, when the random variable is in the multiple form as in our problem, the optimal price under demand uncertainty in PSNP is greater than the riskless price as shown in Lemma 2 of Petruzzi and Dada (1999). Thus, incorporating demand uncertainty into PSNP causes the firm to increase the selling price. In addition to examining the impact of exchange-rate uncertainty (rather than demand uncertainty) on the optimal choice of selling prices, our model differs from that of Petruzzi and Dada (1999) by featuring an allocation flexibility in distributing products to multiple markets in stage 2. When the source of uncertainty is exchange rate, rather than demand, incorporating uncertainty causes the firm to reduce its selling price in the presence of the allocation flexibility with multiple markets.

5.3 Price under Risk Aversion

We next examine the impact of risk aversion through the VaR constraint in (12) in the original problem (11) – (13). Our analysis intends to find an answer to our research question: Does the firm increase or decrease its selling price as a consequence of the VaR constraint when compared with the optimal price under uncertainty and the riskless price? We denote the optimal manufacturing quantity and price decisions under risk aversion (x^r, p^r), and compare them with (x^u, p^u) and (x^0, p^0).

We first characterize the optimal policies under endogenous pricing and risk aversion. The following proposition shows that the four policies identified in Section 4 continue to be potentially optimal for the model in (11) – (13).

Proposition 5. *The optimal price and production quantities for the four potentially optimal policies are as described in Table 4:*

Policy	Optimal Price, p^r	Optimal Production Quantity, x^r
PHB	$p^{PHB} = \dfrac{cx-\beta}{d_H + e_\alpha\left(x-d_H\right)}$	$x^{PHB} = \dfrac{\left(1-e_\alpha\right)d_H + \beta/p}{c/p - e_\alpha}$
PHX	$p^{PHX} = -\dfrac{d_x - cd_{x_p} + d_m\theta}{d_{x_p} + d_{m_p}\theta}$	$x^{PHX} = d_x^{PH} = -p\left[d_{x_p} + d_{m_p}\theta\right] + cd_{x_p} - d_m\theta$
PHA	$p^{PHA} = \dfrac{cx-\beta}{d_H + e_\alpha\left(x-d_H\right)}$	$x^{PHA} = \dfrac{\left(1-e_\alpha\right)d_H + \beta/p}{c/p - e_\alpha}$
TD	$p^{TD} = p^0$	$x^{TD} = x^0$

Table 4. Optimal price and quantity expressions for alternative policies.

Simultaneous optimization of price and quantity decisions under demand risk is examined extensively in the PSNP literature. This literature widely reports that risk aversion causes the firm to reduce the initial inventory quantity, and often, to increase the selling price. Agrawal and Seshadri (2000) report that the optimal price under risk aversion is greater than the risk-neutral price under a multiplicative demand random error term, and is less than the risk-neutral price under an additive demand random error term. They argue that the price increase under risk aversion is a consequence of scaling stemming from the multiplicative form of randomness. In our problem, exchange-rate random variable is also described with a multiplicative term, and has a similar scaling effect in the firm's profit with a slight revision. While the profit margin in foreign market scales up with exchange-rate, the profit margin in the domestic market does not scale up or down, and remains unchanged with exchange rate. Thus, one would intuit that the optimal price choice under risk aversion would be greater than that under a risk-neutral objective function. However, we show that three of the four combinations can be experienced under risk aversion:

$p^r > p^u$ and $x^r < x^u$,

$p^r < p^u$ and $x^r < x^u$,

$p^r < p^u$ and $x^r > x^u$.

However, the case where $p^r > p^u$ and $x^r > x^u$ does not occur under the VaR constraint. It is important to highlight that the VaR constraint in (12) creates non-linearity in the objective function, and therefore, policies PHA and PHB require a line search in order to determine the optimal decisions. Thus, the optimal price and quantity decisions cannot be characterized in closed-form expressions. However, the results in Proposition 5 enable us to determine their values. The online supplement provides numerical examples that demonstrate the existence of the above three results. See examples B4(a) – (c) and B5 for the case when the optimal policy is TD in the absence of (12), and B6(a) – (c) and B7 when the optimal policy is PHX in the absence of (12).

The consequence of the above finding is that our results differ from those reported under demand risk using a multiplicative random demand error term. When the source of uncertainty is exchange rate rather than demand, the optimal price under risk aversion can decrease below the optimal risk-neutral price, and thus, it is possible to observe $p^r < p^u$. This result differs from earlier publications that examined risk aversion under demand uncertainty using a multiplicative random error term. Moreover, the optimal manufacturing quantity in our problem can also increase due to the reduction in price, and thus we can experience $x^r > x^u$ under risk aversion.

Proposition 4(g) has shown that the optimal price under uncertainty is less than or equal to the riskless price: $p^u \leq p^0$. Consider the case when $p^u = p^0$. Proposition 5 identifies that the optimal price and quantity under policy TD are equal to those under the riskless price, i.e., $p^{TD} = p^0$ and $x^{TD} = x^0$. Thus, $p^u = p^{TD} = p^0$ and $x^u = x^{TD} = x^0$. We have already established that the optimal price under risk aversion can be greater or less than the optimal under uncertainty. As a consequence of $p^u = p^0$, the optimal price under risk aversion can also be greater or less than the riskless price.

We interpret a lower probability of violating the VaR constraint and a lower expected profit in the lower α-percentile of the exchange-rate distribution as smaller risk aversion. The next proposition shows that production hedging policies continue to have smaller risk exposure than policy TD from both VaR and CVaR perspectives under endogenous pricing. It also shows that policy PHX is risk free when the ratio of the domestic market demand to the foreign market demand is greater than or equal to the ratio of cost to price.

Proposition 6. (a) *Policy* PHX *is risk-free if* $p^{PHX}\,(d_H^{PHX}/d_F^{PHX}) \geq c$; (b) *For a given loss* β *where* $\beta \geq 0$, $P[\Pi^j(x^j, p^j) < -\beta] \leq P[\Pi^{TD}(x^{TD}, p^{TD}) < -\beta]$ *for* j = PHA, PHB; (c) *For a given* α, *suppose that the VaR under* TD *corresponds to a loss, i.e.,* $\Pi(x^{TD}, p^{TD} \mid e = e_\alpha) \leq 0$. *Then,* $E[\Pi^j(x^j, p^j) \mid \tilde{e} \leq e_\alpha] \geq E[\Pi^{TD}(x^{TD}, p^{TD}) \mid \tilde{e} \leq e_\alpha]$ *for* j = PHA, PHB.

In sum, the combination of the price-setting flexibility and risk aversion leads to a higher potential for eliminating the TD policy and a greater possibility of utilizing a production hedging policy. In the absence of risk aversion, when the firm switches from the traditional policy of manufacturing the total demand to a production hedging policy, it reduces its optimal selling price in order to extend the benefits of allocation flexibility based on exchange-rate fluctuations. Decreasing price increases the demand in both markets; thus, the firm might actually manufacture a greater amount under production hedging in comparison to when it manufactures the total demand. Under risk aversion, the firm can further decrease the selling price under exchange-rate risk. The reduced price increases demand in markets, and can lead to a higher manufacturing quantity. Thus, production hedging, under endogenous pricing and risk aversion, may not be all detrimental to consumers as they might obtain the product at a lower price and have a larger amount made available by the firm. Moreover, our analysis demonstrates that production hedging policies continue to have a smaller risk exposure under endogenous pricing.

6. The Impact of Financial Hedging

This section presents the impact of incorporating financial hedging into the model presented in Section 4. Can financial hedging eliminate production hedging? We next present the benefits of financial hedging and its interaction with production hedging.

We employ an analysis using currency futures contracts as the financial instrument, and thus, we use $s = h$; a similar analysis can be replicated using options contracts where $s \le h$ with no change in our main conclusion. Each unit of financial contract has a unit cost of c_h (also referred to as the premium) and a strike (or, exercise) price of e_s where the seller requires some nominal positive margin δ on the sale. Thus, $c_h = c_h^o(e_s) + \delta$ where $c_h^o(e_s)$ denotes the cost of a financial contract where the expected profit for the financial institution is exactly zero, i.e.,

$$c_h^o\left(e_s\right) = \int_{e_l}^{e_s} \left(e_s - e\right) f\left(e\right) de - \int_{e_s}^{e_h} \left(e - e_s\right) f\left(e\right) de \,. \tag{15}$$

Without loss of generality, we assume that the firm purchases financial contracts where the strike price equals the mean price (i.e., $e_s = \bar{e}$) and therefore $c_h^o(\bar{e}) = 0$. Thus, $c_h = \delta$.

Stage 1 objective function is expressed in (1), and we revise the VaR constraint in (2) in order to accommodate deterministic demand:

$$P_{(\tilde{e})}\left[-cx - c_h h + \pi_2\left(y_H^*, y_F^*, s^*, \overline{z}_H, \overline{z}_F \,\middle|\, x, p, h, e\right) < -\beta\right] \le \alpha \,. \tag{16}$$

In stage 2, regardless of the realized exchange rate, futures contracts are exercised based on the strike price e_s, i.e., $s = h$. Thus, the second-stage objective function can be written as:

$$\pi^*\left(x, p, h, e\right) = \max_{\substack{(y_H, y_F) \ge 0 \\ y_H + y_F \le x \\ s = h}} E\left[\pi_2\left(y_H, y_F, s, \overline{z}_H, \overline{z}_F \,\middle|\, x, p, h, e\right)\right] = \begin{bmatrix} p \min\left\{y_H, d_H\left(p\right)\right\} \\ + pe \min\left\{y_F, d_F\left(p\right)\right\} \\ + \left(e_s - e\right) h \end{bmatrix} \tag{17}$$

where the term $(e_s - e)h$ in (17) is the return from financial contracts at the time of expiration.

Proposition 6 has shown that production hedging policies lead to smaller risk exposure. When policy TD violates the risk constraint in (16), the firm switches to policy PHA. When PHX violates (16), it alters the policy to PHB. Financial hedging enables the firm to stick with policies TD and PHX without having to switch to policies PHA and PHB, respectively. This is because the firm can now buy a sufficient number of financial hedging contracts in stage 1 and satisfy the VaR constraint in (16).

Proposition 7. *Suppose* $e_\alpha < 1$. *If the VaR constraint is not satisfied*

(a) $$\left\{\begin{array}{l} under\ \text{TD}\ and\ if\ \ \delta < p - c\left[\dfrac{\left(1-\frac{p}{c}e_\alpha\theta\right)-e_\alpha}{(1-\theta)-e_\alpha}\right],\ then\ h^* = \dfrac{-\pi\left(x^{TD} = d_H + d_F, h = 0, e_\alpha\right)-\beta}{p\left(1-e_\alpha\right)-c_h}; \\ under\ \text{PHX}\ \left(where\ d_F > d_H\right)\ and\ if\ \ \delta < p - c,\ then\ h^* = \dfrac{-\pi\left(x^{PHX} = d_F, h = 0, e_\alpha\right)-\beta}{p\left(1-e_\alpha\right)-c_h}; \end{array}\right\}$$

otherwise $h^* = 0$; (b) *When* $\delta = 0$, $E[\Pi^{PHA}(x^{PHA}, h^*)] = E[\Pi^{TD}(x^{TD}, h = 0)]$ *and* $E[\Pi^{PHB}(x^{PHB}, h^*)] = E[\Pi^{PHX}(x^{PHX}, h = 0)]$.

Proposition 7(b) shows that, when the financial institution providing the financial hedging instrument does not make a profit, i.e., $\delta = 0$, the firm can eliminate the need to alter its optimal policy choice as a consequence of the VaR constraint. If the TD policy is optimal in the absence (16), the firm can buy a sufficient number of financial hedging instruments as prescribed in Proposition 7(a) in the presence of (16) and obtain the same expected profit instead of having to switch to policy PHA. When $\delta = 0$, financial hedging eliminates policy PHA (i.e., the change in expected profit due to production hedging is positive without financial hedging and negative with financial hedging). Thus, financial hedging and production hedging act as substitutes, i.e., financial hedging decreases the value of production hedging. Next, consider the problem setting where the optimal risk-neutral policy is PHX. If $\delta = 0$ and PHX violates (16), then the firm can purchase a sufficient number of financial hedging contracts as prescribed in Proposition 7(a) and obtain the same profit in the absence of (16). When $\delta = 0$, financial hedging eliminates the need to switch from policy PHX to PHB (i.e., the gain in expected profit due to production hedging increases under financial hedging). In this case, financial hedging and production hedging are complements, i.e., financial hedging increases the value of production hedging. We emphasize that financial hedging does not eliminate production hedging. The next proposition formalizes when financial hedging and production hedging act as complements and substitutes.

Proposition 8. (a) *Financial hedging and production hedging are complements if* $(x^u, p^u) = (x^{PHX}, p^{PHX})$, *and financial hedging cannot eliminate production hedging from being the optimal solution even if* $\delta = 0$; (b) *When* $\delta = 0$, *financial hedging and production hedging are substitutes if* $(x^u, p^u) = (x^{TD}, p^{TD})$.

Proposition 8(a) shows that, even in the least costly financial hedging scenario with $\delta = 0$, financial hedging cannot always eliminate production hedging from being the optimal policy. Consider the event that policy PHB is the optimal policy under risk aversion in the absence of financial hedging. When the firm is enabled to engage in financial hedging, Proposition 8(a) shows that the firm can switch its policy choice from PHB to PHX in the presence of financial hedging. While financial hedging helps improve expected profit (because the expected profit is higher under PHX than PHB), it does not eliminate production hedging. As a result, in this scenario, production hedging and financial hedging are both utilized, and therefore, production hedging and financial hedging behave in a complementary manner. Our finding is consistent with Mello et al. (1995) where financial hedging is utilized to reduce the firm's agency costs stemming from the outstanding debt. In our model, however, the complementary effect comes from the need to restrict the downside risk from the stochastic exchange rate.

Let us next consider the scenario when policy TD is optimal in the risk-neutral setting, but PHA becomes optimal under risk aversion in the absence of financial hedging. Proposition 8(b) indicates that, when the risk-neutral optimal policy is TD, financial hedging enables the firm to manufacture the total demand, and switch policy PHA to policy TD. Thus, financial hedging eliminates the need to switch to production hedging, and therefore, financial hedging acts as a substitute to production hedging in this scenario. Our finding in Proposition 8 is similar to the finding in Dong and Tomlin (2012) where inventory and insurance can play both complementary and substitute roles under different settings for a firm that faces disruption risks. While Dong and Tomlin (2012) limit their description of operational hedging to the inventory quantity decision, our model offers a more comprehensive view of operational hedging with simultaneous price and quantity decisions under exchange-rate risk.

7. Existence of Production Hedging under Demand Uncertainty

This section shows that production hedging exists under demand uncertainty for the model developed in Section 3. We consider the case when the two market demand random error terms are identically and independently distributed (i.i.d.). Under demand uncertainty, when each market is analyzed independently for any given p, the firm would manufacture $x_H^*(p) = G_H^{-1}((p - c)/p)$ for the home market and $x_F^*(p) = G_F^{-1}((p - c)/p)$ for the foreign market. We first describe production hedging in the presence of demand uncertainty. Solutions that feature a smaller production amount than the sum of the two independent quantities resemble the principle of production hedging. Therefore, letting x^* and p^* denote the optimal quantity and price, we define optimal solutions where $x^* < x_H^*(p^*) + x_F^*(p^*)$ as production hedging solutions under stochastic demand.

We note that production hedging can continue to be optimal under demand uncertainty. Our purpose in this section is to show that several properties and insights continue to apply under demand uncertainty. We focus on the structural properties pertaining to production hedging policies and associated managerial insights, and we limit consideration to settings where the VaR constraint is not binding; we do not characterize all of the potentially optimal solutions under demand uncertainty. We present a detailed solution approach for the second-stage problem under demand uncertainty employing Lagrangian relaxation in Appendix C of the online supplement. As can be seen in this solution approach, the second-stage allocation decisions can be classified in three regions: $R1 = \{e: e_l \le e \le e_1(x)\}$, $R2 = \{e: e_1(x) \le e \le e_2(x)\}$, and $R3 = \{e:$

$e_2(x) \le e \le e_h\}$ where $e_1(x)$ is the point that the firm's second-stage profit from the first unit allocated to the foreign market is equal to the next unit allocated to the domestic market, and $e_2(x)$ is the point that the firm's second-stage profit from the first unit allocated to the foreign market is equal to the next unit allocated to the domestic market.

$$(y_H^*, y_F^*) = \begin{cases} (x, 0) & \text{if } e \in R1 \\ \left(y_H^*(x,e), x - y_H^*(x,e)\right) & \text{if } e \in R2 \\ (0, x) & \text{if } e \in R3 \end{cases}$$

where $y_H^*(x, e)$ is the optimal amount that can be allocated to the home market for a given initial manufacturing quantity x and realized exchange-rate e. Its value is determined by equating the marginal return from an additional unit allocated to the domestic market with that from allocating the additional unit to the foreign market. Note that under deterministic demand, for all exchange rate realizations, we only observe the allocation schemes where all products are either allocated to the domestic market as in region $R1$ or where all products are allocated to the foreign market as in region $R3$. The rationing scheme in region $R2$ does not exist under deterministic demand, and is a consequence of the stochastic demand setting.

Our first main finding from the analysis of the deterministic demand revealed that the firm would benefit from manufacturing a smaller amount. We next examine whether the reduction in production quantity continues to be the prevailing behavior under stochastic demand. The next proposition shows that, when demand uncertainty is introduced to the problem, the firm manufactures even a smaller amount under production hedging than it does under deterministic demand. Moreover, the newsvendor ratio that determines the target fractile for each market's stochastic demand is restricted from above by the amount of exchange-rate uncertainty defined with θ. We restrict our proof to the case when the two market demands follow i.i.d. random variables with symmetric pdf for all random variables, but it holds true at every price level that makes production hedging optimal.

Proposition 9. (a) *The newsvendor ratio under production hedging policies is less than or equal to* θ*;* (b) *For a symmetric pdf for exchange rate and demand uncertainty, with demand in each market following i.i.d. random variables and the mean of the stochastic demand equals the deterministic demand, when production hedging is optimal under deterministic demand with* $\theta \le 1/3$ *and price remains fixed, the optimal production amount under production hedging with stochastic demand is less than or equal to that under deterministic demand*.

For any reasonable continuous pdf representing exchange-rate uncertainty, the value of θ is less than 0.25. For example, the value of θ is 0.25 for a uniform distribution on a support of [0, 2], its value is less than 0.25 for distributions that are centered around mean with the same support (see Lemma A1 in the online supplement). The value of θ is maximized when random exchange rate follows a two-point distribution with at $e = \{0, 2\}$ with probability 0.50 at each realization; in this case, $\theta = 1$. The consequence of Proposition 9 is that, for any reasonable pdf representing the uncertainty in the exchange rate with θ less than 1/3, the firm will operate at a newsvendor fractile less than 50% under production hedging policies. The condition where θ is less than 1/3 is a sufficient condition, and our finding holds true for an even larger set of distributions. Proposition 9 allows us to show that, in the presence of demand uncertainty, the firm will manufacture an even smaller amount than the amount it would produce under deterministic demand.

Our second main finding from the analysis of the deterministic demand revealed that the optimal price gets reduced under production hedging. Would price reduction continue under stochastic demand? It is known from Petruzzi and Dada (1999) that when demand is described with a linear function supplemented by an additive random error term, the optimal price choice in PSNP is smaller than that under deterministic demand. While our problem in (1) – (3) does not follow the PSNP structure identically, we observe a similar behavior under linear demand with i.i.d. additive random error terms.

Proposition 10. *When each market follows an identical linear demand function with a symmetric and additive i.i.d. random variables where the mean of the stochastic demand equals the deterministic demand and the random exchange rate follows a symmetric pdf with* $\theta \leq 1/3$, *the optimal price under production hedging in the presence of stochastic demand is less than that under deterministic demand.*

In conclusion, our analysis demonstrates that our main findings continue to hold under demand uncertainty. First, we observe that, at any price level under production hedging, the firm's optimal production quantity is below that under deterministic demand; thus, the firm manufactures even less under stochastic demand. Second, we observe that, under production hedging, price can continue to get reduced with the introduction of demand uncertainty.

8. Conclusions and Managerial Insights

This paper investigates a risk-averse firm's production planning, pricing, and financial hedging decisions under exchange-rate and demand uncertainty. It employs a value-at-risk measure to limit the firm's realized losses in amount and in probability.

Our analysis leads to three main conclusions. First, we show that production hedging is not just capable of maximizing expected profit, but also is an effective risk-mitigation approach. Production hedging reduces the firm's risk exposure. Manufacturing a quantity less than the total demand enables the firm to benefit from the flexibility to sell a bigger portion of its limited supply in markets that maximize revenue based on the realized exchange rate. We show that, regardless of whether risk is defined in the form of a value-at-risk or a conditional-value-at-risk perspective, production hedging policies lead to smaller risk exposure than the traditional policy of producing and satisfying the global demand. Moreover, the domestic market provides a downside protection under production hedging without having to lose on the upside potential, which resembles a currency option commonly seen in financial hedging. Production hedging becomes even more desirable under increasing levels of (i) exchange-rate uncertainty, (ii) demand uncertainty, (iii) risk aversion, and (iv) unit manufacturing cost.

Second, our paper provides a comprehensive set of results pertaining to the optimal price decision. In the absence of risk aversion, we show that the firm prefers to reduce the optimal selling price under production hedging. Reduction in price leads to an increase in demand in both markets, and the firm can enjoy greater benefits from the allocation flexibility based on the realized exchange rate. We also show that incorporating risk aversion can lead to both an increase and a decrease in the optimal price choice. It is important to highlight that, unlike the common expectation, risk aversion can further reduce the optimal price. With the reduction in price, the firm might end up manufacturing a larger quantity than what it would produce in the absence of uncertainty. Thus, production hedging is not always detrimental to consumers as they access a larger amount of the product at a lower price.

Third, we show that financial hedging can be both a substitute and a complement to production hedging. When the firm's value at risk exceeds the tolerable loss, financial hedging serves as a complement to recover the lost profit from having to switch policies. However, financial hedging cannot always eliminate production hedging from being the optimal choice.

Our paper demonstrates the existence of production hedging in the presence of demand uncertainty. However, it does not characterize the complete set of potentially optimal solutions. Future research should examine the problem under demand uncertainty in depth in order to characterize all potentially optimal policies.

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Online Supplement

Risk Mitigation of Production Hedging

Appendix A – Proofs of Technical Results

Proposition A1. *When* $e_l \leq e < c/p$, *then* $\Pi^j(x^j) > \Pi^{TD}(x^{TD})$ for j = PHB, PHX, PHA.

Proof of Proposition A1. Suppose $e < 1$ as we are deriving profit for each production policy when exchange is realized unfavorable to the firm and therefore the firm prioritize the home market over the foreign market. The following are the profits for each production policy when $e < 1$:

$\Pi^{TD}(x = d_H + d_F \mid e < 1) = p(d_H + ed_F) - c(d_H + d_F)$

$\Pi^{PHA}(d_x < x < d_H + d_F \mid e < 1) = pd_H + pe(x - d_H) - cx$

$\Pi^{PHX}(x = d_H \geq d_F \mid e < 1) = pd_H - cd_H$ and $\Pi^{PHX}(x = d_F \geq d_H \mid e < 1) = pd_H + pe(d_F - d_H) - cd_F$

$\Pi^{PHB}(d_H < x < d_F \mid e < 1) = pd_H + pe(x - d_H) - cx$

We derive the condition where each production hedging policy yields a higher profit compared to the TD policy by solving $\Pi^{PH} > \Pi^{TD}$ for e. Π^{PH} is used as a general term to express profits for each production hedging policy. ❑

Proof of Proposition 1. Part (a): For any price level, the second-order derivative of the objective function with respect to x is zero, implying that the objective function is linearly increasing in the production amount. For this reason, it is sufficient to consider the first-order derivatives to complete the proof.

Parts (b-i) and (b-ii): When $0 \leq x \leq d_m$, $E\left[\Pi(x)\right] = -cx + \int_{e_l}^{\bar{e}} pxf(e)de + \int_{\bar{e}}^{e_h} pexf(e)de$; $\partial E[\Pi(x)] / \partial x = -c + p(1+\theta)$. This implies that if $p < c/(1+\theta)$, the optimal production amount is $x^* = 0$; otherwise, if $p \geq c/(1+\theta)$, then the optimal production amount is at least d_m if (2) is satisfied. When $d_m \leq x \leq d_x$, if the home demand is larger than the foreign demand at a price level p, then the objective function can be written as follows:

$$E\left[\Pi\left(d_F < x < d_H\right)\right] = -cx + \int_{e_l}^{\bar{e}} pxf(e)de + \int_{\bar{e}}^{e_h}\left[ped_F + p\left(x - d_F\right)\right] f(e)de,\ \partial E\left[\Pi(x)\right]/\partial x = -c + p$$

or, alternatively if the foreign demand is higher than the domestic demand at p, then,

$$E\left[\Pi\left(d_H < x < d_F\right)\right] = -cx + \int_{e_l}^{\bar{e}}\left[pd_H + pe\left(x - d_H\right)\right] f(e)de + \int_{\bar{e}}^{e_h} pexf(e)de,\ \partial E\left[\Pi(x)\right]/\partial x = -c + p\,.$$

Both cases imply that if $p < c$, the optimal production amount is $x^* = d_m$ if (2) is satisfied; otherwise, if $p \geq c$, then the optimal production amount is at least d_x if (2) is satisfied. Since we restrict our analysis to where $p \geq c$, the firm will always manufacture more than d_m. Now, consider the case when $d_x \leq x \leq d_H + d_F$.

$$E\left[\Pi\left(d_x \le x < d_H + d_F\right)\right] = -cx + \int_{e_l}^{\bar{e}} \left[pd_H + pe\left(x - d_H\right)\right] f\left(e\right) de + \int_{\bar{e}}^{e_h} \left[ped_F + p\left(x - d_F\right)\right] f\left(e\right) de,$$

$$\partial E\left[\Pi\left(d_x \le x < d_H + d_F\right)\right] / \partial x = -c + p\left(1 - \theta\right).$$

This implies that if $p < c/(1-\theta)$, then the optimal production amount is $x^* = d_x$ if (2) is satisfied; otherwise, if $p \ge c/(1-\theta)$, then the optimal production amount is $d_H + d_F$ if (2) is satisfied. If $d_H \ge d_F$, $x^* = d_x$ always satisfies (2) for $c \le p < c/(1-\theta)$ which we label as the PHX policy. However for the same price region, if $d_H < d_F$ and $p < (cd_F - \beta)/(d_H(1-e_\alpha)+d_F e_\alpha))$, then $x^* = d_x$ violates (2). Thus, if $d_H < d_F$ for $c \le p <$ $\min\{\max\{c, (cd_F - \beta)/(d_H(1-e_\alpha)+ d_F e_\alpha)\}, c/(1-\theta)\}$, then it is optimal to produce the maximum amount while satisfying (2). Because (2) is binding, we have $x^* = ((1-e_\alpha)d_H + (\beta/p))/((c/p) - e_\alpha)$ which we label as the PHB policy. If $d_H < d_F$ for $\min\{\max\{c, (cd_F - \beta)/(d_H(1-e_\alpha)+ d_F e_\alpha)\}, c/(1-\theta)\} \le p \le c/(1-\theta)$, then $x^* = d_x$ since (2) is satisfied, which we label as the PHX policy.

Parts (b-iii) and (b-iv): If $p \ge c/(1-\theta)$, then the optimal production amount is $x^* = d_H + d_F$ if (2) is satisfied. However, if $p < ((d_H + d_F)c - \beta)/(d_H + d_F e_\alpha)$, then (2) is violated. Thus, if $c/(1-\theta) < p < ((d_H + d_F)c - \beta)/(d_H + d_F e_\alpha)$ then it is optimal to produce the maximum amount while satisfying (2). Because (2) is binding, we have $x^* = ((1-e_\alpha)d_H + (\beta/p))/((c/p) - e_\alpha)$ which we label as the PHA policy. If $\max\{c/(1-\theta), ((d_H + d_F)c - \beta)/(d_H + d_F e_\alpha)\} \le p$, then $x^* = d_H + d_F$ since (2) is satisfied which we label as the TD policy. Thus for a given price level, there are four potentially optimal production policies.

Part (c): For $c \le p < c/(1-\theta)$, it is already proven that it is optimal to produce $x^* = d_x$. When $d_H > d_F$ in this case, the PHX policy always satisfies (2) because the firm can sell all of its products in the home market at low exchange rate realizations without a chance for a loss. ❑

Proof of Proposition 2. Part (a): Let e^{TD} satisfy $\Pi(x^{TD} \mid e = e^{TD}) = (p-c)d_H + (pe^{TD} - c)d_F = -\beta$, i.e., $P[\Pi(x^{TD}) < -\beta] = P[\tilde{e} \le e^{TD}]$. First, suppose that $p \ge c$. Then for any $x \in [0, d_H]$, we have $\Pi(x \mid e = e^{TD}) = (p - c)x \ge 0 \ge -\beta = \Pi(x^{TD} \mid e = e^{TD})$ and for any $x \in [d_H, d_H + d_F]$, we have $\Pi(x \mid e = e^{TD}) = (p-c)d_H + (pe^{TD} - c)(x - d_H) \ge (p-c)d_H + (pe^{TD} - c)d_F = -\beta = \Pi(x^{TD} \mid e = e^{TD})$.

Now suppose that $p < c$. If $pe^{TD} \ge c$, then the preceding approach, but with the priority on the foreign market instead of the domestic market, can be used to conclude $\Pi(X \mid e = e^{TD}) \ge -\beta = \Pi(x^{TD} \mid e = e^{TD})$ for any $x \in [0, d_H + d_F]$. Alternatively, if $pe^{TD} < c$, then there are losses in both markets at $e = e^{TD}$, and it is clear that the loss under the optimal allocation of x to foreign and domestic markets is not more than $\Pi(x^{TD} \mid e = e^{TD}) = -\beta$ for any $x \in [0, d_H + d_F]$. For a given $x \in [0, d_H + d_F]$, let e^x satisfy $\Pi(x \mid e = e^x) = -\beta$, i.e., $P[\Pi(x) < -\beta] = P[\tilde{e} \le e^x]$. From $\Pi(x \mid e = e^{TD}) \ge -\beta = \Pi(x^{TD} \mid e = e^{TD})$ for any $x \in [0, d_H + d_F]$, it follows that $e^x \le e^{TD}$, which in turn implies $P[\Pi(x) < -\beta] \le P[\Pi(x^{TD}) < -\beta]$for any $x \in [0, d_H + d_F]$.

Part (b): Due to $\Pi(x^{TD} \mid e) \leq \Pi(x^{TD} \mid e = e_\alpha) \leq 0$ for all $e \leq e_\alpha$, it follows from the analysis presented in part (a) that $\Pi(x \mid e) \geq \Pi(x^{TD} \mid e)$ for any $x \in [0, d_H + d_F]$ and $e \leq e_\alpha$ (e.g., $\Pi(x^{TD} \mid e) \leq 0$ plays the role of $-\beta$ and e plays the role of e^{TD}). Therefore, $E[\Pi(x|\tilde{e} \leq e_\alpha)] \geq E[\Pi(x^{TD}|\tilde{e} \leq e_\alpha)]$ for any $x \in [0, d_H + d_F]$. ❑

Proof of Proposition 3. Part (a): Proposition 1 shows that the TD policy is optimal when the selling price is greater than or equal to max$\{c/(1-\theta), ((d_H + d_F)c - \beta)/(d_H + d_F e_\alpha)\}$. Note that as θ increases, the value of $c/(1-\theta)$ also increases. Thus, the TD policy is less likely to be optimal for greater values of θ.

Part (b): TD policy is optimal when the selling price is greater than or equal to max$\{c/(1-\theta), ((d_H + d_F)c - \beta)/(d_H + d_F e_\alpha)\}$. Note that as the firm becomes more risk-averse, the value of α and/or β decreases. As α decreases, the value of e_α also decreases correspondingly, which results to an increase for $((d_H + d_F)c - \beta)/(d_H + d_F e_\alpha)$. Also as β decreases the value of $((d_H + d_F)c - \beta)/(d_H + d_F e_\alpha)$ increases. Thus, for decreasing values of α and/or β, the interval of selling price values that satisfy max$\{c/(1-\theta), ((d_H + d_F)c - \beta)/(d_H + d_F e_\alpha)\}$ becomes smaller. As a result, with higher risk aversion, the TD policy is less likely to be optimal, and one of the production hedging policies is more likely to be optimal.

Part (c): As the unit manufacturing cost c increases, max$\{c/(1-\theta), ((d_H + d_F)c - \beta)/(d_H + d_F e_\alpha)\}$ also increases which is the threshold point where the selling price needs to be greater than or equal to for the TD policy to be optimal. Thus, the TD policy is less likely to be optimal for greater values of c. ❑

Proof of Proposition 4. Part (a): Proposition 1 shows that if $p \geq c$ and (2) (or (12)) is ignored then there are only two potentially optimal policies: Policies TD and PHX. The same applies to a price-setting firm under a risk-neutral setting.

Part (b): From the first- and second-order conditions of (7), we derive the following, respectively:

$$\partial E\left[\prod^{TD}\right] / \partial p = d_H + (p-c)d_{H_p} + d_F + (p-c)d_{F_p},$$

$$\partial^2 E\left[\prod^{TD}\right] / \partial p^2 = \left[2d_{H_p} + (p-c)d_{H_{pp}}\right] + \left[2d_{F_p} + (p-c)d_{F_{pp}}\right] \leq 0.$$

Thus, policy TD is concave in p and have a unique optimal solution. We find the optimal price by solving

$$\partial E\left[\prod^{TD}\right] / \partial p = d_H + (p-c)d_{H_p} + d_F + (p-c)d_{F_p} = 0$$

for p. The optimal production quantity expression for TD can be derived by rearranging the optimal price expression. From the first- and second-order conditions of (9) where $d_x \geq d_m$, we derive the following, respectively:

$$\partial E\left[\prod^{PHX}\right] / \partial p = d_x + (p-c)d_{x_p} + \theta\left(d_m + p d_{m_p}\right),$$

$$\partial^2 E\left[\prod^{PHX}\right] / \partial p^2 = 2d_{x_p} + (p-c)d_{x_{pp}} + \theta\left(2d_{m_p} + p d_{m_{pp}}\right) \leq 0.$$

Thus, policy PHX is concave in p and have a unique optimal solution. We find the optimal price by solving

$$\partial E\left[\prod^{PHX}\left(x = d_x \geq d_m\right)\right] / \partial p = d_x + (p-c)d_{x_p} + \theta\left(d_m + p d_{m_p}\right) = 0$$

for p. The optimal production quantity expression for PHX can be derived by rearranging the optimal price expression.

Part (c): Recall the objective functions of policies TD and PHX. Substituting the optimal price expressions that maximize in Table 3, we get the following expected profit expressions:

$$E[\Pi^{TD}(p^{TD})] = (d_H^{TD} + d_F^{TD})^2/[-(d_{Hp}^{TD} + d_{Fp}^{TD})] \tag{18}$$

$$E[\Pi^{PHX}(p^{PHX})] = [(d_x^{PHX} + \theta d_m^{PHX})^2 - cd_{xp}^{PHX}(d_x^{PHX} + \theta d_m^{PHX})]/[-(d_{xp}^{PHX} + \theta d_{mp}^{PHX})] \tag{19}$$

Expected profit expressions in (18) – (19) can be rewritten as:

$$E[\Pi^{TD}(p^{TD})] = p^{TD}(d_x^{TD}(p^{TD}) + p^{TD}d_m^{TD}(p^{TD}))/\varepsilon^{TD}(p^{TD}) \tag{20}$$

$$E[\Pi^{PHX}(p^{PHX})] = p^{PHX}(d_x^{PHX}(p^{PHX}) + \theta d_m^{PHX}(p^{PHX}))/\varepsilon^{PHX}(p^{PHX}) - c\{\theta[d_x(p^{PHX})d_{mp}(p^{PHX}) - d_{xp}(p^{PHX})d_m(p^{PHX})]/[d_{xp}(p^{PHX}) + \theta d_{mp}(p^{PHX})]\} \tag{21}$$

For the TD policy in (20), the optimal price solves

$$(p^{TD} - c)/p^{TD} = 1/\varepsilon^{TD}(p^{TD}). \tag{22}$$

For the PHX policy in (21), the optimal price solves

$$(p^{PHX} - c(d_{xp}(p^{PHX})/(d_x^{PHX}(p^{PHX}) + \theta d_m^{PHX}(p^{PHX}))))/p^{PHX} = 1/\varepsilon^{PHX}(p^{PHX}). \tag{23}$$

The following inequalities on optimal prices are not possible simultaneously: $p^{TD} < c/(1-\theta)$ *and* $p^{PHX} > c/(1-\theta)$. If $p^{TD} < c/(1-\theta)$, then from Proposition 1 we have $E[\Pi^{TD}(p^{TD})] < E[\Pi^{PHX}(p^{TD})]$. If $p^{PHX} \geq c/(1-\theta)$, then $E[\Pi^{PHX}(p^{PHX})] \leq E[\Pi^{TD}(p^{PHX})]$. But by definition $E[\Pi^{PHX}(p^{TD})] < E[\Pi^{PHX}(p^{PHX})]$ and $E[\Pi^{TD}(p^{PHX})] < E[\Pi^{TD}(p^{TD})]$, which implies

$$E[\Pi^{TD}(p^{TD})] < E[\Pi^{PHX}(p^{TD})] < E[\Pi^{PHX}(p^{PHX})] \text{ and } E[\Pi^{PHX}(p^{PHX})] < E[\Pi^{TD}(p^{PHX})] \leq E[\Pi^{TD}(p^{TD})],$$

which is a contradiction.

Part (d): Part (c) shows that $p^{TD} < c/(1-\theta)$, then from Proposition 1 we have $E[\Pi^{TD}(p^{TD})] < E[\Pi^{PHX}(p^{TD})]$. Substituting the condition in (3) into (22) at the price point of $c/(1-\theta)$ provides: $\varepsilon^{TD}(c/(1-\theta)) > 1/\theta$.

Part (e): Part (c) shows that if $p^{TD} < c/(1-\theta)$ then p^{PHX} cannot be greater than $c/(1-\theta)$ and therefore $c \leq p^{PHX} < c/(1-\theta)$. Thus, $p^{TD} < c/(1-\theta)$ is a sufficient condition for $(x^u, p^u) = (x^{PHX}, p^{PHX})$. If $p^{TD} \geq c/(1-\theta)$, then p^{PHX} needs to be less than $c/(1-\theta)$ in order to still be the optimal policy; otherwise, policy TD is the optimal policy. However, when $p^{TD} \geq c/(1-\theta)$, $p^{PHX} < c/(1-\theta)$ is not sufficient to assure that policy PHX is optimal. In this case, because both optimal prices are in their respective optimal price regions, the expected profits must be evaluated and compared. Thus, if $p^{TD} \geq c/(1-\theta)$, then $p^{PHX} < c/(1-\theta)$ and $E[\Pi(x^{TD}, p^{TD})] < E[\Pi(x^{PHX}, p^{PHX})]$ in order for PHX to be optimal, i.e., $(x^u, p^u) = (x^{PHX}, p^{PHX})$.

Part (f): Part (c) shows that if $p^{PHX} > c/(1-\theta)$ then p^{TD} cannot be less than $c/(1-\theta)$ and therefore $p^{TD} \geq c/(1-\theta)$. Thus, $p^{PHX} > c/(1-\theta)$ is a sufficient condition for $(x^u, p^u) = (x^{TD}, p^{TD})$. If $p^{PHX} < c/(1-\theta)$, then p^{TD} must be greater than $c/(1-\theta)$ in order to still be the optimal policy; otherwise, policy PHX is the optimal policy. However, when $p^{PHX} < c/(1-\theta)$, $p^{TD} \geq c/(1-\theta)$ is not sufficient to assure that policy TD is optimal. In this case, because both optimal prices are in their respective optimal price regions, the expected profits must be evaluated and compared. Thus, if $p^{PHX} < c/(1-\theta)$, then $p^{TD} \geq c/(1-\theta)$ and $E[\Pi(x^{TD}, p^{TD})] > E[\Pi(x^{PHX}, p^{PHX})]$ in order for TD to be optimal, i.e., $(x^u, p^u) = (x^{TD}, p^{TD})$.

Part (g): Table 3 shows that $p^0 = p^{TD}$ and $p^u = p^{TD}$ or $p^u = p^{PHX}$. Part (e) and (f) shows that $p^{PHX} < c/(1-\theta)$ is a necessary condition for policy PHX to be optimal, and $p^{TD} \geq c/(1-\theta)$ is a necessary condition for policy TD to be optimal. Thus, we know that $p^{PHX} \leq p^{TD}$ and this proves that $p^u \leq p^0$. ❑

Proof of Proposition 5. Table 3 provides the optimal price and production quantity for TD and PHX where (12) is ignored. We now consider (12) for the risk-averse firm. If TD is optimal from a risk-neutral perspective but (12) is violated, the firm switches to the PHA policy.

PHA policy: We find the optimal price and production quantity for PHA by solving $\Pi^{PHA}(d_x < x < d_H + d_F, e = e_\alpha) = pd_H + pe_\alpha (x - d_H) - cx = -\beta$. By rearranging the terms, we find the binding optimal price and production quantity for PHA. If PHX is optimal from a risk-neutral perspective but (12) is violated, the firm switches to the PHB policy.

PHB policy: We find the optimal price and production quantity for PHB by solving $\Pi^{PHB}(d_H < x < d_F, e = e_\alpha) = pd_H + pe_\alpha (x - d_H) - cx = -\beta$. By rearranging the terms, we find the binding optimal price and production quantity for PHB. ❑

Proof of Proposition 6. Part (a): Proposition 1 shows that the PHX policy is risk-free when $d_H \geq d_F$, which continues to hold under endogenous pricing (i.e., $P[\Pi^{PHX}(x^{PHX} = d_H \geq d_F, p^{PH}) < -\beta] = 0$). Here we derive conditions where the PHX policy continues to be risk-free when $d_F \geq d_H$ by showing $e^{PHX} \leq 0 \leq e_l$ where e^{PHX} satisfies $\Pi^{PHX}(x^{PHX}=d_F \geq d_H, p^{PHX} \mid e = e^{PHX}) = -\beta$. We derive e^{PHX} by solving

$$\Pi^{PHX}(x^{PHX}=d_F \geq d_H, p^{PHX} \mid e = e^{PHX}) = -cd_F^{PHX} + p^{PHX} d_H^{PHX} + p^{PHX}e^{PHX}(d_F^{PHX} - d_H^{PHX}) = -\beta.$$

Thus, $e^{PHX} = (cd_F^{PHX} - p^{PHX} d_H^{PHX} - \beta) / (p^{PHX}(d_F^{PHX} - d_H^{PHX}))$. The numerator of e^{PHX} shows that if $p^{PHX}(d_H^{PHX}/d_F^{PHX}) \geq c$, then $e^{PHX} \leq 0$ for any given loss β where $\beta \geq 0$.

Part (b): Suppose $\Pi^{TD}(x^{TD}, p^{TD} \mid e = e_\alpha) < -\beta$. We know by definition that $\Pi^{PHA}(x^{PHA}, p^{PHA} \mid e = e_\alpha) = \Pi^{PHB}(x^{PHB}, p^{PHB} \mid e = e_\alpha) = -\beta$. Thus, $P[\Pi^j(x^j, p^j) < -\beta] \leq P[\Pi^{TD}(x^{TD}, p^{TD}) < -\beta]$ for j = PHA, PHB.

Part (c): Suppose that VaR constraint is violated for the TD policy and the firm switches to the PHA policy. The profits for each policy where $e = e_\alpha$ are as follows:

$$\Pi^{TD}(x^{TD}, p^{TD} \mid e = e_\alpha) = -c(d_H^{TD} + d_F^{TD}) + p^{TD} d_H^{TD} + p^{TD}e_\alpha d_F^{TD} < -\beta$$

$$\Pi^{PHA}(x^{PHA}, p^{PHA} \mid e = e_\alpha) = -cx^{PHA} + p^{PHA}d_H^{PHA} + p^{PHA}e_\alpha(x^{PHA} - d_H^{PHA}) = -\beta.$$

Now we evaluate the profits for each policy where $e = e_\alpha - \Delta$ for any $\Delta \in (0, e_\alpha]$

$$\Pi^{TD}(x^{TD}, p^{TD} \mid e = e_\alpha - \Delta) = \Pi^{TD}(x^{TD}, p^{TD} \mid e = e_\alpha) - \Delta\, p^{TD}d_F^{TD} < -\beta - \Delta\, p^{TD}d_F^{TD}.$$

$\Pi^{PHA}(x^{PHA}, p^{PHA} \mid e = e_\alpha - \Delta) = \Pi^{PHA}(x^{PHA}, p^{PHA} \mid e = e_\alpha) - \Delta\, p^{PHA}(x^{PHA} - d_H^{PHA}) = -\beta - \Delta\, p^{PHA}(x^{PHA} - d_H^{PHA})$.

Thus, a sufficient condition for $E[\Pi^{PHA}(x^{PHA}, p^{PHA}) \mid \tilde{e} \le e_\alpha] \ge E[\Pi^{TD}(x^{TD}, p^{TD}) \mid \tilde{e} \le e_\alpha]$ is

$$p^{TD} d_H(p^{TD}) > p^{PHA}(x^{PHA} - d_H(p^{PHA})).$$

Note that $(p^{TD} - c)[d_H(p^{TD}) + d_F(p^{TD})]$ is the maximum unconstrained profit, maximized, and thus

$$\begin{aligned}(p^{TD} - c)[d_H(p^{TD}) + d_F(p^{TD})] &> (p^{PHA} - c)[d_H(p^{PHA}) + d_F(p^{PHA})] \\ &> (p^{PHA} - c)[d_H(p^{PHA}) + (x^{PHA} - d_H(p^{PHA}))] \\ &= (1 - e_\alpha)p^{PHA}(x^{PHA} - d_H(p^{PHA})) - \beta.\end{aligned}$$

We can also note that

$$(p^{TD} - c)[d_H(p^{TD}) + d_F(p^{TD})] < (1 - e_\alpha)p^{TD} d_F(p^{TD}) - \beta.$$

Therefore

$$(1 - e_\alpha)p^{TD} d_F(p^{TD}) - \beta > (1 - e_\alpha)p^{PHA}(x^{PHA} - d_H(p^{PHA})) - \beta,$$

which, due to $e_\alpha < 1$, yields

$$p^{TD} d_F(p^{TD}) > p^{PHA}(x^{PHA} - d_H(p^{PHA})). \quad \square$$

Proof of Proposition 7. We first present the proof for part (b): The VaR constraint under policy TD will be violated when the realized loss at the realized exchange rate $e = e_\alpha$ exceeds β, and the firm will engage in financial hedging when $-c(d_H + d_F) + pd_H + pd_F e_\alpha = \pi(x^{TD} = d_H + d_F, h = 0, e_\alpha) < -\beta$. Solving for h provides the optimal number of contracts necessary to satisfy the VaR constraint: $h^{TD} = [-\pi(x^{TD} = d_H + d_F, h = 0, e_\alpha) - \beta]/(e_s - e_\alpha - c_h)$. The VaR constraint under policy PHX will be satisfied when $d_H \ge d_F$, and can be violated when $d_F > d_H$. Thus, we consider only the case when $d_F > d_H$. The VaR constraint is violated under policy PHX when the realized loss at the realized exchange rate $e = e_\alpha$ exceeds β, and the firm will engage in financial hedging when $-cd_F + pd_H + p(d_F - d_H)e_\alpha = \pi(x^{PHX} = d_F, h = 0, e_\alpha) < -\beta$. Solving for h provides the optimal number of contracts necessary to satisfy the VaR constraint: $h^{PHX} = [-\pi(x^{PHX} = d_F, h = 0, e_\alpha) - \beta]/(e_s - e_\alpha - c_h)$.

Part (a): It is shown above that when $h = h^{TD}$, the TD policy satisfies the VaR constraint. Thus, it is sufficient to evaluate the expected profit at a strike price equal to the exchange rate.

$$\begin{aligned}E\left[\Pi\left(x^{TD}, h^{TD}\right)\right] &= -c\left(d_H + d_F\right) - c_h h^{TD} + \int_{e_l}^{e_s}\left[h^{TD}\left(e_s - e\right) + pd_H + ped_F\right]f\left(e\right)de + \int_{e_s}^{e_h}\left[pd_H + ped_F\right]f\left(e\right)de \\ &= \left(p - c\right)d_H + \left(p\bar{e} - c\right)d_F = E\left[\Pi\left(x^{TD}, h = 0\right)\right] \ge E\left[\Pi\left(x^{PHI}, h = 0\right)\right].\end{aligned}$$

Thus, the TD policy with $h = h^{TD}$ dominates the PHA policy. The PHX policy is only at risk when $d_F \ge d_H$, so we prove only the case when $x^{PHX} = d_F$. It is already shown that when $h = h^{PHX}$, the PHX policy satisfies the VaR constraint. Thus, it is sufficient to evaluate the expected profit at a strike price equal to the exchange rate.

$$E\left[\Pi\left(x^{PHX}=d_F,h^{PHX}\right)\right]=-cd_F-c_h h^{PHX}+\begin{Bmatrix}\int_{e_l}^{e_s}\left[h^{PHX}\left(e_s-e\right)+pd_H+pe\left(d_F-d_H\right)\right]f\left(e\right)de\\+\int_{e_s}^{\bar{e}}\left[pd_H+pe\left(d_F-d_H\right)\right]f\left(e\right)de+\int_{\bar{e}}^{e_h}\left[ped_F\right]f\left(e\right)de\end{Bmatrix}$$

$$=pd_H\theta+\left(p\bar{e}-c\right)d_F=E\left[\Pi\left(x^{PHX},h=0\right)\right]\geq E\left[\Pi\left(x^{PHB},h=0\right)\right].$$

Thus, the PHX policy with $h = h^{PHX}$ dominates the PHB policy. ❑

Financial Hedging under CVaR Risk Measure

We continue to consider the case when $e_\alpha < 1$. We examine the setting when risk aversion is described with a CVaR risk measure using the following constraint (replacing (12)):

$$E[\Pi(x, p, h) \mid e \leq e_\alpha] \geq -\beta. \tag{24}$$

Because c_h was defined as in (15), it is important to observe that for a given pair of (x, p) decisions we have

$$E[\Pi(x, p, 0)] = E[\Pi(x, p, h)] \text{ for all } h > 0.$$

We denote the optimal number of financial hedging instruments, denoted h^{CVaR}, that a firm should commit for a pair of (x, p) decisions under (24). Consider the event that a pair of (x, p) decisions violates (25). It is easy to see that the optimal number of financial hedging instruments under (24) for this pair of (x, p) decisions satisfies $E[\Pi(x, p, h^{CVaR}) \mid e \leq e_\alpha] = -\beta$.

Proposition A2. (a) *For a given α and β, the optimal financial hedging quantity h^* that satisfies the VaR constraint cannot satisfy* (24). (b) *For a given α and β, the optimal number of financial hedging instruments acquired under* (24) *is greater than that under the VaR constraint in* (12), *i.e.,* $h^{CVaR} > h^*$.

Proof of Proposition A2. (a) Proposition 7(a) shows that when $e = e_\alpha$ and $h = h^*$, the profit equals to $-\beta$ and therefore satisfies the VaR constraint; i.e., $\Pi(x, p, h = h^* \mid e = e_\alpha) = -\beta$. This implies that the profit when $e < e_\alpha$ is less than $-\beta$; i.e., $\Pi(x, p, h = h^* \mid e < e_\alpha) < -\beta$. Thus, a CVaR constraint using the same risk parameter α and β, $E[\Pi(x, p, h \mid e \leq e_\alpha) \geq -\beta$, cannot be satisfied when $h = h^*$; i.e., $E[\Pi(x, p, h = h^* \mid e \leq e_\alpha) < -\beta$. (b) It follows from part (a) that the firm needs additional number of hedging instruments under (24) in order to compensate for the realized profits that are less than $-\beta$ for the realized values of exchange rate in the region $e < e_\alpha$. Thus, $h^{CVaR} > h^*$. ❑

Proof of Proposition 8. Part (a): When $(x^u, p^u) = (x^{PHX}, p^{PHX})$ but violates (12), Proposition 7 shows that the firm does not switch to the PHB policy as it is no longer an optimal production policy candidate in the presence of financial hedging given that $\delta = 0$. Thus, when $(x^u, p^u) = (x^{PHX}, p^{PHX})$ but violates (12), the firm will use financial hedging along with production hedging and keep the same optimal price and production quantity as (x^u, p^u), i.e., (x^r, p^r) = (x^{PHX}, p^{PHX}). Thus, when (x^u, p^u) = (x^{PHX}, p^{PHX}), financial hedging and production hedging are complements.

Part (b): When (x^u, p^u) = (x^{TD}, p^{TD}) but violates (12), Proposition 7 shows that the firm does not switch to the PHA policy as it is no longer an optimal production policy candidate in the presence of financial hedging given that δ = 0. Thus, when (x^u, p^u) = (x^{TD}, p^{TD}) but violates (12), the firm will use financial hedging and continue with the optimal price and quantity under TD policy, i.e., (x^r, p^r) = (x^{TD}, p^{TD}). Thus, when (x^u, p^u) = (x^{TD}, p^{TD}), financial hedging and production hedging are substitutes. ❑

Proof of Proposition 9. Part (a): The newsvendor ratio is (p – c)/p, which is equal to 1 – (c/p) and is increasing with respect to p. Proposition 1 shows that the maximum optimal price under production hedging is $p = c/(1-\theta)$. Thus, by inserting this maximum price into the newsvendor ratio: $1 - [c/(c/(1-\theta))] = \theta$. Thus, the newsvendor ratio under production hedging policies is less than or equal to θ.

Part (b): Let us begin our analysis with the deterministic demand case and examine the condition that ensures that policy TD is not optimal; alternatively said, when this condition does not hold, we know that PHX is optimal. This can be seen from the following expected profit expression for

$d_H\left(\bar{z}\right) = d_F\left(\bar{z}\right) < x < d_H\left(\bar{z}\right) + d_F\left(\bar{z}\right)$:

$$E\left[\Pi\left(x\right)\right] = -cx + p\int_{e_l}^{1}\left[d_H\left(\bar{z}\right) + e\left(x - d_H\left(\bar{z}\right)\right)\right]f\left(e\right)de + p\int_{1}^{e_h}\left[ed_F\left(\bar{z}\right) + \left(x - d_F\left(\bar{z}\right)\right)\right]f\left(e\right)de .$$

We know that policy TD is not optimal when

$$\frac{\partial E\left[\Pi\left(x\right)\right]}{\partial x} = -c + p\left(\int_{e_l}^{1} ef\left(e\right)de + \int_{1}^{e_h} f\left(e\right)de\right) = -c + p\left(1-\theta\right) = K < 0 .$$

Combined with $p > c$, we know that PHX is the optimal policy, and $x^* = d_H\left(\bar{z}\right) = d_F\left(\bar{z}\right)$ under deterministic demand.

We next examine the same problem under stochastic demand using i. i. d. random variables. Our proof will show that the derivative of the expected profit under stochastic demand evaluated at $x^D = d_H\left(\bar{z}\right) = d_F\left(\bar{z}\right)$. Let us first define two exchange-rate points: $e_1(x)$ is the point where the firm's second-stage profit from the first unit allocated to the foreign market is equal to the next unit allocated to the domestic market, and $e_2(x)$ is the point where the firm's second-stage profit from the first unit allocated to the foreign market is equal to the next unit allocated to the domestic market. Thus, at $x = x^D$,

$$e_1(x^D):\ \frac{\partial\pi_2\left(y_H = x^D, y_F = 0\right)}{\partial y_H} = p\left[1 - G_H\left(\bar{z}\right)\right] = \left.\frac{\partial\pi_2\left(y_H = x^D, y_F = 0\right)}{\partial y_F}\right|_{y_F=0} = pe \tag{26}$$

$$e_2(x^D):\ \frac{\partial\pi_2\left(y_H = 0, y_F = x^D\right)}{\partial y_F} = pe\left[1 - G_F\left(\bar{z}\right)\right] = \left.\frac{\partial\pi_2\left(y_H = 0, y_F = x^D\right)}{\partial y_H}\right|_{y_H=0} = p . \tag{27}$$

It can be seen from $G_H(\bar{z}) = G_F(\bar{z}) = ½$ that $e_1(x^D) = ½$ and $e_2(x^D) = 2$.

Let us define the second-stage objective function in each region as $\pi_{2R1}(y_H, y_F \mid e \in R1)$, $\pi_{2R2}(y_H, y_F \mid e \in R2)$, and $\pi_{2R3}(y_H, y_F \mid e \in R3)$. We know from the definition of these threshold points that

$$\pi_{2R1}(y_H, y_F \mid e = e_1(x)) = \pi_{2R2}(y_H, y_F \mid e = e_1(x)), \text{ and} \tag{28}$$

$$\pi_{2R2}(y_H, y_F \mid e = e_2(x)) = \pi_{2R3}(y_H, y_F \mid e = e_2(x)). \tag{29}$$

Let us define $z(x)$ as the z value that makes $x = d_H(z_H) = d_F(z_F) = d(z)$. For our x^D, we have $z(x^D) = \bar{z}$. In region 2, for a given e, we have $y_H^*(x^D, e)$ that bring the same second-stage expected return from the domestic and foreign markets; thus,

$$p[1 - G_H(z_H(y_H^*(x^D, e))] = pe[1 - G_H(z(x^D) - z_H(y_H^*(x^D, e))]. \tag{30}$$

The expected profit function can be written as follows:

$$\begin{aligned} E\left[\Pi(x)\right] = &-cx + \int_{e_l}^{e_1(x)} p\left\{\int_{z_{Hl}}^{z(x)} d_H(z_H) g_H(z_H) dz_H + \int_{z(x)}^{z_{Hh}} x g_H(z_H) dz_H\right\} f(e) de \\ &+ \int_{e_1(x)}^{e_2(x)} p\left\{\begin{array}{l} \int_{z_{Hl}}^{z_H(y_H^*(x,e))} d_H(z_H) g_H(z_H) dz_H + \int_{z_H(y_H^*(x,e))}^{z_{Hh}} y_H^*(x,e) g_H(z_H) dz_H \\ +e\left(\int_{z_{Fl}}^{z(x)-z_H(y_H^*(x,e))} d_F(z_F) g_F(z_F) dz_F + \int_{z(x)-z_H(y_H^*(x,e))}^{z_{Fh}} \left(x - y_H^*(x,e)\right) g_F(z_F) dz_F\right)\end{array}\right\} f(e) de. \\ &+ \int_{e_2(x)}^{e_h} pe\left\{\int_{z_{Fl}}^{z(x)} d_F(z_F) g_F(z_F) dz_F + \int_{z(x)}^{z_{Fh}} x g_F(z_F) dz_F\right\} f(e) de \end{aligned}$$

Due to (26), (27), (28), (29), and (30), the marginal expected profit evaluated at $x = x^D$ can be written as follows:

$$\begin{aligned} \frac{\partial E\left[\Pi(x^D)\right]}{\partial x} = &-c + \int_{e_l}^{e_1(x^D)} p\left[1 - G_H(\bar{z})\right] f(e) de + \int_{e_1(x^D)}^{e_2(x^D)} pe\left[1 - G_H\left(\bar{z} - z_H\left(y_H^*\right)\right)\right] f(e) de \\ &+ \int_{e_2(x^D)}^{e_h} pe\left[1 - G_F(\bar{z})\right] f(e) de \end{aligned}.$$

We add and subtract $\int_{e_l}^{1} pef(e)de$ and $\int_{1}^{e_h} pf(e)de$, and rewrite the above first-order condition as follows:

$$\begin{aligned} \frac{\partial E\left[\Pi(x^D)\right]}{\partial x} = &K + p\int_{e_l}^{e_1(x)} \left[\left[1 - G_H(\bar{z})\right] - e\right] f(e) de + p\int_{e_1(x)}^{1} \left[e\left[1 - G_H\left(\bar{z} - z_H\left(y_H^*\right)\right)\right] - e\right] f(e) de \\ &+ p\int_{1}^{e_2(x)} \left[e\left[1 - G_H\left(\bar{z} - z_H\left(y_H^*\right)\right)\right] - 1\right] f(e) de + p\int_{e_2(x)}^{e_h} \left[e\left[1 - G_F(\bar{z})\right] - 1\right] f(e) de \end{aligned}.$$

We know that $G_H\left(\bar{z}\right) = G_F\left(\bar{z}\right)$ = ½, and $1 - G_H\left(\bar{z} - z_H\left(y_H^*\right)\right) \geq 1 - G_H\left(\bar{z}\right)$ = ½. Therefore,

$$\frac{\partial E\left[\Pi\left(x^D\right)\right]}{\partial x} \leq K + p\int_{e_l}^{1/2}\left[\frac{1}{2} - e\right] f\left(e\right)de + p\int_{1/2}^{1}\left[e\frac{1}{2} - e\right] f\left(e\right)de + p\int_{1}^{e_h}\left[e\frac{1}{2} - 1\right] f\left(e\right)de.$$

Adding and subtracting $p\int_{1/2}^{e_h}\left[\frac{1}{2} - e\right] f\left(e\right)de$ to the right hand side of the above inequality provides

$$\frac{\partial E\left[\Pi\left(x^D\right)\right]}{\partial x} \leq K + p\left[-\frac{1}{2} + \frac{1}{2}\int_{1/2}^{1}\left[e - 1\right] f\left(e\right)de + \frac{3}{2}\theta\right].$$

We already know $K \leq 0$, and $\frac{1}{2}\int_{1/2}^{1}\left[e - 1\right] f\left(e\right)de \leq 0$, and therefore, the first-order condition is non-positive when $-\frac{1}{2} + \frac{3}{2}\theta \leq 0$, or when $\theta \leq 1/3$. Thus, as a consequence, when $\theta \leq 1/3$, the optimal production amount under stochastic demand is less than that under deterministic demand. ❑

Lemma A1. (a) *For a symmetric continuous distribution around the mean of* 1, $\theta < 0.50$; (b) *For a uniform distribution on a support* [0, 2], $\theta = 0.25$; (c) *For a distribution centered around the mean of* 1 *on the same support* [0, 2], $\theta < 0.25$.

Proof of Lemma A1. We will use definition of $\theta = \int_{1}^{e_h} ef\left(e\right)de - \int_{1}^{e_h} f\left(e\right)de$ in the proof. (a) For a symmetric distribution around the mean of 1, $\int_{1}^{e_h} f\left(e\right)de = 0.50$. Observe that $\int_{1}^{e_h} ef\left(e\right)de < \int_{e_l}^{e_h} ef\left(e\right)de = \bar{e} = 1$. Thus, $\theta = \int_{1}^{e_h} ef\left(e\right)de - \int_{1}^{e_h} f\left(e\right)de < 1 - 0.50 = 0.50$. (b) For a the uniform distribution, $\theta = \int_{1}^{2} ef\left(e\right)de - \int_{1}^{2} f\left(e\right)de = \frac{e^2}{4}\Big|_1^2 - 0.5 = [1 - 0.25] - 0.50 = 0.25$. (c) Using the definition of θ, one can interpret it as the sum of $(e - 1)$ values multiplied by the value of pdf $f(e)$. Consider points e_1 and e_2 from the uniform distribution on the support of [0, 2], where $e_2 > e_1 > 1$. On a uniform distribution $f(e_2) = f(e_1)$, and from part (b), we know that $\theta = 0.25$. We describe the pdf of the distribution that is centered around the mean with the same support of [0, 2] with $f_c(e)$, and its value of θ with θ_c. For a distribution that is centered around the mean on the same support, $f_c(e_2) < f(e_2) = f(e_1) < f_c(e_1)$. Therefore, we have $[e_2 \times f_c(e_2) - e_1 \times f_c(e_1)] < [e_2 \times f(e_2) - e_1 \times f(e_1)]$. Thus, $\theta_c = \int_{1}^{e_h} ef_c\left(e\right)de - \int_{1}^{e_h} f_c\left(e\right)de < \theta = \int_{1}^{e_h} ef\left(e\right)de - \int_{1}^{e_h} f\left(e\right)de = 0.25$. ❑

Proof of Proposition 10. We begin our proof with the deterministic demand case. We describe demand in each market as $d_H(p_H, \tilde{z}_H) = d_H(p_H, \tilde{z}_F) = d(p, \tilde{z}) = a - bp + \tilde{z}$. Under deterministic demand, when production hedging is optimal, the firm produces $x^D = d(p, \bar{z})$, and the expected profit function under deterministic demand can be expressed in terms of price:

$$E\left[\Pi\left(p,\bar{z}\right)\right] = -cd\left(p,\bar{z}\right) + p\int_{e_l}^{1}\left[d\left(p,\bar{z}\right)\right]f\left(e\right)de + p\int_{1}^{e_h}\left[ed\left(p,\bar{z}\right)\right]f\left(e\right)de .$$

From Proposition 4, we know that the above expected profit expression is concave in p, and the first-order condition provides the optimal price.

$$\begin{aligned}\frac{\partial E\left[\Pi\left(p,\bar{z}\right)\right]}{\partial p} &= \left(-b\right)\left[-c + p\int_{e_l}^{1} f\left(e\right)de + p\int_{1}^{e_h} ef\left(e\right)de\right] + \left[d\left(p,\bar{z}\right)\right]\left(\int_{e_l}^{1} f\left(e\right)de + \int_{1}^{e_h} ef\left(e\right)de\right). \\ &= \left(-b\right)\left[-c + p\left(1+\theta\right)\right] + \left[a - bp + \bar{z}\right]\left(1+\theta\right)\end{aligned}$$

Equating the above first-order condition to zero provides the optimal price: $p^D = \dfrac{a + b\left(\dfrac{c}{1+\theta}\right) + \bar{z}}{2b}$. We also use the condition that equates the first-order condition to zero in our proof; thus, we have

$$\left(-b\right)\left[-c + p\left(1+\theta\right)\right] + \left[a - bp + \bar{z}\right]\left(1+\theta\right) = 0.$$

We next consider the stochastic demand case where the firm continues to follow production hedging. We know from Proposition 9 that, at any price where production hedging is optimal, the firm will manufacture a production amount that is less than the production quantity with deterministic demand, and thus $x(p) = d\left(p,\hat{z}\right) \le d\left(p,\bar{z}\right)$ and $\hat{z}\left(p\right) \le \bar{z}$ for all $p \le c/(1-\theta)$. For this proof, we write the exchange rate functions that delineate regions $R1$, $R2$, and $R3$ in terms of price rather than quantity. We express price p that satisfies $x^D = d\left(p,\bar{z}\right)$ with p^D.

We again make the same observations as in the proof of Proposition 9 regarding the marginal returns from each unit allocated to the domestic market versus the foreign market, at $x = d\left(p,\hat{z}\right)$.

$$e_1(p):\ \frac{\partial \pi_2\left(y_H = d\left(p,\hat{z}\right), y_F = 0\right)}{\partial y_H}\frac{\partial y_H\left(p\right)}{\partial p} = \frac{\partial \pi_2\left(y_H = d\left(p,\hat{z}\right), y_F = 0\right)}{\partial y_F}\frac{\partial y_F\left(p\right)}{\partial p}\Bigg|_{y_F=0} \tag{31}$$

$$e_2(p):\ \frac{\partial \pi_2\left(y_H = 0, y_F = d\left(p,\hat{z}\right)\right)}{\partial y_F}\frac{\partial y_F\left(p\right)}{\partial p} = \frac{\partial \pi_2\left(y_H = 0, y_F = d\left(p,\hat{z}\right)\right)}{\partial y_H}\frac{\partial y_H\left(p\right)}{\partial p}\Bigg|_{y_H=0}. \tag{32}$$

We define the second-stage objective function in each region as $\pi_{2R1}(y_H, y_F \mid e \in R1)$, $\pi_{2R2}(y_H, y_F \mid e \in R2)$, and $\pi_{2R3}(y_H, y_F \mid e \in R3)$. We know from the definition of these threshold points that

$$\pi_{2R1}(y_H, y_F \mid e = e_1(p)) = \pi_{2R2}(y_H, y_F \mid e = e_1(p)), \text{ and} \tag{33}$$

$\pi_{2R2}(y_H, y_F \mid e = e_2(p)) = \pi_{2R3}(y_H, y_F \mid e = e_2(p)).$ (34)

In region $R2$, for a given e, we have $y_H^*(p, e)$ that bring the same second-stage expected return from the domestic and foreign markets, and we describe the z value corresponding to $y_H^*(p, e)$ with $z(y_H^*(p, e))$. We have

$$\frac{\partial \pi_2\left(y_H = d\left(p, z\left(y_H^*\left(p,e\right)\right)\right), y_F\right)}{\partial y_H}\frac{\partial y_H\left(p\right)}{\partial p} = \frac{\partial \pi_2\left(y_H = d\left(p, z\left(y_H^*\left(p,e\right)\right)\right), y_F\right)}{\partial y_F}\frac{\partial y_F\left(p\right)}{\partial p}. \tag{35}$$

The expected profit function can be written in terms of price p and stocking factor $\hat{z}$:

$$\begin{aligned} E\left[\Pi\left(p,\hat{z}\right)\right] = & -cd\left(p,\hat{z}\right) + \int_{e_l}^{e_1(p)} p\left\{\int_{z_l}^{\hat{z}(p)} d\left(p,z\right)g\left(z\right)dz + \int_{\hat{z}(p)}^{z_h} d\left(p,\hat{z}\right)g\left(z\right)dz\right\} f\left(e\right)de \\ & + \int_{e_1(p)}^{e_2(p)} p\left\{\begin{array}{l} \displaystyle\int_{z_l}^{z\left(y_H^*(p,e)\right)} d\left(p,z\right)g\left(z\right)dz + \int_{z\left(y_H^*(p,e)\right)}^{z_h} d\left(p, z\left(y_H^*\left(p,e\right)\right)\right)g\left(z\right)dz \\ +e\left(\begin{array}{l} \displaystyle\int_{z_l}^{\hat{z}(p)-z\left(y_H^*(p,e)\right)} d\left(p,z\right)g\left(z\right)dz \\ \displaystyle+\int_{\hat{z}(p)-z\left(y_H^*(p,e)\right)}^{z_h} \left(d\left(p,\hat{z}\right) - d\left(p, z\left(y_H^*\left(p,e\right)\right)\right)\right)g\left(z\right)dz \end{array}\right) \end{array}\right\} f\left(e\right)de. \\ & + \int_{e_2(p)}^{e_h} pe\left\{\int_{z_l}^{\hat{z}(p)} d\left(p,z\right)g\left(z\right)dz + \int_{\hat{z}(p)}^{z_h} d\left(p,\hat{z}\right)g\left(z\right)dz\right\} f\left(e\right)de \end{aligned}$$

Let (p^*, z^*) be the optimal solution to the stochastic problem under production hedging, i.e., $\frac{\partial E\left[\Pi\left(p^*,z^*\right)\right]}{\partial p} = \frac{\partial E\left[\Pi\left(p^*,z^*\right)\right]}{\partial z} = 0$. Therefore, given that $p^* \le c/(1 - \theta)$, when $z^* = \hat{z}\left(p^*\right) \le \overline{z}$, then we can show that $\frac{\partial E\left[\Pi\left(p^D,z^*\right)\right]}{\partial p} \le 0$, which implies $p^* \le p^D$.

Due to (31), (32), (33), (34), and (35), the marginal expected profit evaluated at $p = p^D$ can be written as follows:

$$\frac{\partial E\left[\Pi\left(p^D\right)\right]}{\partial p}=(-b)\left[-c+p^D\int_{e_l}^{e_1\left(p^D\right)} f(e)de+p^D\int_{e_1\left(p^D\right)}^{e_2\left(p^D\right)} ef(e)de+p^D\int_{e_2\left(p^D\right)}^{e_h} ef(e)de\right]$$

$$+\left[\begin{array}{l}\displaystyle\int_{e_l}^{e_1\left(p^D\right)}\left\{\int_{z_l}^{\bar z} d\left(p^D,z\right)g(z)dz+\int_{\bar z}^{z_h} d\left(p^D,\bar z\right)g(z)dz\right\}f(e)de\\ \displaystyle+\int_{e_1\left(p^D\right)}^{e_2\left(p^D\right)}\left\{\begin{array}{l}\displaystyle\int_{z_l}^{z\left(y_H^*\left(p^D,e\right)\right)} d\left(p^D,z\right)g(z)dz+\int_{z\left(y_H^*\left(p^D,e\right)\right)}^{z_h} d\left(p^D,z\left(y_H^*\left(p^D,e\right)\right)\right)g(z)dz\\ \displaystyle+e\left(\int_{z_l}^{\bar z-z\left(y_H^*\left(p^D,e\right)\right)} d\left(p^D,z\right)g(z)dz+\int_{\bar z-z\left(y_H^*(p,e)\right)}^{z_h}\left(d\left(p^D,\bar z\right)-d\left(p^D,z\left(y_H^*\left(p^D,e\right)\right)\right)\right)g(z)dz\right)\end{array}\right\}f(e)de\\ \displaystyle+\int_{e_2\left(p^D\right)}^{e_h} e\left\{\int_{z_l}^{\bar z} d\left(p^D,z\right)g(z)dz+\int_{\bar z}^{z_h} d\left(p^D,\bar z\right)g(z)dz\right\}f(e)de\end{array}\right]$$

Observe that

$$\left[\begin{array}{l}\displaystyle\int_{e_l}^{e_1\left(p^D\right)}\left\{\int_{z_l}^{\bar z} d\left(p^D,z\right)g(z)dz+\int_{\bar z}^{z_h} d\left(p^D,\bar z\right)g(z)dz\right\}f(e)de\\ \displaystyle+\int_{e_1\left(p^D\right)}^{e_2\left(p^D\right)}\left\{\begin{array}{l}\displaystyle\int_{z_l}^{z\left(y_H^*\left(p^D,e\right)\right)} d\left(p^D,z\right)g(z)dz+\int_{z\left(y_H^*\left(p^D,e\right)\right)}^{z_h} d\left(p^D,z\left(y_H^*\left(p^D,e\right)\right)\right)g(z)dz\\ \displaystyle+e\left(\begin{array}{l}\displaystyle\int_{z_l}^{\bar z-z\left(y_H^*\left(p^D,e\right)\right)} d\left(p^D,z\right)g(z)dz\\ \displaystyle+\int_{\bar z-z\left(y_H^*(p,e)\right)}^{z_h}\left(d\left(p^D,\bar z\right)-d\left(p^D,z\left(y_H^*\left(p^D,e\right)\right)\right)\right)g(z)dz\end{array}\right)\end{array}\right\}f(e)de\\ \displaystyle+\int_{e_2\left(p^D\right)}^{e_h} e\left\{\int_{z_l}^{\bar z} d\left(p^D,z\right)g(z)dz+\int_{\bar z}^{z_h} d\left(p^D,\bar z\right)g(z)dz\right\}f(e)de\end{array}\right]$$

$$\leq d\left(p^D,\bar z\right)\left(\int_{e_l}^{1} f(e)de+\int_{1}^{e_h} ef(e)de\right)=d\left(p^D,\bar z\right)(1+\theta)$$

and

$$\int_{e_l}^{e_1\left(p^D\right)} f(e)de+\int_{e_1\left(p^D\right)}^{e_2\left(p^D\right)} ef(e)de+\int_{e_2\left(p^D\right)}^{e_h} ef(e)de\leq\int_{e_l}^{1} f(e)de+\int_{1}^{e_h} ef(e)de=(1+\theta),$$

therefore, the following inequality holds:

$$\frac{\partial E\left[\Pi\left(p^D\right)\right]}{\partial p}\Big|_{p=p^D} \le (-b)\left[-c+p^D\left(1+\theta\right)\right]+d\left(p^D,\bar{z}\right)\left(1+\theta\right)=0\,.$$

Thus, as a consequence, when production hedging is optimal in the deterministic demand setting, the optimal price under stochastic demand is less than that under deterministic demand. ❑

Appendix B – Examples and Potential Extensions

This section shows our findings are robust under modeling extensions. Before proceeding with our analysis, let us first introduce an example that demonstrates that production hedging leads to a smaller risk exposure than the TD policy.

Example B1. *A firm manufactures a product at* c = $95/*unit in the US. The product can sell in the US for* $100/*unit and in Europe for* €100/*unit*. *The* €*-to-*$ *exchange-rate* ($/€) *can take three scenarios:* € *depreciates to* $0.60/€ *with probability* 0.30, *takes on its expected value* $1.00/€ *with probability* 0.40, *and appreciates to* $1.40/€ *with probability* 0.30. *The firm makes a sure profit of* $5/*unit from sales in the US and an expected profit of* $5/*unit from sales in Europe.*

The demand values (*at* $100 *and* €100 *prices*) *are* d_H = 5 *units in the US and* d_F = 5 *units in Europe. If the firm manufactures the total demand,* x^{TD} = 10, *it generates* $25 (= $5/*unit* × 5 *units of demand*) *of profit in the US*, $25 (= €5/*unit* × $1/€ × 5 *units of demand*) *of expected profit in Europe, and the combined expected profit from following policy TD is*

$E[\Pi^{TD}(x = 10)]$ = – $95(10) + ($100×5 + €100×$0.60/€ × 5)(0.30) + ($100×5 + €100×$1.00/€×5)(0.40) + ($100×5 + €100×$1.40/€×5)(0.30) = $50.

However, the firm can generate a higher expected profit by implementing a production hedging policy. Note that PHX and PHB converge to the same policy where $x^{PHX} = x^{PHB} = 5$ *units*.

$E[\Pi^{PHX}(x = 5)]$ = – $95(5) + ($100×5)(0.30) + ($100×5)(0.40) + (€100×$1.40/€×5)(0.30) = $85.

Next, we introduce the risk perspective and the VaR constraint. Consider the case that the same firm's managers would not want to lose more than $100, 20% *of the time, i.e.,* $\beta = 100$, $\alpha = 0.20$. *i.e.,* $P[\Pi(x) < -100] < 0.20$. *Let us evaluate the losses when* € *depreciates.*

$\Pi^{TD}(x = 10 \,|\, e = \$0.6/€)$ = – $95(10) + ($100×5 + €100×$0.60/€×5) = – $150.

This would violate the VaR constraint as the firm loses $150 (> β = 100) *with a* 30% *chance* (> α = 0.20).

Next, consider the same risk concern and the VaR constraint under the production hedging policy.

$\Pi^{PHX}(x = 5 \,|\, e = \$0.6/€)$ = – $95 (5) + ($100×5) = $25.

The firm does not lose any money under production hedging and satisfies the VaR constraint.

B.1. Impact of Violating Anti-Dumping Law with Different Prices

What if the firm does not comply with the anti-dumping law restriction in Article VI.1.(a) with equal values of p_H and p_F (= $p_F\bar{e}$)? What happens when p_F is not equal to p_H? The next example shows that production hedging policies continue to be prevalent under unequal prices.

Example B2. *We consider a firm that manufactures a product at* c = $60/*unit in the US, and sells it in the US and Europe. The demand is described as* $d_H(p) = 130 - 1.5p$ *in the US and as* $d_F(p) = 100 - p$ *in Europe. The* €-*to*-$ *exchange rate* ($/€) *follows a uniform distribution on a support of* [0, 2].

For the risk-neutral firm (*i.e.,* $\beta = \infty$), *the optimal policy choice is* PHX *with optimal selling prices and manufacturing quantities of* $p_H = 69$, $p_F = 73.5$ *and* $x = 26.5$, *generating* $E[\Pi^{PHX}(x = 26.5, p_H = 69, p_F = 73.5)]$ = $813.2.

Note that there is no need to switch to another policy or purchase financial hedging contracts even if the firm becomes extremely risk-averse (*i.e.,* $\alpha = 0$ *and* $\beta = 0$) *because the* PHX *policy is risk-free as the firm can set different prices. Thus, even if the firm has the ability to set different prices in each market, production hedging can continue to evolve as the optimal choice as it maximizes the expected profit and eliminate the risk exposure completely.*

We find that if a firm can set different prices in different countries, production hedging becomes even more effective in mitigating exchange rate risks and it also enables the firm to gain greater profits.

B.2. Minimum Allocation Requirement

Our model enables the firm to not make any shipments to a market in Stage 2. How would our result change if the firm is forced to satisfy a minimum allocation amount described as k_i (> 0) for in each market $i = H, F$? This requirement replaces the non-negativity constraint in Stage 2 of the model with $y_i \geq k_i$ for $i = H, F$. It can be easily seen that policy PHX would increase the total production amount from the maximum of the two demand values to the sum of the maximum of the two demand values and the minimum allocation requirement in the market that has smaller demand. Thus, minimum allocation requirements cannot eliminate production hedging policies.

B.3. Transportation and Localization Costs

Our model ignores the presence of transportation and/or localization costs in each local market. Let us define t_i as the combination of transportation and/or localization costs in order to sell the product in market $i = H, F$. In the second stage, the firm will discard the product when the realized exchange rate is extremely low, i.e., $e < t_F/p$. In this case, the firm is better off by discarding the product (with zero return) rather than selling it in the foreign market; we term this as allocation hedging. It can be easily seen that with higher t_F values, the firm will switch from the TD policy to production hedging policies. Thus, incorporating transportation and/or localization costs into the model makes the finding regarding the risk mitigation of production hedging policies more pronounced.

B.4. Demand Influenced by Exchange Rate

From a Purchasing Power Parity (PPP) Theory perspective, one can argue that the demand for a product in a foreign market can be influenced by exchange rate fluctuations. When the foreign-to-domestic exchange rate is low (high), demand is lower (higher) due to the fact that consumers in the foreign market have less (more) purchasing power. Including a PPP perspective results in defining the demand function as $d_i(p, e)$, where foreign demand is positively correlated with e and domestic demand is negatively correlated with e. When the exchange rate realization e is low (high), the firm prefers to sell in the home (foreign) market, which due to PPP, is also when home (foreign) market demand is high. The allocation decision naturally targets the market with high purchasing power, and as a consequence, the profit opportunity under production hedging policies are enhanced relative to TD. In conclusion, production hedging policies are even more desirable as they mitigate the exchange-rate risk better than the TD policy when demand is affected by the exchange rate.

B.5. Postponed Pricing

We next examine whether the firm would continue to follow production hedging in order to mitigate the exchange-rate risk under a postponed pricing scheme. In this revised model, the firm determines the optimal quantity to be manufactured (along with the number of currency options) in Stage 1; the optimal price decisions are made along with the optimal allocation quantities after realizing the exchange rate in Stage 2. We impose the same anti-dumping law restriction $p = p_H = p_F e$ at the time of determining prices. The demand in the foreign market changes with exchange-rate realization, and becomes $d_F(p/e)$. Appreciation in exchange rate results in a lower selling price in the foreign market with higher demand, and depreciation leads to higher price in the foreign market with lower demand. Because the optimal manufacturing amount at $\bar{e} = 1$ resembles our TD policy, we use it as the benchmark between the TD and production hedging policies. The following example demonstrates that production hedging policies continue to be effective risk mitigation strategies under postponed pricing.

Example B3. *The demand in each market is described as* $d_H(p) = 105 - p$ *in the US and as* $d_F(p) = 105 - p$ *in Europe. The firm manufactures a product at* c = \$95/unit *in the US. The €-to-\$ exchange-rate* (\$/€) *can take three scenarios: € depreciates to* \$0.60/€ *with probability* 0.30, *takes on its expected value* \$1.00/€ *with probability* 0.40, *and appreciates to* \$1.40/€ *with probability* 0.30.

We take the benchmark TD policy as the amount that the firm would manufacture at the expected exchange rate. Using this definition of the TD policy, we find that $x^{TD} = 10$ *and* $E[\Pi(x^{TD} = 10)] = 134$. *In the optimal solution we get* $x^* = 12.28$ *with the expected profit of* \$138.79. *The second-stage allocation decisions differ at each realization of this optimal choice.*

Scenario 1: Exchange rate depreciates with $e = 0.6$. *The optimal price becomes* $p = p_H = 92.72$ *and* $p_F = 92.72/0.6 = 154.53$, *resulting in demand values* $d_H(92.72) = 105 - 92.72 = 12.28$ *and* $d_F(154.53) = 0$. *The* *allocation decisions are* $y_H = 12.28$ *and* $y_F = 0$. *Thus, all products are allocated to and sold in the home market.*

Scenario 2: Exchange rate takes on its expected value with $e = 1.0$. *The optimal price becomes* $p = p_H = p_F = 98.86$, *resulting in demand values* $d_H(98.86) = d_F(98.86) = 105 – 98.86 = 6.14$. *The allocation decisions are* $y_H = y_F = 6.14$. *Thus, both markets are equally served.*

Scenario 3: Exchange rate appreciates to $e = 1.4$. *The optimal price becomes* $p = p_H = 129.80$ *and* $p_F = 129.80/1.4 = 92.72$, *resulting in demand values* $d_H(129.80) = 0$ *and* $d_F(92.72) = 105 – 92.72 = 12.28$. *The allocation decisions are* $y_H = 0$ *and* $y_F = 12.28$. *Thus, all products are allocated to and sold in the foreign market.*

This example demonstrates that production hedging continues to be more desirable under the postponed pricing setting.

In sum, extensions such as independent pricing, minimum market allocation requirements, transportation and/or localization costs, the effects of exchange rate on the demand function, and postponed pricing make our finding that production hedging is a prevalent exchange-rate risk mitigation strategy.

B.6. Impact of VaR on Optimal Price and Quantity

When the risk constraint is not satisfied under the TD policy, the firm switches to the PHA policy, and when it is not satisfied under the PHX policy, the firm switches to PHB policy. However, violating the risk constraint does not always lead to a monotone behavior in price and manufacturing quantity decisions when compared with the risk-neutral optimal decisions. Specifically, the optimal price and optimal quantity under the PHA and PHB policy can be increasing or decreasing when compared to the levels of that under the TD and PHX policies where the risk constraint is non-binding. Thus, as the firm becomes more risk averse, the firm can increase or decrease its selling price and production quantity.

Examples B4 and B5 illustrate the various changes in the optimal price and manufacturing quantity decisions due to the VaR constraint for a firm whose risk-neutral optimal policy is TD. In these examples, the VaR constraint gets violated at the TD policy, and the firm is forced to switch to a PHA policy.

Example B4(a). *Manufacturing cost of the product is* $c = \$52$*/unit in the US, and sells it in the US and Europe. The demand is described as* $d_H(p) = 100 – p$ *in the US and as* $d_F(p) = 130 – 1.5p$ *in Europe. The €-to-\$ exchange rate* (\$/€) *follows a uniform distribution on a support of* [0, 2]. *The optimal production policy, price and expected profit for a risk-neutral firm* (*i.e.,* $\beta = \infty$) *is the following:* $E[\Pi^{TD}(p^{TD} = 72, x^{TD} = 50)] = \$1000.00$. *Thus, we have* $p^u = 72$ *and* $x^u = 50$.

Example B4(b). *For a risk-averse firm where* $\alpha = 0$ *and* $\beta = 500$, *the optimal production policy, price and expected profit is the following:* $E[\Pi^{PHA}(p^{PHA} = 72.8, x^{PHA} = 47.7)] = \$997.61$. *This example demonstrates that a risk-averse firm*

(1) <u>*increases optimal price*</u> *because* $p^r = p^{PHA} = 72.5 > p^u = p^{TD} = 72$, *and*

(2) <u>*decrease production quantity*</u> *because* $x^r = x^{PHA} = 47.7 < x^u = x^{TD} = 50$.

Example B4(c). *For an extremely risk-averse firm where* $\alpha = 0$ *and* $\beta = 0$ (*i.e., the firm prefers to eliminate the risk of losing money at every realization of the exchange rate*), *the optimal production policy, price and expected profit is the following:* $E[\Pi^{PHA}(p^{PHA} = 70, x^{PHA} = 40.4)] = \$982.69$. *This example demonstrates that a risk-averse firm can*

(1) <u>*decrease optimal price*</u> *because* $p^r = p^{PHA} = 70 < p^u = p^{TD} = 72$, *and*

(2) <u>*decrease production quantity*</u> *because* $x^r = x^{PHA} = 40.4 < x^u = x^{TD} = 50$.

Example B5. *Manufacturing cost of the product is* $c = \$63$*/unit in the US, and sells it in the US and Europe. The demand is described as* $d_H(p) = 118 - p$ *in the US and as* $d_F(p) = 100 - p$ *in Europe. The* €-*to*-$ *exchange rate* ($/€) *follows a uniform distribution on a support of* [0, 2]. *The optimal production policy, price and expected profit for a risk-neutral firm* (*i.e.,* $\beta = \infty$) *is the following:* $E[\Pi^{TD}(p^{TD} = 86, x^{TD} = 46)] = \$1058$. *Thus, we have* $p^u = 86$ *and* $x^u = 46$. *For an extremely risk-averse firm where* $\alpha = 0$ *and* $\beta = 0$ (*i.e., the firm prefers to eliminate the risk of losing money at every realization of the exchange rate*), *the optimal production policy, price and expected profit is the following:* $E[\Pi^{PHA}(p^{PHA} = 85.9, x^{PHA} = 46.2)] = \$1054.51$. *This example demonstrates that a risk-averse firm can*

(1) <u>*decrease optimal price*</u> *because* $p^r = p^{PHA} = 85.9 < p^u = p^{TD} = 86$, *and*

(2) <u>*increase production quantity*</u> *because* $x^r = x^{PHA} = 46.2 > x^u = x^{TD} = 46$.

Examples B6 and B7 illustrate the various changes in the optimal price and manufacturing quantity decisions due to the VaR constraint for a firm whose risk-neutral optimal policy is PHX. In these examples, the VaR constraint gets violated at the PHX policy, and the firm is forced to switch to a PHB policy.

Example B6(a). *Manufacturing cost of the product is* $c = \$70$*/unit in the US, and sells it in the US and Europe. The demand is described as* $d_H(p) = 100 - p$ *in the US and as* $d_F(p) = 120 - p$ *in Europe. The* €-*to*-$ *exchange rate* ($/€) *follows a uniform distribution on a support of* [0, 2]. *The optimal production policy, price and expected profit for a risk-neutral firm* (*i.e.,* $\beta = \infty$) *is the following:* $E[\Pi^{PHX}(p^{PHX} = 86, x^{PHX} = 34)] = \$845.00$. *Thus, we have* $p^u = 86$ *and* $x^u = 34$.

Example B6(b). *For a risk-averse firm where* $\alpha = 0.1$ *and* $\beta = 820$, *the optimal production policy, price and expected profit is the following:* $E[\Pi^{PHB}(p^{PHB} = 86.1, x^{PHB} = 33.7)] = \$841.38$. *This example demonstrates that a risk-averse firm can*

(1) <u>*increase optimal price*</u> *because* $p^r = p^{PHB} = 86.1 > p^u = p^{PHX} = 86$, *and*

(2) <u>*decrease production quantity*</u> *because* $x^r = x^{PHB} = 33.7 < x^u = x^{PHX} = 34$.

Example B6(c). *For an extremely risk-averse firm where* $\alpha = 0$ *and* $\beta = 0$ (*i.e., the firm prefers to eliminate the risk of losing money at every realization of the exchange rate*), *the optimal production policy, price and* *expected profit is the following:* $E[\Pi^{PHB}(p^{PHB} = 79.7, x^{PHB} = 23.1)] = \$628.67$. *This example demonstrates that a risk-averse firm can*

(1) <u>*decrease optimal price*</u> *because* $p^r = p^{PHB} = 79.7 < p^u = p^{PHX} = 86$, *and*

(2) <u>*decrease production quantity*</u> *because* $x^r = x^{PHB} = 23.1 < x^u = x^{PHX} = 34$.

Example B7. *Manufacturing cost of the product is* $c = \$70$/*unit in the US, and sells it in the US and Europe. The demand is described as* $d_H(p) = 130 - 1.5p$ *in the US and as* $d_F(p) = 100 - p$ *in Europe. The €-to-\$ exchange rate (\$/€) follows a uniform distribution on a support of* [0, 2]. *The optimal production policy, price and expected profit for a risk-neutral firm (i.e.,* $\beta = \infty$*) is the following:* $E[\Pi^{PHX}(p^{PHX} = 73.6, x^{PHX} = 26.4)] = \$455.68$. *Thus, we have* $p^u = 73.6$ *and* $x^u = 26.4$. *For a risk-averse firm where* $\alpha = 0.05$ *and* $\beta = 350$, *the optimal production policy, price and expected profit is the following:* $E[\Pi^{PHB}(p^{PHB} = 73.3, x^{PHB} = 26.7)] = \$455.49$. *This example demonstrates that a risk-averse firm can*

(1) <u>*decrease optimal*</u> *because* $p^r = p^{PHB} = 73.3 < p^u = p^{PHX} = 73.6$, *and*

(2) <u>*increase production quantity*</u> *because* $x^r = x^{PHB} = 26.7 > x^u = x^{PHX} = 26.4$.

Appendix C – Demand Uncertainty

Section 6 has identified the number of financial hedging contracts that equate the realized profits at α probability to the firm's tolerated loss of β. We obtain a similar result when demand uncertainty is incorporated into the model. For each production quantity decision that is made in the first-stage, there is a corresponding number of financial hedging contracts, defined as $h^*(x)$ that satisfies

$$P_{(\tilde{e},\tilde{z}_H,\tilde{z}_F)}\left[-cx - c_h h^*(x) + \pi^*\left(x, p, h^*(x), e\right) < -\beta\right] = \alpha .$$

Using $h^*(x)$, when $\delta = 0$, the firm can elevate its expected profit to the level of the expected profit that can be obtained for $(x, p, h = 0)$ in the absence of a risk constraint. Thus, in the rest of our analysis, we assume that $\delta = 0$, and the firm engages in $h^*(x)$ units of financial hedging contracts.

Under demand uncertainty, the second-stage allocation problem is expressed in (3). Because the first-order derivatives with respect to allocation decisions are non-negative, i.e.,

$$\frac{\partial \pi\left(y_H, y_F \mid x, p, h, e\right)}{\partial y_H} = p\left[1 - G_H\left(y_H\right)\right] \geq 0, \text{ and } \frac{\partial \pi\left(y_H, y_F \mid x, p, h, e\right)}{\partial y_F} = pe\left[1 - G_F\left(y_F\right)\right] \geq 0,$$

the value of both allocation decision variables would increase up to the point that $y_H + y_F = x$ as long as $x \leq G_H^{-1}(1) + G_F^{-1}(1)$. A Lagrangian approach can be employed in order to determine the optimal solution. We divide the demand probability space into five regions: $P(\text{A}) = P[0 < x < d_m]$, $P(\text{B1}) = P[d_F < x < d_H]$, $P(\text{B2}) = P[d_H < x < d_F]$, $P(\text{C}) = P[d_x < x < d_H + d_F]$, and $P(\text{D}) = P[x > d_H + d_F]$. Let λ be the Lagrangian multiplier for the constraint $y_H + y_F \leq x$, and $E[\lambda_j]$ j = A, B1, B2, C, D represents the expectation of the Lagrangian multiplier in the specified probability regions. $E[\lambda_j]$ is continuous and increasing in in j = A, B1, B2, C, and is equal to zero in j = D. Thus, the optimal solution satisfies $E[\lambda(X)] = E[\lambda_A]\, P(A) + E[\lambda_{BH}]\, P(B1) + E[\lambda_{BF}]\, P(B2) + E[\lambda_C]\, P(C) = c$.

The optimal allocation decisions can be classified in three regions. With smaller manufacturing quantities and low realizations of exchange rates, the allocation is made only to the home market, satisfying $G_H(y_H) = (p - \lambda)/p$ or $\lambda = p(1 - G_H(y_H)) \equiv \lambda(y_H)$. With smaller manufacturing quantities but higher realizations of exchange rates, the allocation is made only to the foreign market, satisfying $G_F(y_F) = (p - \lambda)/p$ or $\lambda = pe(1 - G_F(y_F)) \equiv \lambda(y_F)$. In the rest of the scenarios, allocation occurs to both markets satisfying $G_H^{-1}((p - \lambda)/p) + G_F^{-1}((pe - \lambda)/pe) = x$. Figure C1 shows how the optimal second-stage decisions vary according to the realized value of the exchange-rate e for a given pair of (x, p) for independent uniform demand distributions at each market. In Figure C1, the line curve Line L1 is obtained by solving $1 - G_H(x) = e(1 - G_F(0)) = e$, curve L2 is obtained by solving $1 - G_H(0) = 1 = e(1 - G_F(x))$ at each exchange rate realization e.

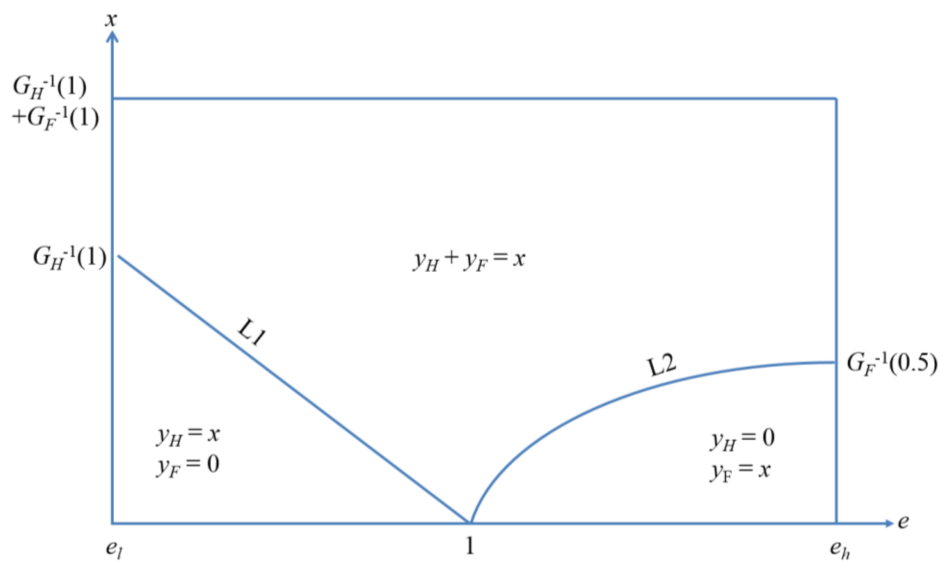

Figure C1. Optimal second-stage decisions for a given x and p and uniformly distributed demand.