

Direct Trade and the Third-Wave Coffee: Sourcing and Pricing a Specialty Product under Uncertainty

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Academic/Practical Relevance: Third-wave coffee movement is an initiative that promotes the production of high-quality coffee through a sourcing practice called Direct Trade (DT) that creates intimate relations with plantations, growing regions, and farmlands.

Problem definition: We study sourcing and pricing decisions of an agricultural processor that sells a specialty product targeted toward the quality-sensitive segment of consumers while operating under two sources of uncertainty: (1) randomness in crop yield and (2) randomness in market prices for a similar but inferior product in global markets. The processor initially leases farmland, and at the end of the growing season, observes the realized values of the crop supply and the market price of the commercial product. The firm then determines the amount of crop supply to be processed into the specialty product and the selling price of its specialty product.

Methodology: We use stochastic programming with recourse.

Results: The paper makes three contributions. First, it develops a model that examines production and pricing decisions for a processor who engages in Direct Trade and operates under yield and market price of commercial coffee uncertainty. Second, our analysis shows that a conservative sourcing policy coined as the underinvestment policy can emerge as an optimal decision where the processor intentionally reduces the amount of leased farmland guaranteeing to operate under supply shortage. Higher degrees of uncertainty in either source of uncertainty encourage underinvestment. Third, we show that the processor should invest in agricultural regions where the crop yield is negatively correlated with the global commercial yield.

Managerial implications: Direct Trade can be an exemplary sourcing practice for premium agricultural products. Our policy recommendations encourage investments in farmlands in less-populated regions, and leads to a higher amount of specialty coffee at lower prices, benefiting consumers with a higher consumer surplus.

Keywords: direct trade, sourcing, pricing, yield uncertainty, price uncertainty

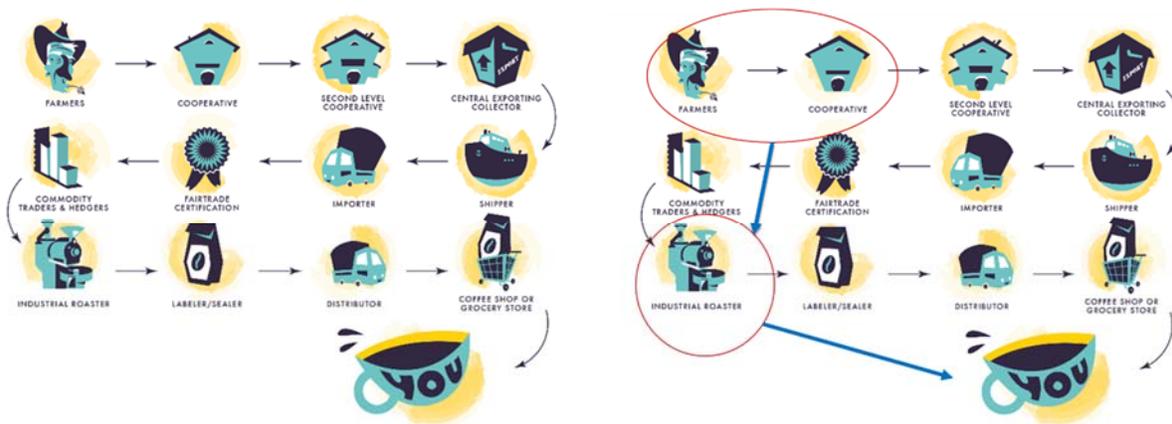
1. Introduction

Coffee is one of the most popular beverages worldwide. The United States Department of Agriculture estimates the global annual coffee consumption at a record of 150.8 million bags (60 kilograms, or approximately 132 pounds, per each bag) in its 2016/2017 forecast review¹. While coffee consumption is expected to continue to experience a steady growth, consumers seem to increasingly demand high quality coffees around the world (Stabiner 2015, Craymer 2015). There are a few coffee roasters in the United States that have already targeted this trending specialty coffee market, and are believed to be reshaping the coffee industry (Strand 2015). The pioneer coffee roasters in the specialty coffee industry are

¹ <http://apps.fas.usda.gov/psdonline/circulars/coffee.pdf>

Intelligentsia, Stumptown, and Counter Culture. These three roasters are referred to as the Big Three of the Third-Wave Coffee, an initiative aimed at enhancing the quality of the produced coffee. Third Wave Coffee is considered as a specialty food product rather than a commodity. This is exemplified by the recent acquisitions of Intelligentsia and Stumptown by Peet’s Coffee at high prices, indicating an increase in the interest and practice in the specialty coffee market.

In order to be able to offer superior coffee, these leading coffee roasters rely heavily on direct communication and close collaboration with the growers, which has given rise to a new form of sourcing practices called Direct Trade. Through Direct Trade, specialty coffee roasters can improve the quality of coffee beans by providing guidance and resources to the growers, and closely monitoring the growing and harvesting process. Figure 1 demonstrates how Direct Trade departs from the traditional sourcing practice where coffee beans are traditionally sourced through coffee traders and exchanges.



(a) Typical coffee supply chain

(b) Direct Trade coffee supply chain

Figure 1. A schematic view of the coffee supply chain. Source: Forbes.com.

Inspired by the business model of the Big Three, a growing number of coffee roasters have started to leave the coffee exchanges and engage in Direct Trade. The departure from the traditional sourcing practice results in two separate coffee markets according to Wernau (2015). We refer to these markets as specialty (superior) coffee market, and commercial (inferior) coffee market in this paper.

Our study is motivated by the recent trend of engaging in Direct Trade in the coffee industry. We focus on the specialty coffee supply chain where a coffee roaster adopts Direct Trade sourcing in order to target the quality-sensitive segment of consumers. Our analysis captures the interaction between the specialty coffee market and the commercial coffee market from the consumer choice perspective. We explore the impact of the supply uncertainty caused by engaging in Direct Trade, and the uncertainty in the global commercial coffee market on the coffee roaster’s sourcing and pricing decisions. Commercial coffee is

known as a commodity with remarkable price fluctuations due to the extreme variation in global yield. Figure 2 demonstrates the daily prices of the International Coffee Organization (ICO) composite indicator of the commercial coffee beans from March 2013 to March 2015. It exhibits a wide range of prices with more than 80% difference between the maximum (October 2014) and the minimum price (November 2013). The climate changes such as the El Niño phenomenon are expected to continue to bring about drastic fluctuations in the price of the commodity coffee beans and the retail price of the commercial (roasted) coffee (Cooke 2014, Cosgrave 2014, Osborn 2015, Woody 2015). In this paper, we incorporate the variation in the market price of the commercial coffee into our analysis and examine its impact on the sourcing and pricing decisions of a specialty coffee roaster.

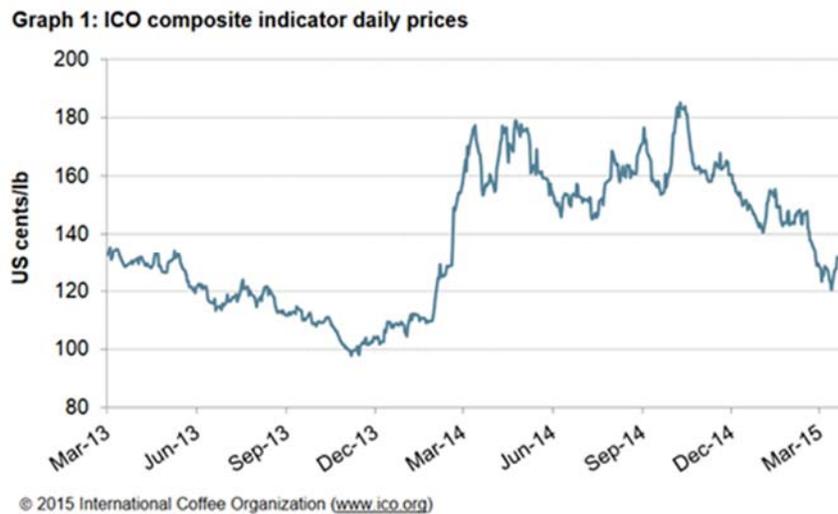


Figure 2. Fluctuating price of commercial coffee beans

We provide insights into managing a specialty-coffee supply chain, which is an emerging phenomenon in the coffee industry, by addressing the following research questions:

1. What is the impact of yield uncertainty stemming from Direct Trade on sourcing decisions of agribusinesses in a specialty coffee supply chain?
2. How does the uncertainty in the commercial coffee market influence the coffee roaster's level of investment in Direct Trade?
3. How should a coffee roaster set the price for its specialty product based on the market price of the commercial coffee considering the consumer choice between the specialty and the commercial coffees?
4. What is the impact of correlation between the specialty and the global commercial crop supply?

Our study makes three contributions: First, to the best of our knowledge, our study is the first to develop a mathematical model capturing the main driving forces that influence the sourcing and pricing decisions in a specialty coffee supply chain. Incorporating the uncertainty in commercial coffee market as well as the variation in the Direct Trade yield into our model, we identify the optimal sourcing and pricing decisions of a specialty coffee roaster.

Second, we show that a conservative sourcing policy may emerge as an effective policy under uncertainty, which would never be optimal in a deterministic setting. Our analysis demonstrates that either yield uncertainty or the fluctuations in the price of the commercial coffee may lead to a conservative sourcing policy where the coffee roaster intentionally underinvests in Direct Trade. This sourcing policy—which we refer to as the “underinvestment policy”—can be considered as an operational hedging approach that helps the coffee roaster mitigate the negative consequences of the potential uncertainties. This result complements the earlier literature on global production planning, which identifies “production hedging” (deliberately producing less than demand) as an effective operational hedging policy (Kazaz 2005, Park et al. 2016) under exchange-rate uncertainty in manufacturing settings. Our work shows a similar hedging approach for agribusinesses that operate under uncertainty pertaining to crop yield and market price of a competing inferior product. In our findings, the emerged hedging policy can be interpreted as “capacity hedging” which involves underinvesting in agricultural farmland. Under this conservative behavior, the firm utilizes its pricing lever in order to shrink the market size at all realizations of the two random variables. Thus, the market size served is significantly smaller than what the firm would have served in the absence of uncertainty. Our finding should not be perceived as lowering the initial investment in Direct Trade alone. When the underinvestment policy is implemented, the firm ensures to underserve the potential target market at *any* realization of uncertainty. Thus, it is a significantly more conservative policy than those reported in earlier (and similar) publications.

Our third contribution relates to the impact of correlation between the specialty Direct Trade supply and the global supply of commercial coffee beans. Our analysis indicates that higher degrees of positive correlation between the two random variables representing the specialty yield and the price of the commercial coffee improve the expected profit. Therefore, the specialty coffee roaster benefits from investing in agricultural regions where the specialty coffee yield is highly negatively (positively) correlated with the global commercial yield (price of the commercial coffee). This result suggests two important implications: (1) It encourages the specialty coffee roasters to invest more in new developing agricultural regions away from the large and developed areas which make the most contribution to the global commercial yield. This strategy can potentially improve lives of growers in those small developing regions. (2) More negative (less positive) correlation between the specialty yield and the global commercial yield reduces the likelihood of underinvestment, and thus allows the processor to invest more freely in DT. This

policy recommendation leads to a higher amount of coffee at lower prices, benefiting consumers with a higher level of consumer surplus.

The remainder of this paper is organized as follows. Section 2 describes how our work departs from the current literature. Section 3 introduces our model, Section 4 presents its analysis; it identifies the optimal investment and pricing decisions, characterizes the underinvestment policy, and examines the impact of uncertainty on Direct Trade investment decisions. Section 5 demonstrates the possibility of adopting the underinvestment policy by specialty coffee roasters, using certain sets of parameters inspired by practice. Section 6 provides concluding remarks. All proofs are relegated to the appendix.

2. Literature Review

This study examines sourcing and pricing decisions of an agricultural firm under random yield and market-price for a similar (and inferior) product. The impact of supply uncertainty on sourcing strategies is extensively examined in supply chain literature. Yano and Lee (1995) and Tang (2006) provide a comprehensive review of this literature in manufacturing environments. Our paper is related with the stream of research that explores operational hedging strategies for managing supply uncertainty. In this literature, supply diversification is proven to be an effective approach toward mitigating supply risk: Gerchak and Parlar (1990), Parlar and Wang (1993), Anupindi and Akella (1993), and more recently, Tomlin and Wang (2005), Dada et al. (2007), Burke et al. (2009), Jain et al. (2014), and Tan et al. (2016) demonstrate the benefits of dual/multiple sourcing in managing supply uncertainty. However, in these studies the firm (buyer) does not have pricing power, and thus, the selling price is considered as exogenous. Our work characterizes an operational hedging policy that differs from supply diversification, and it develops a new policy that underserves the target market by lowering the level of investment in farmland capacity in the presence of monopolistic power.

There are a few studies that consider a firm's price-setting ability in the presence of supply uncertainty. Li and Zheng (2006), and Feng (2010) investigate joint inventory and pricing policies in multi-period manufacturing settings. Tang and Yin (2007) consider pricing and purchase quantity decisions under supply uncertainty. Kazaz and Webster (2015) consider a single-period Newsvendor model under supply and demand uncertainty and explore how the source of uncertainty influence the tractability of the problem and the optimal decisions. Our study departs from these papers by considering the consumer choice between two products and the effect of uncertainty in the competing market on sourcing, production and pricing decisions of an agricultural processor.

Sourcing with Direct Trade resembles agricultural supply chain literature where agricultural firms lease farmland in order to grow products and experience yield uncertainty. Kazaz (2004), Kazaz and Webster (2011) examine the impact of yield-dependent trading cost on sourcing and production planning of an agricultural firm which sells a single product. Our study departs from these publications by considering two

products—one superior and one inferior product— with two associated consumer segments. While our main focus is exploring the sourcing and pricing decisions of the superior product, our analysis captures the consumer’s choice between the two products, which affects the demand for the superior product.

Noparumpa et al. (2011) examine the interrelationships among three flexibilities: Downward substitution, price setting, and fruit trading under supply and quality uncertainty. While we do not incorporate quality uncertainty into our model, we capture the impact of uncertainty in the price of an inferior product determined by the global market on sourcing and pricing of a superior product by an agricultural processor.

Our model’s structure is similar to that of Noparumpa et al. (2015) where a winemaker has to determine how much of its wine to be sold in advance in the form of wine futures, the price of wine futures, and the amount of wine to be sold later in the retail chain. Our study differs from their work in terms of the decisions, and the environmental factors influencing sourcing and pricing decisions: Our paper explores the impact of the interaction between two different consumer segments on allocation decisions of an agricultural processor. Our study also captures the farmland investment decisions made by the processor prior to the selling season and incorporates yield uncertainty stemming from such a sourcing practice.

There is a recent stream of literature that considers simultaneous sourcing of agricultural inputs (Boyabatli et al. 2011, Boyabatli 2015). However, this body of research investigates multi-product supply management where the crop yield is assumed to be deterministic using two or more different agricultural inputs. Our study is structurally different from these papers as we emphasize developing effective policies for sourcing a single agricultural input under yield and market-price uncertainty.

In sum, motivated by an emerging agribusiness practice in the coffee industry, our study features a new agricultural supply chain framework. In order to target the fast-growing niche segment of quality-sensitive consumers, a small number of coffee roasters have drastically changed their sourcing policy and engaged in Direct Trade, which in turn has led to exposure to supply risk. Our study investigates the sourcing and pricing decisions of such agricultural processors and provides insights into managing the supply chain of the associated agricultural products.

3. Model

In this section, we present the model developed to explore the global sourcing decisions of a firm operating in an agricultural environment where yield uncertainty influences the firm’s crop supply. The firm sells a finished product (i.e., roasted coffee) which requires processing an agricultural product (i.e., green coffee beans) as input to be converted to the final product. The main target market of the firm (processor) is the quality-sensitive segment of consumers who prefer specialty coffees and are willing to pay a premium price (hereafter “specialty segment”). The processor (coffee roaster) engages in Direct Trade (DT) in order to be able to source quality coffee beans and offer superior processed (roasted) coffee to the specialty segment. An inferior alternative for consumers is the Non-Direct-Trade (NDT) roasted coffee which is extensively

offered by a large number of coffee roasters, and is desired by a massive population of consumers who consider coffee as a necessity product. As such, we consider the NDT coffee as a commercial/commodity product whose price is exogenously determined by the enormous global market.

We develop a two-stage stochastic model with recourse to examine the processor's sourcing and pricing decisions in the presence of yield uncertainty of the DT crop and price uncertainty of the commercial coffee. The operating environment and the sequence of decisions can be described as follows: Prior to the growing season, which corresponds to stage 1 of our model, the processor determines the amount of farmland to grow DT coffee beans, denoted Q . Along with the leasing decision, the processor provides instructions and technical assistance to the growers in order to ensure the desired quality of the specialty coffee crop. This initial investment costs l per unit of farmland, and includes the costs of monitoring and other activities associated with operations. At the end of the growing season, corresponding to stage 2 of our model, the processor realizes (1) the DT crop yield to be processed and sold as a specialty coffee, and (2) the global market price for the commercial coffee. The DT crop supply harvested at the end of the growing season is expressed as Qy (stochastically proportional to the lease quantity) where $y \in [y_l, y_h]$ is the realization of a random variable Y with mean \bar{y} , standard deviation σ_y , probability density function (pdf) $g(y)$, and cumulative distribution function (cdf) $G(y)$. The market (retail) price of the commercial coffee, Φ , is considered random with realization $\varphi \in [\varphi_l, \varphi_h]$, mean $\bar{\varphi}$, standard deviation σ_φ , pdf $h(\varphi)$ and cdf $H(\varphi)$. The uncertainty in the price of the commercial coffee represents the uncertainty in the commercial market which stems from the fluctuations in the global yield of the coffee beans and other uncertain economic factors. In our base model, the DT crop yield and the price of the commercial coffee are considered independent random variables, because the amount of crop yield from DT is assumed to be relatively small compared to the amount of the commodity coffee's global yield. We later relax this assumption and analyze the impact of correlation on the investment decision and the expected profit of the processor. We also assume throughout the paper that the support of Y and Φ remain unchanged as the level of uncertainty associated with each variable alters (except for the case of no uncertainty where the random variable is replaced with its mean).

Upon the realizations of the DT yield and the market price of the commercial coffee, the processor uses its monopolistic power within the specialty segment, and sets the selling price, denoted p , for its offered specialty coffee targeted toward this segment. An equivalent interpretation is that the processor determines the quantity of the specialty coffee to be sold, denoted q , in this stage. The processing cost is r per unit of product, and includes a per pound payment to the farmers at the time of harvesting as well as importing, shipment, roasting, and packaging costs. The extra DT crop is processed and salvaged at s where $r \leq s < r + (l/\bar{y})$. The first inequality implies that the salvage value of the processed crop is greater than or equal to the processing cost, and the second inequality indicates that the salvage value is less than the total unit cost

from processing leasing the farmland (otherwise the firm leases an infinite amount of farmland). These conditions represent the operating environment for the processor, and our conclusions do not depend on these inequalities. It is also important to note that a shortage in the DT coffee supply cannot be compensated by sourcing from the spot market because the coffee beans in the spot market would not be considered a DT product. Figure 3 illustrates the sequence of decisions and realizations.

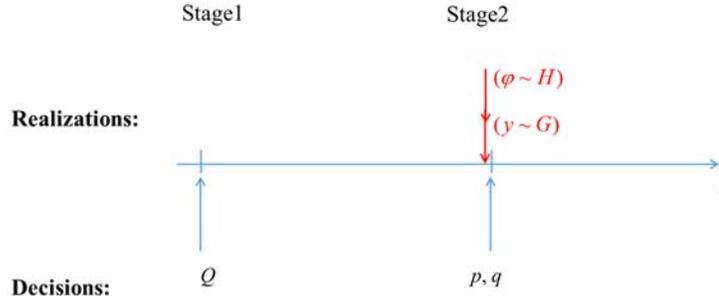


Figure 3. Sequence of decisions and realizations.

The first-stage objective function in our model maximizes the expected profit, denoted $\Pi(Q)$, from the leasing decision Q in the presence of yield (Y) and market-price of commercial coffee (Φ) uncertainty. It can be expressed as follows:

$$\max_{Q \geq 0} \Pi(Q) = -IQ + E_{Y, \Phi} [\pi^*(Q, Y, \Phi)]$$

where $\pi^*(Q, y, \varphi)$ is the optimal second-stage profit for a given pair of realized values y of Y and φ of Φ .

The second-stage profit depends on the pricing decision of the processor, which affects the demand for the specialty product.

We utilize a multinomial logit (MNL) model to capture the consumer choice between the DT specialty (superior) coffee and the NDT commercial (inferior) coffee based on the prices and the consumers' valuation for each product. At the beginning of the selling season, equivalent to the end of the growing season and corresponding to stage 2 of our model, the consumers of the specialty segment consider three options: (1) purchase specialty coffee at price p , (2) purchase commercial coffee at price φ , (3) do not purchase.

The utility associated with each purchasing alternative is driven by the difference between the valuation of the consumer for that particular product and the price of the product. Specifically, the consumer's valuation for the specialty coffee is $V_s = v_s + \varepsilon_s$ where ε_s is a random variable with $E[\varepsilon_s] = 0$, and the consumer surplus – the difference between valuation and the price – determines the random utility of

purchasing the specialty product, i.e., $U_s = V_s - p = v_s + \varepsilon_s - p$. The average utility of purchasing the specialty product among consumers is then $u_s = E[U_s] = v_s - p$.

Similarly, the random utility from purchasing the commercial coffee can be expressed as $U_c = V_c - \varphi = v_c + \varepsilon_c - \varphi$, where ε_c is a random variable with $E[\varepsilon_c] = 0$. Thus, the average utility of purchasing the commercial product is $u_c = E[U_c] = v_c - \varphi$. In order to reflect the preferences of the specialty segment, we assume $v_s > v_c$.

Finally, the random utility of the no-purchase option is $U_0 = \varepsilon_0$, which results in zero average utility of this alternative among consumers because ε_0 is a random variable with $E[\varepsilon_0] = 0$.

Among the three options, the consumers of the specialty segment choose the option associated with the highest utility. Therefore, the probability of purchasing the specialty product can be expressed as:

$$P[U_s > \max\{U_c, U_0\}] = P[\max\{\varepsilon_c, \varepsilon_0\} - \varepsilon_s < v_s - p].$$

Assuming that ε_s , ε_c , and ε_0 are identically and independently distributed (i.i.d.) Gumbel random variables with zero mean and scale parameter β , we note that $\max\{\varepsilon_c, \varepsilon_0\}$ is a Gumbel random variable with $E[\max\{\varepsilon_c, \varepsilon_0\}] = \beta \ln 2$, and scale parameter β , and $\max\{\varepsilon_c, \varepsilon_0\} - \varepsilon_s$ is a logistic random variable. As a result, the probability of purchasing the specialty coffee is:

$$P[U_s > \max\{U_c, U_0\}] = \frac{e^{(v_s - p)/\beta}}{1 + e^{(v_c - \varphi)/\beta} + e^{(v_s - p)/\beta}}$$

which corresponds to the fraction of consumers in the specialty segment who purchase the specialty coffee. Thus, the demand for the specialty coffee is determined by the MNL model as follows:

$$q(p | \varphi) = M \times P[U_s > \max\{U_c, U_0\}] = M \frac{e^{(v_s - p)/\beta}}{1 + e^{(v_c - \varphi)/\beta} + e^{(v_s - p)/\beta}} \quad (1)$$

where M is the market potential for the specialty coffee (i.e., the maximum consumer population who may potentially buy the specialty product).

Consequently, the inverted demand function can be expressed as:

$$p(q | \varphi) = v_s + \beta \ln \left[\frac{M - q}{k(\varphi)q} \right] \quad (2)$$

where

$$k(\varphi) = 1 + e^{(v_c - \varphi)/\beta} \quad (3)$$

The second-stage optimal profit can then be expressed as:

$$\pi^*(Q, y, \varphi) = \max_{q \leq Q_y} [\pi(q | Q, y, \varphi)]$$

where

$$\pi(q | Q, y, \varphi) = p(q | \varphi)q + s(Qy - q) - rQy = \left(v_s - s + \beta \ln \left[\frac{M - q}{k(\varphi)q} \right] \right) q + (s - r)Qy \quad (4)$$

is the second-stage profit function.

4. Analysis

4.1. Pricing and Processing Decisions

We begin our analysis by investigating the pricing and processing decisions of the firm after realizing the DT crop yield and the market price for the commercial coffee in stage 2 of our model. We determine the optimal price ($p^*(\varphi, y)$) for the specialty DT product, which provides the corresponding quantity to be sold ($q^*(\varphi, y)$). These results utilize the Lambert W function (Coreless et al. 1996) where $W(x)$ is the value of w satisfying $x = we^w$.

Proposition 1. Let $\alpha(\varphi) = \frac{W\left(e^{(v_s - s)/\beta - 1} / k(\varphi)\right)}{1 + W\left(e^{(v_s - s)/\beta - 1} / k(\varphi)\right)}$, then

$$(p^*(\varphi, y), q^*(\varphi, y)) = \begin{cases} \left(v_s + \beta \ln \left[\frac{M - Qy}{k(\varphi)Qy} \right], Qy \right) & \text{if } Qy < M\alpha(\varphi) \\ \left(s + \beta \left[1 + W\left(e^{(v_s - s)/\beta - 1} / k(\varphi) \right) \right], M\alpha(\varphi) \right) & \text{if } Qy \geq M\alpha(\varphi) \end{cases},$$

and

$$\pi^*(Q, y, \varphi) = \begin{cases} Qy \left(v_s - r + \beta \ln \left[\frac{M - Qy}{k(\varphi)Qy} \right] \right) & \text{if } Qy < M\alpha(\varphi) \\ M\beta W\left(e^{(v_s - s)/\beta - 1} / k(\varphi) \right) + (s - r)Qy & \text{if } Qy \geq M\alpha(\varphi) \end{cases}.$$

The value of $\alpha(\varphi)$ is the optimal fraction of the specialty segment to be targeted by the processor, and the value of $M\alpha(\varphi)$ is the optimal market size to be served (optimal demand). We observe that the processor sets different selling prices for the specialty coffee contingent upon whether the crop supply (Qy) is sufficient for serving the optimal market size.

The next proposition shows how the optimal price of the specialty coffee (optimal “specialty price”, hereafter) changes with the realized market price of the commercial coffee (realized “commercial price”, hereafter). Throughout this paper, we use “increase”, “decrease”, and “concave” in their weak sense.

Proposition 2. The optimal specialty price ($p^*(\varphi, y)$) increases in the realized commercial price (φ). However, the magnitude of the increase is less than that of φ , i.e., $0 < \partial p^*(\varphi, y) / \partial \varphi < 1$.

The processor charges a higher specialty price at higher levels of realized commercial price. We note that the demand for the specialty coffee depends on both the specialty price set by the processor (p) and the realized commercial price determined by the global market (φ). Proposition 2 states that the optimal

specialty price takes the same direction as the realized commercial price. In other words, if the commercial price increases (decreases), the specialty price should increase (decrease) as well. Consequently, specialty price and commercial coffee price have counter-balancing effects on the demand. The next proposition identifies the dominant effect and clarifies how the optimal target market size changes with respect to the realized commercial price.

Proposition 3. *The optimal fraction of the specialty segment to be targeted by the processor ($\alpha(\varphi)$) and thus the optimal processing quantity ($q^*(\varphi, y)$) increases in the realized commercial price (φ).*

Proposition 3 shows that, even though the processor raises the specialty price as commercial price increases, its effect on the market allocation is dominated by the effect of commercial price. Consequently, the optimal targeted fraction of the specialty segment expands if the realized commercial price increases. This implies that the processor benefits from targeting a larger specialty market while charging a higher price as the commercial coffee becomes more expensive.

4.2. Investment Decision

We next examine the processor's investment decision at the beginning of the growing season corresponding to stage 1 of our model. From Proposition 1, we use the optimal second-stage profit expressions to write the first-stage expected profit function:

$$\begin{aligned} \Pi(Q) = & -(l + r\bar{y})Q + \iint_{U(Q)} Qy \left(v_s + \beta \ln \left[\frac{M - Qy}{k(\varphi)Qy} \right] \right) g(y)h(\varphi) dy d\varphi \\ & + \iint_{O(Q)} \left[M\beta W \left(\frac{e^{(v_s-s)/\beta-1}}{k(\varphi)} \right) + sQy \right] g(y)h(\varphi) dy d\varphi \end{aligned} \quad (5)$$

where $U(Q) = \{(y, \varphi) \mid Qy < M\alpha(\varphi)\}$ and $O(Q) = \{(y, \varphi) \mid Qy \geq M\alpha(\varphi)\}$ describing underage and overage in DT crop supply, respectively.

The following proposition indicates that there is always a unique optimal amount of farmland to be leased for any given probability distributions of yield and commercial price.

Proposition 4. (a) *The first-stage expected profit function $\Pi(Q)$ is concave in the leasing amount Q , and the optimal amount of farmland to lease, denoted Q^* , is the unique solution to the below first order condition (FOC):*

$$\iint_{U(Q^*)} y \left(v_s + \beta \ln \left[\frac{M - Q^*y}{k(\varphi)Q^*y} \right] - \beta \frac{M}{M - Q^*y} \right) g(y)h(\varphi) dy d\varphi + \iint_{O(Q^*)} sy g(y)h(\varphi) dy d\varphi = l + r\bar{y} \quad (6)$$

(b) *The optimal expected profit is*

$$\Pi(Q^*) = M\beta \left[\iint_{U(Q^*)} \frac{Q^*y}{M - Q^*y} g(y)h(\varphi) dy d\varphi + \iint_{O(Q^*)} W \left(\frac{e^{(v_s-s)/\beta-1}}{k(\varphi)} \right) g(y)h(\varphi) dy d\varphi \right] \quad (7)$$

Note that the main objective of engaging in DT is to serve the quality-sensitive specialty segment, because sourcing through DT allows the processor to ensure the desired quality by providing instructions while closely collaborating with the growers. The growers, on the other hand, have to have sufficient incentives to take part in DT. The processors typically pay the farmer a premium price². As a result, sourcing through DT is costly for the processors, which creates a downward pressure on the leasing decision and prevents the processor from overinvesting in DT. On the other hand, the flexibility to charge a premium price for the specialty coffee segment creates an incentive to increase investment in farmland associated with DT. The FOC (6) captures these opposing driving forces and determines the optimal leasing quantity. The two terms on the left-hand side express the marginal revenue of leasing farmland when the crop supply falls short of the optimal target market (i.e., $Qy \leq M\alpha(\varphi)$) and when there is sufficient supply (i.e., $Qy > M\alpha(\varphi)$), respectively, while the right-hand side is the (expected) marginal cost of sourcing and processing the crop supply of coffee beans.

4.3. Optimal Investment Policy

In this section, we examine the optimal investment policies under yield and commercial price uncertainty. Before proceeding with the optimal policy structure pertaining to the leasing decision of the processor, we establish three benchmarks that are beneficial for characterizing the optimal behavior. We begin with the first benchmark associated with the amount of farmland that would be leased in the absence of uncertainty. It should be observed that when there is no uncertainty associated with the DT crop supply nor the commercial market, the processor invests in the exact amount of farmland yielding the amount of supply that fulfills the optimal target market $M\alpha(\bar{\varphi})$. Our first benchmark, denoted Q^D , establishes the amount of farmland to be leased when the processor ignores the uncertainty by setting $Q\bar{y} = M\alpha(\bar{\varphi})$, and thus,

$$Q^D = M\alpha(\bar{\varphi}) / \bar{y}.$$

We establish a second benchmark that helps characterize the processor's potentially conservative behavior. It can be observed that the lowest market size occurs when the market price random variable takes its lowest value at the lower support of its pdf, φ_l . It can also be observed that the highest crop yield for the processor is observed when the random yield variable takes its highest value over the support of its pdf, y_h . Our second benchmark establishes the minimum amount of farmland lease that results in just sufficient crop yield at its highest realization (y_h) to cover the minimum market demand ($M\alpha(\varphi_l)$). We describe this benchmark as:

$$Q^L = M\alpha(\varphi_l) / y_h.$$

² Intelligentsia Coffee website (<http://www.intelligentsiacoffee.com/content/direct-trade>)

Finally, the third benchmark represents a high leasing quantity that at lowest realization of yield (y_l) can fulfill the maximum market demand ($M\alpha(\varphi_h)$). We define this last benchmark as:

$$Q^H = M\alpha(\varphi_h) / y_l.$$

Remark 1. $Q^L < Q^D < Q^H$.

Next, we use these three benchmark quantities to characterize four distinct DT investment policies, and then identify the subset of policies that can potentially be optimal. We denote the optimal amount of farmland to be leased by Q^* , and describe the set of four different investment policies as follows:

- (1) Underinvestment policy: $0 < Q^* < Q^L$,
- (2) Moderately-low-investment policy: $Q^L \leq Q^* < Q^D$,
- (3) Moderately-high investment policy: $Q^D \leq Q^* < Q^H$,
- (4) Overinvestment policy: $Q^* \geq Q^H$.

Figure 4 illustrates these four investment policies.

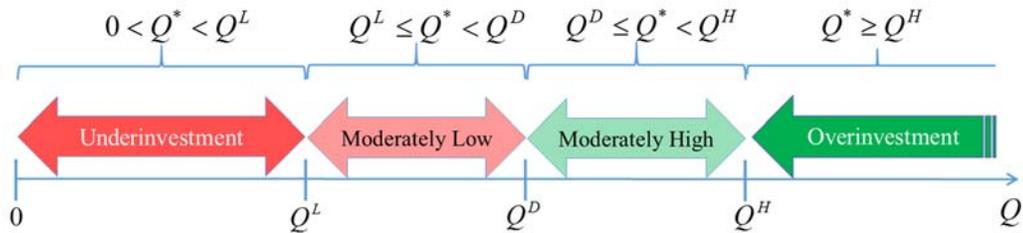


Figure 4. DT Investment policies.

Our analysis indicates that, although the overinvestment policy is never optimal, under yield and commercial price uncertainty, the processor deliberately underinvests in DT under certain conditions. We describe this conservative sourcing behavior as the “underinvestment policy” because the processor reduces the leasing amount of farmland to the extent that it is guaranteed to operate under limited supply. Therefore, at any realization of the yield and the commercial price random variables, (1) the processor increases the selling price of the specialty coffee in order to reduce the demand for specialty coffee and match the realized crop supply; and (2) it serves a market demand that is smaller than the smallest possible market that is optimal to target at the end of the growing season. In other words, by lowering the investment level below a threshold according to the underinvestment policy, the processor chooses to always operate under supply shortage, and never fully utilizes the potential specialty coffee demand at the realized market price φ , but rather uses its pricing flexibility to reduce the market demand to match the realized crop supply.

It is worth noting that, as mentioned above, when there is no uncertainty associated with the DT crop supply or the commercial market, the processor invests in the exact amount of farmland equal to Q^D ,

yielding the amount of supply that fulfills the target specialty coffee demand that is equal to $M\alpha(\bar{\varphi})$.

Therefore, the processor has no incentive to underinvest in DT when the supply and the commercial market conditions are deterministic. Our analysis, however, shows that the underinvestment policy can be optimal under either of yield and commercial price uncertainty.

In the global manufacturing network literature, Kazaz et al. (2005) are the first to introduce an effective operational hedging policy called production hedging that involves producing less than the total demand and thus underserving the markets. Park et al. (2016) build on this work by showing that the production hedging policy is a viable risk mitigation approach. While in both these studies exchange-rate uncertainty is the main driver of underserving the markets, our work indicates that yield uncertainty as well as the price uncertainty in the competing market can result in a similar operational policy in an agricultural environment. Moreover, production hedging leads to higher profits because of the firm's flexibility to allocate supplies to markets with appreciating currencies. In our model, however, higher profits are obtained in the underinvestment policy because of the firm's flexibility to set prices. In sum, the underinvestment policy explored in this paper can be considered as a "capacity hedging" policy where the low amount of leased farmland proactively restricts the production quantity such that the firm ends up underserving the specialty market, regardless of the realizations of yield and the commercial price.

Next, we identify the conditions that determine the unique optimal investment policy, and show how the three potentially optimal policies are obtained by reviewing these optimality conditions. The optimality conditions are:

$$\text{OC1: } \iint_A y \left(v_s + \beta \ln \left[\frac{y_h - \alpha(\varphi_l)y}{k(\varphi)\alpha(\varphi_l)y} \right] - \beta \frac{y_h}{y_h - \alpha(\varphi_l)y} \right) g(y)h(\varphi)dyd\varphi < l + r\bar{y}$$

$$\text{OC2: } \iint_{U(Q^D)} y \left(v_s + \beta \ln \left[\frac{M - Q^D y}{k(\varphi)Q^D y} \right] - \beta \frac{M}{M - Q^D y} \right) g(y)h(\varphi)dyd\varphi + \iint_{\alpha(Q^D)} syg(y)h(\varphi)dyd\varphi < l + r\bar{y}$$

Proposition 5. *Let A represent the two-dimensional support of y and φ (i.e.,*

$$A = \{(y, \varphi) \mid y_l \leq y \leq y_h, \varphi_l \leq \varphi \leq \varphi_h\}.$$

- (a) *It is optimal for the processor to underinvest in DT (i.e., $Q^* < Q^L$) if and only if OC1 holds;*
- (b) *It is optimal for the processor to invest in a moderately-low amount of DT (i.e., $Q^L \leq Q^* < Q^D$) if and only if OC1 does not hold and OC2 holds;*
- (c) *It is optimal for the processor to invest in a moderately-high amount of DT (i.e., $Q^D \leq Q^* < Q^H$) if and only if OC2 does not hold;*
- (d) *The processor should never overinvest in DT, i.e., $Q^* < Q^H$.*

Proposition 5 identifies the conditions that lead to each investment policy. OC1 establishes that the processor may be better off by lowering its investment level such that it ensures the specialty crop supply falls short of the optimal market size to be targeted by the processor at any realization of the yield and the commercial price. Note that according to Proposition 3, the smallest portion of the specialty segment is targeted by the processor when the commercial price is realized at its lowest level (φ_l). In addition, we observe that when the processor faces supply shortage, it charges a market clearing price. OC1 implies that the processor benefits from operating under supply shortage and charging market clearing price than investing more on DT when the marginal cost of additional supply (right-hand side of the inequality) overweighs its expected marginal revenue (left-hand side of the inequality) even when the highest possible realization of the DT supply (Q_{y_h}) can barely fulfill the lowest possible level of optimal demand (i.e., $M\alpha(\varphi_l)$). Proposition 5 also suggests that, under uncertainty, the processor may increase or decrease its leasing quantity relative to the optimal leasing quantity in deterministic environment. However, our numerical analysis presented in Section 6 demonstrates that, using parameter values representative of the real-world environment, the amount of DT investment decreases in uncertainty, and therefore, even the moderately-high investment policy may not emerge as an optimal solution. It is also noteworthy that Proposition 5 formally shows that under uncertainty, the processor should not overinvest in DT, reflecting the dominant downward impact of uncertainty on the amount of DT investment.

The following section focuses on the underinvestment policy and examines the driving forces of this highly conservative sourcing practice in more details.

4.4. Underinvestment Policy

We first establish the optimal investment decision and profit under the underinvestment policy, and then explore how the processor's investment decision is affected by the variation in specialty crop yield stemming from DT and the fluctuations in the price of the commercial coffee.

Remark 2. *When it is optimal for the processor to underinvest, the optimal leasing quantity, denoted Q_U^* , is the unique solution satisfying the following condition:*

$$\iint_A y \left(v_s + \beta \ln \left[\frac{M - Qy}{k(\varphi)Qy} \right] - \beta \frac{M}{M - Qy} \right) g(y)h(\varphi)dyd\varphi = l + r\bar{y}, \quad (8)$$

and the expected profit for the underinvestment policy can be expressed as:

$$\Pi_U(Q_U^*) = M\beta \int_{y_l}^{y_h} \frac{Q_U^* y}{M - Q_U^* y} g(y)dy. \quad (9)$$

4.4.1. Impact of uncertainties on the investment policy, profit and consumer surplus

In this section, we investigate how the investment policy is affected by the yield variation and the commercial price fluctuations. We particularly study whether these uncertainties encourage or hinder the

underinvestment policy, and whether their effects reinforce or counterbalance the opposing forces in the optimal lease decision. The following proposition refers to a two-point distribution. We note that for given support and mean, the variance of a random variable is maximized under a two-point distribution, which

for random price and yield are $P[\Phi = \varphi_l] = \frac{\varphi_h - \bar{\varphi}}{\varphi_h - \varphi_l}$, $P[\Phi = \varphi_h] = 1 - P[\Phi = \varphi_l]$, $P[Y = y_l] = \frac{y_h - \bar{y}}{y_h - y_l}$, $P[Y = y_h] = 1 - P[Y = y_l]$.

Proposition 6. (a) *If for a given level of commercial price uncertainty (yield uncertainty), the underinvestment policy is optimal under a two-point distribution of random yield (random commercial price), there exists a threshold for the degree of yield uncertainty (commercial price uncertainty) above which the underinvestment policy is optimal. (b) When the underinvestment policy becomes optimal at a higher level of yield uncertainty (commercial price uncertainty), the expected profit is lower compared to the case with lower levels of yield uncertainty (commercial price uncertainty) where the underinvestment policy is not optimal.*

The above proposition implies that both types of uncertainties faced by the processor lead to the underinvestment policy becoming a more viable sourcing policy. Specifically, as long as the underinvestment policy is optimal under the highest level of yield or commercial price variation (under a two-point distribution), the processor eventually implements the underinvestment policy as variation increases. The firm continues to implement this policy as the variation further increases. Additionally, the fact that underinvestment is associated with a lower expected profit suggests that this policy is adopted under unfavorable circumstances caused by the uncertainty in yield and commercial price.

We note that under the underinvestment policy, the marginal revenue in the left-hand side of (8) decreases as the variation in yield or commercial price increases. Consequently, at higher levels of yield or commercial price uncertainty, the processor eventually restricts its DT investment to such low levels that result in serving a specialty coffee demand that is strictly less than the minimum market size that is optimal to serve at the end of the growing season. This finding implies that, as the operating environment becomes more uncertain, the underinvestment policy allows the processor to exercise its monopolistic power over the entire specialty supply (by charging a market-clearing price), and at the same time, save on the opportunity cost of facing overage at higher investment levels.

We established above that the variation in the specialty yield as well as the uncertainty in the commercial market encourage the adoption of the underinvestment policy. We next investigate which uncertainty is the main driver of this behavior. Are both yield and commercial price uncertainties necessary for the underinvestment policy to become an effective sourcing policy? Specifically, we examine whether either of these uncertainties by itself result in the underinvestment policy.

Proposition 7. *Either yield uncertainty or commercial price uncertainty by itself can lead to the underinvestment policy.*

Our analysis demonstrates that either source of uncertainty in this operating environment can cause the processor to adopt the underinvestment policy. In other words, neither yield nor commercial price uncertainty can be considered necessary or the primary driver of the underinvestment policy; they both contribute significantly to the optimality of this sourcing policy.

The next proposition identifies how the degree of uncertainty in yield and commercial price random variables affect the amount of DT investment when the processor implements the underinvestment policy.

Proposition 8. *When the processor adopts the underinvestment policy, as the degree of yield uncertainty or commercial price uncertainty increases, (a) the optimal leasing quantity decreases, and (b) the expected profit decreases.*

The above proposition, along with the findings in propositions 6 and 7, suggest that the optimal leasing amount and the expected profit exhibit a diminishing behavior under an increasingly uncertain operating environment. This is an important insight because when the underinvestment policy is not optimal, the analytical investigation is inconclusive about the exact behavior of the leasing quantity and the expected profit with respect to uncertainty. Propositions 6 and 7 imply that higher degrees of uncertainty eventually lead to the underinvestment policy with lower levels of investment in DT farmland and expected profit. Proposition 8 complements these findings by showing that, when the underinvestment policy is optimal, additional uncertainty reduces the optimal amount of investment and the expected profit. We can, therefore, conclude that, when the underinvestment policy is the prevailing choice for the processor, the overall leasing quantity and the expected profit decreases in yield and commercial price uncertainty.

It is also worth elaborating on the impact of the underinvestment policy on the consumer surplus. As mentioned before, when the underinvestment policy is implemented, the processor always operates under supply shortage, and utilizes its pricing power to charge a higher market-clearing price. This suggests a decline in the consumer surplus as the consumers (of the specialty segment) have to pay higher prices compared to the case where the processor does not adopt the underinvestment policy. As a result, the circumstances that lead to the underinvestment policy being optimal can hurt both parties—the processor and the consumers. The next section discusses an investment strategy that helps the processor to mitigate the likelihood of the underinvestment policy, and can potentially improve the lives of growers around the world.

4.4.2. Impact of correlation

In this section, we relax the independence assumption between the two random variables describing yield and market price uncertainty, and explore the impact of correlation between the specialty crop supply obtained through DT and the price of the commercial coffee set by the global market. We note that the

global yield of the commercial coffee beans, negatively impacts the commercial coffee price, and therefore, any positive (negative) correlation between the specialty yield and the global yield results in a negative (positive) correlation between the random yield and the random commercial price.

The next proposition provides insights into how the correlation between the DT yield and the commercial price affects the sourcing policy.

Proposition 9. *(a) If the underinvestment policy is optimal under a certain level of positive (negative) correlation between the DT yield and the global yield, it is also optimal under more positive (less negative) levels of correlation. (b) When the underinvestment policy is optimal under all levels of correlation, the expected profit increases (decreases) as the correlation becomes more negative (positive).*

The finding in Proposition 9 implies that the underinvestment policy is more (less) likely to be optimal under stronger positive (negative) levels of correlation between the specialty DT yield and the commercial yield. Therefore, a lower degree of correlation between the specialty yield and the commercial yield allows the processor to invest more freely in DT. Recall that propositions 6, 7, and 8 imply either type of uncertainty faced by the processor is undesirable in this environment; both forms of uncertainty negatively affect the processor’s expected profit and the overall investment amount in DT. Proposition 9 suggests that positive (negative) correlation exacerbates (diminishes) the adversarial effects of the variation in the specialty yield and the uncertainty in the commercial market, and diminishes (improves) the amount of investment in DT and the associated expected profit.

The consequence of the above result is that it is beneficial for the coffee roasters to invest more in the agricultural regions with the crop yield that is more strongly and negatively correlated with the global commercial coffee yield. It is well known that Brazil has the highest share of coffee beans supply in the global market. Our analysis suggests that investing in agricultural regions with strong negative (or weak positive) correlation with Brazil’s coffee yield—as much as the geographical properties necessary for high-quality coffee allows—is advantageous to the third-wave coffee roasters who utilize DT. In sum, this strategy encourages investment in small and developing agricultural areas, and potentially improves the lives of growers in the less popular coffee-growing regions. In a similar vein, a roaster may direct a portion of its technical assistance to growers on actions that help reduce the negative effect of weather and disease on yield.

5. Numerical Analysis

In this section, we present numerical analyses designed to shed light into the impact of uncertainty on the processor’s investment decision using parameter values representative of the real-world setting.

Considering the complexity of the analytical expressions capturing two different types of uncertainty, we utilize two symmetric three-point distributions that represent random yield and random commercial price. We choose three-point distributions because (1) they simplify the optimal decision computation; and, (2)

they enable us to demonstrate effectively the impact of the degree of uncertainty by adjusting the level of variation. For the latter reason, we vary the degree of uncertainty within a full range of standard deviation values—from zero standard deviation corresponding to deterministic case to the highest degree of standard deviation corresponding to two-point distribution—by changing the end-point probabilities.

We first describe our source of information for random yield in DT. We determine the fluctuations in random yield by considering the fluctuations in the price of commercial coffee beans as depicted in Figure 2 as a proxy to infer the variation in commercial coffee yield; we think that the fluctuations in commercial yield represents the random yield in DT. We establish a support as follows: The mean is scaled to 1, and the upper support is approximately 80% higher than the lower support – as Figure 2 suggests – in the symmetric three-point distribution. Therefore, the lower and upper support points are 0.7 and 1.3, respectively.

We determine the commercial price of coffee from data provided by the International Coffee Organization (ICO: www.ico.org) which publishes the annual average retail price of roasted coffee in selected importing countries. Figure 5 illustrates the retail price of roasted coffee in the United States from 2010 to 2015. It shows that the retail price for coffee ranges from \$4 to \$6 per pound for the commercial coffee. Therefore, we use a symmetric three-point distribution with lower support and upper support points at \$4 and \$6 per pound, respectively, and a mean of \$5 per pound.

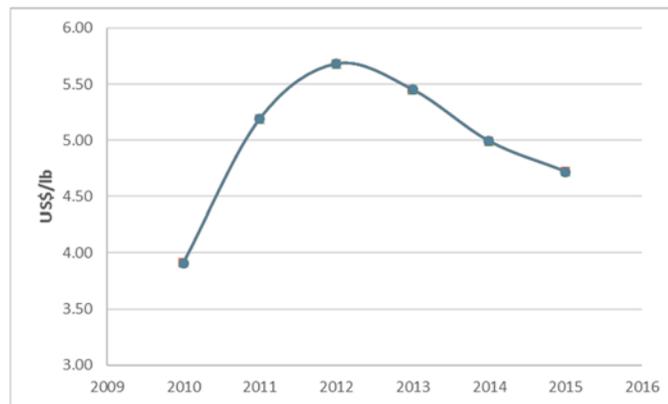


Figure 5. Retail price of roasted coffee beans in the US.

Next, we describe the choice of parameter values for the consumer valuation of the specialty product (v_s), and β . We utilize the retail price of the DT coffee on the Intelligentsia Coffee, and Stumptown Coffee Roasters websites, and consider the common mid-range prices of \$16.00-\$18.00 per 12 oz. of DT coffee. We convert these prices to per pound prices of \$21.30-24.00 range, and set the value of v_s at \$26.00 reflecting the specialty segment’s willingness to pay for the DT coffee, higher than the range of prices that the roaster charges. We set the specialty segment’s valuation of the commercial coffee at the mean

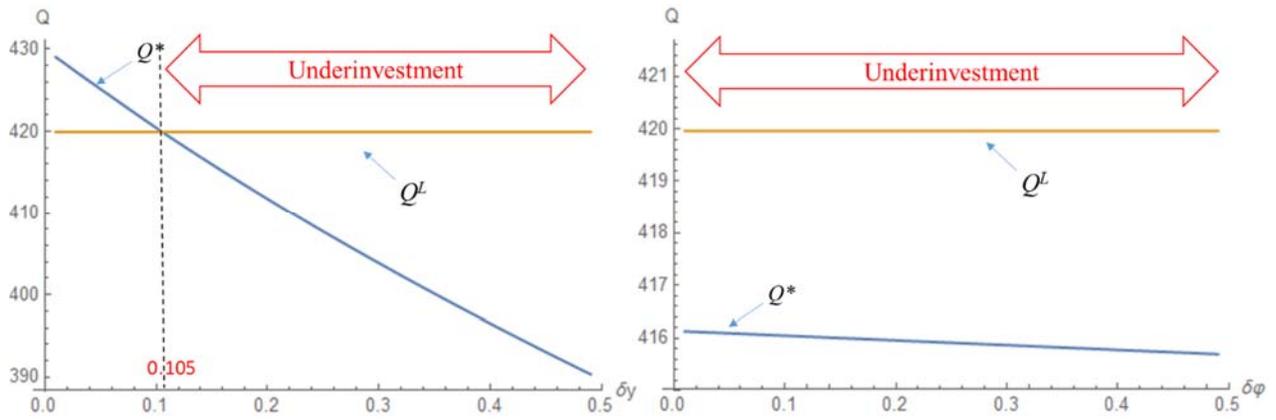
commercial price, i.e., $v_c = \$5.00$, and the salvage value of the DT coffee at $\$7.00$, i.e., $s = \$7.00$ which is above the highest possible commercial coffee price ($\$6.00$). We note that the salvage value of the DT coffee being greater than the commercial price suggests a premium even for the dated DT coffee, and lower salvage values make our underinvestment policy more pronounced. We then choose a range of values for β such that the optimal unconstrained price resulting from our analytical model conforms to the range of prices charged by Intelligentsia and Stumptown ($\$21.30$ - $\$24.00$ per pound). Note that the unconstrained optimal price is a lower bound for the range of possible optimal prices. Therefore, we conduct the analysis and illustrate the results using three values of 6, 8, and 10 for β , which result in unconstrained optimal prices of $\$20.50$, $\$21.78$, and $\$23.45$, respectively.

It can be observed from the condition in (6) that when the mean of DT yield is scaled to 1, the investment decision depends on the summation of the two cost terms (on the right-hand side of the equation). We set $l = \$9.00$ per area of the farmland that yields on average one pound of coffee, and $r = \$4.00$, which adds up to $\$13.00$ as the total unit cost of coffee. It can also be observed that higher levels of total cost make the underinvestment policy more likely to be adopted by the processor. We choose $M = 1000$ for the market potential.

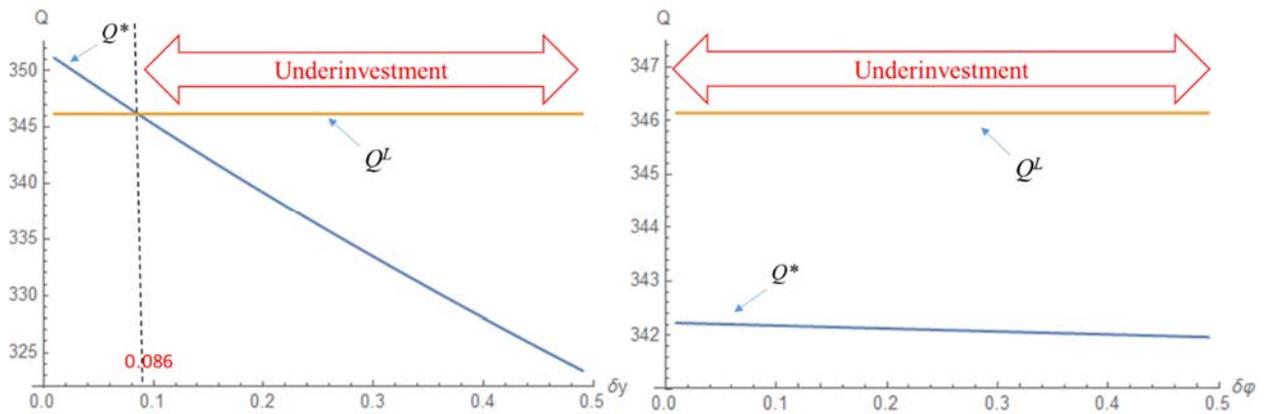
In order to examine the impact of uncertainty on the investment decision, and particularly the likelihood of the underinvestment policy, we vary the end-point probabilities of the yield and commercial price uncertainty—denoted by δ_y and δ_p , respectively—from 0.01 to 0.49, and investigate the changes in the investment decision. Figure 6 illustrates the results.

Several observations can be made from the numerical analyses presented in Figure 6. First, the underinvestment policy is the prevailing policy under a fair degree of yield and commercial price uncertainty. Figure 6(c) where price is at its highest level features the only set of parameters under which the underinvestment does not appear to be optimal.

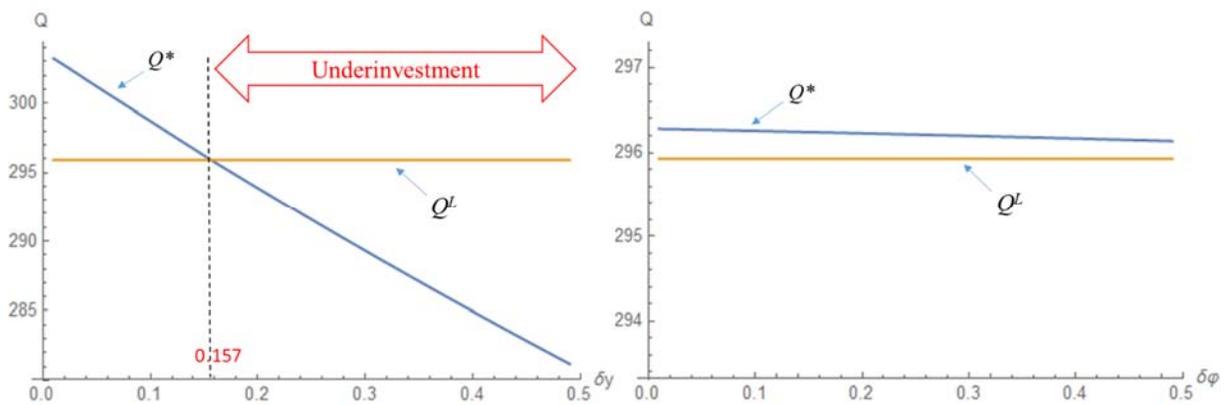
Second, yield uncertainty in DT has a stronger effect on the investment decisions than the commercial price uncertainty. This phenomenon can be observed by comparing the left figure with the right one in each part of Figure 6. We note that the target market for the processor is the niche segment of consumers who prefer the specialty product over the commercial one and are willing to pay the premium. Therefore, the fluctuations in the commercial price may not influence the specialty consumers' choice significantly, and consequently, the effect of variations in the commercial market can be considered an indirect cross-market effect. However, the DT yield directly influences the crop supply targeted toward the specialty segment, and thus, the processor's profit. In other words, yield uncertainty in DT has a direct own-market effect which influences the investment decision more strongly. In addition, at higher levels of farmland invested in DT, yield uncertainty amplifies the variation in the crop supply, which causes the processor to operate under higher degrees of uncertainty.



(a) $\beta = 6, p^0 = \$20.50, Q^L = 419.96, Q^D = 555.60$



(b) $\beta = 8, p^0 = \$21.78, Q^L = 346.14, Q^D = 458.70$



(c) $\beta = 10, p^0 = \$23.45, Q^L = 295.93, Q^D = 392.15$

Figure 6. Investment decision at different levels of uncertainty (p^0 = unconstrained optimal price per pound). In each figure, either δ_y or δ_ϕ varies while the other one is set at 0.15).

Third, we do not observe moderately-high investment policy as the optimal solution within the chosen ranges of parameters which are inspired by the real-world environment. As mentioned before, our analytical examination is inconclusive about the behavior of the investment decision with respect to uncertainty. We conjecture that increasing degrees of uncertainty—whether DT yield or commercial price uncertainty—lead to lower levels of leasing quantity, and therefore, the moderately-high investment policy does not emerge in our numerical analyses. It is worth remembering that Proposition 5 formally shows that the overinvestment policy is never optimal.

6. Conclusions and Managerial Insights

This paper examines sourcing and pricing decisions of an agricultural processor under yield uncertainty of the agricultural input required for the offered specialty product and the price uncertainty of the competing commercial product. Our study is motivated by a recent trend in the coffee industry and it makes three main contributions.

First, we develop an analytical model which captures a trending supply chain framework where Direct Trade is an emerging sourcing practice. In order to target the quality-sensitive segment of consumers, a small number of coffee roasters have drastically changed their sourcing policy and engaged in Direct Trade, which involves direct communication and close collaboration with the growers, but results in increased exposure to crop supply risk. We identify the optimal sourcing decisions under yield and commercial price uncertainty in the first stage of our proposed model, and the optimal pricing policy upon realizations of the specialty crop supply and the commercial product's price.

Second, we analyze the impact of yield and commercial price uncertainty, and demonstrate that a viable sourcing policy—which would never be adopted in a deterministic setting—emerges as a result of uncertainty. This policy requires the processor to deliberately underinvest in Direct Trade. This conservative sourcing policy highlights the significant impact of the variation in the specialty yield as well as the uncertainty in the commercial market on the investment decisions of the processor.

Third, our analysis indicates that negative correlation between the specialty yield and the global commercial yield counterbalances the detrimental effects of uncertainty and improves the Direct Trade investment decision and the expected profit, overall. Consequently, our study suggests that the specialty coffee roaster benefits from investing in agricultural regions where the yield is more negatively correlated with the global yield. This strategy promotes investment in developing agricultural regions, and potentially improves the lives of growers in the less popular coffee-growing regions. In addition, this sourcing policy reduces the likelihood of underinvestment, and it leads to a greater amount of specialty coffee supply at lower prices, benefiting consumers with a higher consumer surplus.

While Direct Trade is commonly used in specialty coffee industry, it can be utilized as an exemplary sourcing practice for other premium agricultural products. Our findings, therefore, can be generalized for various agricultural businesses.

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Appendix

Proof of Proposition 1. The first- and second-order derivatives of the second-stage profit function (4) with respect to q are respectively:

$$\frac{d\pi(q)}{dq} = v_s - s + \beta \ln \left[\frac{M-q}{k(\varphi)q} \right] - \frac{\beta M}{M-q}$$

$$\frac{d^2\pi(q)}{dq^2} = -\frac{\beta M}{q(M-q)} - \frac{\beta M}{(M-q)^2} = -\frac{\beta}{q} \left[\frac{M}{(M-q)} \right]^2 < 0$$

Thus, as shown in Proposition 1 in Noparumpa et al. (2015) for a similar problem, $\pi(q)$ is concave in q . The unconstrained optimal processing quantity $q^0(\varphi)$ can then be obtained from the first order condition (FOC) (We use superscript “0” for the optimal decisions associated with the problem where there is no supply

constraint). That is, $q^0(\varphi)$ solves $v_s - s + \beta \ln \left[\frac{M-q}{k(\varphi)q} \right] - \frac{\beta M}{M-q} = 0$ which implies $\frac{M-q}{k(\varphi)q} = e^{\frac{M}{M-q} \frac{v_s-s}{\beta}}$. We

rearrange to get $\frac{q}{M-q} e^{\frac{q}{M-q}} = e^{(v_s-s)/\beta-1} / k(\varphi)$ which conforms to the Lambert W function structure. Thus,

$\frac{q}{M-q} = W \left(e^{(v_s-s)/\beta-1} / k(\varphi) \right)$ which implies

$$q^0(\varphi) = M \frac{W \left(e^{(v_s-s)/\beta-1} / k(\varphi) \right)}{1 + W \left(e^{(v_s-s)/\beta-1} / k(\varphi) \right)} = M\alpha(\varphi) \quad (10)$$

Substituting (10) into (2), we get

$$p^0(\varphi) = v_s + \beta \ln \left[\frac{1}{k(\varphi)W \left(e^{(v_s-s)/\beta-1} / k(\varphi) \right)} \right] = v_s - \beta \ln \left[k(\varphi)W \left(e^{(v_s-s)/\beta-1} / k(\varphi) \right) \right].$$

Note that by the definition of the Lambert W function $W(z)e^{W(z)} = z$, which implies $k(\varphi)W(z)e^{W(z)} = k(\varphi)z$, and thus $\ln[k(\varphi)W(z)] + W(z) = \ln[k(\varphi)z]$. Let $z = e^{(v_s-s)/\beta-1} / k(\varphi)$. It follows that $\ln \left[k(\varphi)W \left(e^{(v_s-s)/\beta-1} / k(\varphi) \right) \right] + W \left(e^{(v_s-s)/\beta-1} / k(\varphi) \right) = \ln \left[e^{(v_s-s)/\beta-1} \right] = (v_s - s) / \beta - 1$. Therefore, we can re-write $p^0(\varphi)$ as

$$\begin{aligned} p^0(\varphi) &= v_s - \beta \left[(v_s - s) / \beta - 1 - W \left(e^{(v_s-s)/\beta-1} / k(\varphi) \right) \right] \\ &= s + \beta \left[1 + W \left(e^{(v_s-s)/\beta-1} / k(\varphi) \right) \right] \end{aligned}$$

Therefore, if $Qy \geq q^0(\varphi)$, $q^*(\varphi) = q^0(\varphi)$, $p^*(\varphi) = p^0(\varphi)$, and the optimal second-stage profit is

$$\pi^*(Q, y, \varphi) = M\beta W \left(e^{(v_s-s)/\beta-1} / k(\varphi) \right) + (s-r)Qy$$

which results from substituting (10) into (4).

Otherwise, if $Qy < q^0(\varphi)$, due to concavity of $\pi(q)$, $q^*(\varphi) = Qy$, and equations (2) and (4) yield $p^*(\varphi)$ and $\pi^*(Q, y, \varphi)$, respectively. \square

Proof of Proposition 2. If $Qy < M\alpha(\varphi)$, taking the first-order derivative of $p^*(\varphi, y)$ expressed in Proposition 1 with respect to φ , we get

$$\frac{\partial p^*(\varphi, y)}{\partial \varphi} = \beta \frac{-k'(\varphi)}{k(\varphi)} = \frac{e^{(v_c-\varphi)/\beta}}{1+e^{(v_c-\varphi)/\beta}} = 1 - \frac{1}{k(\varphi)} \quad \text{if } Qy < M\alpha(\varphi) \quad (11)$$

where the last two equalities use the definition of $k(\varphi)$ by (3).

If $Qy \geq M\alpha(\varphi)$, we have

$$\frac{\partial p^*(\varphi)}{\partial \varphi} = \beta \frac{dW(z)}{dz} \cdot \frac{dz(\varphi)}{d\varphi} \quad (12)$$

where $z(\varphi) = e^{(v_s - s)/\beta - 1} / k(\varphi)$. In order to find $dW(z)/dz$, we take the derivative of both sides of

$W(z)e^{W(z)} = z$ with respect to z (Noparumpa et al. 2015) to get $W'(z)[e^{W(z)} + W(z)e^{W(z)}] = 1$ which yields

$$\frac{dW(z)}{dz} = \frac{W(z)}{z[1+W(z)]} = \frac{\alpha(\varphi)}{z(\varphi)} \quad (13)$$

We then find the derivative of $z(\varphi)$ with respect to φ as

$$\frac{dz(\varphi)}{d\varphi} = \frac{-k'(\varphi)}{k(\varphi)^2} e^{(v_s - s)/\beta - 1} = \frac{-k'(\varphi)z(\varphi)}{k(\varphi)} = \frac{z(\varphi)}{\beta} \left(1 - \frac{1}{k(\varphi)}\right) \quad (14)$$

Therefore, substituting (13) and (14) into (12), we get

$$\frac{\partial p^*(\varphi, y)}{\partial \varphi} = \alpha(\varphi) \left(1 - \frac{1}{k(\varphi)}\right) \quad \text{if } Qy \geq M\alpha(\varphi) \quad (15)$$

Note that by definition, $\alpha(\varphi) < 1$ (Proposition 1), and $k(\varphi) > 1$ (equation (3)). Therefore, (11) and (15) imply that $\partial p^*(\varphi, y)/\partial \varphi < 1$. \square

Proof of Proposition 3. From Proposition 1, we can re-write $\alpha(\varphi)$ expression as $\alpha(\varphi) = W(z) / [1 + W(z)]$

where $z = e^{(v_s - s)/\beta - 1} / k(\varphi)$. By chain rule, we have

$$\frac{d\alpha(\varphi)}{d\varphi} = \frac{d\alpha(W)}{dW} \cdot \frac{dW(z)}{dz} \cdot \frac{dz(\varphi)}{d\varphi} \quad (16)$$

The first derivative on the right-hand side (RHS) is

$$\frac{d\alpha(W)}{dW} = \frac{1}{[1 + W(z)]^2} \quad (17)$$

Using the derivations in the proof of Proposition 2, we substitute (17), (13), and (14) into (16) to get

$$\frac{d\alpha(\varphi)}{d\varphi} = \frac{W(z)}{\beta[1 + W(z)]^3} \left(1 - \frac{1}{k(\varphi)}\right)$$

which is positive. The results follow. \square

Proof of Proposition 4. (a) The first-order derivative of the expected profit function (5) with respect to Q is

$$\frac{\partial \Pi(Q)}{\partial Q} = -(l + r\bar{y}) + \iint_{U(Q)} y \left(v_s + \beta \ln \left[\frac{M - Qy}{k(\varphi)Qy} \right] - \beta \frac{M}{M - Qy} \right) g(y)h(\varphi) dy d\varphi + \iint_{\alpha(Q)} syg(y)h(\varphi) dy d\varphi$$

and the second-order derivative with respect to Q is:

$$\frac{\partial^2 \Pi(Q)}{\partial Q^2} = \iint_{U(Q)} \frac{-\beta y}{Q} \left(\frac{M}{M - Qy} \right)^2 g(y)h(\varphi) dy d\varphi < 0.$$

Consequently, the expected profit function is concave in Q and (6) is the FOC which yields the unique optimal leasing quantity.

(b) By substituting Q^* into (5) and rearranging, the optimal expected profit can be expressed as:

$$\begin{aligned}
\Pi(Q^*) &= -(l + r\bar{y})Q^* \\
&+ Q^* \left[\iint_{U(Q^*)} y \left(v_s + \beta \ln \left[\frac{M - Q^* y}{k(\varphi)Q^* y} \right] - \beta \frac{M}{M - Q^* y} \right) g(y)h(\varphi) dy d\varphi + \iint_{O(Q^*)} syg(y)h(\varphi) dy d\varphi \right] \\
&+ M\beta \left[\iint_{U(Q^*)} \frac{Q^* y}{M - Q^* y} g(y)h(\varphi) dy d\varphi + \iint_{O(Q^*)} W \left(\frac{e^{(v_s - s)/\beta - 1}}{k(\varphi)} \right) g(y)h(\varphi) dy d\varphi \right]
\end{aligned} \tag{18}$$

Substituting the FOC (6) into (18), the first two terms cancel out, which results in (7). \square

Proof of Remark 1. $Q^D = M\alpha(\bar{\varphi})/\bar{y}$ is larger than $Q^L = M\alpha(\varphi_l)/y_h$, because $\bar{y} < y_h$, and according to proposition 3, $\bar{\varphi} > \varphi_l$ implies $\alpha(\bar{\varphi}) > \alpha(\varphi_l)$. Similarly, $Q^D < Q^H = M\alpha(\varphi_h)/y_l$ because $\bar{y} > y_l$ and $\alpha(\bar{\varphi}) < \alpha(\varphi_h)$ \square

Proof of Proposition 5. (a) By definition, the processor underinvests in DT if and only if $Q^* < Q^L$. Therefore, the first-order derivative of the (first-stage) expected profit function with respect to Q evaluated at $Q^L = M\alpha(\varphi_l)/y_h$ must be negative (Note that in this case $Qy < M\alpha(\varphi)$ for all realizations of y and φ):

$$\frac{\partial \Pi(Q)}{\partial Q} \Big|_{\frac{M\alpha(\varphi_l)}{y_h}} = -(l + r\bar{y}) + \iint_A y \left(v_s + \beta \ln \left[\frac{y_h - \alpha(\varphi_l)y}{k(\varphi)\alpha(\varphi_l)y} \right] - \beta \frac{y_h}{y_h - \alpha(\varphi_l)y} \right) g(y)h(\varphi) dy d\varphi < 0$$

which results in OC1.

(b) First, note that due to concavity of the expected profit function in Q , the left-hand side (LHS) of OC2 is smaller than that of OC1 (since $Q^D > Q^L$). Therefore, if OC1 holds, OC2 holds as well, but not vice versa. For $Q^* \geq Q^L$ to hold, OC1 must not hold according to part (a). In addition, for $Q^* < Q^D$ to hold, the first-order derivative of the (first-stage) expected profit function with respect to Q evaluated at Q^D must be negative, which results in OC2.

(c) For $Q^* \geq Q^D$ to hold, OC2 must not hold according to part (b). Also, for $Q^* < Q^H$ to hold, the first-order derivative of the (first-stage) expected profit function with respect to Q evaluated at $Q^H = M\alpha(\varphi_h)/y_l$ must be negative (Note that in this case $Qy > M\alpha(\varphi)$ for all realizations of y and φ), i.e.,

$$\frac{\partial \Pi(Q)}{\partial Q} \Big|_{\frac{M\alpha(\varphi_h)}{y_l}} = -(l + r\bar{y}) + \iint_A syg(y)h(\varphi) dy d\varphi = (s - r)\bar{y} - l < 0$$

which always holds because the salvage value is lower than the total expected unit sourcing cost, i.e., $s < l/\bar{y} + r$.

(d) The results directly follow from the proof of part (c). \square

Proof of Remark 2. (a) When the processor adopts the underinvestment policy, the crop supply always falls short of the optimal demand and the processor sets a market-clearing price in the second stage, which results in the optimal second-stage profit as expressed below:

$$\pi_U^*(Q, y, \varphi) = Qy \left(v_s - r + \beta \ln \left[\frac{M - Qy}{k(\varphi)Qy} \right] \right)$$

Therefore, the first-stage expected profit function for a Q associated with underinvestment policy can be expressed as:

$$\Pi_U(Q) = -(l + r\bar{y})Q + \iint_A Qy \left(v_s + \beta \ln \left[\frac{M - Qy}{k(\varphi)Qy} \right] \right) g(y)h(\varphi) dy d\varphi \tag{19}$$

From Proposition 4, we can infer that $\Pi_U(Q)$ is concave in Q , and thus the below FOC yields the optimal leasing quantity (Q_U^*).

$$\frac{\partial \Pi_U(Q)}{\partial Q} = -l - r\bar{y} + \iint_A y \left(v_s + \beta \ln \left[\frac{M - Qy}{k(\varphi)Qy} \right] - \beta \frac{M}{M - Qy} \right) g(y)h(\varphi) dy d\varphi = 0 \tag{20}$$

(b) Note that by substituting Q_U^* into (19) and rearranging, the optimal expected profit can be written as:

$$\begin{aligned} \Pi_U(Q_U^*) &= -(l + r\bar{y})Q_U^* + Q_U^* \left[\iint_A y \left(v_s + \beta \ln \left[\frac{M - Q_U^* y}{k(\varphi) Q_U^* y} \right] - \beta \frac{M}{M - Q_U^* y} \right) g(y) h(\varphi) dy d\varphi \right] \\ &\quad + \iint_A M \beta \frac{Q_U^* y}{M - Q_U^* y} g(y) h(\varphi) dy d\varphi \end{aligned}$$

Substituting (20) into (19), the first two terms cancel out, and we get (9):

$$\Pi_U(Q_U^*) = \iint_A M \beta \frac{Q_U^* y}{M - Q_U^* y} g(y) h(\varphi) dy d\varphi = M \beta \int_{y_l}^{y_h} \frac{Q_U^* y}{M - Q_U^* y} g(y) dy. \quad \square$$

Proof of Proposition 6. (a) OC1 can be expressed as $-l - r\bar{y} + E_\Psi [E_Y (n(Y, \Phi))] < 0$, where

$$\begin{aligned} n(y, \varphi) &= y \left(v_s + \beta \ln \left[\frac{y_h - \alpha(\varphi_h) y}{k(\varphi) \alpha(\varphi_h) y} \right] - \beta \frac{y_h}{y_h - \alpha(\varphi_h) y} \right) \\ &= y \left(v_s + \beta \ln [y_h - \alpha(\varphi_h) y] - \beta \ln [k(\varphi) \alpha(\varphi_h) y] - \beta \frac{y_h}{y_h - \alpha(\varphi_h) y} \right) \end{aligned} \quad (21)$$

Note that $n(y, \varphi)$ is concave in y . This is because

$$\begin{aligned} \frac{\partial n(y, \varphi)}{\partial y} &= v_s + \beta \ln \left[\frac{y_h - \alpha(\varphi_h) y}{k(\varphi) \alpha(\varphi_h) y} \right] - \beta \frac{y_h}{y_h - \alpha(\varphi_h) y} + y \beta \left(\frac{-\alpha(\varphi_h)}{y_h - \alpha(\varphi_h) y} - \frac{1}{y} - \frac{\alpha(\varphi_h) y_h}{(y_h - \alpha(\varphi_h) y)^2} \right) \\ &= v_s + \beta \ln \left[\frac{y_h - \alpha(\varphi_h) y}{k(\varphi) \alpha(\varphi_h) y} \right] - \beta \frac{y_h}{y_h - \alpha(\varphi_h) y} - \beta \left(\frac{y_h}{y_h - \alpha(\varphi_h) y} \right)^2 \end{aligned}$$

and thus,

$$\frac{\partial^2 n(y, \varphi)}{\partial y^2} = -\beta \left(\frac{\alpha(\varphi_h)}{y_h - \alpha(\varphi_h) y} + \frac{1}{y} + \frac{\alpha(\varphi_h) y_h}{(y_h - \alpha(\varphi_h) y)^2} + \frac{2y_h^2 \alpha(\varphi_h)}{(y_h - \alpha(\varphi_h) y)^3} \right) < 0$$

which is negative because by definition, $y_h > y$, and $0 < \alpha(\varphi) < 1$ (Proposition 1), and thus $y_h > \alpha(\varphi_h) y$ which implies the first and the last term within the parenthesis are positive. As a result, by the definition of second-order stochastic dominance, $E_Y[n(Y_2, \Phi)] < E_Y[n(Y_1, \Phi)]$ if Y_2 is a mean-preserving spread of Y_1 . This implies that, as the degree of yield uncertainty increases, the LHS of OC1 decreases, which means OC1 eventually holds as the variation increases, if it holds when the random yield features the highest possible variation, i.e., two-point distribution.

We take an analogous approach to prove the LHS of OC1 decreases in commercial price uncertainty.

We rewrite OC1 as $-l - r\bar{y} + E_Y [E_\Psi (n(Y, \Phi))] < 0$, where $n(y, \varphi)$ is defined by (21), and show that $n(y, \varphi)$ is concave in φ . We have

$$\frac{\partial n(y, \varphi)}{\partial \varphi} = \beta y \left(-\frac{k'(\varphi)}{k(\varphi)} \right) = y \left(1 - \frac{1}{k(\varphi)} \right)$$

and thus,

$$\frac{\partial^2 n(y, \varphi)}{\partial \varphi^2} = y \frac{k'(\varphi)}{k(\varphi)^2} = \frac{-y}{\beta} \left(1 - \frac{1}{k(\varphi)} \right) < 0$$

which is negative since by definition, $k(\varphi) > 1$ (equation (3)). Therefore, the results follow.

(b) We demonstrate that $\Pi_U(Q_U^*) < \Pi(Q^*)$. Rewriting (7) and (9), we express the difference as:

$$\begin{aligned}
\Pi(Q^*) - \Pi_U(Q_U^*) &= M\beta \int_{\phi_l}^{\phi_h} \left[\int_{y_l}^{M\alpha(\phi)/Q^*} \frac{Q^*y}{M-Q^*y} g(y) dy + \int_{M\alpha(\phi)/Q^*}^{y_h} W\left(\frac{e^{(v_s-s)/\beta-1}}{k(\phi)}\right) g(y) dy \right] h(\phi) d\phi \\
&\quad - M\beta \int_{\phi_l}^{\phi_h} \int_{y_l}^{y_h} \frac{Q_U^*y}{M-Q_U^*y} g(y) h(\phi) dy d\phi \\
&= M\beta \int_{\phi_l}^{\phi_h} \left[\int_{y_l}^{M\alpha(\phi)/Q^*} \left(\frac{Q^*y}{M-Q^*y} - \frac{Q_U^*y}{M-Q_U^*y} \right) g(y) dy \right. \\
&\quad \left. + \int_{M\alpha(\phi)/Q^*}^{y_h} \left(W\left(\frac{e^{(v_s-s)/\beta-1}}{k(\phi)}\right) - \frac{Q_U^*y}{M-Q_U^*y} \right) g(y) dy \right] h(\phi) d\phi,
\end{aligned}$$

and make two observations. First, the first term within the bracket is positive because $Q_U^* < Q^*$. Second, the second term within the bracket is also positive, because $Q_U^*y < M\alpha(\phi)$ for any realizations of y and ϕ by the definition of underinvestment and thus,

$$\frac{Q_U^*y}{M-Q_U^*y} < \frac{M\alpha(\phi)}{M-M\alpha(\phi)} = \frac{\alpha(\phi)}{1-\alpha(\phi)} = W\left(\frac{e^{(v_s-s)/\beta-1}}{k(\phi)}\right)$$

As a result, $\Pi(Q^*) - \Pi_U(Q_U^*) > 0$. \square

Proof of Proposition 7. When the commercial price is deterministic and the processor encounters only yield uncertainty, we can replace ϕ with $\bar{\phi}$ in all expressions of Proposition 1 including the optimal second-stage profit function expressions as follows:

$$\pi^*(Q, y, \bar{\phi}) = \begin{cases} Qy \left(v_s - r + \beta \ln \left[\frac{M - Qy}{k(\bar{\phi})Qy} \right] \right) & \text{if } Qy < M\alpha(\bar{\phi}) \\ M\beta W \left(e^{(v_s-s)/\beta-1} / k(\bar{\phi}) \right) + (s-r)Qy & \text{if } Qy \geq M\alpha(\bar{\phi}) \end{cases}$$

Consequently, the first-stage expected profit function can be written as

$$\Pi_{\bar{\phi}}(Q) = -(l+r\bar{y})Q + \int_{y_l}^{M\alpha(\bar{\phi})/Q} Qy \left(v_s + \beta \ln \left[\frac{M - Qy}{k(\bar{\phi})Qy} \right] \right) g(y) dy + \int_{M\alpha(\bar{\phi})/Q}^{y_h} \left[M\beta W \left(\frac{e^{(v_s-s)/\beta-1}}{k(\bar{\phi})} \right) + sQy \right] g(y) dy$$

and thus,

$$\frac{\partial \Pi_{\bar{\phi}}(Q)}{\partial Q} = \int_{y_l}^{M\alpha(\bar{\phi})/Q} y \left(v_s + \beta \ln \left[\frac{M - Qy}{k(\bar{\phi})Qy} \right] - \beta \frac{M}{M - Qy} \right) g(y) dy + \int_{M\alpha(\bar{\phi})/Q}^{y_h} syg(y) dy$$

Therefore, the processor underinvests if and only if $Q^* < M\alpha(\bar{\phi})/y_h$, or

$$\frac{\partial \Pi_{\bar{\phi}}(Q)}{\partial Q} \Big|_{\frac{M\alpha(\bar{\phi})}{y_h}} = -l - r\bar{y} + \int_{y_l}^{y_h} y \left(v_s + \beta \ln \left[\frac{y_h - \alpha(\bar{\phi})y}{k(\bar{\phi})\alpha(\bar{\phi})y} \right] - \beta \frac{y_h}{y_h - \alpha(\bar{\phi})y} \right) g(y) dy < 0. \quad (22)$$

When the yield is deterministic, and the processor encounters only commercial price uncertainty, the optimal second-stage profit function becomes (note that $\alpha(\phi)$ is an increasing function of ϕ):

$$\pi^*(Q, \bar{y}, \phi) = \begin{cases} M\beta W \left(e^{(v_s-s)/\beta-1} / k(\phi) \right) + (s-r)Q\bar{y} & \text{if } \phi \leq \hat{\phi}(Q) \text{ (or } Q\bar{y} \geq M\alpha(\phi)) \\ Q\bar{y} \left(v_s - r + \beta \ln \left[\frac{M - Q\bar{y}}{k(\phi)Q\bar{y}} \right] \right) & \text{if } \phi > \hat{\phi}(Q) \text{ (or } Q\bar{y} < M\alpha(\phi)) \end{cases}$$

The first-stage expected profit function can then be written as:

$$\Pi_{\bar{y}}(Q) = -(l + r\bar{y})Q + \int_{\varphi_l}^{\hat{\varphi}(Q)} \left[M\beta W\left(\frac{e^{(v_s-s)/\beta-1}}{k(\varphi)}\right) + sQ\bar{y} \right] h(\varphi) d\varphi + \int_{\hat{\varphi}(Q)}^{\varphi_h} Q\bar{y} \left(v_s + \beta \ln \left[\frac{M - Q\bar{y}}{k(\varphi)Q\bar{y}} \right] \right) h(\varphi) d\varphi$$

and,

$$\frac{\partial \Pi_{\bar{y}}(Q)}{\partial Q} = \int_{\varphi_l}^{\hat{\varphi}(Q)} s\bar{y}h(\varphi) d\varphi + \int_{\hat{\varphi}(Q)}^{\varphi_h} \bar{y} \left(v_s + \beta \ln \left[\frac{M - Q\bar{y}}{k(\varphi)Q\bar{y}} \right] - \beta \frac{M}{M - Q\bar{y}} \right) h(\varphi) d\varphi$$

Therefore, the processor underinvests if and only if $Q^* < M\alpha(\varphi_l) / \bar{y}$, or

$$\begin{aligned} \frac{\partial \Pi_{\bar{y}}(Q)}{\partial Q} \Big|_{\frac{M\alpha(\varphi_l)}{\bar{y}}} &= -l - r\bar{y} + \int_{\varphi_l}^{\varphi_h} \bar{y} \left(v_s + \beta \ln \left[\frac{1 - \alpha(\varphi_l)}{k(\varphi)\alpha(\varphi_l)} \right] - \beta \frac{1}{1 - \alpha(\varphi_l)} \right) h(\varphi) d\varphi \\ &= -l - r\bar{y} + \int_{\varphi_l}^{\varphi_h} \bar{y} \left(v_s - \beta \ln \left[k(\varphi)W\left(e^{(v_s-s)/\beta-1} / k(\varphi)\right) \right] - \beta \left[1 + W\left(e^{(v_s-s)/\beta-1} / k(\varphi)\right) \right] \right) h(\varphi) d\varphi < 0 \end{aligned}$$

Note that by the definition of the Lambert W function $W(z_l)e^{W(z_l)} = z_l$, which implies

$k(\varphi_l)W(z_l)e^{W(z_l)} = k(\varphi_l)z_l$, and thus $\ln[k(\varphi_l)W(z_l)] + W(z_l) = \ln[k(\varphi_l)z_l]$. Let $z_l = e^{(v_s-s)/\beta-1} / k(\varphi_l)$. It

follows that $\ln \left[\left(\frac{k(\varphi_l)}{k(\varphi)} \right) k(\varphi)W\left(e^{(v_s-s)/\beta-1} / k(\varphi_l)\right) \right] + W\left(e^{(v_s-s)/\beta-1} / k(\varphi_l)\right) = \ln \left[e^{(v_s-s)/\beta-1} \right] = (v_s - s) / \beta - 1$,

or $\ln \left[k(\varphi)W\left(e^{(v_s-s)/\beta-1} / k(\varphi_l)\right) \right] = (v_s - s) / \beta - 1 + W\left(e^{(v_s-s)/\beta-1} / k(\varphi_l)\right) - \ln \left[k(\varphi_l) / k(\varphi) \right]$. Thus, we can simplify the above condition to

$$\frac{\partial \Pi_{\bar{y}}(Q)}{\partial Q} \Big|_{\frac{M\alpha(\varphi_l)}{\bar{y}}} = -l + (s - r)\bar{y} + \int_{\varphi_l}^{\varphi_h} \bar{y} \beta \ln \left[k(\varphi_l) / k(\varphi) \right] h(\varphi) d\varphi < 0. \quad (23)$$

As a result, conditions (22) and (23) show that uncertainty in either yield or commercial price by itself can lead to the underinvestment policy. \square

Proof of Proposition 8. (a) The FOC for the underinvestment policy (8) can be re-written as:

$$-l - r\bar{y} + E_{Y,\Psi} [n_U(Q, Y, \Phi)] = 0 \quad (24)$$

where $n_U(Q, y, \varphi) = y \left(v_s + \beta \ln \left[\frac{M - Qy}{k(\varphi)Qy} \right] - \beta \frac{M}{M - Qy} \right)$. Due to a similar argument presented in the proof

of proposition 6, if we show that $n_U(Q, y, \varphi)$ is concave in y and φ , the LHS of (24) decreases in yield and commercial price uncertainty, respectively, and thus, the optimal leasing quantity must decrease in the level of uncertainty in order to satisfy the FOC (due to concavity of the expected profit function).

We observe that $n_U(Q, y, \varphi)$ is indeed concave in both y and φ , because

$$\begin{aligned} \frac{\partial n_U(Q, y, \varphi)}{\partial y} &= v_s + \beta \ln \left[\frac{M - Qy}{k(\varphi)Qy} \right] - \beta \frac{M}{M - Qy} + y\beta \left(-\frac{M}{y(M - Qy)} - \frac{MQ}{(M - Qy)^2} \right) \\ &= v_s + \beta \ln \left[\frac{M - Qy}{k(\varphi)Qy} \right] - \beta \frac{M}{M - Qy} - \beta \left(\frac{M}{M - Qy} \right)^2 \\ &= v_s + \beta \ln \left[\frac{M - Qy}{k(\varphi)Qy} \right] - \beta \frac{M(2M - Qy)}{(M - Qy)^2} \end{aligned}$$

$$\frac{\partial^2 n_U(Q, y, \varphi)}{\partial y^2} = -\beta \left(\frac{M}{y(M-Qy)} + \frac{MQ(3M-Qy)}{(M-Qy)^3} \right) = \frac{-\beta M^2 (M+Qy)}{y(M-Qy)} < 0$$

and,

$$\frac{\partial n_U(Q, y, \varphi)}{\partial \varphi} = \beta y \left(-\frac{k'(\varphi)}{k(\varphi)} \right) = y \left(1 - \frac{1}{k(\varphi)} \right)$$

$$\frac{\partial^2 n_U(Q, y, \varphi)}{\partial \varphi^2} = y \frac{k'(\varphi)}{k(\varphi)^2} < 0.$$

(b) When the underinvestment policy is optimal, the first-stage expected profit function in (19) can be expressed as:

$$-(l+r\bar{y})Q + E_{Y,\Psi} [m_U(Q, Y, \Phi)], \text{ where } m_U(Q, Y, \Phi) = Qy \left(v_s + \beta \ln \left[\frac{M-Qy}{k(\varphi)Qy} \right] \right).$$

We show $m_U(Q, y, \varphi)$ is concave in both y and φ , and thus due to a similar argument developed in the proof of proposition 6, the first-stage expected profit decreases in the yield and commercial price variation. Note that

$$\frac{\partial m_U(Q, y, \varphi)}{\partial y} = Q \left(v_s + \beta \ln \left[\frac{M-Qy}{k(\varphi)Qy} \right] - \beta \frac{M}{M-Qy} \right),$$

$$\frac{\partial^2 m_U(Q, y, \varphi)}{\partial y^2} = -Q\beta \left(\frac{M}{(M-Qy)y} + \frac{MQ}{(M-Qy)^2} \right) = -\frac{Q\beta}{y} \left(\frac{M}{M-Qy} \right)^2 < 0$$

and,

$$\frac{\partial m_U(Q, y, \varphi)}{\partial \varphi} = \beta Qy \left(-\frac{k'(\varphi)}{k(\varphi)} \right) = Qy \left(1 - \frac{1}{k(\varphi)} \right),$$

$$\frac{\partial^2 m_U(Q, y, \varphi)}{\partial \varphi^2} = Qy \frac{k'(\varphi)}{k(\varphi)^2} < 0$$

Hence, the results follow. \square

Proof of Proposition 9. (a) Note that OC1 can be re-written as:

$$-(l+r\bar{y}) + v_s \bar{y} + \beta E_Y \left[Y \left(\ln \left[\frac{y_h}{\alpha(\varphi_l)Y} - 1 \right] - \frac{y_h}{y_h - \alpha(\varphi_l)Y} \right) \right] - \beta E_{Y,\Phi} [Y\Lambda] < 0$$

or

$$-(l+r\bar{y}) + v_s \bar{y} + \beta E_Y \left[Y \left(\ln \left[\frac{y_h}{\alpha(\varphi_l)Y} - 1 \right] - \frac{y_h}{y_h - \alpha(\varphi_l)Y} \right) \right] - \beta (\rho_{Y\Lambda} \sigma_Y \sigma_\Lambda + \bar{y} \mu_\Lambda) < 0 \quad (25)$$

where $\Lambda = \ln[k(\Phi)] = \ln \left[1 + e^{(v_c - \Phi)/\beta} \right]$ with mean μ_Λ and standard deviation σ_Λ , and $\rho_{Y\Lambda}$ is the correlation coefficient between Y and Λ , i.e., $\rho_{Y\Lambda} = \text{Cov}(Y, \Lambda) / \sigma_Y \sigma_\Lambda$. Keeping the marginal distributions of Y and Φ fixed (so the marginal of Λ is fixed, as well as the moments of Y , Φ , and Λ), we observe that as the correlation between Y and Φ increases (becomes more positive)—i.e., the correlation between DT yield and the commercial yield is more negative—the correlation between Y and Λ decreases (becomes more negative). This inflates the left-hand side of (25), which implies it is less likely that the underinvestment policy becomes optimal. The results follow.

(b) When the underinvestment policy is optimal, using the notation and definitions introduced in part (a) of the proof we express the expected profit (19) as:

$$\begin{aligned}
\Pi_U(Q) &= Q \left\{ -(l + r\bar{y}) + v_s \bar{y} + \beta E_Y \left[Y \ln \left[\frac{M}{QY} - 1 \right] \right] - \beta E_{Y,\Phi} [Y\Lambda] \right\} \\
&= Q \left\{ -(l + r\bar{y}) + v_s \bar{y} + \beta E_Y \left[Y \ln \left[\frac{M}{QY} - 1 \right] \right] - \beta (\rho_{Y\Lambda} \sigma_Y \sigma_\Lambda + \bar{y} \mu_\Lambda) \right\}
\end{aligned}$$

and note that with the same argument presented in part (a), as the correlation between the specialty yield (Y) and the commercial yield increases, the correlation between Y and Φ decreases. This implies the correlation between Y and Λ increases, which results in a lower expected profit. \square