Surface versus Air Shipment of Humanitarian Goods under Demand Uncertainty

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The combination of insufficient funds and limited information regarding the demand in regions of desperate need presents a great challenge to many humanitarian organizations. This paper examines how a humanitarian organization can minimize the expected shortage in delivering relief aid to regions of need, either though surface or air transportation, in the presence of demand uncertainty with a budget constraint.

The paper makes four contributions. First, we show that when there is reserved supply for air transportation, it is optimal to provide a higher service level through surface shipment to regions with greater demand variance. Second, we show that the demand variation plays a significant role in the allocation of funds between surface and air shipments. The reaction of the humanitarian organization to higher degrees of demand uncertainty can be determined by the optimal level of inventory purchased for surface shipment. If the optimal inventory for surface shipment is less (greater) than the mean demand, then we show that increasing degrees of demand uncertainty leads to increasing (decreasing) reliance on the air shipment option with greater (smaller) levels of inventory reserved for air transportation and decreasing (increasing) levels of inventory reserved for surface shipment. Third, we demonstrate that a humanitarian organization should focus its resources and efforts to improve the demand forecast in one region as opposed to evenly allocating resources to all regions. Fourth, we show that the expected amount of shortages reduces with a higher number of regions to serve due to a risk-pooing effect.

1. Introduction

World Health Organization reports that nearly 20 million children under the age of five suffer from severe acute malnutrition annually. Efforts to eradicate severe acute malnutrition are critical because this ill condition can directly lead to child death, or act as an indirect cause of common childhood illnesses such as diarrhea and pneumonia which dramatically increase the fatality rate of children. World Health Organization estimates that approximately 1 million children die every year from severe acute malnutrition. To combat the rapidly deteriorating nutritional health status of children, United Nations Children's Fund (UNICEF) along with other humanitarian organizations procure and distribute ready-to-use therapeutic food (RUTF) to countries of desperate need that are highly concentrated in the Horn of Africa such as Kenya, Ethiopia, and Somalia. RUTF is the third largest commodity group for procurement at UNICEF after vaccines and pharmaceuticals. Despite the resources dedicated to its purchase and distribution, UNICEF reports that only 10 to 15 percent of children in need are provided with RUTF. RUTF supply chain continues to struggle due to the lack of effective planning. Our work is motivated by the challenges experienced in the RUTF supply chain, and responds to the desperate need to plan the acquisition and distribution effectively so that the shortages are minimized.

There are many challenges and complexities that UNICEF and other humanitarian organizations need to overcome in order to effectively meet the uncertain demand for RUTF. One of the main challenges for the humanitarian organization (HO), UNICEF in this particular case, is that the demand for humanitarian goods in different countries is extremely hard to predict. Moreover, historical data has not been used in the most efficient manner and is often not visible by multiple organizations to produce a common shared forecast. Thus, different stakeholders produce their own forecasts, leading to further uncertainty in the supply chain.

UNICEF prepares for this demand uncertainty through two types of response mechanisms:

(1) Slow onset demand: Slow onset corresponds to the *proactive* preparation by stocking humanitarian goods in advance of the season. Slow onset demand is relatively predictable because the slowness in demand generally provides sufficient advance warning relative to lead-time of the need. For this reason, slow onset utilizes the surface shipment option as it is less expensive than air shipment. Swaminathan et al. (2009) describe that RUTF is procured from Nutriset, and is shipped via sea transportation from France to the ports of various countries located in the Horn of Africa. Komrska et al. (2013) report that the sea shipment cost of RUTF is $4.58 per carton from France to the Horn of Africa.

(2) Rapid onset demand: Rapid onset corresponds to *reactive* preparation through emergency shipments because rapid onset demand is unpredictable. Rapid onset utilizes the air shipment option as it provides a faster response time than the surface shipment. The cost of air, however, is significantly higher than surface transportation: $36.92 per carton. Rapid onset shipment is utilized when (i) there is an endemic that exceeds the expectations, and (ii) when the inventory built through slow onset prior to the epidemic is insufficient to meet the demand. Thus, rapid onset demand is first satisfied by slow onset inventory to the extent possible; when the slow onset inventory is insufficient, air shipment is utilized to cover the shortfall.

Humanitarian organizations often have to create separate funds for slow onset demand and rapid onset demand, and determine the funds to be allotted for each purpose. UNICEF, for example, determines the necessary budget for slow onset demand and rapid onset shipments at the beginning of each planning cycle (corresponding to one year). UNICEF reserves a specific fund called the Emergency Programme Fund (EPF) in order to provide rapid response to rising emergency needs. As a result, the funds for rapid onset shipments come from a different budget than the funds allocated for slow onset response. Figure 1 shows that UNICEF allocated $821.5 million in 2015 for slow onset shipments of all humanitarian goods, and $15.2 million for its EPF program that provides the funds for rapid onset shipments.

The financial flows within UNICEF's RUTF programs are described in Komrska et al. (2013); we next elaborate how these funds are used for the procurement and distribution decisions of RUTF. Figure 2 describes the annual financial planning for the slow onset and rapid onset response mechanisms at UNICEF. According to this financial planning, UNICEF Programme Division makes two financial decisions at the beginning of each planning cycle: (1) Determine the amount of money to be allocated to local

UNICEF Country Offices for the procurement and surface shipment of RUTF in advance of the season (i.e., the amount of funds allocated for slow onset response mechanism); and, (2) determine the amount of money to be allocated for UNICEF EPF for the purchase and air shipment of RUTF in rapid onset response mechanism. Thus, UNICEF Programme Division allocates a fixed budget to each UNICEF Country Office and to EPF at the beginning of the annual planning cycle. The funds allocated from the UNICEF Programme Division to local UNICEF Country Offices enables these local UNICEF officers to procure RUTF during the year. Because UNICEF Country Offices do not receive credit (or other benefits) for not spending the allotted budget, they spend the total budget on the humanitarian goods according to the UNICEF Programme Division's procurement plans. The Programme Division uses demand projections from Country Offices along with historical data to develop forecasts of demand. These forecasts account for both slow onset and rapid onset demand in the upcoming quarter. UNICEF's EPF funds, on the other hand, are used to satisfy rapid onset demand via air transport when there is insufficient local supply. Our paper sheds light into how much money should be allocated for the slow onset procurement and surface shipment at UNICEF Programme Division and how much money should be allocated for the procurement and air shipment at UNICEF EPF.

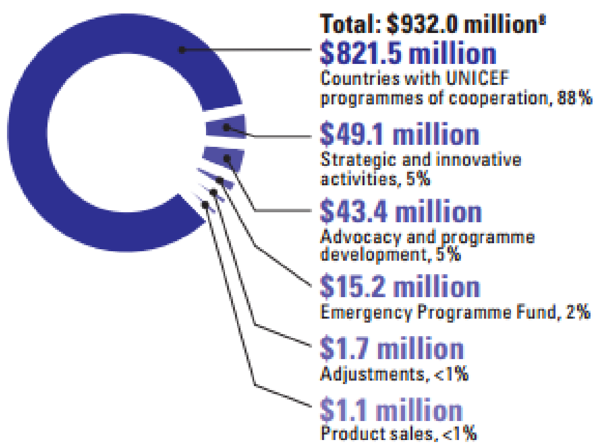

Figure 1. UNICEF's Program Division budget allocation in 2015 between slow onset and rapid onset response mechanisms.

Demand uncertainty, as a consequence of poor forecasting, is known to yield poor results in commercial settings. In humanitarian operations, however, the consequences can be much more devastating than in commercial environments. This is why an HO needs to prepare well for its rapid onset shipments so that it can provide rapid response to the rising human needs. The inefficiencies in the distribution of humanitarian goods can lead to deterioration of health for those who are in desperate need of humanitarian goods. Komrska et al. (2013) identify two reasons for the shortage of RUTF in 2008 that caused 8.4 million to suffer from "hunger emergency" in Ethiopia, Kenya and Somalia: Poor forecasting of demand for RUTF, and lack of reserved emergency funds. Despite the late addition of $8.2 million in emergency funds, the problem was not resolved effectively because it took additional time for UNICEF to get the additional emergency funding approved, and to purchase and ship RUTF to the region in crisis. This example shows that, when not accounted upfront, the effectiveness of an additional financial support is clearly limited, and therefore an HO has to effectively plan for its budget allocation and reserve funds for emergency air shipments at the beginning of the planning cycle.

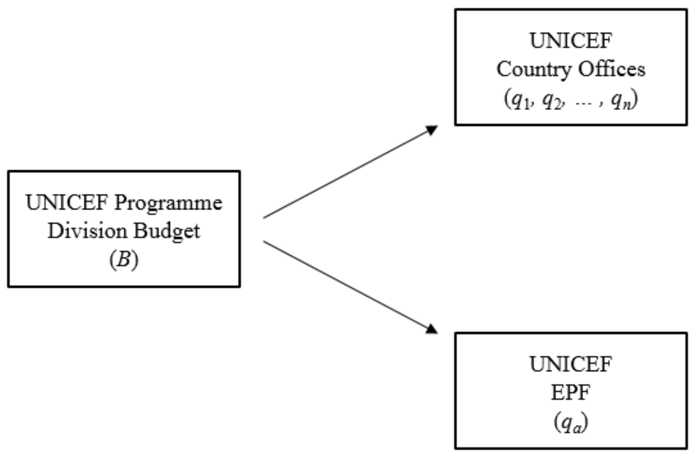

Figure 2. Financial planning at UNICEF Programme Division, and funds allocation to UNICEF Country Offices and Emergency Programme Fund.

Our study provides insight into how critical it is to design an effective inventory and transportation mechanism with surface and air shipment of humanitarian goods, not only for the children suffering from malnutrition in Africa, but also for the humanitarian needs in the recent Syrian refugee crisis. While there has been no purchase of RUTF and ready-to-use supplementary food (RUSF) prior to 2014, UNICEF has reported an initial need of 500,000 children in the refugee camps located in Lebanon, and another 150,000 in the camps located in Turkey in the first six months of 2014. With the recent influx, the number of children in need of RUTF and RUSF in the Turkish refugee camps has increased from 150,000 to numbers exceeding 750,000 in August 2015.[1] Thus, the demand for such humanitarian goods are not easily predictable. The uncertainty in demand is even further exacerbated with the ongoing migration from the refugee-

[1] https://www.humanitarianresponse.info/system/files/documents/files/unicef_syria_monthly_humanitarian_situation_report_140815.pdf.

hosting nations to the European countries. The consequence of these movements make it extremely challenging to make reliable forecasts to provide the less-costly surface shipments for the needs of the Syrian refugees.

Our paper examines how an HO can optimize the expenditure of a limited budget that minimizes the expected shortage of perishable humanitarian goods by making the following decisions in the presence of demand uncertainty: (1) the amount of humanitarian goods to be acquired and distributed through surface transportation to each country, and (2) the amount of humanitarian goods to be acquired and distributed via air shipment when necessary. We build an analytical model in order to determine the optimal stocking decisions for the surface and air shipment alternatives in the presence of demand uncertainty while operating with a limited budget. The model features an objective function that minimizes the total expected shortage. After the realization of demand in each country, HO distributes relief aid that is reserved for air transportation to the regions of need. We examine how different factors, including the unit total landed costs, budget, variation in demand, correlation between demands, and the number of regions to serve, impact the effectiveness of the humanitarian operations. Our analysis provides insights on the types of effort that further improve the effectiveness of humanitarian operations.

Our paper makes four contributions. First, when there is money reserved for air shipments, we show that it is optimal to provide a higher service level through surface transportation to the regions with greater demand variation. However, when the air shipment alternative is costly and is abandoned, it is optimal to provide an equal service level to each region regardless of the variation in their needs. Second, we provide insight regarding the impact of the variation in demand on the optimal service levels through surface and air shipments. We identify that the HO's reaction to higher degrees of demand uncertainty depends on the optimal service level describing the level of inventory purchased for surface shipment. If the optimal service level for surface shipment is negative (implying that the inventory for surface shipment is less than the mean demand), then we show that the HO relies more on the air shipment option by increasing its inventory purchased for air shipment while decreasing its service level for the surface shipment alternative. Alternatively, if the optimal service level for surface shipment is positive (implying that the inventory for surface shipment is greater than the mean demand), then the HO reduces its reliance on the air shipment and increases its service level for the surface shipment option. Third, we demonstrate that the HO should focus its resources and efforts to reduce the demand variation in one region as opposed to equally allocating resources to reduce the variation in all regions. This might seem like a surprising result at first sight, however, eliminating the variation in one region enables the HO to reduce the need for air shipments. Thus, the relief organization can provide a higher service level to the region with demand uncertainty through the less-costly surface shipment alternative. Fourth, when the budget increases linearly in the number of regions to serve, a higher number of regions increases the benefits from the joint planning of surface and air shipments due to the risk-pooling effect. However, correlation between the demands in different regions can reduce the benefits from this model that simultaneously optimizes surface and air shipments.

The paper is organized as follows: Section 2 provides a literature review and describes how our paper departs from earlier publications. Section 3 introduces the general model along with its analysis. Section 4 presents the analysis and develops the technical results. Using data from humanitarian and relief organizations, Section 5 demonstrates the impact of factors influencing the effectiveness of humanitarian operations. Section 6 discusses how the results alter under various extensions. Section 7 provides conclusions and managerial insights. The online supplement contains the appendix with all the proofs and derivations.

2. Literature Review

In this section, we describe how our work is related with the following streams of literature that are close to our study: (1) Procurement and inventory management, (2) transportation/distribution models, and (3) Newsvendor problem.

The literature pertaining to procurement and inventory management is rich for commercial environments, however, the insights are often inapplicable for HOs because of the fundamental differences. Unlike corporations, the objective of an HO would not be profit maximization, but rather making positive impact on humanity such as saving lives, reducing malnutrition, and maximizing service levels for humanitarian goods. Moreover, the problems that HOs confront are often more challenging due to budget limitations. There are a few papers that examine the optimal allocation of limited budget to mitigate the negative impact of disasters. Salmerón and Apte (2010) focus on pre-establishing capacity for warehouses, medical facilities, ramp spaces, and shelters within a limited budget prior to a disaster so that the expected number of casualties is minimized. Under limited budget, Vanajakumari et al. (2016) examine an integrated optimization model, which focuses on the problems encountered in the last mile distribution such as determining the locations for temporary warehouse and inventory level along with the number of trucks and the routing of trucks from the warehouse to the point of distribution. Rather than focusing on the last mile, our paper departs from Salmerón and Apte (2010) and Vanajakumari et al. (2016) by focusing on the upstream distribution of relief aid, which can be perceived as a macro-level problem for the headquarters of an international HO.

Eftekhar et al. (2014) also focus on the macro-level problem for an international HO, but limits the scope to vehicle procurement. Besiou et al. (2014) examine the impact of earmarked funding in the purchasing decision of vehicles in designing the distribution operations both from the perspectives of minimizing total cost and maximizing service levels for an international HO. Natarajan and Swaminathan (2014) focus on a different form of uncertainty associated with the timing and amount of funding for an international HO. To the best of our knowledge, there are no analytical studies that focus on the problem of distributing relief aid on an aggregate level where there are two characteristically different transportation modes. Thus, our paper fills this void in the procurement and inventory management literature. We believe that the insights we provide in this study, when coupled with the insights from the last mile distribution literature, would enable an HO to increase the overall effectiveness and efficiency within a humanitarian supply chain.

The literature on the distribution and transportation of humanitarian goods primarily focuses on the last mile transport; de la Torre et al. (2012) provides a comprehensive review of the literature pertaining to the last mile distribution operations. A majority of these studies develop models associated with the Vehicle Routing Problem (VRP) with various objectives such as minimizing unsatisfied demand (Yi and Özdamar 2007), cost minimization (Huang et al. 2012), minimizing latest arrival time (Campbell et al. 2008), and maximizing travel reliability (Vitoriano et al. 2009). Our paper is similar to these articles as the objective for our study is minimizing shortages. However, our work departs from these publications in two ways: (1) Earlier publications using a VRP approach examine shortages under deterministic demand and our paper considers random demand in the stocking of humanitarian goods; (2) earlier publications focus on the routing decisions, and our work emphasizes the mode of transportation, specifically surface and air transportation, and the amount of inventory dedicated to each of these delivery modes. While the majority of studies focus on distribution through ground transportation, De Angelis et al. (2007) and Barbarosoğlu et al. (2002) study the distribution of humanitarian goods using helicopters.

This paper, if stripped off its context, belongs to the literature associated with Newsvendor Problem using flexibility with a limited budget. Utilizing flexibility in the presence of stochastic demand has been studied extensively. The different forms of flexibility that have been examined under the Newsvendor literature includes flexible resources/process (Van Mieghem 1998, Kouvelis and Tian 2013), dual sourcing (Tomlin and Wang 2005), and responsive pricing (Biller et al. 2006). In our paper, air transportation provides flexibility to HOs due to the short lead-time which enables the HO to respond to demand fluctuations in different countries. To the best of our knowledge, there is no analytical paper that considers air transportation as a flexible resource and investigate how an HO can utilize it to reduce the negative impact of demand uncertainty. Thus, our paper enriches the literature by studying a new form of flexibility.

Pasandideh et al. (2011) and Serel (2012) incorporates a budget constraint while using the Newsvendor framework. Our paper differs from these earlier publication as we minimize the expected shortage of demand, rather than minimizing the total expected cost or maximizing the expected profit, under a given limited budget. Although the Newsvendor Problem has been extensively studied in the literature, there are no papers that consider minimizing expected shortage of demand utilizing different shipment modes with a budget constraint to the best of our knowledge. From this perspective, our problem can be considered as the "non-profit" version of the Newsvendor Problem. Moreover, unsatisfied demand in our model is not backlogged. Thus, our treatment of unsatisfied demand is consistent with the lost sales formulation of the Newsvendor Problem. As a result, our model is technically more complex than the traditional Newsvendor Problem with backlogged demand. More importantly, our objective function with the minimization of expected shortage with a cost consideration under a limited budget arises in practice (e.g., late deliveries are not beneficial in solving the crisis).

Despite the many challenges arising due to demand uncertainty in relief aid distribution, most of the studies in the literature employ deterministic demand. There are a few papers that examine the problem of distributing humanitarian goods while incorporating demand uncertainty. Barbarosoğlu and Arda (2004) incorporate demand uncertainty, however, their analysis is limited to analyzing the response to a specific type of disaster, earthquakes, under different demand values with a finite number of scenarios. Gonçalves et al. (2013) present a case study for World Food Programme distributing humanitarian aid in Ethiopia where the optimal supply and distribution amount is numerically derived from different scenarios with various levels of demand. Our paper analyzes a generalized problem for humanitarian aid distribution such that the insights are robust and not limited to a specific type of disaster, country, or form of a random demand function.

In summary, our paper contributes to the literature as follows: (1) Our paper focuses on the problem of distributing relief aid on an aggregate level, (2) our work analyzes two characteristically different transportation modes, specifically surface and air transportation, where the air transportation mode provides flexibility to humanitarian organizations, (3) we minimize expected shortage of demand for the Newsvendor Problem, and (4) our paper analyzes a generalized problem for humanitarian aid distribution.

3. The Model

This section presents the model developed for the managers of an HO that procures and distributes perishable humanitarian goods with a limited budget to regions of need in the presence of demand uncertainty. The objective for the managers of HO is to minimize the total expected shortages during the upcoming planning cycle.

The model is formulated as a two-stage stochastic program. In stage 1, the HO makes a budget allocation decision in stage 1 subject to a budget constraint. Specifically, the HO determines the portion of the available budget to be used for the purchase and transportation costs of humanitarian goods shipped through surface transportation (sea and ground transportation) to each region (e.g., country office). The remainder of the budget is reserved for emergency response, i.e., for the purchase and transportation costs of humanitarian goods to be shipped through air when necessary. We express the unit total landed cost of purchasing and surface transportation with c_s, and the unit total landed cost of purchasing and air shipment with c_a.

We express demand in each region in currency units and net of local supply in the region at the start of the planning cycle. For example, suppose a particular region has 150 units of RUTF in stock, the realized demand in the upcoming planning cycle in 1,150 units, and the purchase and surface transport cost is \$10/unit. Then net demand in RUTF units is 1,000, and \$10,000 in currency units. We scale the unit total landed cost of surface transportation to $c_s = 1$, and adjust the unit total landed cost of air c_a and the regional demand using the same scale.

Let q_i denote the funds allocated to region i where $i = 1, 2, \ldots, n$. We assume that the ratio of landed-cost-via-air to landed-cost-via-surface is the same across regions. This assumption allows us to gain additional insight into structural properties. In addition, data suggest that differences in ratios across regions are small for the setting motivating this work. We discuss the impact of relaxing this assumption in Section 6. Because demand is stated in purchase and surface transport funding needs by region, we express the quantity of demand that can be satisfied by air transport in the same unit, and denote as q_a. The managers of HO need to account for both transportation modes in their budgetary planning. Therefore, for fixed budget B, the decision $\mathbf{q} = (q_1, q_2, \ldots, q_n, q_a) \geq 0$ must satisfy $\sum_{i=1}^{n} q_i + c_a q_a \leq B$. Our budget B can be considered as the amount of money that is free from the fixed costs that might incur as a result of operations in each country. Because there is no cost associated with over-stocking, HO will exhaust its entire budget in each planning cycle. Thus, the budget consumption for the relief organization always strictly equals B, i.e.,

$$\sum_{i=1}^{n} q_i + c_a q_a = B \,. \tag{1}$$

In stage 2, after the realization of demand in each country, the HO realizes the additional amount of relief aid necessary in excess of local supply in each region. The HO still has to ration the amount of emergency relief q_a to the countries that experience demand exceeding the amount q_i. Our model does not allow transshipment of humanitarian goods that are already shipped through surface. Transshipment of goods that are already sent through surface is not practical for several reasons: (1) Poor transportation infrastructure between these nations; (2) regional autonomy and self-interest (e.g., hesitancy to reduce stock in home country to satisfy need in a different country); and (3) the perishable nature of the humanitarian goods considered in our paper.

The overage cost in our model indirectly involves the purchasing cost of humanitarian goods, shipping them via surface transportation, while not being able to satisfy the needs at other regions. Specifically, if the HO ships more than the realized demand for a country through surface transportation, the HO not only wastes the unit total landed cost of surface shipment c_s for each unit that exceeds demand but also loses the opportunity to satisfy demand in a country that lacks supply of humanitarian goods.

We describe random demand in each region with D_i and its realization with d_i for $i = 1, \ldots, n$. We express the expected demand and the standard deviation of demand in each region with μ_i and σ_i, respectively. We standardize the surface transportation quantity q_i in order to compare the service levels in each region. Let z_i denote the standardized stocking factor for each region through surface transportation where $z_i = (q_i - \mu_i)/\sigma_i$. From the value of z_i, HO can determine the appropriate service level through the surface shipment alternative for region i. We index the regions in order of increasing uncertainty, i.e.,

$$\sigma_1 \le \sigma_2 \le \ldots \le \sigma_n,$$

and we let

$$\mu = \sum_{i=1}^{n} \mu_i$$

$$\sigma = \sum_{i=1}^{n} \sigma_i .$$

Note that while q_i must be nonnegative, z_i can take on negative values. The values of z_i cannot be smaller than $-\mu_i/\sigma_i$ (i.e., $z_i \ge -\mu_i/\sigma_i \Leftrightarrow q_i = \mu_i + z_i\sigma_i \ge 0$). We express the random demand in region i as $D_i = \mu_i + Z_i\sigma_i$, where Z_i represents the standardized random error term for region i. We assume that the pdf for the random demand in each region has the same functional form, but not necessarily has identical parameter values. Thus, the pdf of the standardized random error term Z_i, denoted $\phi(z)$, is common across regions. The cdf of Z_i is $\Phi(z)$ with support $\Omega = [z_l, z_h]$ where $\mu_i + z_l\sigma_i \ge 0$ for all i (i.e., random demand is nonnegative). We make no other assumptions regarding the distribution of Z_i.

The problem of selecting $\mathbf{q}$ to minimize the expected shortage subject to (1) can be expressed as follows:

$$\min_{\mathbf{q}\ge 0}\left\{ E\left[\left(\sum_{i=1}^{n}(D_i - q_i)^+ - q_a\right)^+\right] : \sum_{i=1}^{n} q_i + c_a q_a = B \right\}.$$

The problem can be rewritten in terms of the stocking factor vector $\mathbf{z} = (z_1, \ldots, z_n)$:

$$\min_{\mathbf{z}\ge -(\mu_1/\sigma_1,\ldots,\mu_n/\sigma_n),\, q_a \ge 0} \left\{ E\left[S(\mathbf{z}, q_a)\right] : \sum_{i=1}^{n}(\mu_i + z_i\sigma_i) + c_a q_a = B \right\} \tag{2}$$

where

$$S(\mathbf{z}, q_a) = \left(\sum_{i=1}^{n}\left((\mu_i + Z_i\sigma_i) - (\mu_i + z_i\sigma_i)\right)^+ - q_a\right)^+ = \left(\sum_{i=1}^{n}\sigma_i (Z_i - z_i)^+ - q_a\right)^+ .$$

To eliminate trivial solutions, we consider the two cases when the budget (i) can cover the sum of the minimum demand in each region through surface transportation, and (ii) cannot cover the overall maximum demand through surface transportation. Thus, we have

$$\sum_{i=1}^{n}\left(\mu_i + z_l\sigma_i\right) < B\,, \tag{3}$$

$$\sum_{i=1}^{n}\left(\mu_i + z_h\sigma_i\right) > B\,. \tag{4}$$

The case of insufficient budget to cover the sum of minimum demands leads to an intuitive policy of the HO making investments only in the surface transportation with no money reserved for the air shipment alternative. Similarly, the case of excess budget also leads to an intuitive policy as the maximum demand in each region can be satisfied through surface transportation and there would be no shortages.

4. The Analysis

This section presents the analysis of the problem setting with an arbitrary number of regions, denoted *n*. The analysis of the model presented in Section 3 is highly complex and it is not possible to solve for q_i for $i = 1,\ldots, n$ and q_a simultaneously with closed-form expressions. As can be seen from propositions A1 and A2 in the online supplement, the optimal stocking factors for surface and air shipment decisions cannot be characterized in closed-form expressions even with independent demand and equal demand variations.

We begin our analysis by examining the optimal surface transportation decisions for a given level of budget reserved for air shipments. The following result helps characterize the optimal service levels using the surface shipment option for a given amount of relief aid reserved for air transportation q_a.

Proposition 1.

(a) *If* $q_a = 0$, *then*

$$z_i^* = (B - \mu)/\sigma \text{ for all } i. \tag{5}$$

(b) *If* $q_a > 0$ *and* $\sigma_1 = \sigma_2 = \ldots = \sigma_n$, *then*

$$z_i^* = (B - \mu - c_a q_a)/\sigma \text{ for all } i. \tag{6}$$

(c) *Suppose that* $q_a > 0$ *and* $\sigma_i < \sigma_{i+1}$ *for some i. If nonnegativity constraints* ($q_i \geq 0$, $i = 1, \ldots, n$) *are non-binding*, *then*

$$z_1^* \leq z_2^* \leq \ldots \leq z_n^*. \tag{7}$$

Proposition 1(a) states that if there is no relief aid allocated for air shipment, then it is optimal for the HO to equalize the shortage probabilities across regions by assigning equal service levels to all regions. If the standard deviations are equal in all *n* regions, then Proposition 1(b) indicates that it is optimal to equalize the shortage probabilities net of air deliveries across the regions. When regions have unequal variances, however, Proposition 1(c) shows that the HO would prioritize the regions with higher demand variance, and assigns a greater service level through the surface shipment alternative. If $q_a > 0$, then unless some regions receive no surface shipments in an optimal solution, it is optimal for the relief organization to transport more (in terms of service levels) through surface transportation to regions with greater volatility. It is important to highlight that the above finding in Proposition 1(c) is robust as it is not restricted to a specific form of demand uncertainty or a probability density function.

From equations (5) and (6) in Proposition 1, it is easy to see the impact of increasing mean demand on the optimal service level decisions. Higher degrees of mean demand (μ) lead to a reduction in the optimal service level decisions for the surface shipment alternative. Thus, the HO's reaction to higher degrees of mean demand is identical to the reaction it shows to reduced levels of budget (B) available for the purchase and transportation of humanitarian goods.

Proposition 1(a) and (b) show that humanitarian goods will be allocated to each country at equal service levels. This occurs when there is no budget reserved for air shipment according to Proposition 1(a), and when there is budget reserved for air shipment but each country has equal demand variation according to Proposition 1(b). However, Proposition 1(c) advocates not serving each country at equal service levels in order to provide the maximum coverage and the minimum amount of expected shortages. Thus, when there is budget reserved for air shipments ($q_a > 0$) and when countries have varying demand variation ($\sigma_i \neq \sigma_{i+1}$ for some i) the HO will not make a fair allocation to each country; thus $z_i \neq z_{i+1}$ for some i.

What if HO is forced by country governments and local politicians to provide a fair allocation of humanitarian goods through surface transportation? Let z_f denote the fair allocation of humanitarian goods through surface shipment; each country is served a quantity based on the equal service level designated with z_f. The problem in (2) becomes equivalent to solving

$$\min_{z_f \geq -\max\{\mu_1/\sigma_1,\ldots,\mu_n/\sigma_n\}} \left\{ E\left[\left(\left[\sum_{i=1}^{n} \sigma_i \left(Z_i - z_f \right)^+ - q_a \right] \right)^+ \right] : \mu + z_f \sigma \leq B \right\}.$$

Remark 1. *For given q_a, HO makes fair allocation of humanitarian goods through surface shipment with*

$$z_f^* = (B - \mu - c_a q_a)/\sigma \text{ for all } i. \quad (8)$$

It is important to observe that the fair allocation service level z_f^* in equation (8) is not identical to the optimal service level under the restriction of equal variances in (6). The expression in (8) does not require equal demand variances in each country. It is also worth mentioning that the requirement to serve each country in a fair manner with equal service levels designated with (8) leads to a suboptimal coverage. The fair allocation rule in (8) can result in a higher expected shortage under the following two conditions: (1) there is budget allocated for air shipments (i.e., $q_a > 0$); and, (2) the demand variances are not equal ($\sigma_i \neq \sigma_{i+1}$ for some i). Thus, forcing humanitarian organizations to operate in a fair manner can result in sacrificing a higher number of human suffering.

We extend our derivations using equal demand variations. We already know from expression (1) that HO exhausts its entire budget between surface and air shipment decisions. The consequence of equal demand variations in an arbitrary number of regions, along with (1), is that the problem described in (2) with (n + 1) decision variables (n decision variables for surface shipment decisions and one decision variable for the air shipment decision) reduces to a single-variable optimization problem.

Remark 2. *If* $\sigma_1 = \sigma_2 = \ldots = \sigma_n$, *then* (2) *reduces to the following univariate optimization problem:*

$$\min_z \left\{ E\left[\left(\frac{\sigma}{n} \sum_{i=1}^{n} (Z_i - z)^+ - \frac{B - \mu - z\sigma}{c_a} \right)^+ \right] : \frac{-\max_i \{\mu_i\}}{\sigma / n} \le z \le \frac{B - \mu}{\sigma} \right\}. \tag{9}$$

In the remaining part of the analysis, we employ the assumption $\sigma_1 = \sigma_2 = \ldots = \sigma_n$ in order to arrive at insightful results. We next characterize the shortage of humanitarian goods, and its excess, by using the following two probability sets:

$$\Omega_n^0(z) = \left\{ \frac{\sigma}{n} \sum_{i=1}^{n} (Z_i - z)^+ > \left(\frac{B - \mu - z\sigma}{c_a} \right) \right\} \text{ and } \Omega_n^1(z) = \left\{ \frac{\sigma}{n} \sum_{i=1}^{n} (Z_i - z)^+ \le \left(\frac{B - \mu - z\sigma}{c_a} \right) \right\}.$$

Shortages occur in the probability set $\Omega_n^0(z)$. Some countries might experience higher demand than the surface allocation, i.e., $Z_i > z$ for some i = 1, …, n. When inventory stocked through surface transportation is insufficient, the shortage can be covered through air shipments. However, even the inventory reserved for air shipment can be insufficient to fulfill these various country needs; this occurs when $\frac{\sigma}{n} \sum_{i=1}^{n} (Z_i - z)^+ > \left((B - \mu - z\sigma) / c_a \right)$. The term $\frac{\sigma}{n} \sum_{i=1}^{n} (Z_i - z)^+$ corresponds to the total shortage from surface stocking quantities in the absence of air shipment, and the term $\left((B - \mu - z\sigma) / c_a \right)$ is the amount of inventory reserved for air shipment. In the probability set $\Omega_n^1(z)$, the total amount of inventory from surface and air shipments is sufficient to fulfill all country needs, and therefore, the HO does not experience any shortages of humanitarian goods.

We further partition the probability set of shortages $\Omega_n^0(z)$ into various subsets describing the combination of regions that cause the shortages. Let $C(n, i)$ represents the combination of (n, i); the set $\Omega_n^0(z)$ can be expressed in a total number of probability subsets equivalent to $J = \sum_{i=1}^{n} C(n,i) = 2^n - 1$. Let us briefly describe how these probability subsets are formed. When there are 2 regions to cover (i.e., n = 2), the probability set $\Omega_n^0(z)$ can be partitioned into three subsets. In the first subset, the surface level in country 1 is insufficient to fulfill the demand while the surface level in country 2 satisfies its local demand. Moreover, the inventory reserved for air shipment is insufficient to fulfill all the needs of country 1. In the second probability subset, the surface level in country 1 is sufficient to meet the demand, however, the surface shipment in country 2 does not fulfill the demand and the inventory reserved for air shipment is insufficient to cover the shortages in country 2. In the third probability subset, the inventory from surface shipment in neither country is sufficient, and the inventory reserved for air shipment is insufficient to fulfill the total needs. When the analysis involves three countries (n = 3), the total number of subsets is $J = C(3,1) + C(3,2) + C(3,3) = 3 + 3 + 1 = 7$. Thus, as the number of countries (described with parameter n) in the analysis increases, the total number of probability subsets grows exponentially. We describe the index of the combination of regions that experience shortage from surface shipment with j where $j = 1,\ldots, J$. For a given combination j, let $\Lambda^+(j)$ describe the set of countries where the surface shipment is insufficient to fulfill the realized demand, i.e., $\Lambda^+(j) = \{i\text{: when } Z_i > z\}$. Similarly, let $\Lambda^-(j)$ describe the set of countries where the surface shipment is sufficient to fulfill the demand, i.e., $\Lambda^-(j) = \{i\text{: when } Z_i \leq z\}$. We use probability subset $\Omega_n^{0j}(z)$ in order to describe j^{th} partition of $\Omega_n^0(z)$, where

$$\Omega_n^{0j}(z) = \left\{ \frac{\sigma}{n} \sum_{i=1}^{n} 1_{i \in \Lambda^+(j)} (Z_i - z) > \left((B - \mu - z\sigma)/c_a \right) \right\}.$$

Using the above definition, the next proposition develops the optimal stocking level decisions.

Proposition 2. *The optimal stocking factor z^* for the surface shipment decisions satisfies*

$$\sum_{j=1}^{J} \left[\int \cdots \int_{\Omega_n^{0j}(z^*)} \left[\frac{\sigma}{n} \left(\left(\sum_{i=1}^{n} 1_{i \in \Lambda^+(j)} (-1) \right) + \frac{n}{c_a} \right) \right] \phi(y_1) \ldots \phi(y_n)\, dy_1 \ldots dy_n \right] = 0. \quad (10)$$

Proposition 2 develops the conditions in which the optimal stocking factor for the surface shipment minimizes the expected shortage. In this case the optimal air shipment is equal to $q_a^* = (B - \mu - z^*\sigma)/c_a$.

Proposition 2 also provides insight into the relationship between the number of regions to cover (described with n) and the unit total landed cost of air shipment (described with c_a). Recall that we consider equal demand variances between countries, and thus, the term (σ/n) is the demand variation in each country. The first-order condition presented in (10) sums up the variations in the countries that experience higher realized demand than the surface inventory, i.e., when $i \in \Lambda^+(j)$. Consider the event that the air shipment is significantly more expensive than surface shipment, and c_a is greater than n. In this case, the left-hand side of the expression in (10) is guaranteed to be negative, implying that the first-order condition cannot be satisfied. In this case, HO should increase its surface shipment and avoid reserving funds (and inventory) for air shipment. However, when c_a is less than n, the first-order condition forces HO to reduce the surface level inventory in exchange for inventory reserved for air shipment. Next, consider the event that the unit total landed cost of air shipment is as inexpensive as the surface shipment, i.e., $c_a = c_s = 1$. In this case, the left-hand side of the expression in (10) is always positive, and the first-order condition cannot be satisfied. In this case, HO will minimize the surface shipment inventory and maximize the inventory reserved for air shipments. Thus, the expression (10) hints at the existence of a threshold on the unit total landed cost for air shipment. As the value of c_a is closer to c_s, Proposition 2 recommends reduction in z^*, and as the value of c_a increases and approaches the number of regions to cover (described with n), then Proposition 2 recommends increasing z^*.

The next proposition develops the condition in which the optimal stocking factor z^* that is defined in Proposition 2 takes an intermediate solution.

Proposition 3. *If* $c_a > (B - \mu - z_l\sigma)/(z_h - z_l)\sigma$, *then* $z^* > z_l$ *and* $q_a^* < q_a^{\max} \equiv (B - \mu - z_l\sigma)/c_a$.

Proposition 3 provides condition for the cost of air shipment in which the optimal stocking factor z^* takes in intermediate solution where the surface transportation is away from its two support points $[z_l, z_h]$ and the air shipment amount is less than its potentially maximum value. When the condition in Proposition 3 is not satisfied and $c_a \leq (B - \mu - z_l\sigma)/(z_h - z_l)\sigma$, then the cost of air transportation is exceedingly less costly that the HO uses the entire budget for air shipments and $z^* = z_l$.

We next examine the impact of demand variation on the probability of shortages and the expected amount of shortages. In deriving our technical results, we continue to make use of the derivations involving the partitioning of the probability set $\Omega_n^0(z)$ into smaller subsets. The next proposition shows that the probability of shortages and the expected amount of shortages increase with higher degrees of demand uncertainty expressed with greater values of σ.

Proposition 4. *For any given z, both the probability of shortages described by* $P[\Omega_n^0(z)]$ *and the expected amount of shortages are non-decreasing in* σ *when* $B > \mu$.

The above proposition shows that demand uncertainty expressed with parameter σ expands the feasibility of the probability set $\Omega_n^0(z)$ when $B > \mu$. To understand the impact of demand variation, let us describe a problem setting where the HO has to cover demand in two countries. For a given value of z representing the stocking factor for surface inventory, there are three possible probability regions where shortages can be experienced. In the first probability subset, the realization of random standardized demand in country 1 (denoted z_1) exceeds the surface inventory z and the inventory reserved for air shipments (denoted q_a) while the realized standardized demand in country 2 is below the surface inventory z. Thus, all shortages arrive from the needs in country 1. As the demand variation increases, the mass falling in this region of probability subset is non-decreasing, leading to a potential increase in probability, and therefore, to an increase in expected shortage. In the second probability subset, realized standardized demand in country 2 (denoted z_2) exceeds the sum of surface inventory z and the inventory reserved for air shipments, while the realized standardized demand in country 1 is less than the surface inventory z. In this subset, all shortages arise from the excess demand in country 2. As the demand variation increases, the mass falling in this region of probability subset is non-decreasing. Therefore, the probability of shortages and the expected amount of shortages increase in this subset. The third subset involves the events when the realized standardized demand in each country is greater than the surface level inventory and the sum of the realized demand exceeds the sum of the surface level and air shipment inventories. Increasing demand variation often leads to an increase in this region as well; however, when the variation decreases the mass in this probability region, the movement in the mass is captured in the other two probability subsets. Therefore, the overall probability increases with higher degrees of demand variation, leading to an increased amount of expected shortages. The result in Proposition 4 is general as it is derived for an arbitrary number of regions. In sum, we conclude that higher values of σ imply a higher expected shortage for humanitarian goods. Moreover, higher degrees of demand uncertainty lead to a greater amount of expected shortages.

We next examine the impact of demand uncertainty on the optimal stocking factor decisions. The following lemma shows that the amount of inventory reserved for air shipment can be both increasing and decreasing with higher degrees of demand variation.

Lemma 1. *For a given* $z < 0$ (> 0), $\partial q_a / \partial \sigma > 0$ (< 0).

The above lemma indicates that the behavior of the inventory reserved for air shipment is determined by the sign of the surface level stocking factor z. Specifically, if the stocking factor for the surface shipment option is negative, implying that the HO stocks less than the mean demand in each country, then increasing degrees of demand variation lead to a higher level of inventory reserved for air shipment. In this case, demand variation amplifies the importance of air shipment and its benefits from the flexibility to serve the potentially increasing needs in each country.

As established earlier in equation (1), the HO would be exhausting its entire budget between surface-level inventory and the inventory reserved for air shipments. Using this observation, the next lemma shows that the impact of increasing degrees of demand variation on the inventory transported through surface shipment is the opposite of the impact developed for the inventory reserved for air shipment.

Lemma 2. *If* $\partial q_a / \partial \sigma > 0$ (< 0), *then* $\partial z / \partial \sigma < 0$ (> 0).

The consequence of lemmas 1 and 2 is that higher degrees of demand variation lead to either an increase in air shipment with a reduction in inventory that moves with surface shipment, or a decrease in air shipment with a higher reliance on the inventory stocked with surface shipment. The next proposition shows that it is sufficient to obtain the optimal stocking factor for surface shipment in order to determine which of the behavior will be the prevailing reaction to increasing degrees of demand uncertainty.

Proposition 5. *When* $z^* > 0$ (< 0)*, air shipment decreases* (*increases*) *in* σ *while the stocking factor for surface shipment increases* (*decreases*) *in* σ.

Proposition 5 shows that the demand variation plays a significant role in the allocation of funds between surface and air shipments. The reaction of the humanitarian organization to higher degrees of demand uncertainty can be determined by the optimal level of inventory purchased for surface shipment. If the optimal inventory for surface shipment is less (greater) than the mean demand, then we show that increasing degrees of demand uncertainty leads to increasing (decreasing) reliance on the air shipment option with greater (smaller) levels of inventory reserved for air transportation and decreasing (increasing) levels of inventory reserved for surface shipment.

It is important to highlight that, for an arbitrary number of regions to serve, the optimal stocking levels through surface transportation and the optimal quantity for air shipments cannot be characterized in closed-form expressions. Overall, our model with the objective function of minimizing expected shortages is not tractable under an arbitrary number of regions to serve. In the next section, we provide additional insight regarding the impact of various parameters on the optimal surface and air shipment quantities through the analysis of a two-country setting.

5. Impact of Parameters

This section presents numerical analysis using data from UNICEF and other publications in order to provide answers to the following three questions:

(1) How do various factors, including the difference in the unit total landed cost of each transportation mode, the amount of budget, the degree of demand variation, and the correlation between the demands of various regions, and the number of regions to serve impact the effectiveness of the humanitarian operations?
(2) How should budget be allocated based on the change in each of the above factors?
(3) What are the types of effort that could further improve the effectiveness of the humanitarian supply chain operations?

We use the following data provided by UNICEF and Komrska et al. (2013) while constructing a base case for the RUTF supply chain in serving the Horn of Africa:

- *Number of regions*: We consider that HO serves two regions, i.e., $n = 2$: Niger ($i = 1$) and Ethiopia ($i = 2$). These two regions represent the countries with the greatest needs for RUTF.
- *Unit total landed cost of surface and air transportation modes*: The unit total landed cost combines the purchasing cost with the unit transportation cost. For the procurement cost of RUTF, we use the price provided by Nutriset, the largest supplier for RUTF. The unit procurement cost is approximately equal to \$45/carton.[2] We use a surface transportation cost of \$5/carton indicating the freight cost via sea shipment from France, where Nutriset is located, to the Horn of Africa. This indicates that the unit total landed cost for the surface transportation mode is c_s =

[2] Source: UNICEF Supply Division, July 2015, (http://www.unicef.org/supply/files/RUTF_Pricing_Data_final_July_2015.pdf). Since the price is given in euros as Nutriset is located in France, we apply 1.2 USD/euro exchange rate to derive the price in USD.

\$50/carton. We use a unit air transportation cost of \$35/carton indicating the cost of air shipment. As a consequence, the unit total landed cost for the air transportation mode is c_a = \$80/carton.

- *Demand*: We consider uniform annual demand on the basis of the principle of insufficient reason originally proposed by Pierre Laplace in the 1700s (Luce and Raiffa 1957).[3] Thus, we assume that $D_1 = U \sim [22, 234]$ and $D_2 = U \sim [12, 292]$. The demand for RUTF in Niger and Ethiopia varies quite significantly. The supports for each region is derived from the UNICEF's actual order amount (in '000 cartons) of RUTF for each region from 2005 and 2010 where we use the maximum and minimum demand.[4] (Note: while, to simplify notation, we define demand in our model in terms of currency units (e.g., uncertain Niger demand in currency units is uniform between \$50×22 and \$50×234), we define demand in units of product instead of units of currency in this example to clarify the elements underlying demand.) For the base case, we assume that there is no correlation between the demands in each country.
- *Budget*: In 2012, UNICEF allocated a total of \$88.3 million from its Programme Division funds to the Country Offices in Niger and Ethiopia (\$30 million and \$58.3 million, respectively). However, the exact budget allocated for RUTF is not reported. We assume that 14% of the entire budget is allocated to RUTF, which is the proportion that UNICEF allocated to nutritional humanitarian goods in 2015. Thus, we use B = \$12.5 million (> 14% × \$88.3 million) for the base case which slightly greater than 14% of the sum of Niger and Ethiopia's total budget.

We next examine the impact of the value of relative differences in the unit total landed costs, budget, and demand distribution parameters, both from a variation perspective and the correlation between the demands of each region. We then extend the analysis to a higher number of regions.

5.1 Impact of the Difference in the Unit Total Landed Costs

We first examine the impact of the unit total landed cost (i.e., the sum of the unit costs of purchasing and transportation) for the surface and air shipment alternatives. In our analysis, we keep the sea freight rate constant at t_s = \$5/carton and vary only the air freight cost t_a between \$15 and \$55 per carton, i.e., $t_a = \{15, 25, 35, 45, 55\}$. This enables us to examine the impact of the difference between the unit total landed costs of the surface and air transportation modes. As a consequence of the above unit transportation costs, we have the unit total landed costs for the surface and air shipment alternatives as c_s = \$50/carton and $c_a = \{60, 70, 80, 90, 100\}$ in this numerical analysis.

[3] If all we know about a random variable is that it takes values over a finite range, then we can only use uniform distribution. If we assume any distribution other than uniform, then it implies that we have additional information about the distribution of the random variable.
[4] Source: http://www.unicef.org/supply/files/Overview_of_UNICEF_RUTF_Procurement_in_2010.pdf.

The relative unit cost difference between air and surface transportation plays an important role on how heavily the relief organization relies on the surface and air transportation alternatives. The results are presented in Table 1 where $E[S^*]$ is the expected shortages at the optimal stocking levels.

c_a	\$60	\$70	\$80	\$90	\$100
$E[S^*]$	73,196	77,321	77,414	77,414	77,414
q_a^*	77	8.5	0	0	0
Σq_i^*	157.6	238.1	250.0	250.0	250.0
z_1^*	-0.93	-0.30	-0.21	-0.21	-0.21
z_2^*	-0.81	-0.29	-0.21	-0.21	-0.21

Table 1. Effect of various sea transportation costs on the optimal values.[5]

Table 1 shows that the expected shortage decreases as the air transportation option becomes economical. As air shipment becomes less costly, HO utilizes the air shipment alternative more frequently, and the relief organization becomes more agile and responsive (e.g., c_a = \$60 and \$70). However, the benefits of air transportation is limited with increasing air shipment costs. There exists a threshold of the unit total landed cost, air shipment is perceived to be overly expensive and is not utilized (e.g., c_a = \$80, \$90 and \$100). For the base case (c_a = \$80), we find that it is optimal for UNICEF to not utilize air transportation, which corresponds to not allocating any funds for EPF, due to the following two reasons: (1) relatively expensive air shipment cost, and (2) insufficient amount of budget. The budget of \$12.5 million allocated to RUTF for both countries is not sufficient to even cover the sum of mean demands, $\mu_1 + \mu_2 = 280$, through surface transportation (i.e., 280,000 × \$50 = \$14 million). Thus, even if all RUTF are distributed through the cheaper surface shipments, the expected shortage of RUTF is extremely high. We find that the expensive but flexible air transportation mode only becomes valuable when an HO has sufficient amount of budget.

Note that the discrepancy between the surface shipment amounts dedicated to Niger and Ethiopia (equivalently, service levels between these two regions) becomes greater as more supplies are reserved for air transportation. Consistent with our finding in Proposition 1(a), this difference completely disappears when the air transportation option becomes expensive and is abandoned from the optimal allocation of budget.

5.2 Impact of Budget

We next examine the impact of the available budget on the optimal stocking decisions, and the expected shortage. Table 2 shows how critical it is for HOs to secure greater amounts of budget in order to minimize the shortages of supply in regions of need.

[5] Highlighted column in tables represent the base case for the computational study.

B	\$10,000,000	\$12,500,000	\$15,000,000	\$17,500,000	\$20,000,000
$E[S^*]$	108,004	77,414	51,906	31,425	15,712
q_a^*	0	0	0	7.2	22.5
Σq_i^*	200.0	250.0	300.0	338.5	364
z_1^*	-0.56	-0.21	0.14	0.40	0.55
z_2^*	-0.56	-0.21	0.14	0.42	0.62

Table 2. Effect of various amount of budgets on the optimal values.

It is common to observe HOs relentlessly working to secure a sufficient amount of budget. Table 2 demonstrates how significantly budget affects the expected shortage. Through our numerical analysis, we find that if the budget is increased by 60% (i.e., \$7.5 million is added to the total budget) for RUTF, then the expected shortage could be reduced by 80%. Again, we observe that it makes sense to reserve funds for EPF, which corresponds to rapid onset emergencies, only when UNICEF can allocate adequate amount of budget to each UNICEF Country Office.

Donations are not always provided in cash to an HO. Corporations and government agencies often donate to HOs by providing funds in the form of charter flights for shipments. To gain insights on the impact of charter flights, we answer the following question: If an additional fund of \$7.5 million were provided in the form of charter flights for RUTF shipments as opposed to cash, how would this affect the expected shortage? From Table 2, we know that an addition budget of \$7.5 million in the form of cash would reduce the expected shortage by 80%. An additional budget of \$7.5 million in the form of charter flights is equivalent to shipping 93,750 cartons of RUTF through air transportation which would reduce the expected shortage by 74%. This shows that donations in the form of charter flights are quite effective but not as effective as providing additional funds with cash because the HO cannot use the additional funds in the most effective manner, which also explains why earmarked donations have limited benefits.

5.3 Impact of the Degree of Distortion in Demand Volatility

In this subsection, we examine the impact of the distortions in demand variations between two countries. It is rather straightforward to see that an increase in demand fluctuations causes an increase of shortages. This is because higher variation in demand leads to a greater chance of a mismatch between supply and demand. In this case, HO would prefer to utilize air transportation over surface movement of humanitarian goods.

While the negative consequences of increasing demand variation is obvious, it is not that clear how the degree of distortion in demand variation, denoted δ, affects the performance of an HO under a constant total variation. In order to examine the impact of the distortion in demand variance, we keep the sum of the demand variances constant, i.e., $0.5(\sigma_1^2 + \sigma_2^2) = \bar{\sigma}^2$. Let $\delta \geq 0$ where $\delta = \sigma_2^2 - \bar{\sigma}^2 = \bar{\sigma}^2 - \sigma_1^2$

given that $\sigma_2^2 \geq \sigma_1^2$. To gain more insightful results from the base case, we increase the budget to B = \$20 million while keeping other parameters the same, so that we can observe how the air transportation funds change with regard to distortion in demand variance.

We begin our analysis with $\delta = 0$ which represents the case with the amount of distortion where variation in each country are equal. We increase the degree of distortion to δ = 2,562 representing the case with the maximum amount of distortion.[6] Table 3 tabulates the results pertaining to the impact of the increasing degrees of distortion in demand variation.

δ	0	697	1,394	1,978	2,562
D_1	U[4, 252]	U[13, 243]	U[22, 234]	U[30, 225]	U[40, 216]
D_2	U[28, 276]	U[20, 284]	U[12, 292]	U[6, 298]	U[0, 304]
$E[S^*]$	16,090	15,889	15,712	15,110	14,588
q_a^*	23.3	23.1	22.5	22.2	21.8
Σq_i^*	362.8	363	364	364.5	365
z_1^*	0.58	0.56	0.55	0.54	0.53
z_2^*	0.58	0.60	0.62	0.64	0.66

Table 3. Effect of degree of distortion in demand on the optimal values.

One would intuit that increasing levels of distortion in demand variation would lead to an increase in the amount of expected shortages. However, Table 3 demonstrates that the expected amount of shortages decreases along with the necessity for air shipment as the degree of distortion in demand increases. The rationale behind this rather surprising result becomes clearer when we examine the case with the highest degree of distortion in demand variation corresponding to $\delta = 2562$. In this case, the demand variance in D_1 is minimized while the demand variance is maximized in D_2. Comparing to other scenarios with less amount of distortion in demand, we find that the air transportation option is used least. This is because the value of the flexible (but expensive) air transportation mode decreases as the demand in one country becomes more deterministic. Thus, with the need for the expensive air transportation mode decreasing, the expected shortage diminishes as the HO can acquire and ship greater amounts of RUTF.

Consistent with our finding in Proposition 1(b), when the demand variances are equal in different countries corresponding to the scenario where $\delta = 0$, we can observe that the optimal stocking factor for surface shipments are identical with high service levels, $z_1^* = z_2^* = 0.58$.

Table 3 provides insight regarding how HOs should allocate resources and efforts in forecasting the needs in each region. One would expect that equally dividing the efforts to improve demand accuracy by

[6] If $\delta > 2562$, then the lower support of D_2 becomes negative.

reducing the variation in demand would return the highest benefits. However, the results in Table 3 indicate that an HO should allocate all resources to reduce the demand variation, when possible, in one region. This action would reduce the need to rely on the more expensive air shipment option, and enables HO to utilize the cheaper surface shipment alternative. This can be seen from the comparison of the expected shortages under the two extreme distortion scenarios with $\delta = 2562$ and $\delta = 0$: The expected amount of shortages is roughly halved at the maximum degree of distortion in demand variation.

5.4 Impact of Number of Countries to Serve

We next examine the impact of the number of countries a relief organization has to serve. We increase the number of countries to be served from two countries to six countries, i.e., $n = \{2, 3, 4, 5, 6\}$. In order to solely focus on the impact of the number of countries to serve, we standardize the demand for each country where the demand for each country follows an independent and identical uniform distribution as $D_i = U \sim [50, 150]$. We allot \$5 million for each country so that the budget would be sufficient to transport the mean demand ($\mu_i = 100$) through surface transportation (i.e., 100,000 × \$50 = \$5 million). All other parameters follow the base case scenario as described earlier. The results are tabulated in Table 4.

n	2	3	4	5	6
B	\$10,000,000	\$15,000,000	\$20,000,000	\$25,000,000	\$30,000,000
$E[S^*]$	25,000	7,047	5,050	3,513	2,289
q_a^*	0	27.8	24.1	17.1	8.0
q_i^*	100	85.2	93.98	96.58	98.67
z_i^*	0.00	-0.51	-0.21	-0.12	-0.05

Table 4. Effect of number of regions to serve on the optimal values.

Table 4 shows that HO can significantly reduce the amount of expected shortages when serving more countries with the proviso that the budget increases linearly with the total expected demand. The intuition behind the reduction in the amount of expected shortages stems from the fact that the aggregated level of demand fluctuations decreases due to the "law of large numbers." Even if the demand in each country follows a unique distribution, the aggregated total demand would centralize around the mean and resemble a Normal distribution and its properties. This observation leads to smaller expected amount of shortages. This scenario also demonstrates that efficiency can be vastly improved by coordinating various relief organizations with similar missions while integrating their available funds to act as if there is a single organization budget.

Table 4 illuminates a surprising insight regarding the optimal allocation of budget to air transportation with a higher number of countries to serve. The optimal amount of humanitarian goods acquired for air transportation shows a non-monotonic behavior where it first increases then continues to decrease with respect to the number of countries to serve. This is because there are two opposing factors that conflict as the number of countries increase. As the number of countries that the HO needs to support increases, the value of flexibility that the air shipment option provides increases, which favors allocating more budget to air transportation. On the other hand, as the number of countries to serve increases, the aggregate demand becomes more centralized around the expected total demand, featuring a smaller overall variation to benefit from air shipments, which works against allocating more budget to air transportation. Due to these two opposing forces, we observe a non-monotonic behavior pertaining to the reserved budget for air transportation as the number of countries increases.

5.5 Impact of Correlation

Our preceding analysis has considered independent demand distributions. We next examine the effect of correlation between the demands in the two regions. We present the results for only positive correlation as it represents the typical case where regions are close enough that they are both affected by natural disasters or other detrimental events. Let ρ represent the correlation parameter where $0 \leq \rho \leq 1$. Note that the shape of the ellipse changes with the degree of correlation. We consider a bivariate normal distribution for the joint probability density function $f(d_1,d_2)$ of the two regions:

$$f\left(d_1, d_2 \middle| \rho\right) = \frac{1}{2\pi\sigma_1\sigma_2\sqrt{1-\rho^2}} \exp\left[-\frac{\frac{(d_1-\mu_1)^2}{\sigma_1^2} - \frac{2\rho(d_1-\mu_1)(d_2-\mu_2)}{\sigma_1\sigma_2} + \frac{(d_2-\mu_2)^2}{\sigma_2^2}}{2\left(1-\rho^2\right)} \right]. \tag{11}$$

For equation (11), the mean demand for both country equals to 100 (i.e., $\mu_1 = \mu_2 = 100$) and the standard deviation for both country equals to 50 (i.e., $\sigma_1 = \sigma_2 = 50$). The transportation costs for each mode is equal to the base case. To gain more insightful results regarding how the budget reserved for air transportation changes with respect to correlation, we consider a budget that is sufficient, B = \$12.5 million, so that the HO reserves budget for air transportation when there is no correlation between demands (i.e., ρ = 0).

ρ	0	0.25	0.5	0.75	1
$E[S^*]$	20,834	21,316	21,399	21,399	21,399
q_a^*	19.3	7.3	0	0	0
Σq_i^*	219.1	238.2	250.0	250.0	250.0
z_1^*	0.19	0.38	0.50	0.50	0.50
z_2^*	0.19	0.38	0.50	0.50	0.50

Table 5. Effect of correlation on the optimal values.

Table 5 shows that positive correlation regarding the demand in each region has an adverse impact on minimizing expected shortages. This implies the fact that having no correlation serves as the best-case scenario for an HO. When there is no demand correlation between regions (i.e., $\rho = 0$), it is optimal for the HO to reserve budget for air transportation in the case of rapid onset emergencies. However, our numerical analysis shows that when $\rho \geq 0.5$, it is optimal to not reserve any budget for air shipments and distribute the entire budget to each Country Office for RUTF to be shipped by surface transportation. We can also observe that the optimal stocking factor for both regions are equal regardless of the correlation. This is because the demand variances are equal for both regions.

The result regarding the impact of correlation on minimizing expected shortages is certainly noteworthy because the result is not quite intuitive. We can better understand why positive correlation has an adverse impact on minimizing expected shortages by focusing on the air shipment budget. If the demands in both regions are perfectly positively correlated ($\rho = 1$), there is no value to be gained from the flexible air transportation mode because you either have high or low demand realization in both regions. Under this scenario, it is similar to restricting the HO to operate with only one shipment option. However, if there is no correlation between the demands in each region, there is value to the flexible air transportation mode given that the HO has sufficient budget and the air shipment is not excessively costly. Under this scenario, the HO can freely choose between two different shipment modes as opposed to be restricted to only utilize surface transportation. This numerical result sheds light onto how HOs should allocate its budget regarding how much demand correlation there is among the regions of interest. If there is a great amount of correlation between demands, it is better to reserve less for emergency funds. On the other hand, if there is not much correlation, it is better to reserve more for emergency funds.

6. Potential Extensions

In this section, we present several potential extensions that are not incorporated into the model presented in Section 3. The model in Section 3 assumes that the ratio of landed-cost-via-air to landed-cost-via-surface is the same across regions. We begin our discussion in this section by relaxing the assumption of having equal air-to-surface cost ratios for different regions. Through numerical illustrations, we demonstrate the impact of featuring differing costs in surface and air shipment options to the countries of service. Our model in Section 3 ignores the impact of uncertainty that can be influencing the surface transportation option. We then provide a discussion regarding the impact of lead-time uncertainty in surface transportation on the optimal stocking levels for the surface and air shipment options.

6.1 Impact of different transportation costs

To examine the impact of different surface and air transportation costs for different countries, we compare the optimal values of the base case we developed in Section 5 (i.e., the highlighted columns in Table 1, 2, and 3) with the optimal values obtained by only modifying transportation costs. The following are the parameters we use for the base case: (1) total landed cost of surface transportation for both regions is $c_{s1} = c_{s2} = c_s = \$50$; (2) total landed cost of air transportation for both regions is $c_{a1} = c_{a2} = c_a = \$80$; (3) random demand in each region follows a Uniform Distribution with $D_1 = U \sim [22, 234]$ and $D_2 = U \sim [12, 292]$; and, (4) total budget is B = \$12.5 million.

To better understand the impact of different transportation costs, we analyze three different numerical examples that illustrate the reactions in the expected shortages and on the optimal stocking decisions for the surface and shipment decisions. Example 1 shows the impact of varying air shipment costs. Example 2 highlights the influence of varying surface transportation costs. Example 3 combines the impact of varying unit shipping costs both in surface and air shipment options. In each example, we increase and decrease the cost parameters by 20% from their base values.

Example 1. *We examine the case where both regions have equal total landed cost of surface transportation* (c_s = \$50)*, but different total landed cost of air transportation. We keep the total landed cost of air transportation in region* 1 *constant at* c_{a1} = \$80*, and increase/decrease the total landed cost of air transportation in region* 2 *by* 20%*, i.e.,* c_{a2} = {\$64, \$80, \$96}. *The following table shows the impact of varying air shipment cost on the expected amount of shortages* $E[S^*]$*, amount of humanitarian goods reserved for air shipment* q_a^**, and optimal stocking factors* z_1^* *and* z_2^* *for the surface shipment option.*

c_s	c_{a1}	c_{a2}	$E[S^*]$	q_a^*	z_1^*	z_2^*
		\$64	77,361	10	-0.29	-0.31
\$50	\$80	\$80	77,414	0	-0.21	-0.21
		\$96	77,414	0	-0.21	-0.21

Table 6. Optimal values for Example 1.

Table 6 shows that when the unit total landed cost of air shipment to a country is reduced (from \$80 to \$64), the HO increases its the amount of inventory reserved for air shipment and reduces the amount of humanitarian goods sent through surface shipment. The increase in air shipment can be seen from the fact that q_a^* increases from zero in the base case (when c_{a2} = \$80) to 10 units in the lower unit air shipment cost to country 2 (when c_{a2} = \$64). It should be noted that the reduction in the air shipment cost to country 2 causes a decrease in the surface level stocking factors in both countries. In the base case, the optimal stocking factors for the surface shipment alternative are $z_1^* = z_2^* = -0.21$. When the unit cost of air shipment to country 2 is reduced from c_{a2} = \$80 to c_{a2} = \$64, both stocking factor values decrease: $z_1^* = -0.29$ and $z_2^* = -0.31$. Notice that the reduction in the stocking factor for country 2's surface shipment is larger than that of country 1. Because the air shipment is relatively less costly to country 2, the HO reduces its surface shipment stocking factor for this country and relies more heavily on the relatively less costly air shipment. We also observe from the results in Table 6 that when the cost of air shipment to country 2 exceeds a certain threshold, e.g., when c_{a2} = {\$80, \$96}, the HO abandons the air shipment alternative. Thus, the optimal values for surface and air shipment are identical under c_{a2} = \$96 with those developed under c_{a2} = \$80.

Example 2. *We examine the case where both regions have equal total landed cost of air transportation* (c_a = \$80)*, but different total landed cost of surface transportation. We keep the total landed cost of surface transportation in region* 1 *constant at* c_{s1} = \$50*, and increase/decrease the total landed cost of surface transportation in region* 2 *by* 20%*, i.e.,* c_{s2} = {\$40, \$50, \$60}*. The following table shows the impact of varying surface shipment cost on the expected amount of shortages* $E[S^*]$*, amount of humanitarian goods reserved for air shipment* q_a^**, and optimal stocking factors* z_1^* *and* z_2^* *for the surface shipment option.*

c_a	c_{s1}	c_{s2}	$E[S^*]$	q_a^*	z_1^*	z_2^*
		\$40	60,518	0	-0.19	0.19
\$80	\$50	\$50	77,414	0	-0.21	-0.21
		\$60	90,883	0	-0.15	-0.53

Table 7. Optimal values for Example 2.

Table 7 illustrates the impact of the unit total landed cost for surface shipment on the optimal decisions and the expected amount of shortages. It should be immediately recognized in Example 2 that it is optimal to not reserve any budget for air transportation in all three cases. However, the optimal stocking factor decisions are influenced for the surface shipment option. Comparing the base case (where c_{s2} = \$50) with the case where c_{s2} = \$40, we observe that if the surface transportation cost is lower in region 2, the HO increases the stocking factor extensively in region 2 (where z_2^* increases from -0.21 to 0.19) with a slight increase in the stocking factor for region 1 (where z_1^* increases from -0.21 to -0.19). One might wonder why the HO increases its stocking factor for the surface shipment to country 1 when there is no change in the unit cost of the either transportation options to country 1. The reduction in the unit cost of the surface shipment to country 2 is similar to extending the budget for the HO. The savings from the surface cost (when c_{s2} = \$40 instead of c_{s2} = \$50) is rationed as surface inventory between the two countries in order to maximize the coverage and minimize the expected shortage of humanitarian goods. Comparing the base case (where c_{s2} = \$50) with the case where c_{s2} = \$60, we observe a similar phenomenon. The increasing surface shipment cost to country 2 can be perceived as reducing the budget. In this case, the HO significantly reduces the stocking factor for the surface shipment alternative to country 2 by decreasing z_2^* from -0.21 to -0.53. From this significant reduction for surface inventory assigned to country 2, the HO elevates its inventory allocation to the surface shipment alternative in country 1 by increasing z_1^* from -0.21 to -0.15. The increase in the stocking factor for country 1 is because the transportation cost becomes relatively cheaper for region 1 when c_{s2} = \$60. As a result, we conclude that the optimal stocking factor decisions do not exhibit a monotone behavior under unequal surface shipment costs.

Example 3. *We examine the case where both regions have different total landed cost of surface and air transportation. We keep the total landed cost of surface and air transportation in region* 1 *constant at* c_{s1}

= \$50 *and* c_{a1} = \$80*, respectively. We vary the total landed cost of surface and air transportation options in region* 2 *by* 20%*, i.e.,* c_{s2} = {\$40, \$50, \$60} *and* c_{a2} = {\$64, \$80, \$96}*. The following table shows the impact of varying surface and air shipment costs on the expected amount of shortages* $E[S^*]$*, amount of humanitarian goods reserved for air shipment* q_a^**, and optimal stocking factors* z_1^* *and* z_2^* *for the surface shipment option.*

c_{s1}	c_{a1}	c_{s2}	c_{a2}	$E[S^*]$	q_a^*	z_1^*	z_2^*
		\$40	\$64	60,518	0	-0.19	0.19
\$50	\$80	\$50	\$80	77,414	0	-0.21	-0.21
		\$60	\$96	90,234	21.5	-0.31	-0.78

Table 8. Optimal values for Example 3.

Table 8 shows that when both surface and air shipment costs to one country decreases, both countries obtain a higher stocking factor for the surface shipment alternative. Comparing the base case (where c_{s2} = \$50 and c_{a2} = \$80) to the case where c_{s2} = \$40 and c_{a2} = \$64, we make two observations. First, it is optimal to not reserve any budget for air transportation (i.e., $q_a^* = 0$); this is because c_{a2} = \$64 and c_{a2} = \$80 are both exceedingly expensive so that the HO abandons the air transportation option in both cases, and therefore, the different costs do not affect the optimal solution. Second, the optimal values for the stocking factor decisions expressed with z_1^* and z_2^* are identical to the optimal decisions obtained under the case where c_{s2} = \$40 in Table 7. We again observe a similar effect as if the HO has an increased budget, and thus, both z_1^* and z_2^* increase from the savings created from the less expensive surface shipment option. An interesting result is obtained by increasing both of the shipment costs to country 2. Comparing the base case (where c_{s2} = \$50 and c_{a2} = \$80) to the case where c_{s2} = \$60 and c_{a2} = \$96, we observe that it is optimal to reserve a portion of the budget for air transportation, i.e., $q_a^* = 21.5$. This is because now c_{a1} = \$80 becomes relatively cheaper when c_{s2} = \$60. In this setting, the HO reduces its stocking factor for both countries with a significant reduction in the now more expensive country 2: z_1^* decreases from -0.21 to -0.31 and z_2^* decreases from -0.21 to -0.78. Thus, increasing both cost terms to a specific country leads to an increased level of reliance on air shipments with reduced surface inventory shipments.

6.2 Impact of random lead-time for surface transportation

We next discuss how our results would change if we were to incorporate random lead-time into the surface transportation of humanitarian goods into our model. Swaminathan et al. (2009) explain that the sea-based surface transportation of humanitarian goods shows a wide range of lead times mainly because of the delays at the ports of departure, transshipment, and arrival. Congestion at the port is the most common reason for a delay. Other reasons that can cause a delay in surface transportation include issues regarding regulatory paperwork when crossing borders, port strikes, violence due to political reasons, and blocked roads due to either natural or man-made disasters. The lead-time for air shipments is significantly more stable when compared to surface transportation. The aforementioned factors causing delays in surface shipments can be avoided through air shipments.

If there is a greater amount of uncertainty regarding the lead-time for distributing humanitarian goods only for surface transportation, this would favor reserving a greater amount of budget for the air shipment option while reducing the budget allotment for the surface shipment alternative to all countries. Thus, incorporating randomness in the lead-time of the surface shipment alternative into the model would make our results associated with the air shipment alternative more pronounced. As a result, we conclude that incorporating lead-time uncertainty into our model would result in recommending humanitarian organizations to reserve a higher level of funds for air shipment.

6.3 Competition between countries for humanitarian goods

We next elaborate on each local country government's desire to influence the HO's allocation of budget to assign a higher amount of money and inventory for surface shipment; we do not expect local governments to advocate for a higher fund allocation for air shipment. Recall that UNICEF's Programme Division funds are used for the purchase and shipment of surface inventory to each country. UNICEF's Programme Division funds are allocated between each country's local UNICEF offices. The model presented in Section 3 with the objective function described with (2) determines the level of funds that will be rationed to each country without giving any preference to any country government's pressure. This model leads to the optimal choice of z_i^* that determines the amount of money to be allotted to UNICEF's local country offices. These vector values $\mathbf{z}^* = (z_1^*, \ldots, z_n^*)$, along with the optimal value of q_a^*, minimize the total expected shortage of humanitarian goods. We denote the minimum expected shortage by $E[S^*]$ where $E[S^*] = E[S(\mathbf{z}^*, q_a^*)]$.

Let us next consider the case when the local country government pressures the HO (e.g., UNICEF) to allot a higher amount of money for the purchase and shipment of humanitarian goods that will be stocked in local country offices through surface shipment. In this case, the model in (2) would be supplemented with each country's local government desire of

$$\max z_i \tag{12}$$

s.t.

$$-\mu_i/\sigma_i \leq z_i \leq \mu_i + z_h\sigma_i \tag{13}$$

Note that (12) and (13) will be replicated for each country $i = 1, \ldots, n$. Let $\hat{\mathbf{z}} = (\hat{z}_i, \ldots, \hat{z}_n)$ denote the optimal solution vector representing the service level of surface shipment in (2) supplemented with (12)-(13) with, $\hat{q}_a$ represent the air shipment, and $E[\hat{S}]$ as and the expected amount of shortages where $E[\hat{S}] = E[S(\hat{\mathbf{z}}, \hat{q}_a)]$. It is easy to verify that $E[\hat{S}] \geq E[S^*]$.

We conclude that the pressure that can be applied from local country governments to increase the funding for the purchase and surface shipment to one country can result in a higher expected shortage of humanitarian goods. Moreover, because increasing the surface shipment funds to one country requires the reduction from another country's funds for surface shipment and/or the funds for the air shipment, the HO has to determine the optimal fund allocation by ignoring the pressures from local country governments expressed in (12) and (13). In sum, we recommend the HO to ignore local government considerations that are described in (12)-(13) in order to attain the minimum expected shortage with the limited budget.

7. Conclusions

The complexities that humanitarian organizations face in order to coordinate their aid-related supply chains tend to be more challenging compared to conventional operations management. Humanitarian supply chains operate under insufficient amount of information and limited budget. In this paper, we examine the problem of delivering limited amount of perishable relief aid through two kinds of transportation modes, surface and air shipments, to regions of need in the presence of demand uncertainty. Our analysis leads to several insights that can help humanitarian organizations in their efforts to minimize the shortage of supplying essential goods to countries of desperate need.

First, we find that in the presence of reserved funds for air transportation, it is optimal to target a higher service level through surface shipments to regions with greater variance. In contrast, if the air freight cost is excessively costly and when the relief organization abandons this transportation option, it is optimal to provide equal service levels through surface shipments despite the differences of variance in demand.

Second, we show that variation in the demand for humanitarian goods play an important role in the optimal allocation of resources and funds. Increasing the accuracy of forecasts, yielding a smaller degree of uncertainty can be significantly beneficial in reducing the expected amount of shortages. Our work shows the impact of the variation in demand on the optimal stocking decisions for the surface and air shipment alternatives. We show that the HO's reaction to higher degrees of demand uncertainty depends on the optimal service level describing the level of inventory purchased for surface shipment. If the optimal service level for surface shipment is negative (implying that the inventory for surface shipment is less than the mean demand), then we prove that the HO relies more on the air shipment option and decreases its service level for the surface shipment alternative. However, if the optimal service level for surface shipment is positive (implying that the inventory for surface shipment is greater than the mean demand), then the HO reduces its reliance on the air shipment and increases its service level for the surface shipment option.

Third, our analysis shows that an HO should focus its resources and efforts to reduce fluctuations in one region as opposed to evenly allocating resources to reduce fluctuations in all regions. This action not only enables HO to service the region with known or deterministic demand with the surface transportation alternative, but also reduces the need for air shipments by serving the stochastic demand region by allocating a higher number of goods through surface transportation. We also find that there is great potential to reduce expected shortages by integrating distribution operations along with the budget from different humanitarian organizations.

Fourth, our paper demonstrates that the benefits from the combined surface and air shipment planning reduces the total expected shortage when there are more regions to serve. A higher number of regions provides the risk-pooling benefits when the budget increases linearly in the number of regions to serve. Positive correlation between the demands of multiple regions, however, presents a tougher challenge to an HO that is seeking to minimize shortages. The benefits of the responsive air transportation option decreases with respect to correlation, resulting in an increase in the amount of expected shortages.

Our study is originally motivated by the challenges for UNICEF's RUTF supply chain, however, our model is generalizable for other perishable humanitarian goods. It is beneficial for other perishable humanitarian goods that have long lead-times for surface transportation. Medical supplies, pharmaceutical products that are necessary in treating diseases such as malaria in Africa also have long lead-times, and thus, our work is suitable for the acquisition and distribution of these products in order to minimize the expected shortages. Moreover, our model is beneficial in terms of developing a response mechanism to the recent needs of the Syrian refugee crisis by allowing for variations in the needs of humanitarian goods in different regions.

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Online Supplement

Surface versus Air Shipment of Humanitarian Goods under Demand Uncertainty

Appendix

Proposition A1. *Suppose* $n = 2$.

(a) *If* $q_a = 0$, *then*

$$z_1^* = z_2^* = (B - \mu)/\sigma.$$

(b) *If* $q_a > 0$ *and* $\sigma_1 = \sigma_2$ $(= 0.5\sigma)$, *then*

$$z_1^* = z_2^* = (B - \mu - c_a q_a)/\sigma \text{ given that } B - c_a q_a \ge \max_i\{\mu_i\} - \min_i\{\mu_i\}.$$

(c) *If* $q_a > 0$ *and* $\sigma_1 < \sigma_2$, *then*

$$z_1^* \le z_2^* \text{ given that nonnegativity constraints are nonbinding } (i.e., q_1, q_2 \ge 0).$$

Proof of Proposition A1. For a given q_a, we let $k = B - (\mu + c_a q_a)$ and rewrite $B = \mu + z_1\sigma_1 + z_2\sigma_2 + c_a q_a$ as

$$z_2 = (k - z_1\sigma_1)/\sigma_2. \tag{14}$$

From nonnegative $q_i = \mu + z_i\sigma_i \ge 0$ for $i = 1, 2$, we require $z_i \ge -\mu_i/\sigma_i$. Note that

$$z_2 \ge -\mu_2/\sigma_2 \Leftrightarrow z_1 \le (k + \mu_2)/\sigma_1.$$

Substituting the above into (2) yields the following univariate optimization problem:

$$\min_{-\mu_1/\sigma_1 \le z_1 \le (k+\mu_2)/\sigma_1} f(z_1) \tag{15}$$

where $f(z_1)$ is the expected shortage, i.e.,

$$f(z_1) = E\left[\left(\sigma_1(Z_1 - z_1)^+ + \sigma_2\left(Z_2 - \frac{k - z_1\sigma_1}{\sigma_2}\right)^+ - q_a\right)^+\right]$$

$$= \int_{-\infty}^{z_1} \int_{\frac{k - z_1\sigma_1 + q_a}{\sigma_2}}^{\infty} \left[\sigma_2\left(y_2 - \frac{k - z_1\sigma_1}{\sigma_2}\right) - q_a\right]\phi(y_2)\phi(y_1)\,dy_2\,dy_1$$

$$+ \int_{z_1 + \frac{q_a}{\sigma_1}}^{\infty} \int_{-\infty}^{\frac{k - z_1\sigma_1}{\sigma_2}} \left[\sigma_1(y_1 - z_1) - q_a\right]\phi(y_2)\phi(y_1)\,dy_2\,dy_1$$

$$+ \int_{z_1}^{z_1 + \frac{q_a}{\sigma_1}} \int_{\frac{k - y_1\sigma_1 + q_a}{\sigma_2}}^{\infty} \left[\sigma_1(y_1 - z_1) + \sigma_2\left(y_2 - \frac{k - z_1\sigma_1}{\sigma_2}\right) - q_a\right]\phi(y_2)\phi(y_1)\,dy_2\,dy_1$$

$$+\int_{z_1+\frac{q_a}{\sigma_1}}^{\infty}\int_{\frac{k-z_1\sigma_1}{\sigma_2}}^{\infty}\left[\sigma_1(y_1-z_1)+\sigma_2\left(y_2-\frac{k-z_1\sigma_1}{\sigma_2}\right)-q_a\right]\phi(y_2)\phi(y_1)\,dy_2dy_1 .$$

The above integral terms correspond to regions of shortage. The (Z_1, Z_2) space can be divided into the following sets:

$\Omega_0 = \{Z_1 \le z_1 + q_a/\sigma_1$ and $Z_2 \le (k - z_1\sigma_1)/\sigma_2 + q_a/\sigma_2$ and $Z_2 \le (k + q_a - Z_1\sigma_1)/\sigma_2\}$

$\Omega_1 = \{Z_1 \le z_1$ and $Z_2 > (k - z_1\sigma_1)/\sigma_2 + q_a/\sigma_2\}$

$\Omega_2 = \{Z_1 > z_1 + q_a/\sigma_1$ and $Z_2 \le (k - z_1\sigma_1)/\sigma_2\}$

$\Omega_3 = \{Z_1 > z_1$ and $Z_2 > (k - z_1\sigma_1)/\sigma_2$ and $Z_2 > (k + q_a - Z_1\sigma_1)/\sigma_2\}$.

There are no shortages in region Ω_0; shortages are realized in the remaining three regions. Region Ω_1 corresponds to the first integral term in the above expression, Ω_2 corresponds to the second integral term, and Ω_3 corresponds to the last two integral terms.

The first and second derivatives of f with respect to z_1 are

$$f_{z_1} = \sigma_1\left(\int_{-\infty}^{z_1}\int_{\frac{k-z_1\sigma_1+q_a}{\sigma_2}}^{\infty}\phi(y_2)\phi(y_1)\,dy_2dy_1 - \int_{z_1+\frac{q_a}{\sigma_1}}^{\infty}\int_{-\infty}^{\frac{k-z_1\sigma_1}{\sigma_2}}\phi(y_2)\phi(y_1)\,dy_2dy_1\right) \tag{16}$$

$$f_{z_1z_1} = \sigma_1\left(\int_{\frac{k-z_1\sigma_1+q_a}{\sigma_2}}^{\infty}\phi(y_2)\phi(z_1)\,dy_2 + \int_{-\infty}^{\frac{k-z_1\sigma_1}{\sigma_2}}\phi(y_2)\phi\left(z_1+\frac{q_a}{\sigma_1}\right)dy_2\right) > 0. \tag{17}$$

Thus, $f(z_1)$ is convex and is minimized at unique stationary point z_1^{o} (i.e., z_1^{o} solves $f_{z_1}(z_1) = 0$) and the optimal stocking factors are

$$z_1^* = \max\left\{\frac{-\mu_1}{\sigma_1}, \min\left\{z_1^{o}, \frac{k+\mu_2}{\sigma_1}\right\}\right\} \tag{18}$$

$$z_2^* = \frac{k - z_1^*\sigma_1}{\sigma_2}. \tag{19}$$

Proof of part (a): From (16) we see that if $q_a = 0$, then

$f_{z_1}(z_1) = 0 \Leftrightarrow z_1 = (k - z_1\sigma_1)/\sigma_2 = (B - \mu - z_1\sigma_1)/\sigma_2 = z_2$,

which implies

$z_1^{o} = (B - \mu)/\sigma$.

Note that stocking factor constraints $z_i \ge -\mu_i/\sigma_i$ are satisfied at $z_1 = z_2 = z_1^{o} = (B - \mu)/\sigma$ if and only if

$$B \ge \mu - \sigma\min_i\{\mu_i/\sigma_i\}. \tag{20}$$

Thus, if (20) holds, then from (18) and (19) it follows that

$z_1^* = z_2^* = (B - \mu)/\sigma$,

and if (20) doesn't hold, then from (18) and (19) it follows that

$$(z_1^*, z_2^*) = \begin{cases} \left(\frac{-\mu_1}{\sigma_1}, \frac{B-\mu_2}{\sigma_2}\right), \text{ if } \frac{\mu_1}{\sigma_1} \le \frac{\mu_2}{\sigma_2} \\ \left(\frac{B-\mu_1}{\sigma_1}, \frac{-\mu_2}{\sigma_2}\right), \text{ if } \frac{\mu_1}{\sigma_1} \ge \frac{\mu_2}{\sigma_2} \end{cases}.$$

However, because of the assumption $\sum_{i=1}^{n}\left(\mu_i + z_l\sigma_i\right) < B$, (20) always holds. Thus, if $q_a = 0$, then $z_1^* = z_2^* = (B - \mu)/\sigma$.

Proof of part (*b*): From (16) we see that if $\sigma_1 = \sigma_2$ (= 0.5σ), then

$f_{z_1}\left(z_1\right) = 0 \Leftrightarrow z_1 = (k - z_1\sigma_1)/\sigma_1 = (B - \mu - c_a q_a - z_1\sigma_1)/\sigma_1 = z_2$

which implies

$z_1^{o} = (B - \mu - c_a q_a)/\sigma$.

Note that stocking factor constraints $z_i \ge -\mu_i/\sigma_1 = -2\mu_i/\sigma$ are satisfied at $z_1 = z_2 = z_1^{o} = (B - \mu - c_a q_a)/\sigma$ if and only if

$$B - c_a q_a \ge \max_i\{\mu_i\} - \min_i\{\mu_i\} \tag{21}$$

i.e., $\min_i\{\mu_i + z_1^{o}\sigma/2\} = \mu/2 - [\max_i\{\mu_i\} - \min_i\{\mu_i\}]/2 + z_1^{o}\sigma/2 \ge 0 \Leftrightarrow B - c_a q_a \ge \max_i\{\mu_i\} - \min_i\{\mu_i\}$.

Thus, if (21) holds, then from (18) and (19) it follows that

$z_1^* = z_2^* = (B - \mu - c_a q_a)/\sigma$,

and if (21) doesn't hold, then from (18) and (19) it follows that

$$(z_1^*, z_2^*) = \begin{cases} \left(\frac{-\mu_1}{0.5\sigma}, \frac{B-c_a q_a-\mu_2}{0.5\sigma}\right), \text{ if } \mu_1 \le \mu_2 \\ \left(\frac{B-c_a q_a-\mu_1}{0.5\sigma}, \frac{-\mu_2}{0.5\sigma}\right), \text{ if } \mu_1 \ge \mu_2 \end{cases}.$$

For the condition $B - c_a q_a \ge \max_i\{\mu_i\} - \min_i\{\mu_i\}$ to be violated, q_a needs to be great enough that the remaining budget for surface transportation does not cover z_l for each region, which is not an optimal behavior. Thus, $z_1^* = z_2^* = (B - \mu - c_a q_a)/\sigma$ given that $B - c_a q_a \ge \max_i\{\mu_i\} - \min_i\{\mu_i\}$.

Proof of part (*c*): Let $\bar{\sigma} = 0.5\sigma$, $\sigma_1 = \bar{\sigma} - \delta$ and $\sigma_2 = \bar{\sigma} + \delta$ where $\delta \in \left[0, \bar{\sigma}\right]$. Recall the z_1^{o} solves $f_{z_1}\left(z_1\right) = 0$ (i.e., z_1^{o} is the global minimum of convex univariate expected shortage function $f(z_1)$). Let z_2^{o} be the corresponding unconstrained optimal stocking factor for region 2, i.e., $z_2^{o} = (k - z_1^{o}\sigma_1)/\sigma_2$. The proof proceeds through three steps:

(1) We show $z_1^{o}{}'\left(\delta \mid \delta = 0\right) \le 0$.

(2) We show that (1) implies $z_1^{\mathrm{o}}{}'(\delta \mid \delta > 0) \le 0$.

(3) We show that (2) implies $z_1^{\mathrm{o}} \le z_2^{\mathrm{o}}$, which implies $z_1^* \le z_2^*$ when nonnegativity constraints are non-binding.

Step (1): We rewrite (16) as

$$f_{z_1}(z_1,\delta) = (\bar{\sigma}-\delta)\left(\int_{-\infty}^{z_1} \int_{\frac{k-z_1(\bar{\sigma}-\delta)+q}{\bar{\sigma}+\delta}}^{\infty} \phi(y_2)\phi(y_1)\,dy_2dy_1 - \int_{z_1+\frac{q_a}{\bar{\sigma}-\delta}}^{\infty} \int_{-\infty}^{\frac{k-z_1(\bar{\sigma}-\delta)}{\bar{\sigma}+\delta}} \phi(y_2)\phi(y_1)\,dy_2dy_1 \right)$$

and take the partial with respect to δ evaluated at $z_1 = z_1^{\mathrm{o}}$,

$$f_{z_1\delta}(z_1^{\mathrm{o}},\delta) = \frac{\bar{\sigma}-\delta}{\bar{\sigma}+\delta}\left(\begin{array}{l} \left(\dfrac{k-z_1^{\mathrm{o}}\sigma+q_a}{\bar{\sigma}+\delta}\right)\phi\left(\dfrac{k-z_1^{\mathrm{o}}(\bar{\sigma}-\delta)+q_a}{\bar{\sigma}+\delta}\right)\Phi(z_1^{\mathrm{o}}) + \\ \left(\dfrac{k-z_1^{\mathrm{o}}\sigma}{\bar{\sigma}+\delta}\right)\phi\left(\dfrac{k-z_1^{\mathrm{o}}(\bar{\sigma}-\delta)}{\bar{\sigma}+\delta}\right)\left(1-\Phi\left(z_1^{\mathrm{o}}+\dfrac{q_a}{\bar{\sigma}-\delta}\right)\right) \end{array} \right).$$

By the implicit function theorem,

$$z_1^{\mathrm{o}}{}'(\delta) = \frac{-f_{z_1\delta}(z_1^{\mathrm{o}},\delta)}{f_{z_1z_1}(z_1^{\mathrm{o}},\delta)}. \tag{22}$$

Consider the function $z_1^{\mathrm{o}}(\delta)$. At $\delta = 0$, we know from part (b) that $z_1^0(0) = k/\sigma$, which implies

$$k - z_1^{\mathrm{o}}(0)\sigma = 0, \tag{23}$$

and thus

$$f_{z_1\delta}(z_1^{\mathrm{o}},0) = \left(\frac{q_a}{\bar{\sigma}+\delta}\right)\phi\left(\frac{q_a}{\bar{\sigma}}\right)\Phi(z_1^{\mathrm{o}}) \ge 0. \tag{24}$$

From (17), (22), and (24), it follows that

$$z_1^{\mathrm{o}}{}'(\delta \mid \delta = 0) \le 0. \tag{25}$$

Step (2): We see (from (25)) that z_1^0 is initially decreasing in δ, and as z_1^0 decreases, $k - z_1^0\sigma$ increases (becoming positive). And $k - z_1^0\sigma \ge 0$, assures that $f_{z_1\delta}(z_1^{\mathrm{o}},\delta) \ge 0$ for all $\delta \in [0,\bar{\sigma}]$. Therefore

$$z_1^{\mathrm{o}}{}'(\delta) \le 0 \text{ for all } \delta \in [0,\bar{\sigma}]. \tag{26}$$

Step (3): Note that

$z_1^{\mathrm{o}}(0)\sigma = k$ (see (23))

$z_1^{\mathrm{o}}(\delta) \le z_1^0(0)$. (see (26))

Therefore

$$k = z_1^{\mathrm{o}}(\delta)(\bar{\sigma}-\delta) + z_2^{\mathrm{o}}(\delta)(\bar{\sigma}+\delta) \quad \text{(by definition)}$$

$$= z_1^{\mathrm{o}}(\delta)\sigma + \left[z_2^{\mathrm{o}}(\delta) - z_1^{\mathrm{o}}(\delta)\right](\bar{\sigma}+\delta)$$

$$\le z_1^{\mathrm{o}}(0)\sigma + \left[z_2^{\mathrm{o}}(\delta) - z_1^{\mathrm{o}}(\delta)\right](\bar{\sigma}+\delta)$$

$$= k + \left[z_2^{\mathrm{o}}(\delta) - z_1^{\mathrm{o}}(\delta)\right](\bar{\sigma}+\delta),$$

which implies $z_1^{\mathrm{o}} \le z_2^{\mathrm{o}}$. If nonnegativity constraints are nonbinding, then the optimal stocking factors are $z_1^* = z_1^{\mathrm{o}}$ and $z_2^* = z_2^{\mathrm{o}}$ (see (18)). ❑

Proof of Proposition 1. Let $N = \{1, \ldots, n\}$. We may decompose $E[S(\mathbf{z})]$ into two terms as follows:

$$E[S(\mathbf{z}, q_a)] = E\left[\sum_{i \in N \backslash \{l,m\}} \left(\sigma_i (Z_i - z_i)^+ - q_a\right)^+\right] + E\left[\sum_{i \in \{l,m\}} \left(\sigma_i (Z_i - z_i)^+ - \left(q_a - \sum_{i \in N \backslash \{l,m\}} \sigma_i (Z_i - z_i)^+\right)^+\right)^+\right]$$

$$= E\left[S_{N \backslash \{l,m\}}(\mathbf{z}, q_a)\right] + E\left[S_{\{l,m\}}(\mathbf{z}, q_a)\right].$$

The second term in the expression is the expected units short for two arbitrarily selected regions denoted l and m. These two regions are served with air shipments, if needed, after the initial shortages at all other regions have been covered via air to the extent of available supply q. From the optimality of $\mathbf{z}^*$, it follows that for any pair (l, m),

$$E\left[S_{\{l,m\}}(\mathbf{z}^*, q)\right] = \min_{-\mu_l/\sigma_l \le z_l \le (k+\mu_m)/\sigma_l} \left\{E\left[S_{\{l,m\}}(\mathbf{z}, q_a)\right] : z_m = \frac{k - z_l \sigma_l}{\sigma_m}, z_j = z_j^* \ \forall j \in N \backslash \{l,m\},\right\}$$

where $k = B - \mu - c_a q_a - \sum_{i \in N \backslash \{l,m\}} z_i^* \sigma_i$. Therefore, if $q_a = 0$ or if $q_a > 0$ and $\sigma_1 = \sigma_2 = \ldots = \sigma_n$, then it follows from Proposition A1 that $z_l^* = z_m^*$ for any pair (l, m), and thus

$$z_1^* = z_2^* = \ldots = z_n^* = \frac{B - \mu - c_a q_a}{\sigma},$$

providing the results presented in Proposition 1(a) and 1(b). Similarly, if $q_a > 0$, $\sigma_i < \sigma_{i+1}$ for some i, and nonnegativity constraints are nonbinding, then it follows from Proposition A1 that $z_l^* \le z_m^*$ for any pair $l < m$, and thus

$$z_1^* \le z_2^* \le \ldots \le z_n^*,$$

which is Proposition 1(c). ❑

Proof of Proposition 2: The proof utilizes the definition of probability subset $\Omega_n^{0j}(z)$ describing the j^{th} partition of $\Omega_n^0(z)$: $\Omega_n^{0j}(z) = \left\{\frac{\sigma}{n}\sum_{i=1}^{n} 1_{i \in \Lambda^+(j)}(Z_i - z) > \left(\frac{B - \mu - z\sigma}{c_a}\right)\right\}$. When realized demand Z_i in country i exceeds the surface stocking factor z, shortages occur when the difference between the realized demand and stocking factor for the surface shipment $\frac{\sigma}{n}\sum_{i=1}^{n} 1_{i \in \Lambda^+(j)}(Z_i - z)$ is greater than the amount of inventory reserved for air shipment $\left(\dfrac{B-\mu-z\sigma}{c_a}\right)$. Thus, the shortage amount for a given set of realized demand values Z_i in each country $i = 1, \ldots, n$ can be written as:

$$\frac{\sigma}{n}\sum_{i=1}^{n} 1_{i\in\Lambda^+(j)}\left(Z_i - z\right) - \left(\frac{B-\mu-z\sigma}{c_a}\right) = \left(\frac{\sigma}{n}\right)\left[\sum_{i=1}^{n} 1_{i\in\Lambda^+(j)}\left(Z_i - z\right) - n\left(\frac{B-\mu}{c_a\sigma} - \frac{z}{c_a}\right)\right].$$

The expected shortage amount in the j^{th} partition of $\Omega_n^0(z)$, described with $f(z \mid j)$, is

$$f(z \mid j) = \int \ldots \int_{\Omega_n^{0j}(z)} \left(\frac{\sigma}{n}\right)\left[\sum_{i=1}^{n} 1_{i\in\Lambda^+(j)}\left(y_i - z\right) - n\left(\frac{B-\mu}{c_a\sigma} - \frac{z}{c_a}\right)\right]\phi\left(y_1\right)\ldots\phi\left(y_n\right)dy_1\ldots dy_n .$$

Combining all partitions, we obtain the total expected shortage, described as $f(z)$:

$$f(z) = \sum_{j=1}^{J}\left[\int \ldots \int_{\Omega_n^{0j}(z)} \left(\frac{\sigma}{n}\right)\left[\sum_{i=1}^{n} 1_{i\in\Lambda^+(j)}\left(y_i - z\right) - n\left(\frac{B-\mu}{c_a\sigma} - \frac{z}{c_a}\right)\right]\phi\left(y_1\right)\ldots\phi\left(y_n\right)dy_1\ldots dy_n\right].$$

Taking the derivative of $f(z)$ with respect to z leads to the first-order condition

$$\sum_{j=1}^{J}\left[\int \ldots \int_{\Omega_n^{0j}(z)} \left(\frac{\sigma}{n}\right)\left[\sum_{i=1}^{n} 1_{i\in\Lambda^+(j)}\left(-1\right) + \left(\frac{n}{c_a}\right)\right]\phi\left(y_1\right)\ldots\phi\left(y_n\right)dy_1\ldots dy_n\right] = 0$$

and provides the expression in (10) by replacing $\Omega_n^{0j}(z)$ with the set $\Omega_n^{0j}(z^*)$. ❑

Proposition A2. *Suppose* $n = 2$ *and* $\sigma_1 = \sigma_2$.

(a) $z_1^* = z_2^* = (B-\mu)/\sigma$ *and* $q_a^* = 0$ *if and only if*

$c_a - 1 \geq \Phi((B-\mu)/\sigma)$, *or equivalently* $(B-\mu)/\sigma \leq \Phi^{-1}(c_a - 1)$;

(b) *If* $(z_h - z_l)\sigma > (B - \mu - z_l\sigma)/c_a$, *then*

$z_1^* = z_2^* = z^* > z_l$ *and* $q_a^* < (B-\mu-z_l\sigma)/c_a$ *where* z^* *satisfies*

$$\left[\bar{\Phi}\left(z^* + 2g(\sigma)\right)\left(\Phi\left(z^*\right) - \left(c_a - 1\right)\right) - \left(c_a - 1\right)P\left[z^* \leq Z_1 \leq z^* + 2g(\sigma), Z_2 \geq z^* + 2g(\sigma) - \left(Z_1 - z^*\right)\right]\right] = 0 . \quad (27)$$

Proof of Proposition A2. Describing the total expected shortage with $f(z)$, the problem can be expressed as follows:

$$\min_z\left\{f(z) : \frac{-\max_i\{\mu_i\}}{\sigma/2} \leq z \leq \frac{B-\mu}{\sigma}\right\}.$$

where

$$f(z) = E\left[\left(\frac{\sigma}{2}\sum_{i=1}^{2}\left(Z_i - z\right)^+ - \frac{B-\mu-z\sigma}{c_a}\right)^+\right]$$

$$= \int_{z+\frac{2(B-\mu-z\sigma)}{c_a\sigma}}^{\infty} \int_{-\infty}^{z}\left[\frac{\sigma}{2}\left(y_1 - z\right) - \frac{B-\mu-z\sigma}{c_a}\right]\phi\left(y_2\right)\phi\left(y_1\right)dy_2 dy_1$$

$$+\int_{-\infty}^{z}\int_{z+\frac{2(B-\mu-z\sigma)}{c_a\sigma}}^{\infty}\left[\frac{\sigma}{2}(y_2-z)-\frac{B-\mu-z\sigma}{c_a}\right]\phi(y_2)\phi(y_1)\,dy_2dy_1$$

$$+\int_{z}^{z+\frac{2(B-\mu-z\sigma)}{c_a\sigma}}\int_{z+\frac{2(B-\mu-z\sigma)}{c_a\sigma}-(y_1-z)}^{\infty}\left[\frac{\sigma}{2}(y_1+y_2-2z)-\frac{B-\mu-z\sigma}{c_a}\right]\phi(y_2)\phi(y_1)\,dy_2dy_1$$

$$+\int_{z+\frac{2(B-\mu-z\sigma)}{c_a\sigma}}^{\infty}\int_{z}^{\infty}\left[\frac{\sigma}{2}(y_1+y_2-2z)-\frac{B-\mu-z\sigma}{c_a}\right]\phi(y_2)\phi(y_1)\,dy_2dy_1\,.$$

Taking the derivative,

$$f_z(z)=2\sigma\left(\frac{1}{c_a}-\frac{1}{2}\right)\bar{\Phi}\left(z+\frac{2(B-\mu-z\sigma)}{c_a\sigma}\right)\Phi(z)$$

$$-\sigma\left(1-\frac{1}{c_a}\right)\left(\begin{array}{l}\bar{\Phi}\left(z+\frac{2(B-\mu-z\sigma)}{c_a\sigma}\right)\bar{\Phi}(z)+\\ P\left[z\le Z_1\le z+\frac{2(B-\mu-z\sigma)}{c_a\sigma},Z_2\ge z+\frac{2(B-\mu-z\sigma)}{c_a\sigma}-(Z_1-z)\right]\end{array}\right). \tag{28}$$

Proof of part (*a*): Let $z_0 = (B-\mu)/\sigma$, which is the stocking factor for which there is no remaining budget available for air, i.e., $q_a = 0$. Let $\bar{\Phi}(z)=1-\Phi(z)$. From (28) it follows that $f_z(z)\le 0$ for all z when $c_a \ge 2$, i.e., $c_a \ge 2$ implies $z^* = z_0$. Now suppose $c_a < 2$. Note that

$$f_z(z_o)=\sigma\bar{\Phi}(z_0)\left\{2\left(\frac{1}{c_a}-\frac{1}{2}\right)\Phi(z_0)-\left(1-\frac{1}{c_a}\right)\bar{\Phi}(z_0)\right\}$$

$$=\frac{\sigma}{c_a}\bar{\Phi}(z_0)\left\{\Phi(z_0)-(c_a-1)\right\},$$

and thus $f_z(z_o)\le 0$ if and only if $c_a - 1 \ge \Phi(z_0)$. Let

$$\zeta(z)=z+\frac{2(B-\mu-z\sigma)}{c_a\sigma}.$$

Note that $\zeta(z)$ is increasing in z (due to $c_a > 1$) and that $\zeta(z_o) = z_0$, i.e.,

$$\zeta(z)\le z_o \text{ for all } z\le z_o. \tag{29}$$

Now suppose that

$$f_z(z_o)\le 0. \tag{30}$$

Note that

$$f_z(z)\le 2\sigma\left(\frac{1}{c_a}-\frac{1}{2}\right)\bar{\Phi}(\zeta(z))\Phi(\zeta(z))-\sigma\left(1-\frac{1}{c_a}\right)\bar{\Phi}(\zeta(z))^2 \quad \text{(see (28))}$$

$$=\frac{\sigma}{c_a}\bar{\Phi}\left(\zeta\left(z\right)\right)\left\{\Phi\left(\zeta\left(z\right)\right)-\left(c_a-1\right)\right\}$$

$$\leq\frac{\sigma}{c_a}\bar{\Phi}\left(\zeta\left(z\right)\right)\left\{\Phi\left(z_0\right)-\left(c_a-1\right)\right\} \qquad \text{(due to (29))}$$

$$\leq 0. \qquad \text{(due to (30))}$$

Thus, $f_z(z_o) \leq 0$ implies $z^* = z_0$ and $q_a^* = 0$. Furthermore, $f_z(z_o) > 0$ implies $z^* < z_0$ and $q_a^* > 0$.

Proof of part (*b*): Evaluating the above at the left boundary of the support,

$$f_z(z_l) = -\sigma\left(1-\frac{1}{c_a}\right)\left(\begin{array}{l}\bar{\Phi}\left(z_l+\frac{2\left(B-\mu-z_l\sigma\right)}{c_a\sigma}\right)+\\ P\left[z_l\leq Z_1\leq z_l+\frac{2\left(B-\mu-z\sigma\right)}{c_a\sigma}, Z_2\geq z_l+\frac{2\left(B-\mu-z_l\sigma\right)}{c_a\sigma}-\left(Z_1-z_l\right)\right]\end{array}\right).$$

Note that $1 - 1/c_a > 0$ (due to $c_a > 1$). Further, the second parenthetical term is nonnegative and is zero if and only

$$z_h - z_l \leq \frac{B-\mu-z_l\sigma}{c_a\sigma}\;. \qquad (31)$$

If (31) does not hold, then $f_z(z_l) < 0$, which implies $z^* > z_l$.

The optimal stocking factor for the surface transportation can be obtained by equating $f_z(z)$ in (28) to 0, and rearranging the terms.

$$\sigma\left[\begin{array}{l}2\left(\frac{1}{c_a}-\frac{1}{2}\right)\bar{\Phi}\left(z+\frac{2\left(B-\mu-z\sigma\right)}{c_a\sigma}\right)\Phi\left(z\right)\\ -\left(1-\frac{1}{c_a}\right)\left(\begin{array}{l}\bar{\Phi}\left(z+\frac{2\left(B-\mu-z\sigma\right)}{c_a\sigma}\right)\bar{\Phi}\left(z\right)+\\ P\left[z\leq Z_1\leq z+\frac{2\left(B-\mu-z\sigma\right)}{c_a\sigma}, Z_2\geq z+\frac{2\left(B-\mu-z\sigma\right)}{c_a\sigma}-\left(Z_1-z\right)\right]\end{array}\right)\end{array}\right]=0$$

implies that

$$\left[\begin{array}{l}2\left(\frac{1}{c_a}-\frac{1}{2}\right)\bar{\Phi}\left(z+\frac{2\left(B-\mu-z\sigma\right)}{c_a\sigma}\right)\Phi\left(z\right)-\left(1-\frac{1}{c_a}\right)\bar{\Phi}\left(z+\frac{2\left(B-\mu-z\sigma\right)}{c_a\sigma}\right)\left(1-\Phi\left(z\right)\right)\\ -\left(1-\frac{1}{c_a}\right)P\left[z\leq Z_1\leq z+\frac{2\left(B-\mu-z\sigma\right)}{c_a\sigma}, Z_2\geq z+\frac{2\left(B-\mu-z\sigma\right)}{c_a\sigma}-\left(Z_1-z\right)\right]\end{array}\right]=0$$

$$\left[\begin{array}{l}\frac{1}{c_a}\bar{\Phi}\left(z+\frac{2\left(B-\mu-z\sigma\right)}{c_a\sigma}\right)\Phi\left(z\right)-\left(1-\frac{1}{c_a}\right)\bar{\Phi}\left(z+\frac{2\left(B-\mu-z\sigma\right)}{c_a\sigma}\right)\\ -\left(1-\frac{1}{c_a}\right)P\left[z\leq Z_1\leq z+\frac{2\left(B-\mu-z\sigma\right)}{c_a\sigma}, Z_2\geq z+\frac{2\left(B-\mu-z\sigma\right)}{c_a\sigma}-\left(Z_1-z\right)\right]\end{array}\right]=0$$

$$\frac{1}{c_a}\begin{bmatrix}\bar{\Phi}\left(z+\frac{2(B-\mu-z\sigma)}{c_a\sigma}\right)\Phi(z)-(c_a-1)\bar{\Phi}\left(z+\frac{2(B-\mu-z\sigma)}{c_a\sigma}\right)\\ -(c_a-1)P\left[z\le Z_1\le z+\frac{2(B-\mu-z\sigma)}{c_a\sigma},Z_2\ge z+\frac{2(B-\mu-z\sigma)}{c_a\sigma}-(Z_1-z)\right]\end{bmatrix}=0$$

$$\begin{bmatrix}\bar{\Phi}\left(z+\frac{2(B-\mu-z\sigma)}{c_a\sigma}\right)\Phi(z)-(c_a-1)\bar{\Phi}\left(z+\frac{2(B-\mu-z\sigma)}{c_a\sigma}\right)\\ -(c_a-1)P\left[z\le Z_1\le z+\frac{2(B-\mu-z\sigma)}{c_a\sigma},Z_2\ge z+\frac{2(B-\mu-z\sigma)}{c_a\sigma}-(Z_1-z)\right]\end{bmatrix}=0$$

$$\begin{bmatrix}\bar{\Phi}\left(z+\frac{2(B-\mu-z\sigma)}{c_a\sigma}\right)\left(\Phi(z)-(c_a-1)\right)\\ -(c_a-1)P\left[z\le Z_1\le z+\frac{2(B-\mu-z\sigma)}{c_a\sigma},Z_2\ge z+\frac{2(B-\mu-z\sigma)}{c_a\sigma}-(Z_1-z)\right]\end{bmatrix}=0$$

$$\left[\bar{\Phi}(z+2g(\sigma))\left(\Phi(z)-(c_a-1)\right)-(c_a-1)P\left[z\le Z_1\le z+2g(\sigma),Z_2\ge z+2g(\sigma)-(Z_1-z)\right]\right]=0$$

and provides the expression in (27). ❑

Proof of Proposition 3. The expected shortage function for general n case is the following:

$$f(z)=E\left[\left(\frac{\sigma}{n}\sum_{i=1}^{n}(Z_i-z)^+-\frac{B-\mu-z\sigma}{c_a}\right)^+\right].$$

Let Ω_n(z) denote the following event:

$$\Omega_n(z):\sum_{i=1}^{n}Z_i>n\left(z+\frac{B-\mu-z\sigma}{c_a\sigma}\right)\ \&\ Z_i\ge z,\forall i.$$

Then we can rewrite $f(z)$ as

$$f(z)=\frac{\sigma}{n}\begin{Bmatrix}E\left[\sum_{i=1}^{n}Z_i-n\left(z+\frac{B-\mu-z\sigma}{c_a\sigma}\right)|\,\Omega_n(z)\right]P\left[\Omega_n(z)\right]+\\ E\left[\left(\sum_{i=1}^{n}(Z_i-z)^+-n\left(\frac{B-\mu-z\sigma}{c_a\sigma}\right)\right)^+|\text{ not }\Omega_n(z)\right]P\left[\text{not }\Omega_n(z)\right]\end{Bmatrix}.$$

Note that $P[\text{not }\Omega_n(z_l)] = 0$. This is because event "not $\Omega_n(z_l)$" requires $Z_i < z_l$ for some i, which is impossible. Accordingly, the derivative of $f(z)$ evaluated at $z = z_l$ becomes

$$f_z(z_l)=-\frac{\sigma}{n}\left\{n\left(\frac{1}{c_a}-1\right)P\left[\Omega_n(z_l)\right]\right\}=-\sigma\left(\frac{1}{c_a}-1\right)P\left[\Omega_n(z_l)\right].$$

Thus, from $c_a > 1$, it follows that $f_z(z_l) \ge 0$ if and only if

$$P\left[\Omega_n(z_l)\right]=0$$

$$\Leftrightarrow \quad P\left[\sum_{i=1}^{n} Z_i > n\left(z_l + \frac{B-\mu-z_l\sigma}{c_a\sigma}\right)\right] = 0$$

$$\Leftrightarrow \quad P\left[Z_i > z_l + \frac{B-\mu-z_l\sigma}{c_a\sigma}\right] = 0$$

$$\Leftrightarrow \quad z_h - z_l \le \frac{B-\mu-z_l\sigma}{c_a\sigma}.$$

$$\Leftrightarrow \quad c_a \le \frac{B-\mu-z_l\sigma}{(z_h-z_l)\sigma}. \tag{32}$$

If (32) does not hold, then $f_z(z_l) < 0$, which implies $z^* > z_l$. In summary, we have proved that Proposition A2(b) extends to $n > 2$, i.e., suppose $\sigma_1 = \sigma_2 = \ldots = \sigma_n$: if $c_a > \frac{B-\mu-z_l\sigma}{(z_h-z_l)\sigma}$, then $z^* > z_l$ and $q_a^* < \frac{B-\mu-z_l\sigma}{c_a}$. ❑

Proof of Proposition 4: According to $\Omega_n^0(z)$, shortages occur when $Z_i > z$ for some $i = 1,\ldots, n$ and when $K(z,\sigma) = \frac{\sigma}{n}\left[\sum_{i=1}^{n}(Z_i - z)^+ - n\left(\frac{B-\mu-z\sigma}{c_a\sigma}\right)\right] > 0$. Thus, the probability set can be defined as follows: $P[\Omega_n^0(z)] = \{Z_i > z$ for some $i = 1,\ldots, n$ and $K(z, \sigma) > 0\}$. Taking the derivative of $K(z, \sigma)$ with respect to σ provides the result: $\frac{\partial K(z,\sigma)}{\partial\sigma} = \frac{1}{n}\left[\sum_{i=1}^{n}(Z_i - z)^+ - n\left(\frac{B-\mu-z\sigma}{c_a\sigma}\right)\right] + \left(\frac{B-\mu}{c_a\sigma}\right) > 0$ because $\left[\sum_{i=1}^{n}(Z_i - z)^+ - n\left(\frac{B-\mu-z\sigma}{c_a\sigma}\right)\right] > 0$ by definition of the probability condition and $\left(\frac{B-\mu}{c_a\sigma}\right) > 0$ when $B > \mu$. Therefore, the probability $P[\Omega_n^0(z)]$ is non-decreasing in σ. The expected amount of shortages, defined by $f(z, \sigma)$ is equivalent to $f(z,\sigma) = \frac{\sigma}{n}\left[E\left[K(z,\sigma)\right] \middle| \Omega_n^0(z)\right] P\left[\Omega_n^0(z)\right]$. Because $K(z, \sigma)$ and $P[\Omega_n^0(z)]$ are non-decreasing in σ, then, $f(z, \sigma)$ is also non-decreasing in σ. ❑

Proof of Lemma 1. The air shipment is equal to $q_a = (B - \mu - z\sigma)/c_a$. The derivative with respect to σ provides the result: $\partial q_a / \partial\sigma = -z / c_a$, and its sign is the opposite of the sign of z. ❑

Proof of Lemma 2. The proof follows from the result of Lemma 1. The air shipment is equal to $q_a = (B - \mu - z\sigma)/c_a$. Note that HO spends its entire budget; therefore, the inventory for air shipment is a substitute to the inventory purchased for surface shipment. From Lemma 1, if the air shipment is increasing with higher degrees of demand variation, because the budget is constant, then the surface shipment stocking factor must decrease. However, if the air shipment is decreasing with higher degrees of demand variation (according to the result in Lemma 1), then the stocking factor of the surface shipment must increase in order to spend the entire budget. ❑

Proof of Proposition 5. The proof follows from lemmas 1 and 2. When $z^* > 0$, Lemma 1 indicates that the air shipment decreases with higher degrees of demand variation. Because HO spends all of its budget between surface and air shipment inventory, Lemma 2 indicates in this case that the stocking factor increases with higher degrees of variation. When $z^* < 0$, on the other hand, Lemma 1 indicates that the air shipment increases with higher degrees of demand variation. Because of the similar arguments associated with spending the entire budget, Lemma 2 indicates in this case that the stocking factor decreases with higher degrees of variation. ❑