Incentivizing Farmers to Invest in Quality through Quality-Based Payment

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We consider a for-profit cooperative that sets a quality-based payment to its risk-averse farmers in order to incentivize them to invest in quality improvements. The quality of the farmer's harvest is affected by his investments during the growing season. To satisfy both the farmer and the cooperative, the payment must be competitive with the open-market prices. Cooperatives often set payments that mimic the open-market prices, however, this practice fails to incentivize farmers to invest in quality.

Our work is motivated by the evidence of farmer underinvestment in crop quality in the olive oil industry. We define and analyze a model of this system where farmers operate under random yield, quality and open-market price. We find that farmers consistently underinvest in crop quality under the payment policy that mimics the open-market prices because (1) the cooperative can command a higher retail price than a farmer and (2) farmers are risk averse. We propose two alternative payment policies that are new to the agricultural literature, both of which can coordinate farmer decisions with the system but differ in terms of ease of implementation and susceptibility to risk aversion. We identify an easy-to-implement policy that can lead to meaningful gains when introduced in conjunction with crop insurance. We calibrate our model using data from the olive oil industry in Turkey and find a profit improvement of 10-15% over the current open-market payment approach.
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1. Introduction

This paper develops payment policies for a for-profit agricultural cooperative firm that has the goal of incentivizing its member farmers to invest in quality improvement efforts. Our motivation comes from Tariş, the second largest premium olive oil producer in Turkey. According to the 2014 Nielsen report, Tariş commands 25.9% of the retail market share in the country, and has distribution and sales in more than 30 countries. Tariş has long been a union of farmers dating back to 1915 during the Ottoman Empire, and has operated in a similar manner after the formation of the Republic of Turkey in 1923. Since 1935, during the government-controlled period, the company implemented what it calls a "Price Support System" (PSS) where olives are purchased at a pre-determined price. PSS allowed for government interventions such as subsidies, and protected the farmer by providing a guaranteed payment. The farmer continued to operate under crop yield uncertainty influenced by weather conditions and diseases.

The dramatic change in the economic landscape in 2000 led Tariş to become a union of farmers managed by an independent for-profit enterprise. The new management team decided to put effort into marketing, branding, and packaging, while continuing to encourage its farmers to invest in quality improvements and have financial stability in the absence of the government support. Thus, Tariş can be described as a marketing cooperative (see An et al. 2015 for its definition) with the motivation of assuring long-term profitability. In February 2001, Tariş established a revised payment scheme that pays farmers according to prices in the open market (e.g., the international olive oil market). We refer to the present payment scheme as the *open-market payment policy* (OMPP) and describe it below as the benchmark payment policy in our analysis.

Olive is a fundamentally different agricultural product crop than commodities such as rice, wheat and maize. Premium olive oil (with oleic acidity less than or equal to 2%) can only be obtained from olives picked from the trees. Our study focuses on farmers who target premium olive oil, and therefore, olives that produce non-premium olives is out of the context of our study.

Farmers make investments that affect the quality of the crop prior to, and during, the growing season; quality improvement efforts include cleaning branches, pruning branch tips, tending to the trees after harvest, weeding, tilling the land, using sheep manure as fertilizers and investing in additional irrigation. Farmers make these quality investments before the open-market price is known. Thus, farmers operate under three forms of uncertainty: yield, crop quality (governed by oleic acidity in the derived oil), and open-market price. Tariş takes the global open-market price (MFAO, i.e. Mercado de Futuros del Aceito de Oliva a.k.a. The Olive Oil Future Market price) as a basis for the payment to be made for olive oil with 2% oleic acidity, and the payment for higher levels of quality is determined by multiplying the price of olive oil with 2% oleic acidity by a factor that reflects the market value of lower oleic acidity. As a result, each farmer is paid according to the market value of the quality of oil determined by the oleic acidity test. Additional detail on the production and payment process in the olive oil industry in the Online Supplement.

The policy of paying farmers according to the quality-dependent open-market value of their harvest extends beyond Tariş. It is the dominant payment scheme among Mediterranean Sea olive oil producers; it is also used by the other two leading producers in Turkey: Komili and Kırlangıç. These three producers have a combined market share that exceeds 70% in Turkey. Paying farmers according to quality and open market value is also the dominant approach in the coffee industry where "differential pricing" is used for paying producers of beans above the open-market price of commodity-grade beans.

Our research was triggered by the belief of Tariş management that farmers are under-investing in quality. Our study focuses on the general question of how a quality-dependent open-market payment policy affects farmer investments in quality and profits in an agricultural supply chain. Our findings corroborate Tariş' belief. We conclude that farmers who are paid according to the open-market price of their crop under-invest in quality for a combination of two reasons. First, the cooperative is able to command a higher price for the product than a farmer. It is a much larger entity with well-established sales channels, high name recognition among consumers (i.e., manifested in brand equity). The cooperative is able to invest in storage facilities to maintain quality (e.g., temperature control). This allows the cooperative to sell a large harvest gradually over time in order to better align with market demand. Farmers have less ability to spread out sale of their harvest over time. Second, farmers are more averse to risk compared to the cooperative. In addition to being a larger entity with greater access to capital, the cooperative is likely to be more diversified. Tariş, for example, includes farmers who grow olives, cotton, figs, and raisins. Member farmers grow only one of these crops.

We develop a model and identify two alternative payment policies to compensate farmers. We clarify the pros and cons of each policy with respect to risk mitigation and ease of implementation. We also examine the role of crop insurance, both under OMPP and under the two alternative payment policies. We calibrate our model with industry data and evaluate the policies. Our results indicate that the use of an alternative farmer payment policy with the availability of crop insurance can increase profit by 10% – 15%. About one-fifth of this gain can be achieved through insurance alone (i.e., keeping OMPP in place).

Our model and analysis apply to a general setting where (1) terms related to the purchase of a farmer's harvest are established before the growing season and (2) the value of the farmer's harvest is affected by his investment in quality. In this setting, farmer investment may be misaligned with what is best from a system-wide perspective, thus exposing opportunities to investigate the extent to which misalignment exists and alternative trading mechanisms that allow both buyer and farmer to be better off. The linkage between farmer production decisions and harvest value is common in agriculture. Contract farming, which is prevalent in a wide variety of agricultural products, is characterized by a pre-production agreement between a farmer and a buyer on the deliverables (Bijman 2008, Singh 2005). Such contracts often include requirements on crop quality. Furthermore, while we focus on a setting where production decisions affect the value of the harvest through quality, the same basic problem arises in settings where production decisions affect the value of the harvest through yield. One example of this is a Merino wool supply chain where farmers make investments in grazing methods that affect wool yield (de Zegher et al. 2019).

The remainder of this paper is organized as follows. Section 2 presents the related literature. Section 3 introduces the model, and Section 4 provides its analysis. Section 5 demonstrates the financial impact from using our payment policy through data provided by Tariş. Section 6 concludes and provides a summary of managerial insights. All proofs and technical derivations are located in the online supplement.

2. Related Literature

Our study is relevant to various research streams including contract farming, supply chain contracting and coordination, and supply and quality uncertainty. Our work brings novelties to the existing publications in these areas by involving farmer risk-aversion and quality-based payment contracts.

Coordination mechanisms have been widely studied in supply chains, and there have been an extensive set of publications that examine revenue-sharing, quantity-discount (aka, two-part tariff), and buyback contracts (see Cachon and Lariviere 2005, Wang et al. 2004, Webster and Weng 2000). These studies advocate the design of contracts from a coordination perspective (see Cai et al. 2010, Moorthy 1987, Ye et al. 2020), which include the use of a two-part tariff as a remedy to the double marginalization phenomenon identified by Spengler (1950).

Contract farming refers to the production of agricultural produce with advance agreements formalized with contracts. Singh (2005) and Tang et al. (2016) state that these contracts ensure that suppliers would provide an agricultural product of a type, at a specific time and at a previously-agreed price, and in the quantity required to a known buyer. Bijman (2008) presents an overview of the contract farming practices in developing countries. Relying on qualitative surveys, his study asserts that contract farming strengthens vertical coordination in agricultural supply chains from a quality management perspective. Hsu et al. (2019) examine manufacturer's coordination with capacitated dairy suppliers. de Zegher et al. (2019) examine the effects of commodity-based sourcing of agricultural products versus direct sourcing on farmers' incentives to invest in quality.

Huh and Lall (2013) consider the crop allocation problem of a farmer given that a subset of the crops might be traded through contracts; the yields depend on the water availability and the market price is uncertain. Huh et al. (2012) examine a single manufacturer who has a pool of identical local farmers who experience the same rainfall per acre. Their work shows that granting farmers the option of breaking the contract may improve the manufacturer's expected profit. Federgruen et al. (2019) model the farmer selection problem of a manufacturer who offers a pool of farmers a menu of price-quantity contracts that would minimize the sum of expected procurement and transportation costs. In all three works, the focus is on the manufacturer's profits, rather than supply chain coordination, and the yields of the products vary across the growers based on water availability, rather than the quality of the product.

Zhao and Wu (2011), Peng and Pang (2019), Ye et al. (2020) examine coordination mechanisms for contract farming. Zhao and Wu (2011) consider a contract between a buyer and a single farmer whereas Ye et al. (2020) analyze a contract between a buyer and n identical farmers. Peng and Pang (2019) consider a three-level contract-farming supply chain with a risk-averse farmer, a risk-neutral supplier, and a risk-neutral distributor. Zhao and Wu (2011) analyze a revenue sharing contract assuming farmers are risk neutral. Ye et al. (2020) introduce farmer risk aversion—farmers set production quantity to minimize conditional value at risk—and identify a coordinating contract that combines revenue sharing with production cost sharing and a minimum payment. Peng and Pang (2019) again consider risk-averse farmers who need to decide how many acres to cultivate and the amount of inputs to invest in each acre in order to reach a target production yield. The setting motivating our work is substantially different from these works. The acreage containing productive olive trees is fixed—there is no quantity decision; farmers in our setting make decisions that affect the quality of output. The population of member farmers in our model is heterogeneous (e.g., with respect to cost structure and risk aversion), as opposed to homogenous. The wholesale price (i.e., the open-market price) and retail price (i.e., cooperative's selling price) are determined by world market prices and are not under control of the cooperative. Uncertain yield and price-dependent demand are correlated instead of being independent.

The difference in our setting from Zhao and Wu (2011), Peng and Pang (2019), and Ye et al. (2020) leads to a distinctly different research questions and findings. The researchers identify coordinating contracts offered by a profit-maximizing buyer that affects system profit through farmers' *quantity* decision. Contract parameters rely on knowledge of farmers' private information (e.g., cost structure, risk aversion). In contrast, our emphasis is on an easy-to-implement price-*quality* schedule that improves farmers' welfare without damaging the cooperative's long-term financial viability.

Qian and Olson (2020) examine a traditional agricultural cooperative where cooperative profits are distributed among the farmers (as owners of the cooperative) with the well-known free-riding effects. Their study focuses on the yield uncertainty and each farmer's borrowing and payment concerns. In addition, the setting in their study does not feature quality-based payment and an effort to improve quality (or yield). As a result, their cooperative setting is substantially different than Tariş, a for-profit cooperative.

A number of researchers have studied the use of alternative contract forms to coordinate quality (or innovation) efforts in manufacturer-retailer supply chains. Due to the manufacturing environment, there are features considered in this literature that are distinct from our setting. First, quality is often modeled as a binary measure tied to a quality failure—product quality is either acceptable or unacceptable (Gurnani and Erkoç (2008), Jabarzare and Rasti-Barzoki 2020, Lee and Li 2018, Nikoofal and Gümüş 2018). Second, the models capture how improvements in quality lead to increased demand, assuming that differences in quality do not affect the selling price (Lambertini 2018, Ma et al. 2013). Third, manufacturer and retailer decisions are analyzed within a deterministic competitive game framework—each player makes decisions to maximize its profit (Wang and Shin 2015, Yan 2015). In our setting, quality is a continuous measure. Higher levels of quality command higher prices in the market, but do not influence market demand. In addition, our setting departs from the traditional private-industry setting where each party makes self-interested decisions. The cooperative is interested in improving the welfare of its member farmers while maintaining a level of profitability sufficient for long-term viability. Furthermore, we allow for farmer risk aversion in the farmer's quality-investment decision. While there are differences in the motivating application that drive differences in model setups, there are several papers within this stream that have examined similar contract forms. Gurnani and Erkoç (2008) show that a two-part tariff can coordinate the system when the manufacturer determines the wholesale price and the retailer puts effort in promotion in order to increase demand. Ma et al. (2013), conversely, show that a two-part tariff cannot coordinate the supply chain where the manufacturer invests in quality effort and the retail invests in sales effort, both of which affect demand. Both works differ from ours as the improvements in quality affect the retail price, not demand in our setting. Wang and Shin (2015) and Yan (2015) both consider a deterministic supply chain where the price-setting supplier invests in innovation/quality and improvement in innovation/quality allows a price-setting buyer to charge a higher price for the product. While Wang and Shin (2015) find that a revenue-sharing contract could coordinate the system, Yan (2015) finds that only the combination of an effort cost sharing and revenue sharing contract works. This stream is distinct from our setting which features multiple sources of uncertainty and the supplier makes a quality decision given a buyer's payment policy.

In sustainable supply chains, the coordination efforts via two-part tariffs and other contracting mechanisms focus on minimizing the cost and waste in the system. Quality is not a decision variable, but an inherent characteristic of the product (e.g., in the form of product deterioration rate), which might affect the final demand, but not the selling price and/or the reimbursement of the farmer (Zhang and Su, 2020).

Supply uncertainty is widely examined in the operations management literature using a stochastically proportional yield. There is a growing literature that examines price and quantity decisions under supply uncertainty in agriculture: Kazaz (2004), Li and Zheng (2006), Tang and Yin (2007), Öner and Bilgiç (2008), Kazaz and Webster (2011, 2015), Noparumpa et al. (2015, 2016a, 2016b), Kazaz et al. (2016), Goel and Tanrısever (2017), Hekimoğlu et al. (2017), Kazaz (2020), Hekimoğlu and Kazaz (2020), Dong (2021) and Guda et al. (2021). However, these studies focus on a single firm's decisions, and ignore the dynamics between a farmer and a retailer. Our paper contributes to this literature by examining coordination through quality decisions within the buyer-seller setting, and by incorporating the cooperative's payment schedule.

3. Model

This section presents the evolution of the payments made to the farmers and then formulates the objectives of the cooperative and its members. Figure 1 describes the sequence of events. The main notation is summarized in Table 1.

Between February and August, a farmer can exert costly effort prior to harvest to improve the quality of his oil, though the effect of this effort is uncertain. Farmer i exerts effort $x_i \geq 0$ at cost $c_{1i} + c_{2i}x_i$ per input unit (e.g., olive tree). The value of c_{1i} is farmer i's variable cost per tree at the minimum effort ($x_i = 0$) and c_{2i} is the sensitivity of farmer i's cost to increases in effort. It is important to note that the farmer invests in effort x_i in the presence of three forms of uncertainty: quality, yield and the open-market price.

Effort x_i creates an uncertain improvement in quality represented by oleic acidity. We denote the random quality improvement with $\tilde{a}_r$ and describe it as follows:

$$\tilde{a}_r = q(x_i) \times \tilde{a} . \tag{1}$$

Function $q(x_i)$ represents the expected improvement in oleic acidity over the lowest grade corresponding to 2% oleic acidity; $q(x_i)$ is differentiable, increasing, and strictly concave. This definition is consistent with the practice, i.e., there is diminishing marginal return with higher investment in effort. We emphasize that we make no assumption that cost is linear in effort (because a unit of effort can be defined without loss of generality as proportional to cost); we only assume diminishing marginal return from investment in quality. The second term $\tilde{a}$ represents the randomness in the resulting oleic acidity and its mean is $E[\tilde{a}] = 1$. If a farmer exerts no effort, he is expected to obtain the lowest grade of olive oil with 2% oleic acidity; this is assured by $q(0) = 1$; thus, we have $E[q(0) \times \tilde{a}] = 1$ implying no expected improvement in quality in the absence of farmer effort.

N	= number of farmers
t_i	= number of trees on the orchard of farmer i
T	= total number of trees of member farmers
c_{1i}	= fixed cost per tree for farmer i
c_{2i}	= marginal cost of quality effort for farmer i
x_i	= quality effort per tree by farmer i
x_{Oi}^{o}	= optimal effort of farmer i under OMPP if the VaR constraint is relaxed (or non-binding)
x_{Oi}^{*}	= optimal effort of farmer i under OMPP
x_{Si}^{*}	= effort of farmer i that maximizes system profit
$q(x)$	= expected quality given effort x
k_a	= payment multiplier for lower acidity
$\tilde{a}$	= random percentage variation in quality; random quality given effort x is $k_a q(x)\tilde{a}$
$\tilde{y}$	= random yield per tree in the region
$\tilde{p}_M$	= random open-market price at the lowest quality (2% oleic acidity)
$\tilde{\varepsilon}$	= random noise in open-market price independent of regional yield
b	= coefficient influencing correlation among regional yield and open-market price
$\tilde{p}_O(x)$	= random open-market price given effort x
α	= value-at-risk probability
β_i	= tolerable loss per tree for farmer i
$\pi_{\bullet i}(x)$	= expected profit of farmer i given effort x and payment policy •; overscore ~ denotes corresponding random variable, and similarly for other profit functions
$\Pi_{\bullet i}(x)$	= expected contribution of farmer i to cooperative profit given farmer effort x under policy •
$\Pi_{\bullet}(\mathbf{x})$	= total expected cooperative profit given farmer effort $\mathbf{x} = (x_1, \ldots, x_N)$ and policy •
$\Psi_i(x)$	= expected contribution of farmer i to system profit given farmer effort x
$\Psi(\mathbf{x})$	= total expected system given farmer effort $\mathbf{x} = (x_1, \ldots, x_N)$

Table 1. List of notation.

The farmer's yield is uncertain. All member farmers for Tariş are located in Edremit Bay, the region that produces more than 70% of Turkey's olive oil. Farmers in this region get exposed to the same weather conditions and diseases; thus, we assume that the farmer's yield is identical to the yield in the region. The stochastic yield, for all farmers in Edremit Bay, is described with random variable $\tilde{y}$; its realization is y and the mean and standard deviation are $E[\tilde{y}] = \mu_y = 1$ and σ_y, respectively.

The farmer's return from the effort investment is also uncertain. Tariş determines a payment scheme using the open-market prices traded in MFAO. The payment for the lowest quality of olive oil corresponding to 2% oleic acidity is represented with random variable $\tilde{p}_M$; its realization is described with p_M and its mean and standard deviation are $E[\tilde{p}_M] = \mu_{p_M}$ and σ_{p_M}, respectively. Almost all olive oil traded in MFAO is the lowest grade of quality; therefore, the realized value of p_M is taken directly from the open-market prices traded in MFAO.

The random variables $\tilde{p}_M$ and $\tilde{y}$ are correlated. The online supplement provides empirical evidence about the negative and linear relationship using data from MFAO prices and the regional yield provided by the Ayvalık Chamber of Commerce. This negative and linear relationship between the yield and prices is well established in economics and is supported by other publications, e.g., Kazaz (2004) and Liao et al. (2019). We describe the relationship between the open-market price for the lowest quality of olive oil with 2% oleic acidity and the regional yield as follows:

$$\tilde{p}_M = E[\tilde{p}_M] - b(\tilde{y} - 1) + \tilde{\varepsilon} = \mu_{p_M} - b(\tilde{y} - 1) + \tilde{\varepsilon} \tag{2}$$

where $\tilde{\varepsilon}$ is the random error term with $E[\tilde{\varepsilon}] = 0$ that captures all other uncertainties that are independent of the regional yield (e.g., the state of national and world economy, etc.). The value of $-b$ (< 0) creates a negative correlation, i.e., higher yield realizations lead to lower open-market prices for the lowest quality of olive oil represented with 2% oleic acidity.[1]

After observing the realizations of the open-market price p_M in MFAO and the regional yield y, Tariş pays each farmer based on oleic acidity (December in Figure 1). Let a_r represent the realization of the resulting oleic acidity $\tilde{a}_r$ and let k_a represent the payment multiplier for each unit of oleic acidity improvement. Let $p_O(a_r)$ denote the payment for the resulting oleic acidity improvement a_r over the lowest grade of olive oil with 2% oleic acidity. Tariş increases its payment for higher quality olive oil (represented with lower levels of oleic acidity) from the realized open-market price for olive oil with 2% oleic acidity by using the following form:

$$p_O(a_r) = p_M \times k_a \times a_r. \tag{3}$$

After Tariş presses the olives, the derived oil rests in temperature-controlled steel tanks for approximately two months for the residue to settle at the bottom of steel tanks. The oil goes through a final test that reveals the resulting oleic acidity a_r in the derived oil before it gets bottled for retail distribution

[1] The main contributors of olive oil to the MFAO market are located around the Mediterranean Sea. While each country has its own microclimate, the empirical evidence shows that if the yield is low in Turkey, countries that are in proximity (e.g., Greece, Italy and Spain) might also be impacted by a similar climatic effect causing a reduction in the aggregate supply of olive oil and leading to an increase in olive oil prices in MFAO.

(corresponding to January/February in Figure 1). Incorporating (1) into (3), we can express the realized payment to the farmer in terms of the original investment x_i in quality improvement:

$$p_O(x_i) = p_M \times k_a \times q(x_i) \times a. \tag{4}$$

At the time the farmer makes the investment x_i, he makes the decision under the uncertainty pertaining to $\tilde{\varepsilon}$ representing all factors influencing the open-market price for the lowest grade of olive oil with 2% oleic acidity, yield $\tilde{y}$ and the random factor $\tilde{a}$ influencing the resulting oleic acidity. Substituting (2) into (4), we can express the farmer's random payment at the time he makes the decision x_i in terms of these three random variables:

$$\tilde{p}_O(x_i) = \tilde{p}_M \times k_a \times q(x_i) \times \tilde{a} = \left(\mu_{p_M} - b(\tilde{y}-1) + \tilde{\varepsilon}\right) \times k_a \times q(x_i) \times \tilde{a}. \tag{5}$$

We assume that randomness in oleic acidity $\tilde{a}$ and random variable $\tilde{\varepsilon}$ in (2) are independent because oleic acidity is not affected by factors such as the state of the national or world economy. However, we allow for the possibility of correlation between $\tilde{a}$ and $\tilde{y}$, i.e., weather conditions can affect yield and quality through oleic acidity.

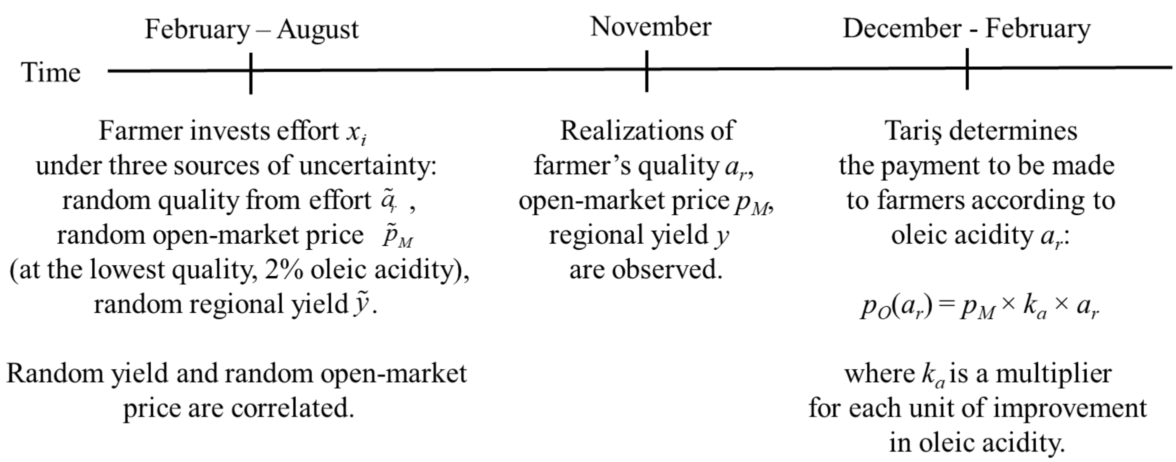


Figure 1. Timeline of events for farmers and the cooperative in olive oil production.

The payment scheme described in (5) is known as the Open-Market Payment Policy (OMPP) and is the underlying practice in Turkey, Greece, Spain and other leading olive oil producing countries. Compared to an additive structure, this payment scheme is more consistent with a multiplicative structure, e.g., the difference between high- and low-quality payments tends to be higher in years when the low-quality price is higher. As a result, we use a multiplicative structure in (5) in order to describe the payments made to farmers for higher quality olive oil. However, it is important to note that our main conclusions are not driven by the functional form of the adjustment in (5). Our main conclusions hold under a general additive / multiplicative structure in adjusting the payment for higher quality olive oil, as discussed in Section 4.4.5 and in greater detail in the Online Supplement.

It is important to highlight that the price expression defined in (5) derives support from observed data: (1) It reflects how farmers are paid under OMPP; and, (2) it affords flexibility to analyze the effects of uncertainties and correlations while featuring analytical tractability.

From (5), the expected OMPP payment made to farmer i for a unit of output can be expressed as:

$$E\left[\tilde{p}_O\left(x_i\right)\right]=\left(\mu_{p_M}-b\left(E\left[\tilde{a}\times\tilde{y}\right]-1\right)\right)\times k_a\times q\left(x_i\right)=\left(\mu_{p_M}-b\sigma_{ay}\right)\times k_a\times q\left(x_i\right)$$

where σ_{ay} represents the covariance of oleic acidity randomness and yield. In the remaining analysis, we simplify the expressions by normalizing the farmer's payment through setting $k_a = 1$ and by adjusting the values of c_{1i} and c_{2i} accordingly.

3.1 Farmer, Cooperative, and System Profit under OMPP

Olive oil farmers have a constant number of olive trees and they cannot change the number of trees immediately. Planting new olive trees requires additional land investment and the new trees will not bear fruit at full capacity for the first 14 to 16 years. Therefore, we consider the case that the olive farmer cannot change its production capacity. As a result, unlike seasonal crops, each olive farmer operates with a fixed production capacity (i.e., number of trees). Let t_i denote the number of trees owned by farmer i where i = 1, …, N.

Recall that $\tilde{y}$ is the random quantity of output (e.g., liters of oil) per unit of input (e.g., olive tree). Thus, at the time a farmer chooses effort x_i, the farmer's random profit per unit of input is the product of yield and the cooperative's payment per unit of output less the quality improvement cost per unit of input:

$$\tilde{\pi}_{Oi}\left(x_i\right)=t_i\left[\tilde{y}\tilde{p}_O\left(x_i\right)-\left(c_{1i}+c_{2i}x_i\right)\right]=t_i\left[\tilde{y}\tilde{p}_M q\left(x_i\right)\tilde{a}-\left(c_{1i}+c_{2i}x_i\right)\right]. \tag{6}$$

We let $\pi_{Oi}\left(x_i\right)$ denote the farmer's expected profit, i.e., $\pi_{Oi}\left(x_i\right)=E\left[\tilde{\pi}_{Oi}\left(x_i\right)\right]$.

A risk-averse farmer is concerned about the possibility of losing money on his harvest, which we model through a value-at-risk (VaR) constraint:

$$\Pr\left(\tilde{\pi}_{Oi}\left(x_i\right)\le -t_i\beta_i\right)\le\alpha. \tag{7}$$

Constraint (7) says that the probability of a loss as large as β_i or more per tree must not be more than α. The value of α is small in practice (e.g., $\alpha \approx$ 5%-10%), i.e., a risk-averse decision maker is concerned about left-tail realizations of profit. For a given α, the larger the value of β_i, the less risk-averse the farmer.

It is important to indicate that this model of risk is consistent with farmer attitudes in the Edremit Bay region. Our original model featured a coherent risk measure, a Conditional Value at Risk (CVaR) description. However, our field study with 33 farmers revealed that their risk attitude can best be described with VaR. In Section 4.4.5 and Online Supplement, we show how our main conclusions continue to hold and/or change under more general risk measures including CVaR.

The farmer's decision problem is

$\max_{x_i} \pi_{Oi}(x_i)$ subject to (7).

There are two points related to our model of the farmer's problem that warrant emphasis. First, in some agricultural settings, a farmer decides the quantity of a crop to plant prior to each growing season, which when aggregated across all farmers, may influence price. As noted above, this feature is not present in our setting because the number of olive trees is fixed over the intermediate term, i.e., trees take 14-16 years to bear fruit at full capacity. Once these trees mature, they remain productive for many years, typically spanning multiple generations of farmers. As a result, production inputs associated with some tree crops (e.g., olives, nuts and citrus, etc.) tend to be relatively stable from season to season due to their extended maturity and long lives. Second, we are studying a relatively small group of farmers who are paid according to the olive oil price in the world market (i.e., Tariş farmers contribute less than 2% of the world's olive oil production). The decisions of this group of farmers regarding their quality improvement efforts are unlikely to affect the world-market price.

We next present the cooperative's profit function. Tariş has N farmers and the production capacity from its member farmers is constant, i.e., $T = \sum_{i=1}^{N} t_i$ is fixed. The cooperative sells olive oil under its own brand, and as a much larger entity with well-established sales channels, brand recognition, and capabilities to maintain the quality of oil via temperature-controlled storage, it is able to sell at a higher price than the farmer. Let m denote the cooperative's markup (net of any variable costs) over the open-market price of oil, where $m > 0$. Recall that $t_i \tilde{y}$ is the total quantity of olive oil from farmer i. Thus, the cooperative's net olive oil sales revenue from farmer i's effort x_i is $t_i \tilde{y}(1+m)\tilde{p}_O(x_i)$ (e.g., typically sold over a period of 6-9 months) and the cooperative's profit from farmer i is $t_i \tilde{y} m \tilde{p}_O(x_i)$. We assume that the cooperative is risk-neutral. The cooperative's expected profit from farmer i and the total expected profit (across all of its member farmers) are:

$$\Pi_{Oi}(x_i) = E\left[t_i \tilde{y} m \tilde{p}_O(x_i)\right], \text{ and}$$

$$\Pi_O(\mathbf{x}) = \sum_{i=1}^{N} \Pi_{Oi}(x_i).$$

The expected system profit associated with farmer i and the total expected system profit are as follows:

$$\Psi_i(x_i) = E\left[t_i\left((1+m)\tilde{y}\tilde{p}_M q(x_i)\tilde{a} - (c_{1i} + c_{2i}x_i)\right)\right] = \Pi_{Oi}(x_i) + \pi_{Oi}(x_i) \tag{8}$$

$$\Psi(\mathbf{x}) = \sum_{i=1}^{N} \Psi_i(x_i).$$

Note that the system profit depends only on farmer efforts $\mathbf{x} = (x_1, \ldots, x_N)$, whereas cooperative and farmer profits include subscript O, reflecting the fact that the allocation of the system profit is affected by how farmers are paid.

4. Analysis

In this section, we describe farmer decisions and profits and compare with system optimal (§4.1) and we characterize properties of a general optimal policy from the cooperative's perspective (§4.2). We then identify two distinct forms of the cooperative's optimal policy (§4.3) and address practical considerations surrounding implementation (§4.4). We conclude by illustrating policy properties for a quadratic quality cost function (§4.6).

4.1 Optimal Expected System Profit and the Farmer's Problem

Let $Z_O = \tilde{y}\tilde{p}_M\tilde{a}$, i.e., Z_O is the random open-market price per unit of input given no effort ($x_i = 0$). Substituting Z_O into (8) and optimizing yields

$$x_{Si}^* = \arg\max \Psi_i\left(x_i\right) = q'^{-1}\left(\frac{c_{2i}}{\left(1+m\right)\mu_{Z_O}}\right), \tag{9}$$

and the optimal quality is $q_{Si}^* = q\left(x_{Si}^*\right)$ (see the Online Supplement for the derivation). The optimal expected system profit is denoted Ψ_S^* and the expected system profit due to farmer i is denoted Ψ_{Si}^*, i.e., $\Psi_{Si}^* = \Psi_i\left(x_{Si}^*\right)$, $\Psi_S^* = \sum_{i=1}^{N}\Psi_i\left(x_{Si}^*\right)$.

Let $\pi_{Oi}\left(\alpha, x_i\right)$ denote the α-fractile of function $\tilde{\pi}_{Oi}\left(x_i\right)$ and let $z_O(\alpha)$ denote the α-fractile of Z_O. With this notation, the farmer's VaR constraint can be rewritten as

$$q\left(x_i\right)z_O\left(\alpha\right) - \left(c_{1i} + c_{2i}x_i\right) \geq -\beta_i \tag{10}$$

and the farmer's problem can be written as

$$\max_{x_i \geq 0}\left\{q\left(x_i\right)\mu_{Z_O} - \left(c_{1i} + c_{2i}x_i\right) : q\left(x_i\right)z_O\left(\alpha\right) - \left(c_{1i} + c_{2i}x_i\right) \geq -\beta_i\right\}.$$

(see the Online Supplement for supporting derivations of content in this section).

It is possible for the risk-averse optimal effort to increase or decrease relative to a risk-neutral decision. The possibility of increasing effort is curious because it goes against the intuition that a risk-averse farmer should be more conservative in his efforts compared to a risk-neutral farmer. As shown in the Online Supplement, the possibility of increased effort under risk aversion can arise only under a risk measure that is not coherent (please see Proposition 11 of Online Supplement proving that effort decreases under risk aversion for any coherent risk measure). The next question is whether conditions leading an increase versus a decrease can be characterized. Proposition 1 identifies a sufficient condition for decreasing effort under risk aversion, a condition that is likely to hold in practice. Under this condition, the main result of Proposition 1 is that the farmer underinvests in quality improvement and, as a consequence, system profit suffers. This can be seen in inequality (13). The farmer, cooperative, and system expected profits at x^*_{Oi} are denoted π^*_{Oi}, Π^*_{Oi}, Ψ^*_{Oi}, $\Pi^*_O = \sum_{i=1}^{N} \Pi^*_{Oi}$, $\Psi^*_O = \sum_{i=1}^{N} \Psi^*_{Oi}$.

Proposition 1. *Define* $x^+_{Oi} = \sup_{x_i} \{ x_i : q(x_i) z_O(\alpha) - (c_{1i} + c_{2i} x_i) \geq -\beta_i \}$ *and* $x^{\circ}_{Oi} = q'^{-1}\left(\frac{c_{2i}}{\mu_{Z_O}} \right)$. *If*

$$z_O(\alpha) < \mu_{Z_O} \tag{11}$$

then

$$x^*_{Oi} = \min \{ x^+_{Oi}, x^{\circ}_{Oi} \} \tag{12}$$

$$x^*_{Si} > x^{\circ}_{Oi} \geq x^*_{Oi},\ q^*_{Si} > q^*_{Oi},\ \Psi^*_{Si} > \Psi_i(x^{\circ}_{Oi}) \geq \Psi^*_{Oi}. \tag{13}$$

In practice, it is likely for inequality (11) to hold. Recall that α is small (e.g., 10% or less). Inequality (11) says that the value of the α-fractile of Z_O is less than the mean of Z_O. For the remainder of the paper we assume that (11) holds.

We note that x°_{Oi} is the optimal solution to the farmer's problem when the VaR constraint (7) is ignored. There are up to two values of x where the VaR constraint is satisfied at equality (see proof of Proposition 1). Equation (12) shows that the farmer's optimal decision is the smaller of unconstrained optimal and the largest value of x that exactly satisfies the VaR constraint. From Proposition 1, we see that farmer underinvestment in quality improvement relative to system optimal is due to low pricing power of the farmer relative to the cooperative. Furthermore, underinvestment may be exacerbated by the farmer's degree of risk aversion as measured by the value of his tolerable loss β_i.

4.2 Cooperative's Problem and Optimal Policy

Under OMPP, Proposition 1 shows that farmers underinvest in quality, which is consistent with the belief of management at Tariş. The proposition raises the question of whether an alternative payment schedule can be developed to improve outcomes. In order to address this question, we step back to consider the cooperative's objectives.

A common mission of an agricultural cooperative is to help its member farmers to be successful, e.g., financially viable over the long term. Tariş shares this mission, going back to the days when it was controlled and supported by government and provided pre-season price guarantees to its member farmers. The cooperative's mission today is to help its member farmers to be successful while assuring its own long-term financial viability as an independent enterprise.

We define the following notation in order to formalize the cooperative's payment problem.

$\Gamma_i(p_M, a_r, y)$ = cooperative payment policy; Γ_i maps the realized price for the lowest quality olive oil with 2% oleic acidity (p_M), resulting oleic acidity a_r (the realization of $q(x_i)\tilde{a}$), and yield (y) to the farmer i payment per unit of oil

$\tilde{\pi}_{\Gamma_i}(x_i) = t_i\left[\tilde{y}\Gamma_i\left(\tilde{p}_M, q(x_i)\tilde{a}, \tilde{y}\right) - \left(c_{1i} + c_{2i}x_i\right)\right]$ = random profit of farmer i

$\pi_{\Gamma_i}(x_i) = E\left[\tilde{\pi}_{\Gamma_i}(x_i)\right]$ = expected profit of farmer i

$\Pi_{\Gamma_i}(x_i) = E\left[t_i\tilde{y}\left((1+m)\tilde{p}_M - \Gamma_i\left(\tilde{p}_M, q(x_i)\tilde{a}, \tilde{y}\right)\right)\right]$ = cooperative expected profit due to farmer i

$\Pi_\Gamma(\mathbf{x}) = \sum_{i=1}^{N}\Pi_{\Gamma_i}(x_i)$ = cooperative expected profit

$\Pi_{\min}$ = cooperative minimum expected profit from the harvest to remain financially viable (a value determined by management)

We assume $0 < \Pi_{\min} < \Psi_S^*$, i.e., the cooperative requires some profit but not the entire optimal system profit to remain viable. The cooperative's payment problem can be formulated as follows.

$$P:\ \max_{\Gamma_i}\left\{\sum_{i=1}^{N}\left(\Pi_{\Gamma_i}\left(x_{\Gamma_i}^*\right) + \pi_{\Gamma_i}\left(x_{\Gamma_i}^*\right)\right)\right\}$$

$$\text{s.t.}\qquad \pi_{\Gamma_i}\left(x_{\Gamma_i}^*\right) \ge \pi_{Oi}^*,\ i = 1, \ldots, N \tag{14}$$

$$\sum_{i=1}^{N}\Pi_{\Gamma_i}\left(x_{\Gamma_i}^*\right) \ge \Pi_{\min} \tag{15}$$

$$x_{\Gamma_i}^* = \arg\max_{x_i}\left\{\pi_{\Gamma_i}(x_i) : \Pr\left(\tilde{\pi}_{\Gamma_i}(x_i) \le -t_i\beta_i\right) \le \alpha\right\}. \tag{16}$$

We see that the cooperative's objective is to maximize expected system profit subject to the requirements that farmers' profits are at least as high as profits under the current open-market payment policy (e.g., individual rationality constraint) and the cooperative is financially viable. One might argue that the right-hand side of (14) could be replaced with a lower value than π_{Oi}^*, e.g., expected profit of the best outside option (that is below the status quo). However, such an approach is not consistent with the cooperative's mission of helping (not hurting) its member farmers, let alone raising challenges with implementation due to farmer resistance and effects on morale and trust.

We next characterize an optimal solution to problem P. We require the following notation.

$$\Psi_{Si}^{**} = t_i\left[(1+m)yp_M aq\left(x_{Si}^*\right) - \left(c_{1i} + c_{2i}x_{Si}^*\right)\right]$$

$$\tilde{\Psi}_{Si}^* = t_i\left[(1+m)\tilde{y}\tilde{p}_M\tilde{a}q\left(x_{Si}^*\right) - \left(c_{1i} + c_{2i}x_{Si}^*\right)\right]$$

$$\overline{\lambda}_i = \pi_{Oi}^* / \Psi_{Si}^*$$

i.e., Ψ_{Si}^{**} is the realized contribution to system profit by farmer *i* under the system-optimal investment by the farmer, $\tilde{\Psi}_{Si}^{*}$ is the random contribution to system profit by farmer *i* under the system-optimal investment by the farmer, and $\overline{\lambda}_i$ is the fraction of farmer profit under OMPP to the expected optimal system profit associated with farmer *i*. Furthermore, let

$$\overline{k}_i = \min_{k\geq 0}\left\{k : k \geq E\left[\left(k - \overline{\lambda}_i \tilde{\Psi}_{Si}^{*} - t_i\beta_i\right)^{+}\right]\right\}.$$

For example, if $\Pr\left(-\overline{\lambda}_i \tilde{\Psi}_{Si}^{*} > t_i\beta_i\right) = 0$, then $\overline{k}_i = 0$. Lemma A1 in the Online Supplement shows that there exists a unique $\overline{k}_i$.

Proposition 2. *A feasible solution to P exists if and only if*

$$\Pi_{\min} \leq \Pi_O^{*} + \Psi_S^{*} - \Psi_O^{*}. \tag{17}$$

If (17) *holds, then following payment policy solves problem P*:

$$\Gamma_i^{*}\left(p_M, a_r, y\right) = \begin{cases} \overline{\lambda}_i\left(1+m\right)p_M a_r + \left[\left(1-\overline{\lambda}_i\right)\left(c_{1i}+c_{2i}x_i\right) - \overline{k}_i\right]/\,y, & \text{if } -\left(\overline{\lambda}_i \Psi_{Si}^{**} - \overline{k}_i\right) \leq t_i\beta_i \\ \overline{\lambda}_i\left(1+m\right)p_M a_r + \left[\left(1-\overline{\lambda}_i\right)\left(c_{1i}+c_{2i}x_i\right) - \overline{\lambda}_i \Psi_{Si}^{**} - t_i\beta_i\right]/\,y, & \text{if } -\left(\overline{\lambda}_i \Psi_{Si}^{**} - \overline{k}_i\right) > t_i\beta_i \end{cases}.$$

The above payment policy yields the optimal expected system profit. Condition (17) ensures that cooperative is financially viable under this policy. All of the increase in expected system profit goes to the cooperative. The following generalization of policy Γ_i^{*} allows the cooperative to share the gain with the farmer:

$$\Gamma_i^{**}\left(p_M, a_r, y, \Delta_i\right) = \Gamma_i^{*}\left(p_M, a_r, y\right) + \frac{\Delta_i}{y},\ \sum_{i=1}^{N}\Delta_i \leq \Psi_S^{*} - \Psi_O^{*} - \left(\Pi_{\min} - \Pi_O^{*}\right),\ \Delta_i \geq 0 \text{ for all } i. \tag{18}$$

The inequality constraints are necessary to assure that the payment policy is feasible; if the first inequality is violated, then the policy violates constraint (15), and if the second inequality is violated, then the policy violates constraint (14). Suppose, for example, that $\Pi_{\min} = \Pi_O^{*}$ (i.e., the cooperative profit under OMPP is the minimum profit for long-term viability). If $\Delta_i = \Psi_{Si}^{*} - \Psi_{Oi}^{*}$ for all *i*, then Γ_i^{**} is feasible, all farmers make quality decisions to maximize system profit, and all of the increase in system profit goes to the farmers. If $\Pi_{\min} \neq \Pi_O^{*}$, then setting $\Delta_i = \left[1 - \left(\Pi_{\min} - \Pi_O^{*}\right)/\left(\Psi_S^{*} - \Psi_O^{*}\right)\right]\left(\Psi_{Si}^{*} - \Psi_{Oi}^{*}\right)$ yields $\sum_{i=1}^{N}\Pi_{\Gamma_i}\left(x_{\Gamma_i}^{*}\right) = \Pi_{\min}$ and gives each farmer the same fraction of the total gain.

4.3 Policy Implementation: RSPP and BMPP

Our conversations with Tariş management have made it clear that ease of implementation of any new farmer payment policy is an important consideration. Policy Γ_i^{**}, which is a three-parameter policy, can be challenging to implement because the payment schedule depends on (1) the farmer's effort-dependent cost per input unit $c_{1i} + c_{2i}x_i$ and (2) the farmer's risk-aversion parameter β_i. In our motivating application, the cooperative has some ability to estimate the farmer's cost $c_{1i} + c_{2i}x_i$ because supplies for maintaining the farm are purchased from the cooperative. However, the cooperative has little insight into each farmer's tolerable loss β_i.

Another potential disadvantage of Γ_i^{**} is the payment function contains a kink at a specific realization of a random variable (that itself is a function of three random primitives), i.e., the payment function changes when the realization of $\bar{\lambda}_i\tilde{\Psi}_{Si}^*$ is above or below $\bar{k}_i - t_i\beta_i$. This potentially makes the payment schedule more complex to communicate with farmers. In the following, we define and analyze two single-parameter payment policies that have implementation advantages over Γ_i^{**}. Both of these policies eliminate the kink by ignoring the farmer's tolerable loss parameter β_i (in effect, setting $\beta i = \infty$). One policy eliminates parameter $\bar{k}_i$ and the other policy eliminates parameter $\bar{\lambda}_i$.

We refer to the first payment policy as the revenue sharing payment policy (RSPP). Compared to policy Γ_i^*, RSPP sets $\bar{k}_i = 0$ and replaces $\bar{\lambda}_i$ with free parameter λ_i:

$$\Gamma_{Ri}\left(p_M, a_r, y\right) = \lambda_i\left(1+m\right)p_M a_r + \left(1-\lambda_i\right)\left(c_{1i} + c_{2i}x_i\right)/\,y \tag{19}$$

where $\lambda_i > 0$. Under RSPP, the farmer receives fraction λ_i of the system profit from his orchard. This policy resembles the revenue sharing contracts of the supply chain literature. As in these contracts, implementation of RSPP requires transparency and/or trust in farmer/cooperative costs and cooperative revenues. Given that transparency/trust is in place, the policy is simple to communicate—the farmer receives fraction λ_i of the farmer's contribution to system profit.

Proposition 3. *Under RSPP, farmer i's random profit function, mean, and variance are*

$$\tilde{\pi}_{Ri}\left(x_i\right) = \lambda_i t_i\left[\left(1+m\right)q\left(x_i\right)Z_O - \left(c_{1i} + c_{2i}x_i\right)\right] \tag{20}$$

$$\pi_{Ri}\left(x_i\right) = \lambda_i t_i\left[\left(1+m\right)q\left(x_i\right)\mu_{Z_O} - \left(c_{1i} + c_{2i}x_i\right)\right] \tag{21}$$

$$V\left[\tilde{\pi}_{Ri}\left(x_i\right)\right] = \left[\lambda_i t_i\left(1+m\right)q\left(x_i\right)\right]^2\sigma_{Z_O}^2\,; \tag{22}$$

if farmers are risk neutral, then there exists an implementation of RSPP in which all farmers select system-optimal effort if and only if

$$\Pi_{\min} \le \Pi_O^* + \Psi_S^* - \Psi_O^*. \tag{23}$$

Condition (23) ensures that cooperative is financially viable under this policy. Since $\Psi_S^* - \Psi_O^* > 0$ the condition as assured to be satisfied if the cooperative is viable under the status quo (OMPP).

Our next result characterizes the effects of farmer risk aversion on RSPP. Let $\Psi_{Si}^*(\alpha)$ denote the α-fractile of farmer i's contribution to system profit given optimal effort x_{Si}^*, i.e., $\Psi_{Si}^*(\alpha) =$

$t_i\left[\left(1+m\right)q\left(x_i\right)z_O\left(\alpha\right)-\left(c_{1i}+c_{2i}x_i\right)\right]$. Let $\sigma^2_{Z_O}$ denote the variance in open market price at the lowest quality, i.e., $\sigma^2_{Z_O}=V\left[Z_O\right]$.

Proposition 4. *Under RSPP, if the farmer is risk averse, then the farmer's VaR constraint is nonbinding at system-optimal effort if and only if*

$$\lambda_i \geq \frac{-t_i\beta_i}{\Psi^*_{Si}\left(\alpha\right)}; \tag{24}$$

and if $\max\left\{\frac{t_i\beta_i}{-\Psi^*_{Si}\left(\alpha\right)},\frac{\pi^*_{Oi}}{\Psi^*_{Si}}\right\} \leq 1-\left(\frac{\Pi_{\min}}{\Pi^*_O}\right)\left(\frac{\Pi^*_{Oi}}{\Psi^*_{Si}}\right)$ *for all i, then setting*

$$\lambda_i \in \left[\max\left\{\frac{t_i\beta_i}{-\Psi^*_{Si}\left(\alpha\right)},\frac{\pi^*_{Oi}}{\Psi^*_{Si}}\right\},1-\left(\frac{\Pi_{\min}}{\Pi^*_O}\right)\left(\frac{\Pi^*_{Oi}}{\Psi^*_{Si}}\right)\right] \text{ for all } i \tag{25}$$

yields an implementation of RSPP in which all farmers select system-optimal effort.

The numerator of the right-hand side of (24) is the farmer's acceptable (negative) profit under the VaR constraint (i.e., the product of number of trees and acceptable loss per tree). The denominator is the α-fractile of the farmer's contribution to system profit at system-optimal effort. Thus, the ratio represents a lower limit on acceptable α-fractile profit as a percent of system profit according to the VaR constraint. The left-hand side is the ratio of system profit paid to the farmer under RSPP, which must dominate the right-hand side to satisfy the VaR constraint. Condition (25) assures that both the farmer's VaR constraint and the cooperative's constraint for long-term financial viability are satisfied.

The next payment policy sets $\overline{\lambda_i}$ = 1 and replaces $\overline{k_i}$ in policy Γ^*_i with free parameter k_i. Setting $\overline{\lambda_i}$ = 1 allows the farmer's payment to reflect the full pricing power of the cooperative. For this reason, we refer to the policy as the brand markup payment policy (BMPP):

$$\Gamma_{Bi}\left(p_M, a_r, y\right)=\left(1+m\right)p_M a_r - k_i / y. \tag{26}$$

Under BMPP, the farmer receives the retail price for the oil from his orchard less a constant t_ik_i. This policy is again easy to understand and communicate. It is akin to a two-part tariff with the cooperative acting like a land-owner and obtaining rent per tree from the farmer. Compared to RSPP, BMPP requires less information sharing and/or trust to implement; sharing of private information is limited to the cooperative. The cooperative shares the retail price of the product net of variable costs. Farmers, who can observe retail prices and are aware of the open-market price, are able to judge the degree to which the cooperative's proposed payment schedule is reasonable. The policy is simple to communicate—the farmer pays the cooperative k_i per tree and is paid the cooperative's net markup over open-market price of the product.

Proposition 5. *Under BMPP, farmer i's random profit function, mean, and variance are*

$$\tilde{\pi}_{Bi}\left(x_i\right)=t_i\left[\left(1+m\right)q\left(x_i\right)Z_O-\left(c_{1i}+c_{2i}x_i\right)-k_i\right] \tag{27}$$

$$\pi_{Bi}(x_i) = t_i\left[(1+m)q(x_i)\mu_{Z_O} - (c_{1i} + c_{2i}x_i) - k_i\right] \tag{28}$$

$$V\left[\tilde{\pi}_{Bi}(x_i)\right] = \left[t_i(1+m)q(x_i)\right]^2 \sigma^2_{Z_O}\ ; \tag{29}$$

if farmers are risk neutral, then there exists an implementation of BMPP in which all farmers select system-optimal effort if and only if

$$\Pi_{\min} \le \Pi^*_O + \Psi^*_S - \Psi^*_O\,. \tag{30}$$

Proposition 6. *Under BMPP, if the farmer is risk averse, then the farmer's VaR constraint is nonbinding at system-optimal effort if and only if*

$$k_i \le \beta_i + \frac{\Psi^*_{Si}(\alpha)}{t_i}\ ; \tag{31}$$

and if $\left(\frac{\Pi_{\min}}{\Psi^*_S}\right)\left(\frac{\Psi^*_{Si}}{t_i}\right) \le \min\left\{\beta_i + \frac{\Psi^*_{Si}(\alpha)}{t_i}, \frac{\Psi^*_{Si} - \pi^*_{Oi}}{t_i}\right\}$ *for all i, then setting*

$$k_i \in \left[\left(\frac{\Pi_{\min}}{\Psi^*_S}\right)\left(\frac{\Psi^*_{Si}}{t_i}\right), \min\left\{\beta_i + \frac{\Psi^*_{Si}(\alpha)}{t_i}, \frac{\Psi^*_{Si} - \pi^*_{Oi}}{t_i}\right\}\right] \text{ for all } i \tag{32}$$

yields an implementation of BMPP in which all farmers select system-optimal effort.

As noted above, condition (30) ensures that the cooperative is financially viable under this policy (e.g., the condition is satisfied if the cooperative is viable under the status quo). The right-hand side of (31) is difference between α-fractile of optimal system profit and the farmer's acceptable loss divided by the number of trees. The left-hand side is the payment per tree, which must dominate the right-hand side to satisfy the VaR constraint. Condition (32) assures that both the farmer's VaR constraint and the cooperative's constraint for long-term financial viability are satisfied.

The literature on manufacturer-retail supply chains has identified three basic contract forms that can coordinate a manufacturer-retailer supply chain in a newsvendor setting featuring a retailer decision on order quantity (e.g., see Cachon and Lariviere 2005): (1) revenue sharing, (2) two-part tariff, (3) buy-back / quantity-flexibility. RSPP has features of a revenue-sharing contract—the buyer agrees to split system profit and the seller chooses quality effort. BMPP has features of a two-part tariff—the buyer's payment function has a term that depends on the seller's quality decision and a term that does not. Buy-back / quantity-flexibility contracts do not apply in our setting (i.e., designed to accommodate specific features of the newsvendor model). Just as in the classical setting, policies RSPP, BMPP, and more generally Γ^{**}_i, specify a payment function that assures marginal profit of the decision-maker(s) is equal to the marginal profit of the system. Thus, in one sense, our general payment policy Γ^{**}_i and Proposition 2 reinforce the fundamental character of incentive-alignment mechanisms in the supply chain literature; while there are differences in the details of incentive-alignment policies for manufacturer-retailer and farmer-cooperative supply chains, the character of levers for incentive alignment are the same. However, our results and analyses address elements that have received little or no attention in the literature: (1) differing risk attitudes between seller and buyer, (2) a buyer purchasing from heterogeneous sellers. These two distinctive features of our model are addressed next.

Although risk aversion of farmers is a phenomenon often observed in practice, many studies either ignore it, or investigate this element within the context of quantity decisions (Huh et al. 2012, Chen and Tang 2015, Peng and Pang 2019, Ye et al. 2020). Despite the fact that VaR or CVaR is frequently used to model risk aversion of suppliers in the literature, to our knowledge, none of the previous studies in agricultural or manufacturing supply chains literature allowed the parameters of the risk function to vary across suppliers. Furthermore, the impact of different forms of payment policies (e.g., revenue sharing, two-part tariff) on mitigating the negative effect of risk aversion has not been discussed before.

The next result characterizes farmer's VaR under RSPP and BMPP given optimal effort when the VaR constraint is relaxed. If VaR is greater than β_i, for example, then the farmer's VaR constraint is binding. The proposition relies on an allocation rule that we explain here. Farmer VaR under RSPP and BMPP is affected by the manner in which the gain from incentive alignment is allocated to individual farmers. To simplify presentation and interpretations, let $\gamma = \Pi_{\min} / \Pi_O^*$, e.g., γ is the cooperative's minimum profit to assure financial viability as a percent of status quo profit. While there is an infinite number of ways for allocating the gain in system profit to farmers, we present results for the case of a "fair" policy, i.e., the gain allocated to each farmer under RSPP and BMPP is equal to the farmer's contribution to increased expected system profit. This means that the values of λ_i and k_i are set so that the increase in expected profit under RSPP and BMPP is

$$\pi_{Ri}\left(x_{Si}^*\right) - \pi_{Oi}^* = \pi_{Bi}\left(x_{Si}^*\right) - \pi_{Oi}^* = \Psi_{Si}^* - \Psi_{Oi}^* - (\gamma - 1)\Pi_{Oi}^* . \tag{33}$$

The farmer's gain is the increase in expected profit from system-optimal effort, $\Psi_{Si}^* - \Psi_{Oi}^*$, augmented by any necessary adjustment to satisfy the cooperative's minimum profit constraint. If $\Pi_{\min} < \Pi_O^*$, for example, then the cooperative shares fraction $1 - \gamma$ of the farmer's profit contribution to cooperative profit under status quo in addition to the gain in profit from the farmer's effort.

Proposition 7. *Under fair allocation of gain to farmers, the policy parameters for RSPP and BMPP are*

$$\lambda_i = 1 - \gamma\Pi_{Oi}^* / \Psi_{Si}^* \tag{34}$$

$$k_i = \gamma\Pi_{Oi}^* / t_i . \tag{35}$$

The farmer's value at risk for each policy when the VaR constraint is relaxed is

$$t_i\beta_{Ri}\left(x_{Si}^*\right) = -\Psi_i\left(\alpha, x_{Si}^*\right) + \left(\frac{\Psi_i\left(\alpha, x_{Si}^*\right)}{\Psi_{Si}^*}\right)\gamma\Pi_{Oi}^* \tag{36}$$

$$t_i\beta_{Bi}\left(x_{Si}^*\right)=-\Psi_i\left(\alpha,x_{Si}^*\right)+\gamma\Pi_{Oi}^* > t_i\beta_{Ri}\left(x_{Si}^*\right). \tag{37}$$

We see that a farmer's value at risk at the system-optimal effort is lower under RSPP than BMPP. There are hints of this result in the expressions for variance in farmer profit in propositions 3 and 5, i.e.,

$$V\left[\tilde{\pi}_{Ri}\left(x_i\right)\right]/V\left[\tilde{\pi}_{Bi}\left(x_i\right)\right]=\lambda_i^2<1$$

(see (22) and (29)). BMPP is an extreme policy in the sense that all of the risk is borne by the farmer; the cooperative's profit is fixed at $\Pi_{\min}$. If farmers are risk neutral or if VaR constraints are not binding at the system-optimal effort for all farmers under BMPP (i.e., $\beta_{Bi}\left(x_{Si}^*\right)<\beta_i$ for all i), then this difference has no effect on system profit—both RSPP and BMPP maximize system profit. However, if this condition is not met, then RSPP yields better performance than BMPP through higher quality efforts by farmers.

To provide a comparison of OMPP with RSPP and BMPP, we similarly consider the farmer's effort decision under OMPP when the VaR constraint is relaxed (i.e., at effort $x_{Oi}^{\circ} \geq x_{Oi}^*$). The farmer's VaR is

$$t_i\beta_{Oi}\left(x_{Oi}^{\circ}\right)=-t_i\left[q\left(x_{Oi}^{\circ}\right)z_O\left(\alpha\right)-\left(c_{1i}+c_{2i}x_{Oi}^{\circ}\right)\right]$$

(see (48)). To compare with RSPP and BMPP, we set $\gamma = 1$ (so expected cooperative profit under RSPP and BMPP is the same as under OMPP) and write in VaR expanded form:

$$t_i\beta_{Ri}\left(x_{Si}^*\right)=-t_i\left[\left(1+m\right)q\left(x_{Si}^*\right)z_O\left(\alpha\right)-\left(c_{1i}+c_{2i}x_{Si}^*\right)\right]\left(1-\Pi_{Oi}^*/\Psi_{Si}^*\right)$$

$$t_i\beta_{Bi}\left(x_{Si}^*\right)=-t_i\left[\left(1+m\right)q\left(x_{Si}^*\right)z_O\left(\alpha\right)-\left(c_{1i}+c_{2i}x_{Si}^*\right)\right]+\Pi_{Oi}^*$$

While $q\left(x_{Oi}^*\right)z_O\left(\alpha\right)<q\left(x_{Si}^*\right)z_O\left(\alpha\right)$ (due to (13) and nonnegative open-market price Z_O), it is also the case that $c_{2i}x_{Oi}^*< c_{2i}x_{Si}^*$. Consequently, depending on the values of parameters and the probability distributions, the farmer's VaR may be higher or lower under OMPP compared to RSPP and BMPP.

4.4 Practical Considerations: Risk Aversion and Farmer Heterogeneity

The optimal policy Γ_i^* in Proposition 2 shows how the negative effects of farmer risk aversion on farmer effort can be eliminated though the selection of policy parameter values. In particular, parameter $\overline{k_i}$ is defined to guarantee that the farmer's maximum loss per tree is no more than the farmer's tolerable loss β_i. In effect, the optimal policy provides insurance for downside risk, while aligning the farmer's incentives with the system. This raises the possibility of introducing insurance as a separate instrument in conjunction with RSPP or BMPP (or OMPP). Aside from addressing the negative effects of farmer risk aversion, insurance can be tailored to each farmer without knowledge of the farmer's risk aversion. For example, consider an insurance policy limits the farmer's loss per tree to no more than β. The farmer is free to select among alternative values of β. Farmer i has incentive to select his tolerable loss β_i, which minimizes his insurance payment while satisfying his VaR constraint. The random payout under the insurance policy depends on β_i and the farmer's profit, which in turn depends on the payment policy, observed yield and quality, and farmer costs. For example, the insurance payout associated with random profit $\tilde{\pi}_i$ is $\max\{-\tilde{\pi}_i - t_i\beta_i, 0\}$. The cost of the insurance to the farmer is the expected payout $E\left[\max\{-\tilde{\pi}_i - t_i\beta_i, 0\}\right]$ plus some markup to cover profit requirements of the provider.

As a risk-neutral entity, the cooperative can provide insurance to farmers with the insurance premium equal to the expected cost, gaining the benefits of increased quality with zero expected cost/profit from the insurance offering. In this case, both cooperative and farmer profits improve when insurance is available. Our conversations with management at Tariş indicate this is their preference (e.g., as opposed to insurance offered through a third-party provider that will extract some surplus). The insurance policy is relatively simple to implement because Tariş has data regarding farmer expenses (i.e., farmers purchase supplies from the cooperative), regional yield, open-market price for the lowest grade of premium oil (with 2% oleic acidity) and the cooperative's payment schedule over time. The data enables Tariş to compute the distribution of farmer profit, from which the premium can be calculated for any loss threshold. As a result, Tariş has begun to offer insurance of this type to farmers in a small village of olive growers on a pilot basis.

The following result formalizes the point that inefficiencies under OMPP arise from a combination of farmer risk aversion and incentive misalignment, and that the use of BMPP or RSPP with insurance eliminates these inefficiencies. We use /I to indicate a payment policy with insurance. The corollary assumes that $\Pi_{\min} - \Pi_O^* \leq \Psi_S^* - \Psi_O^*$. The assumption says that, relative to status quo, the cooperative does not need to extract more than the gain in system profit when farmers select system-optimal effort in order to remain viable. The assumption likely holds in practice, e.g., otherwise the profit requirements in constraints (14) and (15) are simply not sustainable.

Corollary 1. *Suppose* $\Pi_{\min} - \Pi_O^* \leq \Psi_S^* - \Psi_O^*$. $x_{B/Ii}^* = x_{R/Ii}^* = x_{Si}^* > x_{O/Ii}^* = x_{Oi}^{o} \geq x_{Oi}^*$; $\Psi_{B/I}^* = \Psi_{R/I}^* = \Psi_S^* > \Psi_{O/I}^* \geq \Psi_O^*$.

Farmers are not the same. In addition to differences in risk aversion that may be addressed through crop insurance, some farmers are more efficient than others. Farmers may differ in their efficiency along two dimensions: c_{1i} reflects the farmer's cost per tree exclusive of any quality effort (e.g., related to age of trees, location and layout of the farm, etc.); $c_{2i}x$ captures farmer's cost to produce oil of average quality $q(x)$ (e.g., affected by farmer expertise, access to labor, etc.).

In practice, it is often desirable to stipulate a uniform payment schedule, i.e., a single quality-dependent payment schedule that applies to all farmers. It is with respect to this feature that BMPP offers an advantage over RSPP. Notice that the RSPP payment schedule $\Gamma_{Ri}(p_M, a_r, y)$ depends on c_{1i} and c_{2i}. Thus, even when insurance is used to mitigate risk aversion, a single RSPP payment schedule will not lead to system-optimal effort by all farmers (excluding the extreme of identical farmers). However, BMPP is less restricted because $\Gamma_{Bi}(p_M, a_r, y)$ does not depend on farmer's cost parameters.

Proposition 8. (*a*) *If farmers are not identical in their cost efficiencies, then there does not exist a uniform RSPP/I payment schedule that will maximize system profit.* (*b*) *If*

$$\frac{\Pi_{\min}}{T} \le \min_i \left\{ \frac{\Psi^*_{Si} - \pi^*_{Oi}}{t_i} \right\} \tag{38}$$

then the following uniform BMPP/I payment schedule maximizes system profit:

$$\Gamma_{Bi}(p_M, a_r, y) = (1+m) p_M a_r - k / y \text{ for any } k \in \left[\frac{\Pi_{\min}}{T}, \min_i \left\{ \frac{\Psi^*_{Si} - \pi^*_{Oi}}{t_i} \right\} \right]. \tag{39}$$

The left side of condition (38) is the minimum profit per tree for the cooperative to remain financially viable. The expression on the right, $(\Psi^*_{Si} - \pi^*_{Oi}) / t_i$, is the maximum profit per tree that the cooperative could extract from farmer *i* while providing farmer profit of at least π^*_{Oi}. As noted above, the payment schedule given in (39) is simple to communicate: each farmer pays *k* per tree and receives the retail price net of cooperative variable cost for his oil.

If insurance is not available and if the cost structure is similar across farmers, then RSPP will tend to be more attractive than BMPP. In such a setting, the lower variance in farmer profit becomes an advantage. Otherwise, BMPP is likely favored.

We contrast BMPP with a payment policy at some agricultural cooperatives (and cooperatives in general). Some cooperatives, though not Tariş, distribute cooperative profits to its members at the end of each year. Such a policy, on the surface, may appear similar to BMPP, e.g., the cooperative keeps portion *kT* with any remaining profit going to its member farmers. However, there is an important distinction. BMPP pays each farmer according to the quality of his harvest. A year-end profit distribution by the cooperative to farmers according to farm size eliminates this linkage, and thus, it does not align an individual farmer's incentives with the system. A farmer's payoff from a high investment to improve quality is diluted by the lack of investment by other farmers (e.g., free-rider effect). This behavior is observed, for example, in de Zegher et al. (2019) when sheep farmers are paid according to the average value of the output among all farmers. The authors study the positive effects of a change to a direct-sourcing model in which farmers are paid according to the value of their individual outputs.

In summary, the quality-based payment to member farmers in our study eliminates the free-riding effects that are seen in traditional cooperatives. Thus, while the motivation for our work comes from a for-profit cooperative, the insights from our analysis may be relevant for traditional cooperative structures with farmers as owners and profits distributed among the farmers.

Moreover, our conclusions are robust over a broad family of random revenue models that exhibit separability and a set of risk measures that include any coherent risk measure in addition to VaR. That is, given that $\tilde{r}_O(x_i)$ denotes the random open-market price per input unit given farmer effort x_i, consider the following general structure for effort-dependent random open-market price per input unit:

$$\tilde{r}_O(x_i) = q_1(x_i) + q_2(x_i)\, g(\tilde{a}, \tilde{\varepsilon}, \tilde{y}) \tag{40}$$

where q_1 and q_2 are increasing concave functions, and g is any function of the three random primitives for farmer crop quality ($\tilde{a}$), yield in the region ($\tilde{y}$), and remaining unobservable factors that affect the open-market price ($\tilde{\varepsilon}$). (For example, for the model in Section 3, $\tilde{r}_O(x_i) = q(x_i)\tilde{y}\tilde{a}\left(\mu_{p_M} - b(\tilde{y}-1) + \tilde{\varepsilon}\right)$, i.e., $q_1(x_i) = 0$, $q_2(x_i) = q(x_i)$, and $g(a, \varepsilon, y) = ya\left(\mu_{p_M} - b(y-1) + \varepsilon\right)$). It can be shown that the results of the analysis (i.e. farmers underinvesting, risk-averse farmers underinvesting more than risk-neutral farmers, and as evident in Section 5 that it is possible to coordinate the entire system via BMPP or RSPP combined with insurance) hold under general random revenue function (40) with either VaR as the risk measure or any coherent risk measure. For further details on robustness of the results, please see Proposition 11 in Online Supplement and the related discussion.

4.5 Quadratic Cost of Quality

We specify a functional form for the relationship between cost and quality in order to illustrate relationships among decisions and profits in more detail. Tariş does not collect data on the quality-cost relationship. However, drawing on knowledge of the industry, management at Tariş helped us select a quadratic quality cost function. This function captures diminishing marginal return to quality in a plausible manner in the view of management and affords some tractability. Quality as a function of effort x_i is

$$q(x_i) = 1 + x_i^{1/2} \tag{41}$$

and farmer i cost as a function of quality is $C_i(q) = c_{1i} + c_{2i}(q-1)^2$ (obtained by inverting $q(x_i)$ and substituting into cost $c_{1i} + c_{2i}x_i$).

The following proposition shows relationships between OMPP and system optimal decisions and profits associated with farmer i given the quality function defined in (41). Note that expected system profit associated with farmer i given no investment in quality is

$$\Psi_i(0) = t_i\left[(1+m)\mu_{Z_O} - c_{1i}\right] \tag{42}$$

(see (47)). We refer to $\Psi_i(0)$ as *base profit*.

Proposition 9. *The optimal decision, expected quality, and system profit under policies OMPP, OMPP/I, and coordinating policies RSPP/I and BMPP/I are*:

$$x_{Oi}^{*} \le x_{Oi}^{\text{o}} = \left(\frac{\mu_{Z_O}}{2c_{2i}}\right)^2 \le x_{Si}^{*} = \left(\frac{(1+m)\mu_{Z_O}}{2c_{2i}}\right)^2$$

$$q\left(x_{Oi}^{*}\right) \le q\left(x_{Oi}^{\text{o}}\right) = 1 + \frac{\mu_{Z_O}}{2c_{2i}} \le q\left(x_{Si}^{*}\right) = 1 + (1+m)\frac{\mu_{Z_O}}{2c_{2i}} \tag{43}$$

$$\Psi_i\left(x_{Oi}^{*}\right) \le \Psi_i\left(x_{Oi}^{\text{o}}\right) = \Psi_i(0) + t_i(1+2m)\left(\frac{\mu_{Z_O}{}^2}{4c_{2i}}\right) \le \Psi_i\left(x_{Si}^{*}\right) = \Psi_i(0) + t_i(1+m)^2\left(\frac{\mu_{Z_O}{}^2}{4c_{2i}}\right). \tag{44}$$

Expressions (42) – (44) show how the increase in system profit from incentive alignment depends on farmer cost and revenue terms. Suppose that the VaR constraint is nonbinding under OMPP, or that OMPP is offered with insurance (i.e., OMPP/I). Expression (44) shows the difference in system profit and base profit $\Psi_i(0)$ for OMPP and for RSPP/I (equivalently, BMPP/I). We see that incentive alignment increases the difference by a factor of $1 + m^2/(1+2m)$ that depends only on margin. However, the other parameters (c_{1i}, c_{2i}, μ_{Z_O}), in conjunction with m, affect the percentage gain in system profit $\Psi_i\left(x_{Si}^{*}\right)/\Psi_i\left(x_{Oi}^{\text{o}}\right) - 1$. The next proposition presents comparative static results for $\Psi_i\left(x_{Si}^{*}\right)/\Psi_i\left(x_{Oi}^{\text{o}}\right)$.

Proposition 10. *The value of the ratio* $\Psi_i\left(x_{Si}^{*}\right)/\Psi_i\left(x_{Oi}^{\text{o}}\right)$ *is decreasing in* c_{2i} *and is increasing in* c_{1i}; $\Psi_i\left(x_{Si}^{*}\right)/\Psi_i\left(x_{Oi}^{\text{o}}\right)$ *is increasing in* μ_{Z_O} *if and only if*

$$\frac{\mu_{Z_O}}{c_{1i}} \ge \frac{2}{1+m}, \tag{45}$$

and is increasing in m if $\dfrac{\mu_{Z_O}}{c_{1i}} \ge \dfrac{2}{1+m}$.

Note that the gain in system profit over OMPP/I is greater for an efficient farmer (small c_{2i}) than an inefficient farmer (large c_{2i}). This is a consequence of the convexity of the quality cost function, which causes the difference in average quality, $q\left(x_{Si}^{*}\right) - q\left(x_{Oi}^{\text{o}}\right)$, to be decreasing in c_{2i}. On the other hand, if the cost of achieving the lowest quality (c_{1i}) increases, then the percentage gain in system profit increases. However, this is only because all profits shrink as c_{1i} increases; the absolute difference in system profit $\Psi\left(x_{Si}^{*}\right) - \Psi\left(x_{Oi}^{\text{o}}\right)$ is independent of c_{1i}.

The effect of increasing μ_{Z_O} and m is nuanced because increases in these parameters (1) inflate both $\Psi_i\left(x_{Si}^*\right)$ and $\Psi_i\left(x_{Oi}^{\text{o}}\right)$, which puts negative pressure on the percentage gain, and (2) increase the quality difference $q\left(x_{Si}^*\right)-q\left(x_{Oi}^{\text{o}}\right)$, which puts positive pressure on the percentage gain. For the case of μ_{Z_O}, a simple inequality delineates the boundary between where negative or positive pressure dominates. This inequality is a sufficient condition for $\Psi_i\left(x_{Si}^*\right)/\Psi_i\left(x_{Oi}^{\text{o}}\right)$ to be increasing in markup (m), e.g., if the profit gain is increasing in μ_{Z_O}, then it is assured to be increasing in m, but not vice-versa. The reason for this result is related to expression (44). In (44), we see that increases in μ_{Z_O} do not increase the profit gain relative to base profit, but m does. As a consequence, a necessary and sufficient condition for parameter m (as opposed to the sufficient condition in Proposition 10) is very complex and not insightful.

5. Estimating the Value of Insurance and Incentive Alignment in Practice

This section presents the financial impact of insurance and a payment scheme that aligns a farmer's incentives with the system (e.g., via BMPP or RSPP) relative to current practice at Tariş. We explain how we calibrate our model in Section 5.1. Some farmers are more efficient than others (i.e., the cost to improve quality is not the same for all farmers). In Section 5.2, we examine OMPP and system-optimal quality levels and corresponding percentage improvement in system profit associated with farmer *i* over a range of farmer efficiencies. Our analysis illustrates how decisions and profits are influenced by the payment policy and farmer efficiency.

5.1 Model Calibration

We use data from Tariş to estimate parameters and probability distributions in our model. We express all cost and revenue terms in US$. The input unit is the quantity that yields one liter of olive oil on average. The data for the open-market olive oil prices are obtained from MFAO. The sample mean and variance of the open-market price at the lowest quality in the premium category (2% acidity) are μ_{p_M} = 3.8 and σ_{p_M} = 1.06. The data for random yield distribution $\tilde{y}$ are provided by Tariş, and include yield realizations from 2007 to 2015 (see Figure A2 in the online supplement). The probability mass function (pmf) of $\tilde{y}$ is the nine historical realizations with probability 1/9; μ_y = 1 and σ_y^2 = 0.22. The correlation coefficient of observed price-yield data reported in Figure A1 is $\rho_{p_M y}$ = –65.7%. Tariş management believes correlation between uncertain yield $\tilde{y}$ and uncertain quality effect $\tilde{a}$ is insignificant, i.e., $\sigma_{ay} = \sigma_{ay^2}$ = 0. Furthermore, management estimates that the upper limit on uncertain quality effect is a 1% shift in oleic acidity, which translates to a maximum price shift of about 1/7 ≈ 14%. We model $\tilde{a}$ as a uniform random variable on [6/7, 8/7].

The open-market price is clearly bounded, and thus the distribution of $\tilde{\varepsilon}$ should be bounded. The random error parameter $\tilde{\varepsilon}$ is uniformly distributed on [-1.2, 1.2]. Recall that the open-market price for the lowest quality of olive oil with 2% oleic acidity $\tilde{p}_M$ and the regional yield $\tilde{y}$ are correlated; the covariance of these two random variables is $\sigma_{p_M y} = E\left[\left(\tilde{p}_M - \mu_{p_M}\right)\left(\tilde{y} - 1\right)\right] = -b\sigma_y^2$ and the correlation can be expressed as $\rho_{p_M y} = \frac{\sigma_{p_M y}}{\sigma_{p_M}\sigma_y} = \frac{-b\sigma_y}{\sigma_{p_M}} = \frac{-b\sigma_y}{\left(b^2\sigma_y^2 + \sigma_\varepsilon^2\right)^{1/2}}$. We rearrange the correlation expression in order to estimate parameter b (sensitivity of open-market price to regional yield) i.e., $b = \sigma_\varepsilon \Big/ \sigma_y \left[\left(1/\rho_{p_M y}\right)^2 - 1\right]^{1/2}$ = 1.3. As a check on our uniform distribution assumption, we compare histograms of historical and model prices in the online supplement and find suitable comparability (see figures A3 and A4). With these parameters and distributions, the expected open-market price per input unit at the lowest quality level is $\mu_{Z_O} = \mu_{p_M} - b\sigma_y^2$ = 3.8 – 0.3 = 3.5, and the fractile of Z_O at α = 0.10 is $z_O(0.10)$ = 1.9.

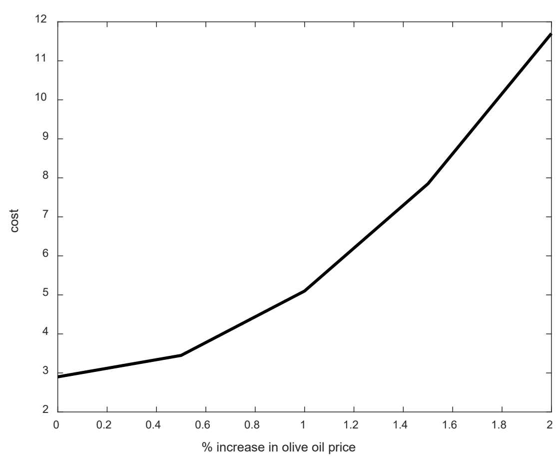


Figure 2. Cost as a function of % increase in the open market olive oil price over the price at the lowest quality, i.e., % price increase = q – 1. The parameter values are c_{1i} = 2.9, c_{2i} = 2.2.

The brand markup parameter m is provided by Tariş, which is the average markup that is net of all variable costs, including bottling, packing, distribution, over all of the oleic acidity levels in the premium category (i.e., oleic acidity ≤ 2%); m = 0.8. Tariş management estimates the typical farmer cost per input unit at the lowest quality level to be \$2.9 (e.g., $\bar{c}_1 = \frac{1}{N}\sum_{i=1}^{N} c_{1i}$ = 2.9).

Recall that the open-market price and the retail price of olive oil depend on quality. Figure 3 illustrates the relationship between cost of the effort by a typical (or average) farmer to improve quality and the corresponding percentage increase in olive oil price relative to the price for the lowest quality olive oil. We shared versions of Figure 2 (computed at different values of average marginal cost parameter $\overline{c}_2$) with Tariş management in order to identify a value of $\overline{c}_2$. Their knowledge and experience suggest that $\overline{c}_2$ = 2.2. In our analysis below, we examine the effect from different values of c_{2i} that span $\overline{c}_2$ = 2.2.

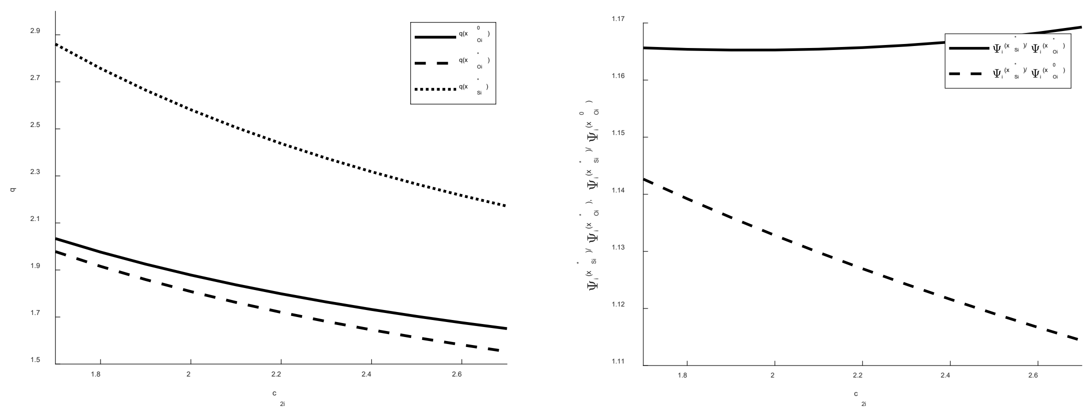
Our work has prompted Tariş to offer insurance to farmers from a small village in the Altınoluk region of Edremit Bay on a pilot basis. Our risk parameters draw on knowledge from this pilot program: 21 of 33 farmers elected to purchase insurance, and the most widely requested insurance coverage corresponds to a loss amount of $\overline{\beta}$ = 0.8 with probability estimated at α = 0.10. We use $\beta_i = \overline{\beta}$ = 0.8 in our numerical illustrations, but we also report results when the farmer's VaR constraint is nonbinding, e.g., a farmer without insurance has a sufficiently high tolerable loss ($\beta_i > \overline{\beta}$) or the farmer has insurance.

5.2 Impact of Incentive Alignment on Quality and Profit for Different Farmer Efficiencies

Recall that $q(x_i)$ is the expected percentage increase in price due to quality effort relative to zero effort to improve quality. Figure 3(a) shows the impact of farmer efficiency on optimal $q(x_i)$ under three settings: (1) optimal quality under OMPP when farmer risk aversion is ignored, $q\left(x_{Oi}^{\mathrm{o}}\right)$ (equivalently, tolerable loss $\beta_i \geq \beta_{Oi} = -\pi_{Oi}\left(0.1, x_{Oi}^{\mathrm{o}}\right)$; (2) optimal quality under OMPP incorporating farmer risk aversion with the VaR constraint, $q\left(x_{Oi}^{*}\right)$; (3) system-optimal quality, $q\left(x_{Si}^{*}\right)$. Note that farmers invest in system-optimal effort under RSPP/I and BMPP/I (i.e., both policies are implementations of an optimal contract). Lower values of c_{2i} represent cost-efficient farmers whereas higher values of c_{2i} correspond to inefficient farmers. Figure 3(a) illustrates relationships in Proposition 9, and the significance of the relationships in a real-world setting. For example, optimal investment in quality is decreasing with farmer inefficiency. The negative relationship between farmer inefficiency and quality is evident in (43) for the case of $q\left(x_{Oi}^{\mathrm{o}}\right)$ and $q\left(x_{Si}^{*}\right)$. In Figure 3, we see the same pattern in $q\left(x_{Oi}^{*}\right)$, i.e., the negative relationship is unaffected by risk aversion. Second, risk aversion (at β_i = 0.8) causes the farmer to decrease the level of quality investment under the OMPP policy (see (43)). Third and most important, Figure 3(a) illustrates the magnitude of quality improvement under incentive alignment (see (43)), e.g., approximately 50% increase in quality over the range of farmer efficiencies. System-optimal quality is significantly higher at every level of farmer efficiency compared to OMPP (with and without risk aversion). This third observation is critical from the perspective of Tariş because it addresses the common problem of under-investment in quality.

Figure 3(b) shows the magnitude of improvement in system profit relative to the current OMPP policy for both risk-neutral and risk-averse farmers. If the farmer is risk neutral (or the VaR constraint is nonbinding), then the gain in system profit ranges between 12% and 14%, with the percentage gain decreasing in farmer inefficiency. From (43) we know that the gain is greater when risk aversion constrains the farmer's investment in quality. Figure 3(b) shows a profit gain of approximately 16.5% under risk aversion over the range of efficiency levels. Interestingly, the percentage gain is not decreasing in farmer inefficiency as it is when the farmer is risk neutral. This is because an inefficient farmer has more to lose from effort to improve quality (due to higher cost), which translates into higher cutbacks in quality effort to satisfy the risk constraint (see the increasing gap between $q\left(x_{Oi}^{o}\right)$ and $q\left(x_{Oi}^{*}\right)$ as a increases, both in absolute and, especially percentage, measures).

Figure 3(b) also exposes the gain from crop insurance with no change in the payment policy. If a risk-averse farmer is offered crop insurance under OMPP, then the percentage gain in system profit ranges from 2% (= 1.165/1.142 – 1) for an efficient farmer (c_{2i} = 1.7) to 4.4% (= 1.165/1.115 – 1) for an inefficient farmer (c_{2i} = 2.7). In other words, approximately 2% to 4% of the 15% gain in profit can be attributed to insurance, with the balance of 12% to 14% due to the alignment of farmer incentives with the system.



(a) (b)

Figure 3. (a) Quality levels under OMPP with risk-neutral and risk averse farmers, and system-optimal quality, as farmer inefficiency (c_{2i}) increases; (b) ratio of system-optimal profit to OMPP system profit for risk-averse and risk-neutral farmers, as farmer inefficiency (c_{2i}) increases.

In our calibration, the VaR constraint is always binding under OMPP for a risk-averse farmer (with $\beta_i = 0.8$; see Figure 3(a)). We find that this is also the case under BMPP if we set the minimum cooperative profit in constraint (15) to be the profit under OMPP (i.e., $\Pi_{\min} = \Pi_O^*$). However, in our calibration, the VaR of the system profit is negative, equivalently, system profit at fractile α is positive, i.e., $\Psi_{Si}^*(\alpha) > 0$ for all $c_{2i} \in [1.7, 2.7]$. Thus, VaR for a farmer under RSPP with profit-share parameter $\lambda \in [0, 1]$ is $\lambda\Psi_{Si}^*(\alpha) > 0$. We observe that in our calibration, insurance is unnecessary for system optimal investment in quality by a risk-averse farmer under RSPP, given that all farmers have the same cost efficiency that is some value between 1.7 and 2.7. This illustrates the main advantage of RSPP over BMPP as noted in Section 4.4.

6. Conclusions

We consider an agricultural cooperative that sets the prices it will pay to its member farmers for different levels of crop quality. Our work is motivated by Tariş, the second largest premium olive oil producer in Turkey. Tariş became an independent entity without government support in year 2000. The change has prompted greater emphasis on improvements that help assure the long-term profitability and financial stability of the cooperative.

Tariş currently makes quality-dependent payments to farmers on the basis of olive oil prices in the open market (i.e., OMPP). Management believes that farmers underinvest in the quality of the crop. Our analysis confirms this belief, and we show that it is a consequence of the cooperative's higher pricing power and its lower sensitivity to risks from yield, quality uncertainty, and open-market price volatility.

We describe and analyze two new payment policies (i.e., BMPP and RSPP), and we show that these policies align a risk-neutral farmer's incentives with the system. We also show that these policies can increase farmer risk. However, when augmented with crop insurance, BMPP and RSPP incentivize farmers to optimally invest in quality, leading to higher system profit with both farmer and cooperative better off. BMPP is easier to implement than RSPP because it is not sensitive to variation in farmer efficiency and does not rely on farmer private information. In other words, if the cooperative offers a single quality-dependent payment schedule to all farmers, then the use of BMPP (RSPP) with insurance leads to system-optimal investment in quality by all (some) farmers. We show that our findings are robust over a broad family of random revenue models and risk measures.

Using industry data, we find the percentage gain in system profit over OMPP due to optimal investment in quality ranges from 10% to 15% (depending on farmer efficiency). Approximately 10% to 20% of the gain stems from mitigating farmer risk aversion (via crop insurance) with the balance of 80% to 90% coming from the alignment of farmer incentives with the system.

We next offer an interpretation of BMPP that serves to both reinforce the intuition into what drives differences between BMPP and OMPP performance and to illuminate the specifics of BMPP implementation. Recall that under OMPP, the quality-dependent price paid to the farmer matches what the farmer would receive in the open market, which is a lower price than what the cooperative can fetch. Under BMPP, the cooperative pays the farmer the full retail value of the farmer's oil from his orchard, then subtracts a constant that is independent of quality and yield. In essence, the cooperative is putting itself in a position that is akin to being a landowner – it is as if the cooperative receives a rent per tree, which is the portion of the payment schedule that is independent of quality and yield, and allows the farmer to invest in the land to maximize his payoff. The "rent" can be set so that cooperative remains financially viable (e.g., comparable to profit under OMPP), and because the farmer is making wiser quality decisions, the farmer is better off. The farmer receives the full retail price for premium olive oil (i.e., the open-market price inflated by the cooperative's brand markup), and implementation boils down to determining an agreeable rent per tree.

The BMPP payment scheme is simple to specify and communicate: Farmers pay a "premium membership fee in return for full retail price." The cooperative identifies a fair and reasonable profit from the region (e.g., sufficient to continue in investing in improvements needed for long-term financial viability with sufficient reserve to weather swings in the market). This value is divided by the number of trees in the region to yield a "fee-per-tree," which can be multiplied by number of trees on an orchard to determine a farmer's annual membership fee. To help with a farmer's cash flow, the payment of the membership fee occurs at the same time a farmer is paid for his oil. However, BMPP does represent a significant change over the current policy, and successful implementation will rely on a high degree of trust between the cooperative and its member farmers.

There are additional benefits associated with offering insurance. The cooperative can collect detailed information about what farmers do in their quality improvement efforts. This would enable the cooperative to educate its member farmers about state-of-the-art farming techniques. It would also lead to a more transparent environment where both the cooperative and its members share information about the costs and revenues in growing olives and producing olive oil. Such information-sharing transparency would result in a stronger dependence and reliance between all parties, and would enable both cooperative and farmer to form common objectives.

We believe that this new payment policy—BMPP—is particularly attractive for those farmers who focus on organic farming using biodynamic methods to improve fruit quality (olives), soil fertility and yields. Certain villages and olive growth regions are marked with certification from the Chamber of Commerce for biodynamic practices. The implementation at these regions can serve as a pilot study for assessing BMPP in practice.

We see two extensions worthy of future research. One worthy extension is to investigate how farmer investments in quality (or yield) affect equilibrium open-market prices. This is especially relevant for market structures comprised of a relatively small number of intermediaries (e.g., cooperatives) setting payment policies for paying farmers. A second worthy extension is to analyze mechanisms (including payment policies) that account for the risk of adulteration in order to receive a higher payment (e.g., artificially inflate the measure quality) or reduce cost (e.g., lower the grade of input). A promising stream of work along this line includes Levi et al. (2020) and Mu et al. (2014, 2016).

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Online Supplement

Incentivizing Farmers to Invest in Quality through Quality-Based Payment

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Olive Oil Production and Payment Process

Immediately after harvest (end of November or early December), Tariş washes and presses the farmer's olives into oil, which is then transferred to steel tanks for storage. Tariş performs an oleic acidity test that reveals the quality of olive oil. While the quality of olive oil is determined by various factors, oleic acidity test results are the dominant and the worldwide measure of quality. In the European Union, olive oil is classified according to the Annex XVI of Regulation (EC) No 1234/2007 which states that olive oil with oleic acidity less than 0.8% is classified as *extra virgin* and olive oil with oleic acidity less than 2% as *virgin*. Table 1 displays the quality classification for premium olive oil used at leading producers of olive oil in Turkey (e.g., Komili, Tariş, Kırlangıç), Greece, Spain, Morocco and Tunisia.

Oleic acidity = 0.3% (also known as "perfect oil" and is obtained very rarely)
0.3% < Oleic acidity ≤ 0.5% (referred to as the highest-grade oil)
0.5% < Oleic acidity ≤ 0.8%
0.8% < Oleic acidity ≤ 1.0%
1.0% < Oleic acidity ≤ 1.5%
1.5% < Oleic acidity ≤ 2.0% (referred to as the lowest-grade oil)

Table A1. Quality classification of "premium olive oil." Note that 0.3% is the theoretical minimal level of oleic acidity; this is difficult to obtain and rarely seen among Tariş member farmers. Olive oil that has more than 2% oleic acidity is classified as *lampante* olive oil, and requires a refining process in order to reduce its fatty acids. Lampante olive oil is not a premium oil.

Establishment of Global Open-Market Prices

Olive oil producers around the world follow the open-market prices established by the Olive Oil Futures Market known as MFAO, which is the benchmark for how prices are moving in international markets. Because the olive oil traded in MFAO is of the lowest quality, all leading producers around the Mediterranean Sea use MFAO prices only to determine the payment for 2% oleic acidity which is the lowest grade of premium olive oil. Figure 1 demonstrates the fluctuations in the open-market prices for 2% oleic acidity in MFAO over a 30-year time interval. The oil is stored for approximately two months after harvest in order to allow residues to settle. In February, when the oil is ready for sale, Tariş takes the MFAO open-market price as a basis for the payment to be made for olive oil with 2% oleic acidity. The payment for higher levels of quality (as listed in Table A1) is determined by multiplying the price of olive oil with 2% oleic acidity by a factor that reflects the market value of lower oleic acidity. As a result, each farmer is paid according to the market value of the quality of oil determined by the oleic acidity test.

Figure A1. Historical prices for the lowest grade of premium olive oil, i.e., 2% oleic acidity, in the world market. Source: The Olive Oil Futures Market known as the Mercado de Futuros del Aceito de Oliva from February 1986 to February 2016.

Empirical Evidence Between the Open-Market Price and the Regional Yield

We provide the details of the correlation between $\tilde{p}_M$ and $\tilde{y}$ and develop the derivations for expression (2) used in the analysis. Recall that the payment to the farmer for the lowest quality of olive oil, with 2% oleic acidity, is equal to the realization of the random MFAO price $\tilde{p}_M$. We obtained nine years of data (2007 – 2015) representing the yield of premium olive oil in the Edremit Bay region from Ayvalik Chamber of Commerce. All member farmers of Tariş are located in Edremit Bay which produces more than 70% of Turkey's olive oil. These farmers experience similar crop yields that is unpredictable prior to the growing season due to variations in temperature, rain and sunshine. Figure A2 displays a scatter plot of realized (p_M, y) values.

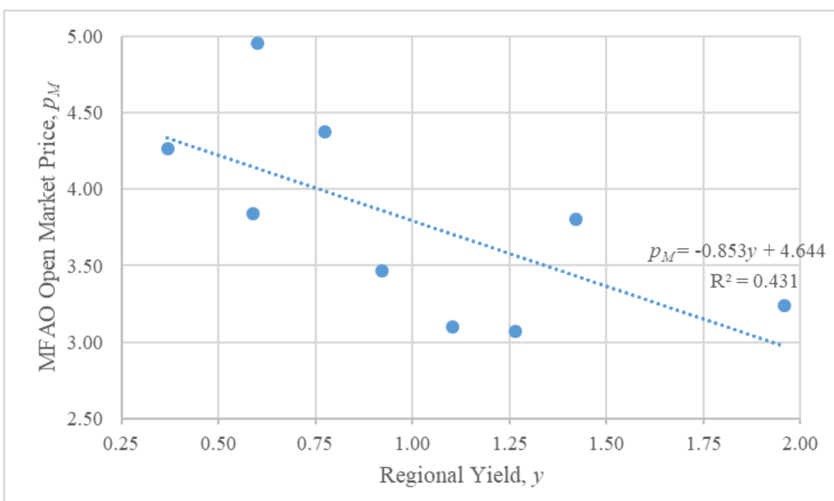

Figure A2. Price for olive oil in MFAO for 2% oleic acidity in February 2008 to 2016 by olive oil yield between 2007 to 2015; each data point is (yield in year x, price in February of year x + 1).

The data in Figure A2 exhibit a negative and linear relationship between the open-market price and the regional yield:

$$p_{Mt} = \beta_0 - \beta_1 y_t + \varepsilon_t \qquad (46)$$

where p_{Mt} is the open-market price in MFAO for the year t harvest, y_t is the realized yield for premium olive oil in the Edremit Bay region in year t, and ε_t is the random error term in year t capturing all unobserved variables. While our data are limited to nine observations, the price model in (46) explains about 43% of the price-yield variation ($R^2 = 43.10\%$) with statistically significant estimates of β_0 and β_1 at 1% and 5%, respectively. The data suggests that prices in MFAO and yield have a negative and linear relationship.

	Coefficient	p-value
Intercept (β_0)	4.644^{***}	9.20×10^{-6}
y_t (β_1)	0.853^{**}	0.05
	$R^2 = 43.10\%$	

Table A2. Regression results of the analysis pertaining to MFAO price and yield relationship in expression (46). *** and ** represent statistical significance at $p \leq 1\%$ and $p \leq 5\%$, respectively.

Histograms of historical and model open-market prices

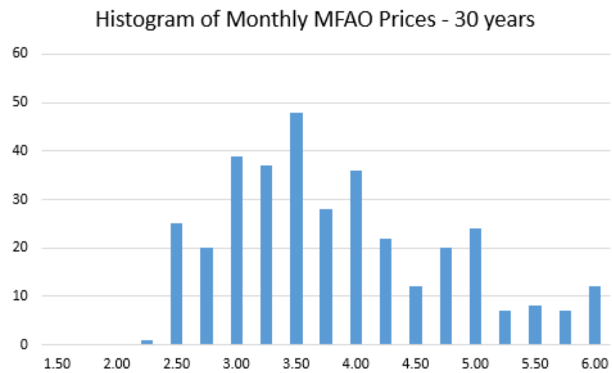

Figure A3. Histogram of observed monthly MFAO price for olive oil with 2% oleic acidity between February 1986 and February 2016 (361 months).

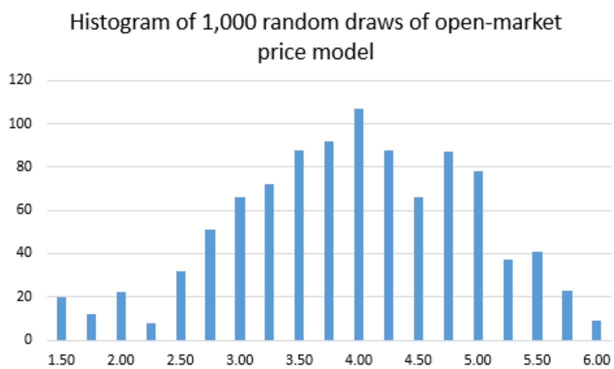

Figure A4. Histogram of 1,000 random draws of our open-market price model.

Additional Derivations

$$\begin{aligned}
\sigma_{p_O(x_i)y} &= E\left[\left(\tilde{p}_O(x_i) - \mu_{p_O(x_i)}\right)(\tilde{y}-1)\right] = q(x_i)E\left[\left(\tilde{a}\tilde{p}_M - \left(\mu_{p_M} - b\sigma_{ay}\right)\right)(\tilde{y}-1)\right] \\
&= q(x_i)E\left[\left((\tilde{a}-1)\tilde{p}_M + \left(\tilde{p}_M - \mu_{p_M}\right) + b\sigma_{ay}\right)(\tilde{y}-1)\right] \\
&= q(x_i)\left(\sigma_{p_M y} + E\left[(\tilde{a}-1)\left(\mu_{p_M} - b(\tilde{y}-1) + \tilde{\varepsilon}\right)(\tilde{y}-1)\right] + E\left[b\sigma_{ay}(\tilde{y}-1)\right]\right) \\
&= q(x_i)\left(\sigma_{p_M y} + \mu_{p_M}\sigma_{ay} - bE\left[(\tilde{a}-1)(\tilde{y}-1)^2\right]\right)
\end{aligned}$$

$$= q(x_i)\Big(\sigma_{p_M y} + \mu_{p_M}\sigma_{ay} - bE\Big[(\tilde{a}-1)\Big(\tilde{y}^2 - \mu_{y^2} - 2(\tilde{y}-1) + \mu_{y^2} - 1\Big)\Big]\Big)$$

$$= q(x_i)\Big(\sigma_{p_M y} + \mu_{p_M}\sigma_{ay} - b\sigma_{ay^2} + 2b\sigma_{ay}\Big)$$

$$\sigma_{p_M y} = E\Big[\big(\tilde{p}_M - \mu_{p_M}\big)(\tilde{y}-1)\Big] = E\Big[\big(\mu_{p_M} - b(\tilde{y}-1) + \tilde{\varepsilon} - \mu_{p_M}\big)(\tilde{y}-1)\Big] = -b\sigma_y^2$$

$$E\big[R_O(x_i)\big] = E\big[\tilde{y}\tilde{p}_O(x_i)\big] = \sigma_{p_O(x_i)y} + \mu_{p_O(x_i)}\mu_y = \sigma_{p_O(x_i)y} + \mu_{p_O(x_i)}$$

$$= q(x_i)\Big(\sigma_{p_M y} + \mu_{p_M}\sigma_{ay} - b\sigma_{ay^2} + 2b\sigma_{ay}\Big) + q(x_i)\big(\mu_{p_M} - b\sigma_{ay}\big)$$

$$= q(x_i)\Big(-b\sigma_y^2 + \mu_{p_M}\sigma_{ay} - b\sigma_{ay^2} + 2b\sigma_{ay}\Big) + q(x_i)\big(\mu_{p_M} - b\sigma_{ay}\big)$$

$$= q(x_i)\Big(\mu_{p_M}\big(1+\sigma_{ay}\big) - b\Big(\sigma_y^2 + \sigma_{ay^2} - \sigma_{ay}\Big)\Big).$$

Derivation of Optimal System Profit

The random and expected open-market price functions per unit of input with effort x_i by farmer i are

$$R_O(x_i) = \tilde{y}\tilde{p}_O(x_i) = \tilde{y}\tilde{p}_M q(x_i)\tilde{a} = q(x_i)Z_O$$

$$E\big[R_O(x_i)\big] = q(x_i)\mu_{Z_O}$$

where

$$\mu_{Z_O} = E\big[\tilde{y}\tilde{p}_M\tilde{a}\big] = E\Big[\tilde{y}\big(\mu_{p_M} - b(\tilde{y}-1) + \tilde{\varepsilon}\big)\tilde{a}\Big] = \mu_{p_M}\big(1+\sigma_{ay}\big) - b\Big(\sigma_y^2 + \sigma_{ay^2} - \sigma_{ay}\Big)$$

(in the above, σ_{ay^2} is the covariance of $\tilde{a}$ and $\tilde{y}^2$). The expected system profit associated with farmer i is

$$\Psi_i(x_i) = E\Big[t_i\big((1+m)R_O(x_i) - (c_{1i} + c_{2i}x_i)\big)\Big] = t_i\Big[(1+m)q(x_i)\mu_{Z_O} - (c_{1i} + c_{2i}x_i)\Big]. \tag{47}$$

It is straightforward to verify that the profit function is concave. Solving the first-order condition for x_i yields (9).

Derivation of Optimal Farmer Profit Under OMPP

Note that

$$\pi_{Oi}(x_i) = t_i\big[q(x_i)\mu_{Z_O} - (c_{1i} + c_{2i}x_i)\big]$$

$$\pi_{Oi}(\alpha, x_i) = t_i\big[q(x_i)z_O(\alpha) - (c_{1i} + c_{2i}x_i)\big] \tag{48}$$

$$\Pr\big(\tilde{\pi}_{Oi}(x_i) \le \pi_{Oi}(\alpha, x_i)\big) = \Pr\Big(t_i\big[q(x_i)Z_O - (c_{1i} + c_{2i}x_i)\big] \le t_i\big[q(x_i)z_O(\alpha) - (c_{1i} + c_{2i}x_i)\big]\Big)$$

$$= \Pr\big(Z_O \le z_O(\alpha)\big) = \alpha.$$

Due to the strict concavity of $q(x_i)$, there are at most two values of x_i that satisfy (10) at equality.

$$x_{Oi}^- = \inf_{x_i}\big\{x_i : q(x_i)z_O(\alpha) - (c_{1i} + c_{2i}x_i) \ge -\beta_i\big\}$$

$$x_{Oi}^+ = \sup_{x_i}\big\{x_i : q(x_i)z_O(\alpha) - (c_{1i} + c_{2i}x_i) \ge -\beta_i\big\}.$$

Note that $\{x_i : q(x_i) z_O(\alpha) - (c_{1i} + c_{2i} x_i) \geq -\beta_i\} \neq \emptyset$; if this was not the case, then the farmer i would not be in the business (i.e., no value of x_i satisfies the VaR constraint). With the above notation and functions, we can rewrite the farmer's decision problem as

$$\max_{x_i \in [x_{Oi}^-, x_{Oi}^+]} q(x_i) \mu_{Z_O} - (c_{1i} + c_{2i} x_i). \tag{49}$$

The optimal investment in quality improvement decision is

$$x_{Oi}^* = \max\{x_{Oi}^-, \min\{x_{Oi}^+, x_{Oi}^{\circ}\}\} \text{ where} \tag{50}$$

$$x_{Oi}^{\circ} = q'^{-1}\left(\frac{c_{2i}}{\mu_{Z_O}}\right) \tag{51}$$

is the optimal solution to the farmer's problem when the VaR constraint (7) is ignored (i.e., first-order condition is $q'(x_i)\mu_{Z_O} - c_{2i} = 0$). From (50) and the risk-neutral optimal solution x_{Oi}°, we see that the risk-averse optimal effort to increase or decrease relative to a risk-neutral decision (follows from the max and min operators that appear in (50)).

Lemma A1. *There exists a unique solution to* $\min_{k \geq 0}\left\{k : k \geq E\left(k - \bar{\lambda}_i \tilde{\Psi}_{Si}^* - t_i \beta_i\right)^+\right\}$.

Proof. Let $X_i = \bar{\lambda}_i \tilde{\Psi}_{Si}^* + t_i \beta_i$. Define $g(k) = E\left(k - \bar{\lambda}_i \tilde{\Psi}_{Si}^* - t_i \beta_i\right)^+ = E\left(k - X_i\right)^+$. Note that $g'(k) = \Pr(X_i \leq k) \in [0, 1]$. If $g(0) = 0$, then it is clear that $\min_{k \geq 0}\left\{k : k \geq E\left(k - \bar{\lambda}_i \tilde{\Psi}_{Si}^* - t_i \beta_i\right)^+\right\} = 0$. Now suppose that $g(0) > 0$. Let $h(k) = k - g(k)$. Note that

$h(0) = -g(0) < 0$

$h'(k) = 1 - g'(k) = \Pr(X_i > k) \geq 0$ for $k \in [0, \infty)$

$h(\infty) = E[X_i] = \pi_{Oi}(x_{Oi}^*) + t_i \beta_i = \pi_{Oi}(x_{Oi}^*) + t_i \beta_i \geq 0$.

The last inequality holds because, by definition, decision x_{Oi}^* satisfies the VaR constraint. That is,

$\pi_{Oi}(x_{Oi}^*) + t_i \beta_i < 0$

$\Rightarrow t_i \beta_i < -\pi_{Oi}(x_{Oi}^*) = -t_i\left[q(x_{Oi}^*)\mu_{Z_O} - (c_{1i} + c_{2i} x_{Oi}^*)\right] < -t_i\left[q(x_{Oi}^*) z_O(\alpha) - (c_{1i} + c_{2i} x_{Oi}^*)\right]$

(see (11)), which violates the VaR constraint. Therefore, $h(k)$ is monotonic increasing on $[0, \infty)$ with endpoints that span the origin, which implies that $h(k)$ crosses zero once as k increases from 0 to ∞. □

Proof of Proposition 1. We first prove the result in (12). Recall that x_{Oi}° is the farmer investment in quality that maximizes expected profit, i.e., $q'(x_{Oi}^{\circ}) = \frac{c_{2i}}{\mu_{Z_O}}$ (see explanation following (51)). The lower bound on quality investment is zero, and we assume without loss of generality that this bound is set low enough that negative investment in quality is never economically desirable (e.g., x_i can be interpreted as the increase in quality investment above the lowest viable level among the population of member farmers), i.e., $x_{Oi}^{\circ} \geq 0$. The result clearly holds if $x_{Oi}^- \leq 0$, i.e., $x_{Oi}^- < x_{Oi}^{\circ}$ and $x_{Oi}^- \leq x_{Oi}^+$ (by definition) and thus $\max\{x_{Oi}^-, \min\{x_{Oi}^+, x_{Oi}^{\circ}\}\} = \min\{x_{Oi}^+, x_{Oi}^{\circ}\}$. Similarly, the result holds if $x_{Oi}^- = x_{Oi}^+$.

Suppose that $0 < x_{Oi}^- < x_{Oi}^+$, which is the only remaining possibility to consider. The values x_{Oi}^- and x_{Oi}^+ are the points at which the following two curves intersect (due to continuity of $q(x_i)$):

$\text{LHS}(x_i) = q(x_i) z_O(\alpha) = c_{2i} x_i + c_{1i} - \beta_i = \text{RHS}(x_i)$.

From the concavity of $q(x_i)$, it follows that LHS(x_i) crosses RHS(x_i) from below to above at $x_i = x_{Oi}^-$ (and if x_{Oi}^+ is finite so that the curves cross again, then LHS(x_i) crosses RHS(x_i) from above to below at $x_i = x_{Oi}^+$). Thus, $q'\left(x_{Oi}^-\right) z_O\left(\alpha\right) \ge c_{2i}$

$$\Rightarrow \quad q'\left(x_{Oi}^-\right) > \frac{c_{2i}}{z_O\left(\alpha\right)}$$

$$> \frac{c_{2i}}{\mu_{Z_O}} \qquad \text{(due to (11))}$$

$$= q'\left(x_{Oi}^{\text{o}}\right)$$

$$\Rightarrow \quad x_{Oi}^- < x_{Oi}^{\text{o}} \qquad \text{(due to the concavity of } q(x_i)\text{)}.$$

Therefore, $x_{Oi}^* = \min\left\{x_{Oi}^+, x_{Oi}^{\text{o}}\right\}$, which is (12). The results in (13) follow from the concavity of $q(x_i)$ and the expressions for x_{Oi}^* and x_{Si}^*. □

Proof of Proposition 2. We first outline the main steps of the proof. We first show that, in the event that the farmer's VaR constraint is not binding, the farmer's quality effort matches system optimal x_{Si}^*. Next, we show that the expected value of the terms in the payment policy function that do not involve $\bar{\lambda}_i$ is equal to zero. This facilitates our next step that is to show that the farmer's expected profit at system-optimal effort is identical to π_{Oi}^*, thereby satisfying constraint (14). Then we show that the farmer's VaR constraint is nonbinding at x_{Si}^*. The above arguments imply that the farmer's optimal expected profit under payment policy $\Gamma_i^*\left(p_M, a_r, y\right)$ is identical to expected profit under OMPP. Furthermore, system profit is maximized so that all of the increase in system profit due to the change in payment policy goes to the cooperative. Thus, there exists a feasible solution to P if and only if (17) holds.

Under policy Γ_i^*, the price paid to the farmer as a function effort is

$$p_{\Gamma_i^*}\left(x_i\right) = \Gamma_i^*\left(p_M, q\left(x_i\right)a, y\right) = \bar{\lambda}_i\left(1+m\right)p_M a q\left(x_i\right) + \left[\left(1-\bar{\lambda}_i\right)\left(c_{1i}+c_{2i}x_i\right) - A\right]/\,y,$$

where A depends on realized optimal system profit, but not on x_i, i.e., $A \in \left\{\bar{k}_i, \bar{\lambda}_i \Psi_{Si}^{**} - t_i\beta_i\right\}$. The farmer's expected profit is

$$\pi_{\Gamma_i^*}\left(x_i\right) = E\left[\bar{\lambda}_i t_i\left(\left(1+m\right)q\left(x_i\right)Z_O - \left(c_{1i}+c_{2i}x_i\right)\right) - A\right] = \bar{\lambda}_i \Psi_i\left(x_i\right) - E\left[A\right],$$

which is proportional to system profit adjusted by a constant. Therefore $\arg\max_{x_i} \pi_{\Gamma_i^*}\left(x_i\right) = x_{Si}^*$.

Suppose that $E\left[\left(\left(-\bar{\lambda}_i\tilde{\Psi}_{Si}^*\right) - t_i\beta_i\right)^+\right] = 0$, i.e., the random loss $-\bar{\lambda}_i\tilde{\Psi}_{Si}^*$ is never more than $t_i\beta_i$. Then $\Pr\left(-\bar{\lambda}_i\tilde{\Psi}_{Si}^* > t_i\beta_i\right) = 0$ and $\bar{k}_i = 0$, which implies $A = 0$. Now suppose that $E\left[\left(\left(-\bar{\lambda}_i\tilde{\Psi}_{Si}^*\right) - t_i\beta_i\right)^+\right] > 0$. In this case, letting $X_i = -\bar{\lambda}_i\tilde{\Psi}_{Si}^* - t_i\beta_i$, we have $\bar{k}_i = E\left[\left(\bar{k}_i + X_i\right)^+\right]$ and

$$E\left[A\right] = \bar{k}_i \Pr\left(\bar{k}_i + X_i \le 0\right) + \left(\bar{k}_i - E\left[\bar{k}_i + X_i \mid \bar{k}_i + X_i > 0\right]\right)\Pr\left(\bar{k}_i + X_i > 0\right) = \bar{k}_i - E\left[\left(\bar{k}_i + X_i\right)^+\right] = 0.$$

Therefore, for any value of $E\left[\left(\left(-\bar{\lambda}_i\tilde{\Psi}_{Si}^*\right) - t_i\beta_i\right)^+\right]$, we have

$$\pi_{\Gamma_i^*}\left(x_{Si}^*\right) = \bar{\lambda}_i\Psi_i\left(x_{Si}^*\right) - E\left[A\right] = \bar{\lambda}_i\Psi_i\left(x_{Si}^*\right) = \pi_{Oi}\left(x_{Oi}^*\right),$$

which implies that constraint (14) is nonbinding at effort x_{Si}^*.

Now consider the VaR constraint in (16). Suppose that $E\left[\left(-\bar{\lambda}_i\tilde{\Psi}^*_{Si}-t_i\beta_i\right)^+\right]=0$. Then, as noted above, $k_i = 0$ and the farmer's random profit at effort x^*_{Si} is

$$\tilde{\pi}_{\Gamma^*_i}\left(x^*_{Si}\right)=\bar{\lambda}_i t_i\left(\left(1+m\right)q\left(x^*_{Si}\right)Z_O-\left(c_{1i}+c_{2i}x^*_{Si}\right)\right)=\bar{\lambda}_i\tilde{\Psi}^*_{Si}.$$

Therefore, $0=E\left[\left(-\bar{\lambda}_i\tilde{\Psi}^*_{Si}-t_i\beta_i\right)^+\right]=E\left[\left(-\tilde{\pi}_{\Gamma^*_i}\left(x^*_{Si}\right)-t_i\beta_i\right)^+\right]$, which implies $\Pr\left(-\tilde{\pi}_{\Gamma^*_i}\left(x^*_{Si}\right)>t_i\beta_i\right)=0$, i.e., the VaR constraint is nonbinding at effort x^*_{Si}. Now suppose that $E\left[\left(-\bar{\lambda}_i\tilde{\Psi}^*_{Si}-t_i\beta_i\right)^+\right]>0$. Then $\bar{k}_i>0$ and from the definition of policy Γ^*_i,

$$\begin{aligned}\tilde{\pi}_{\Gamma^*_i}\left(x^*_{Si}\right)&=\begin{cases}\bar{\lambda}_i t_i\left(\left(1+m\right)q\left(x^*_{Si}\right)Z_O-\left(c_{1i}+c_{2i}x^*_{Si}\right)\right)-\bar{k}_i, & \text{if } -\left(\bar{\lambda}_i\tilde{\Psi}^*_{Si}-\bar{k}_i\right)\le t_i\beta_i\\ \bar{\lambda}_i t_i\left(\left(1+m\right)q\left(x^*_{Si}\right)Z_O-\left(c_{1i}+c_{2i}x^*_{Si}\right)\right)-\bar{\lambda}_i\tilde{\Psi}^*_{Si}-t_i\beta_i, & \text{if } -\left(\bar{\lambda}_i\tilde{\Psi}^*_{Si}-\bar{k}_i\right)>t_i\beta_i\end{cases}\\ &=\begin{cases}\bar{\lambda}_i\tilde{\Psi}^*_{Si}-\bar{k}_i, & \text{if } -\left(\bar{\lambda}_i\tilde{\Psi}^*_{Si}-\bar{k}_i\right)\le t_i\beta_i\\ \bar{\lambda}_i\tilde{\Psi}^*_{Si}-\bar{\lambda}_i\tilde{\Psi}^*_{Si}-t_i\beta_i, & \text{if } -\left(\bar{\lambda}_i\tilde{\Psi}^*_{Si}-\bar{k}_i\right)>t_i\beta_i\end{cases}\\ &\ge -t_i\beta_i\end{aligned}$$

i.e., the VaR constraint is nonbinding.

Through the above arguments we have shown that, under policy Γ^*_i, the farmer's optimal effort is x^*_{Si} and that the farmer's optimal expected profit is $\pi_{\Gamma^*_i}\left(x^*_{Si}\right)=\pi_{Oi}\left(x^*_{Oi}\right)$. Therefore, the cooperative's expected profit is

$$\Pi_{\Gamma^*}=\sum_{i=1}^{N}\Pi_{\Gamma^*_i}\left(x^*_{\Gamma^*_i}\right)=\Psi^*_S-\sum_{i=1}^{N}\pi_{\Gamma^*_i}\left(x^*_{\Gamma^*_i}\right)=\Psi^*_S-\sum_{i=1}^{N}\pi_{Oi}\left(x^*_{Oi}\right)=\Pi^*_O+\Psi^*_S-\Psi^*_O,$$

which is the maximum possible subject to the farmers' individual rationality constraints (16). Thus, there exists a feasible solution to P if and only (17) holds. □

Proof of Proposition 3. Substituting $\Gamma_{Ri}\left(\tilde{p}_M,q\left(x_i\right)\tilde{a},\tilde{y}\right)$ into $\tilde{\pi}_{\Gamma_i}\left(x_i\right)$ yields (20) – (22).

If farmers are risk neutral, then k_i in payment policy Γ^*_i is equal to zero for all i (see Proposition 2). Thus, RSPP is an example of policy Γ^*_i. Therefore, result (17) in Proposition 2 applies, yielding (23). □

Proof of Proposition 4. At system-optimal effort, the farmer's VaR constraint under RSPP is

$$\alpha\ge\Pr\left(\tilde{\pi}_{Ri}\left(x^*_{Si}\right)\le -t_i\beta_i\right)=\Pr\left(\lambda_i\tilde{\Psi}^*_{Si}\le -t_i\beta_i\right),$$

which holds if and only if $\lambda_i\Psi^*_{Si}\left(\alpha\right)\ge -t_i\beta_i$, which can be rewritten as (24).

At effort x^*_{Si}, constraint (14) can be expressed as $\lambda_i\ge\pi^*_{Oi}/\Psi^*_{Si}$. If $\lambda_i\le 1-\left(\Pi_{\min}/\Pi^*_O\right)\left(\Pi^*_{Oi}/\Psi^*_{Si}\right)$ for all i, then $\Pi^*_R=\sum_{i=1}^{N}\Pi_{Ri}\left(x^*_{Si}\right)=\sum_{i=1}^{N}\left(1-\lambda_i\right)\Psi^*_{Si}\ge\left(\Pi_{\min}/\Pi^*_O\right)\sum_{i=1}^{N}\Pi^*_{Oi}=\Pi_{\min}$, and thus constraint (15) is satisfied. Combining the above two inequalities on λ_i with (24), we see that there exists are feasible value of λ_i if $\max\left\{-t_i\beta_i/\Psi^*_{Si}\left(\alpha\right),\pi^*_{Oi}/\Psi^*_{Si}\right\}\le 1-\Pi^*_{Oi}/\Psi^*_{Si}$. If the inequality holds for all i, then λ_i satisfying (25) yields system-optimal effort by all farmers. □

Proof of Proposition 5. Substituting $\Gamma_{Bi}\left(\tilde{p}_M,q\left(x_i\right)\tilde{a},\tilde{y}\right)$ into $\tilde{\pi}_{\Gamma_i}\left(x_i\right)$ yields (27) – (29).

Note that $\pi_{Bi}\left(x_i\right)=\Psi_i\left(x_i\right)-t_ik_i$. Therefore, $\pi_{Bi}'\left(x_i\right)=\Psi_i'\left(x_i\right)$, which implies that risk-neutral farmers select system-optimal effort if individual rationality constraints (see (14)) are satisfied. Satisfying

(14) at equality yields $k_i = \left(\Psi^*_{Si} - \pi^*_{Oi}\right)/t_i$ and $\Pi^*_B = \max_{k_i \le \left(\Psi^*_{Si} - \pi^*_{Oi}\right)/t_i} \sum_{i=1}^{N} \Pi_{Bi}\left(x^*_{Si}\right) = \Pi^*_O + \Psi^*_S - \Psi^*_O$. Therefore, system-optimal effort by all risk-neutral farmers under BMPP is feasible if and only if (30) holds. □

Proof of Proposition 6. At system-optimal effort, the farmer's VaR constraint under BMPP is

$$\alpha \ge \Pr\left(\tilde{\pi}_{Bi}\left(x^*_{Si}\right) \le -t_i\beta_i\right) = \Pr\left(\tilde{\Psi}^*_{Si} - t_i k_i \le -t_i\beta_i\right),$$

which holds if and only if $\Psi^*_{Si}(\alpha) - t_i k_i \ge -t_i\beta_i$, which can be rewritten as (31).

At effort x^*_{Si}, constraint (14) can be expressed as $k_i \le \left(\Psi^*_{Si} - \pi^*_{Oi}\right)/t_i$. If $k_i \ge \left(\Pi_{\min}/\Psi^*_S\right)\left(\Psi^*_{Si}/t_i\right)$ for all i, then $\Pi^*_B = \sum_{i=1}^{N} \Pi_{Bi}\left(x^*_{Si}\right) = \sum_{i=1}^{N} t_i k_i \ge \left(\Pi_{\min}/\Psi^*_S\right)\sum_{i=1}^{N}\Psi^*_{Si} = \Pi_{\min}$, and thus constraint (15) is satisfied. Combining the above two inequalities on k_i with (31), we see that there exists are feasible value of k_i if $\left(\Pi_{\min}/\Psi^*_S\right)\left(\Psi^*_{Si}/t_i\right) \le \min\left\{\beta_i - \Psi^*_{Si}(\alpha)/t_i, \left(\Psi^*_{Si} - \pi^*_{Oi}\right)/t_i\right\}$. If the inequality holds for all i, then k_i satisfying (25) yields system-optimal effort by all farmers. □

Proof of Proposition 7. Note that

$$\pi_{Ri}\left(x^*_{Si}\right) = \lambda_i\Psi^*_{Si} = \Psi^*_{Si} - \gamma\Pi^*_{Oi} = \Psi^*_{Oi} - \Pi^*_{Oi} + \Psi^*_{Si} - \Psi^*_{Oi} - (\gamma - 1)\Pi^*_{Oi} = \pi^*_{Oi} + \Psi^*_{Si} - \Psi^*_{Oi} - (\gamma - 1)\Pi^*_{Oi}$$

$$\pi_{Bi}\left(x^*_{Si}\right) = \Psi^*_{Si} - t_i k_i = \Psi^*_{Si} - \gamma\Pi^*_{Oi} = \pi^*_{Oi} + \Psi^*_{Si} - \Psi^*_{Oi} - (\gamma - 1)\Pi^*_{Oi}$$

and thus (34) and (35) satisfy (33). The farmer's VaR under each policy is

$$t_i\beta_{Ri}\left(x^*_{Si}\right) = -\pi_{Ri}\left(\alpha, x^*_{Si}\right) = -\lambda_i\Psi_i\left(\alpha, x^*_{Si}\right) = -\Psi_i\left(\alpha, x^*_{Si}\right) + \left(\frac{\Psi_i\left(\alpha, x^*_{Si}\right)}{\Psi^*_{Si}}\right)\gamma\Pi^*_{Oi}$$

$$t_i\beta_{Bi}\left(x^*_{Si}\right) = -\pi_{Bi}\left(\alpha, x^*_{Si}\right) = -\Psi_i\left(\alpha, x^*_{Si}\right) + \gamma\Pi^*_{Oi} > t_i\beta_{Ri}\left(x^*_{Si}\right)$$

where the inequality follows from (11), i.e., $z_O(\alpha) < \mu_{Z_O}$ implies

$$\frac{\Psi_i\left(\alpha, x^*_{Si}\right)}{\Psi^*_{Si}} = \frac{(1+m)q\left(x^*_{Si}\right)z_O(\alpha) - \left(c_{1i} + c_{2i}x^*_{Si}\right)}{(1+m)q\left(x^*_{Si}\right)\mu_{Z_O} - \left(c_{1i} + c_{2i}x^*_{Si}\right)} < 1. \square$$

Proof of Corollary 1. The inequalities related to OMPP are given in Proposition 1. The inequalities related to RSPP and BMPP follow from propositions 3 and 5, and the fact that the VaR constraints are not binding when insurance is available. □

Proof of Proposition 8. Part (*a*). A necessary condition for a uniform payment schedule under RSPP/I is $\lambda_i = \lambda$ for all i. Substituting into (19), the payment schedule that incentivizes system-optimal effort by farmer i becomes $\Gamma_{Ri}\left(p_M, a_r, y\right) = \lambda(1+m)p_M a_r + (1-\lambda)\left(c_{1i} + c_{2i}x_i\right)/y$. Note that $\lambda < 1$ is required (due to $\Pi_{\min} > 0$), and thus the price paid depends upon the farmer's cost, which varies by farmer. Therefore, a uniform RSPP/I payment schedule that incentives system-optimal effort by all farmers does not exist.

Part (*b*). For any $k \ge \Pi_{\min}/T$, the cooperative profit is $\Pi^*_B = \sum_{i=1}^{N}\Pi^*_{Bi} = \sum_{i=1}^{N} t_i k = Tk \ge \Pi_{\min}$, i.e., the cooperative's profit constraint (15) is satisfied. Furthermore, for $k \le \left(\Psi^*_{Si} - \pi^*_{Oi}\right)/t_i$, the farmer's profit under system-optimal effort (assured by BMPP/I) is $\pi_{Bi}\left(x^*_{Si}\right) = \Psi^*_{Si} - t_i k \ge \pi^*_{Oi}$, i.e., the farmer's profit constraint (14) is satisfied. Furthermore, constraint (14) is satisfied for all farmers if $k \le \min_i\left\{\left(\Psi^*_{Si} - \pi^*_{Oi}\right)/t_i\right\}$. □

Proof of Proposition 9. Note that $q'(x_i) = \dfrac{1}{2x_i^{1/2}}$, and thus

$$q'\left(x_{Oi}^{o}\right)=\frac{1}{2x_{Oi}^{o\ 1/2}}=\frac{c_{2i}}{\mu_{Z_O}}\Rightarrow\ x_{Oi}^{o}=\left(\frac{\mu_{Z_O}}{2c_{2i}}\right)^2$$

$$q'\left(x_{Si}^{*}\right)=\frac{1}{2x_{Si}^{*\ 1/2}}=\frac{c_{2i}}{(1+m)\mu_{Z_O}}\Rightarrow\ x_{Si}^{*}=\left(\frac{(1+m)\mu_{Z_O}}{2c_{2i}}\right)^2.$$

The remaining results can be obtained through algebra (we omit the details). □

Proof of Proposition 10. Let

$$y=\frac{\left((1+m)\mu_{Z_O}\right)^2}{4c_{2i}\left[(1+m)\mu_{Z_O}-c_{1i}\right]}>0 \text{ and } z=\frac{1+2m}{(1+m)^2}\in(0,\,1).$$

Then

$$\frac{\Psi_i\left(x_{Si}^{*}\right)}{\Psi_i\left(x_{Oi}^{o}\right)}=\frac{1+y}{1+yz}$$

$$\frac{\partial y}{\partial c_{1i}}=\frac{1}{4c_{2i}}\left(\frac{(1+m)\mu_{Z_O}}{\left[(1+m)\mu_{Z_O}-c_{1i}\right]}\right)^2>0$$

$$\frac{\partial y}{\partial c_{2i}}=-\frac{\left((1+m)\mu_{Z_O}\right)^2}{4c_{2i}^{\ 2}\left[(1+m)\mu_{Z_O}-c_{1i}\right]}<0$$

$$\frac{\partial y}{\partial \mu_{Z_O}}=\left(\frac{4c_{2i}(1+m)^2\mu_{Z_O}}{\left(4c_{2i}\left[(1+m)\mu_{Z_O}-c_{1i}\right]\right)^2}\right)\left[(1+m)\mu_{Z_O}-2c_{1i}\right]\geq 0\Leftrightarrow\frac{\mu_{Z_O}}{c_{1i}}\geq\frac{2}{1+m}.$$

Therefore,

$$\frac{\partial}{\partial c_{1i}}\left[\frac{\Psi_i\left(x_{Si}^{*}\right)}{\Psi_i\left(x_{Oi}^{o}\right)}\right]=\frac{\partial}{\partial y}\left[\frac{\Psi_i\left(x_{Si}^{*}\right)}{\Psi_i\left(x_{Oi}^{o}\right)}\right]\frac{\partial y}{\partial c_{1i}}>0$$

$$\frac{\partial}{\partial c_{2i}}\left[\frac{\Psi_i\left(x_{Si}^{*}\right)}{\Psi_i\left(x_{Oi}^{o}\right)}\right]=\frac{\partial}{\partial y}\left[\frac{\Psi_i\left(x_{Si}^{*}\right)}{\Psi_i\left(x_{Oi}^{o}\right)}\right]\frac{\partial y}{\partial c_{2i}}<0$$

$$\frac{\partial}{\partial \mu_{Z_O}}\left[\frac{\Psi_i\left(x_{Si}^{*}\right)}{\Psi_i\left(x_{Oi}^{o}\right)}\right]=\frac{\partial}{\partial y}\left[\frac{\Psi_i\left(x_{Si}^{*}\right)}{\Psi_i\left(x_{Oi}^{o}\right)}\right]\frac{\partial y}{\partial \mu_{Z_O}}\geq 0\Leftrightarrow\frac{\mu_{Z_O}}{c_{1i}}\geq\frac{2}{1+m}.$$

For the sign of $\frac{\partial}{\partial m}\left[\frac{\Psi_i\left(x_{Si}^{*}\right)}{\Psi_i\left(x_{Oi}^{o}\right)}\right]$, note that

$$\frac{\partial y}{\partial m}=\frac{4c_{2i}(1+m)\mu_{Z_O}^{\ 2}}{\left(4c_{2i}\left[(1+m)\mu_{Z_O}-c_{1i}\right]\right)^2}\left[(1+m)\mu_{Z_O}-2c_{1i}\right]\geq 0\Leftrightarrow\frac{\mu_{Z_O}}{c_{1i}}\geq\frac{2}{1+m}$$

$$\frac{\partial yz}{\partial m}=\frac{\partial}{\partial m}\left(\frac{(1+2m)\mu_{Z_O}^{\ 2}}{4c_{2i}\left[(1+m)\mu_{Z_O}-c_{1i}\right]}\right)=\left(\frac{4c_{2i}\mu_{Z_O}^{\ 2}}{\left(4c_{2i}\left[(1+m)\mu_{Z_O}-c_{1i}\right]\right)^2}\right)\left(\mu_{Z_O}-2c_{1i}\right)\geq 0\Leftrightarrow\frac{\mu_{Z_O}}{c_{1i}}\geq 2$$

$$\frac{\partial yz}{\partial y}=\frac{\partial yz/\partial m}{\partial y/\partial m}=\frac{1}{1+m}\left(\frac{\mu_{Z_O}-2c_{1i}}{(1+m)\mu_{Z_O}-2c_{1i}}\right)<0\ \Leftrightarrow\frac{\mu_{Z_O}}{c_{1i}}\in\left(\frac{2}{1+m},2\right)$$

$$\frac{\partial}{\partial m}\left[\frac{\Psi_i\left(x_{Si}^*\right)}{\Psi_i\left(x_{Oi}^{o}\right)}\right]=\frac{1}{1+yz}\frac{\partial y}{\partial m}-\frac{1+y}{\left(1+yz\right)^2}\frac{\partial yz}{\partial m}=\left(\frac{1}{1+y}\right)\left(\frac{\Psi_i\left(x_{Si}^*\right)}{\Psi_i\left(x_{Oi}^{o}\right)}\right)\left[1-\left(\frac{\Psi_i\left(x_{Si}^*\right)}{\Psi_i\left(x_{Oi}^{o}\right)}\right)\frac{\partial yz}{\partial y}\right]\frac{\partial y}{\partial m}.$$

Suppose that $\frac{\mu_{Z_O}}{c_{1i}} \geq 2$. Note that

$$\frac{\Psi_i\left(x_{Si}^*\right)}{\Psi_i\left(x_{Oi}^{o}\right)}=\left(\frac{1+y}{1/z+y}\right)\frac{1}{z}<\frac{1}{z} \qquad \text{(due to } z<1\text{)}$$

$$=1+\frac{m^2}{1+2m}\leq 1+m$$

$$\frac{\partial yz}{\partial y}=\frac{1}{1+m}\left(\frac{\mu_{Z_O}-2c_{1i}}{\left(1+m\right)\mu_{Z_O}-2c_{1i}}\right)\leq\frac{1}{1+m}$$

and thus

$$\left(\frac{\Psi_i\left(x_{Si}^*\right)}{\Psi_i\left(x_{Oi}^{o}\right)}\right)\frac{\partial yz}{\partial y}\leq 1 \text{ which implies } \frac{\partial}{\partial m}\left[\frac{\Psi_i\left(x_{Si}^*\right)}{\Psi_i\left(x_{Oi}^{o}\right)}\right]\geq 0.$$

Now, suppose that $\frac{\mu_{Z_O}}{c_{1i}}\in\left[\frac{2}{1+m},2\right]$. Then $\frac{\partial y}{\partial m}\geq 0$ and $\frac{\partial yz}{\partial y}\leq 0$, and $\frac{\partial}{\partial m}\left[\frac{\Psi_i\left(x_{Si}^*\right)}{\Psi_i\left(x_{Oi}^{o}\right)}\right]\geq 0$. □

Robustness of Results to Alternative Revenue Models and Risk Measures

Our analysis in the previous sections leads to three main conclusions:

1. Under the current policy, farmers under-invest in quality compared to system optimal.
2. Under the current policy, a risk-averse farmer invests less (or the same) in quality compared to a risk-neutral farmer.
3. The use of BMPP with insurance leads to system-optimal investment in quality by farmers with heterogeneous cost structures. And, if all farmers share the same cost structure, then RSPP with insurance leads to system-optimal investment in quality by farmers.

We obtain these three conclusions by using a model of random revenue and a measure of risk that reflects the setting motivating this work. We show that our conclusions are robust over a broad family of random revenue models that exhibit separability and a set of risk measures that include any coherent risk measure in addition to VaR.

Let $\tilde{r}_O\left(x_i\right)$ denote the random open-market price per input unit given farmer effort x_i. Consider the following general structure for effort-dependent random open-market price per input unit:

$$\tilde{r}_O\left(x_i\right)=q_1\left(x_i\right)+q_2\left(x_i\right)g\left(\tilde{a},\tilde{\varepsilon},\tilde{y}\right) \tag{52}$$

where q_1 and q_2 are increasing concave functions, and g is any function of the three random primitives for farmer crop quality ($\tilde{a}$), yield in the region ($\tilde{y}$), and remaining unobservable factors that affect the open-market price ($\tilde{\varepsilon}$).We emphasize that function g places no assumptions on the relationship between the open-market price and regional yield (e.g., a nonlinear relationship is possible), and more generally, no assumptions on how the three random primitives interact to affect open-market price. In addition, the effect of effort on the price of oil could be additive (e.g., $q_2(x_i) = 1$), multiplicative (e.g., $q_1(x_i) = 0$), or any combination of additive / multiplicative effects.

Proposition 11 states that the above three conclusions hold under general random revenue function (52) with either VaR as the risk measure or any coherent risk measure. Our formulation of the farmer's decision problem in Section 3 places a constraint on the maximum value of the risk measure. We note that the following result applies to cases where risk is a constraint and where the risk measure is included in the objective function (e.g., a model with a constraint on the risk measure is equivalent to optimizing the unconstrained Lagrangian with the risk measure in the objective function). The proposition relies on the following notation: $Z = g(\tilde{a}, \tilde{\varepsilon}, \tilde{y})$, z_α is the α-fractile of Z (i.e., $P[Z \le z_\alpha] = \alpha$), and $\mu_Z = E[Z]$.

Proposition 11. (*a*) *The farmer's measure of risk is VaR. If*

$$z_\alpha \le \mu_Z, \text{ then} \tag{53}$$

$$x^*_{Oi} \le x^{\circ}_{Oi} \tag{54}$$

$$x^*_{B/Ii} = x^*_{R/Ii} = x^*_{Si} > x^*_{O/Ii} = x^{\circ}_{Oi} \ge x^*_{Oi}\,;\ \Psi^*_{B/I} = \Psi^*_{R/I} = \Psi^*_S > \Psi^*_{O/I} \ge \Psi^*_O. \tag{55}$$

(*b*) *The farmer's measure of risk is any coherent risk measure* (*i.e., satisfies the properties of monotonicity, sub-additivity, positive homogeneity, and translation invariance; see Artzner et al. 1999*)*. Then* (54) *and* (55) *hold*.

Proof of Proposition 11. Part (*a*). We begin by showing that, if the VaR constraint is binding under OMPP, then the VaR constraint is not satisfied for any $x_i > x^{\circ}_{Oi}$. This result implies that (54) must hold if there exists a feasible solution. Suppose that the VaR constraint is binding, i.e.,

$$q_1(x^{\circ}_{Oi}) + q_2(x^{\circ}_{Oi}) z_\alpha - c_{1i} - c_{2i} x^{\circ}_{Oi} < -\beta. \tag{56}$$

Note that

$$q_1'(x^{\circ}_{Oi}) + q_2'(x^{\circ}_{Oi}) \mu_Z - c_{2i} = 0 \tag{57}$$

(from the FOC). Therefore, for any $x_i > x^{\circ}_{Oi}$,

$$\begin{aligned} q_1(x_i) + q_2(x_i) z_\alpha - c_{1i} - c_{2i} x_i &= q_1(x^{\circ}_{Oi}) + q_2(x^{\circ}_{Oi}) z_\alpha - c_{1i} - c_{2i} x^{\circ}_{Oi} + \int_{x^{\circ}_{Oi}}^{x_i} \left(q_1'(t) + q_2'(t) z_\alpha - c_{2i} \right) dt \\ &\le q_1(x^{\circ}_{Oi}) + q_2(x^{\circ}_{Oi}) z_\alpha - c_{1i} - c_{2i} x^{\circ}_{Oi} + \int_{x^{\circ}_{Oi}}^{x_i} \left(q_1'(t) + q_2'(t) \mu_Z - c_{2i} \right) dt \\ &\qquad \text{(from } z_\alpha \le \mu_Z\text{)} \\ &\le q_1(x^{\circ}_{Oi}) + q_2(x^{\circ}_{Oi}) z_\alpha - c_{1i} - c_{2i} x^{\circ}_{Oi} \qquad \text{(from (57) and concave } q_1, q_2\text{)} \\ &< -\beta_i \qquad \text{(from (56))} \end{aligned}$$

Therefore (54) holds.

The system-optimal effort x^*_{Si} satisfies the FOC

$$(1+m)\left(q_1'(x_i) + q_2'(x_i) E\left[g(\tilde{a}, \tilde{\varepsilon}, \tilde{y})\right]\right) - c_{2i} = 0.$$

If the VaR constraint is nonbinding under BMPP and RSPP (e.g., through insurance), then the farmer selects effort to maximize expected profit.

$$E\left[\tilde{\pi}_B(x_i)\right] = t_i\left[q_1(x_i) + q_2(x_i) E\left[g(\tilde{a}, \tilde{\varepsilon}, \tilde{y})\right] - c_{1i} - c_{2i} x_i - k\right]$$

$$E\left[\tilde{\pi}_R(x_i)\right] = \lambda t_i\left[(1+m)\left(q_1(x_i) + q_2(x_i) E\left[g(\tilde{a}, \tilde{\varepsilon}, \tilde{y})\right]\right) - c_{1i} - c_{2i} x_i\right],$$

The above farmer profit functions are concave and yield the same FOC as the system-optimal problem.

Part (*b*). If the farmer's optimal effort is x°_{Oi} (i.e., the farmer's risk measure does not result in an optimal decision that differs from maximizing expected profit), then (54) and (55) hold (because $x^{\circ}_{Oi} = x^*_{Oi}$). Similarly, (54) and (55) hold if $x^*_{Oi} \le x^{\circ}_{Oi}$.

Now suppose that $x^*_{Oi} > x^{\circ}_{Oi}$. Let $\rho(\cdot)$ be a coherent risk measure. Then for random payoff Z, we have

$$\rho(\lambda Z) = \lambda \rho(Z) \text{ for any } \lambda \ge 0 \quad \text{(positive homogeneity)} \tag{58}$$

$$\rho(\lambda + Z) = \rho(Z) - \lambda \text{ for any } \lambda \quad \text{(translation invariance).} \tag{59}$$

From (58) and (59) it follows that

$$\rho\left(\tilde{\pi}_O(x_i)\right) = \rho\left(t_i\left[q_1(x_i) + q_2(x_i) Z - c_{1i} - c_{2i} x_i\right]\right) = t_i\left[q_2(x)\rho(Z) - \left(q_1(x_i) - c_{1i} - c_{2i} x_i\right)\right].$$

Note that

$$q_2\left(x_{Oi}^{o}\right)\mu_Z + q_1\left(x_{Oi}^{o}\right) - c_{1i} - c_{2i}x_{Oi}^{o} \geq q_2\left(x_{Oi}^{*}\right)\mu_Z + q_1\left(x_{Oi}^{*}\right) - c_{1i} - c_{2i}x_{Oi}^{*} \quad (60)$$

$$q_j\left(x_{Oi}^{*}\right) \geq q_j\left(x_{Oi}^{o}\right), j = 1, 2 \quad (61)$$

((60) follows from the definition of x_{Oi}^{o}, and (61) holds because q_j is increasing in x_i). From (60) and (61) it follows that

$$q_1\left(x_{Oi}^{*}\right) - c_{1i} - c_{2i}x_{Oi}^{*} \leq q_1\left(x_{Oi}^{o}\right) - c_{1i} - c_{2i}x_{Oi}^{o}.$$

Therefore

$$\rho\left(\tilde{\pi}_{Oi}\left(x_{Oi}^{*}\right)\right) = q_2\left(x_{Oi}^{*}\right)\rho(Z) - \left(q_1\left(x_{Oi}^{*}\right) - c_{1i} - c_{2i}x_{Oi}^{*}\right)$$
$$\geq q_2\left(x_{Oi}^{o}\right)\rho(Z) - \left(q_1\left(x_{Oi}^{o}\right) - c_{1i} - c_{2i}x_{Oi}^{o}\right) = \rho\left(\tilde{\pi}_{Oi}\left(x_{Oi}^{o}\right)\right),$$

i.e., decision x_{Oi}^{*} is riskier than decision x_{Oi}^{o}, and results in lower expected profit. Therefore $x_{Oi}^{*} > x_{Oi}^{o}$ cannot be optimal, and thus $x_{Oi}^{*} \leq x_{Oi}^{o}$. □

Proposition 11(*a*) generalizes the results in Proposition 1 to a family of random revenue models. Note that (53) is a generalization of (11), and is likely to hold in practice because α is small (e.g., the 10%-fractile of *Z* is likely to be less than the mean of *Z*).

For the general family of revenue models, Proposition 11(*b*) shows that our main results extend to any coherent risk measure (e.g., CVaR, entropic value at risk, etc.), whether risk is included in the farmer's objective function or specified as a constraint. We emphasize that this result in 4(*b*) does not rely on inequality (53). In other words, as noted in Section 4, it is not possible for risk-averse optimal effort to be larger than the risk-neutral optimal effort under any coherent risk measure. The possibility of an increase in effort can only arise under a risk measure that is not coherent. However, as we see in (53), the probability distribution of random variable $Z = g\left(\tilde{a}, \tilde{\varepsilon}, \tilde{y}\right)$ would have to be highly skewed (assuming small α) for (53) not to hold.

To recap, farmer risk aversion under OMPP puts downward pressure on farmer investment in quality, moving the farmers and the cooperative further away from system optimal. The use of insurance with OMPP mitigates this pressure. However, misalignment of farmer incentives under OMPP reduces supply chain profit, and misalignment is eliminated under BMPP/I and RSPP/I.

Our model in Section 3 applies to the setting of our motivating application. However, our main findings are neither an artifact of the price-quality relationship in (5) nor the use of a VaR constraint for farmer risk aversion. Our main findings continue to hold under generalized forms of random revenue and any coherent risk measure.