Capacity Reservation and Sourcing under Exchange-Rate Uncertainty

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Abstract. We study a firm's capacity reservation and sourcing decisions under exchange-rate and demand uncertainty. The firm initially reserves capacity from one domestic and one international supplier in the presence of exchange-rate and demand uncertainty. After observing exchange rates, the firm determines the amount of capacity to utilize for sourcing under demand uncertainty.

The paper makes four contributions. First, it identifies the set of optimal capacity reservation policies for sourcing activities: One onshore, two offshore, and two dual sourcing policies. The first dual sourcing policy is a defensive action where the firm rations limited capacity between the two sources. The second dual sourcing policy is an opportunistic approach as it features excess capacity investment in order to benefit from currency fluctuations. The analysis shows how the optimal sourcing policy changes with increasing degrees of exchange-rate volatility. Second, while earlier publications classify cost as an order qualifier, we find that characterization of a dominant sourcing strategy is more nuanced under exchange rate uncertainty. In particular, a buyer might not reserve capacity at a supplier who has a lower (expected) cost and choose to work only with a supplier who has a higher unit expected cost. Third, the paper shows that risk aversion reduces the likelihood of single sourcing, specifically offshore sourcing, and increases the likelihood of dual sourcing. Fourth, the study demonstrates that financial hedging can eliminate the negative consequences of risk aversion and make the policy findings more pronounced.

Keywords: *exchange-rate uncertainty, capacity reservation, dual sourcing, onshore sourcing, offshore sourcing.*

1. Introduction

This paper examines the impact of exchange-rate uncertainty on capacity reservation and sourcing decisions. In recent decades, firms have dramatically increased their sourcing from abroad in order to take advantage of the lower costs. When acquisition and operational costs are denominated in foreign currencies, however, fluctuations in exchange rates can make an originally less costly (expensive) supplier an expensive (less costly) one later. Our study investigates the impact of sourcing cost uncertainty caused by fluctuations in exchange rates on sourcing policies using capacity reservation contracts. Some firms adopt dual sourcing, with one supplier in the home country and another abroad, in order to mitigate risks. Our study shows when to use single and dual sourcing policies and identifies the conditions that lead to each policy. It shows how uncertainties in demand, exchange-rate and risk aversion influence the firm's optimal sourcing decisions.

Earlier literature has identified two main reasons to engage in dual sourcing: (i) When the two sources have unequal and positive lead times under demand uncertainty (e.g., Allon and Van Mieghem 2010); and, (ii) when the reliability of the two sources is random (e.g., Dada et al. 2007). Our work contributes to this literature by identifying a third reason: Exchange-rate uncertainty can lead to dual sourcing in the context of capacity reservation contracts.

Our work is motivated by a furniture company in the United States that specializes in school and library furniture. The company sells primarily in its domestic market and faces three seasons of demand corresponding to each school semester. The firm sources most of its products using capacity reservation contracts from either a domestic or an international supplier that is located in Europe or Asia depending on the product. The company prepares for each selling season by initially reserving capacity with its suppliers approximately four months before the selling season begins. At this time point, the firm pays an initial reservation fee for each unit of capacity reserved at its suppliers which gives the buying firm the right to order production later. Depending on the product, the capacity reservation fee ranges between 2 to 5% of the selling price of the furniture firm. The monetary value of the capacity reservation fee is perceived to be approximately equal to the margin of the supplier. As the selling season approaches (e.g., one month before), the firm observes the realization of the random exchange rate and determines how much to order from each supplier in preparation for the selling season. The quantity ordered is limited by the amount of capacity reserved earlier. At this time point, a second payment is made to the suppliers (in the currency of the supplier) based on the actual amount ordered. The payment to the international supplier is influenced by the realized value of the exchange rate. Fluctuating Euro and appreciating Chinese Yuan in recent years motivated the managers of this furniture company to revisit their sourcing policies. The firm's reliance on a single supplier posed more risks when the currency used in the sourcing arrangements appreciated against the US dollar. The experience at the firm serves as the foundation for our study. Specifically, the managers of the furniture company wondered when it would be ideal to use dual sourcing to mitigate the risks stemming from currency fluctuations and the type and degree of capacity reservation decisions under single and dual sourcing arrangements.

Capacity reservation contracts are desirable for the buying firms because they guarantee availability before the season begins. These buying firms continue to have the flexibility of not utilizing the supplier when appreciation in the exchange rate makes the sourcing cost expensive. Capacity reservation contracts are also beneficial for the suppliers as they lead to more efficient production plans. Because the reservation fee is often close to the profit margin of the supplier, suppliers do not engage in strategic behavior under capacity reservation contracts. These reservation fees act as advance payments, covering potential profits, and the suppliers do not have to worry about keeping the facility idle and not earning money in the future.

Sourcing with capacity reservation contracts necessitates formulating the problem as a two-stage stochastic program. The model reflects the operating environment at the furniture company motivating our study. It is general as it can be used in environments where reservation and order decisions separated in time that gives way to fluctuating costs due to changing economic factors such as exchange rates. In the first stage, the firm determines the amount of capacity to be reserved at its suppliers in the presence of exchange-rate and demand uncertainty. In stage 2, after observing the exchange rate, the firm decides how much to order from each supplier; the order quantities are limited by the first-stage reservation decisions.

Our model is general and is not limited to the furniture company that provides the motivating example. It applies to a variety of manufacturing settings with long lead times where a firm outsources its production activities to contract manufacturers serving as suppliers. Capacity reservation contracts are widely used in other industries such as telecommunication, electronics, semi-conductor equipment manufacturing and apparel (Cohen et al. 2003, Erkoç and Wu 2005, Özer and Wei 2006, Peng et al. 2012, Cachon and Lariviere 2001). Custom-designed products may not be procured from the spot market, and therefore, buying firms who operate under long lead-times prefer to reserve capacity in advance of the selling season. Our model helps buying firms determine the most effective use of the domestic and foreign suppliers; it also prescribes when to engage in dual sourcing.

Our analysis considers sourcing agreements made with three types of costs. The firm initially pays a per-unit capacity reservation fee to the supplier in order to reserve capacity for production in the future (stage 1). Later when the season approaches (stage 2), there is another per unit operational cost paid to the supplier for the production order. Payments to the foreign supplier in stage 1 and stage 2 are paid in the foreign currency. In addition to the production cost, the buying firm incurs a transportation cost that includes duties and localization costs. The most common form of transportation agreement is free-on-board (FOB) in global logistics where the buying firm pays for the transportation cost. Therefore, our model includes a transportation cost in the domestic currency of the buying firm. It is important to note that our results continue to hold when the transportation costs are paid in the foreign currency. The sourcing cost is expressed in terms of the total landed cost which is equal to the sum of the costs from capacity reservation, production, transportation (inclusive of duties and localization costs).

Earlier literature reports that cost is an order qualifier, i.e., a supplier who has the lowest total landed cost would always get a positive order quantity. Our study shows that lower total landed cost is neither necessary nor sufficient to warrant ordering from a supplier. In other words, a supplier that has a lower expected total landed cost may not receive any capacity reservation contracts under exchange-rate uncertainty. The reason for this result is that the buying firm may prefer to reserve capacity at a source that has a higher expected total landed cost with the hope that the exchange rate devaluation makes this source a desirable one at the time of production. We develop the necessary and sufficient conditions that lead to this important finding and demonstrate its existence in practice using data from the motivating firm.

The paper makes four contributions. First, it shows that there are five potentially optimal policies: one onshore sourcing, two offshore sourcing and two dual sourcing policies. The two dual sourcing policies are characteristically different. In the first dual sourcing policy, the firm takes a conservative action that mitigates the negative consequences of currency fluctuations by splitting a constant total capacity between the domestic and foreign suppliers. In the second dual sourcing policy, the firm reserves extra capacity in order to benefit from exchange-rate fluctuations. Our study shows how the firm switches from one optimal sourcing policy to another with increasing degrees of exchange-rate volatility. Second, cost is *not* an order qualifier under exchange-rate uncertainty; the buying firm may reserve capacity only at the foreign supplier with the higher expected sourcing cost. Moreover, the behavior can be optimal even if the expected total landed cost from the foreign source is higher than the selling price. This finding adds a dimension to the literature that shows a high-cost supplier will not be selected over a low-cost supplier. Third, the introduction of risk aversion makes dual sourcing policies more desirable than single sourcing policies. Fourth, financial hedging helps the firm mitigate the negative consequences of risk aversion and potentially replicates the expected profit from each policy identified in the risk-neutral setting. Therefore, we conclude that our results are robust as the policy findings continue to hold under risk aversion and financial hedging.

The remainder of the paper is structured as follows. Section 2 describes the related literature. Section 3 presents the model; Section 4 analyzes it and discusses the results. Section 5 isolates the impact of exchange-rate uncertainty and volatility. Section 6 presents numerical illustrations from the furniture maker that motivated our study and demonstrates the existence of our theoretical findings. Section 7 addresses four extensions to our model: (1) impact of risk aversion, (2) use of financial hedging to mitigate risk, (3) the ability to order more units than what is reserved, (4) the impact of non-homogenous leadtimes. Section 8 provides concluding remarks. All proofs and derivations are relegated to the online appendix.

2. Literature Review

This paper studies global sourcing policies under random exchange rate and demand. There are practitioner and academic publications that draw attention to the use of capacity reservation contracts (Li and Kouvelis 1999, Pesen 2003, Serel et al. 2001, Wu et al. 2005). The main drivers for the use of capacity reservation contracts involve securing the ability to produce closer to the selling season in the context of long lead times. This literature notes that capacity reservation contracts are intended to reduce the negative impact of demand uncertainty. Contract terms are such that the supplier is paid a capacity reservation fee that is less than or equal to her margin in order to attract commitment from the buyer. This fee warrants the supplier to deviate from a strategic behavior.

There is a vast literature that investigates different aspects of sourcing decisions with an emphasis on dual sourcing. One stream of research examines dual sourcing under asymmetric and/or stochastic lead time (Fuduka 1964, Whittemore and Saunders 1977, Moinzadeh and Nahmias 1988, Moinzadeh and Schmidt 1991, Tagaras and Vlachos 2001, Veeraraghavan and Scheller-Wolf 2008, Lu and Van Mieghem 2009, Allon and Van Mieghem 2010, Wu and Zhang 2014). Jain et al. (2014) complements these publications by showing empirically that switching from a single to a dual sourcing policy reduces the inventory investment by almost 11%. This literature shows that dual sourcing emerges as an optimal policy only in the presence of unequal and positive lead-times in combatting demand uncertainty. These studies assume a deterministic exchange rate; our paper does not feature asymmetric or stochastic lead-times. Our work contributes to this literature by showing that there is another reason for dual sourcing: Exchange-rate uncertainty also leads to dual sourcing in the presence of capacity reservation contracts.

Another stream of literature explores the impact of reliability on sourcing decisions (Yano and Lee 1995, Tomlin and Wang 2005, Yang et al. 2009, Yang et al. 2012, Kouvelis and Lee 2013) and several publications focus on the cost and reliability trade-off (Dada et al. 2007, Federgruen and Yang 2008, 2009, 2011, Burke et al. 2009, Dong et al. 2017). Dada et al. (2007) consider a manufacturer ordering from suppliers that may be unreliable and prove that a more expensive supplier will be selected only if all less-expensive suppliers are selected, regardless of supplier reliability. Burke et al. (2009) and Federgruen and Yang (2009, 2011) show that this finding continues to hold under a more general settings with unreliable suppliers. The finding is consistent with Hill (2000) who argues that cost is an order qualifier whereas reliability acts as an order winner. Our results show that the principle of cost as an order qualifier is more nuanced in the presence of exchange rate uncertainty. In effect, exchange rate uncertainty introduces uncertainty in unit cost from the foreign supplier. Previous literature has focused on the case of deterministic sourcing cost. We find, for example, that it can be optimal to sole source from the supplier with higher expected cost. That is, a buyer might reserve capacity only at the expensive source. Moreover, we show that utilizing the expensive supplier continues to be prevalent even when the expected profit margin (from using this expensive supplier) is negative. This finding adds a new dimension to our understanding of the role of cost in sourcing strategies.

Another stream of literature examines the impact of exchange-rate and demand uncertainty in global operations (Kogut and Kulatilaka 1994, Huchzermeier and Cohen 1996, Kazaz et al. 2005, Ding et al. 2007, Dong et al. 2010, Chen et al. 2015, Park et al. 2016 and 2017). These papers focus on the impact of exchange-rate uncertainty in the revenues generated from sales in multiple markets. Their emphasis is on the production and distribution decisions related with the downstream of the supply chain. Our study, on the other hand, examines capacity reservation and sourcing decisions and focuses on the sourcing from the upstream in the supply chain.

The major driver of global sourcing practice is well acknowledged to be cost reduction (Feng and Lu 2012, Fox et al. 2006, Zhang et al. 2012, Shunko et al. 2014). Li and Wang (2010) and Chen et al. (2014) examine the trade-offs between the expensive domestic sourcing and low-cost offshore sourcing under exchange-rate risk. Demand uncertainty in their models is realized at the same time as exchange-rate is revealed. In our model, however, the firm continues to make its second-stage ordering decisions under demand uncertainty. As a consequence of the difference in the timing of demand realization, earlier publications do not develop dual sourcing policies with the characterization of rationing capacity between the two suppliers in order to mitigate exchange-rate risk and/or investing in excess capacity in order to benefit from currency fluctuations. Kouvelis (1998) justifies the use of an expensive supplier when the firm does not want to incur a switching cost. Our study, on the other hand, shows that the expensive foreign supplier might be the *only* source utilized even in the absence of switchover costs. Contrary to the common belief that offshore sourcing is utilized because of low sourcing costs associated with foreign suppliers, our paper contributes to this literature by showing that it can be optimal to sole source from a foreign supplier with higher expected cost.

Finally, our results provide insights into the ongoing debate on whether offshore sourcing is still economically viable (Ferreira and Prokopets 2009 and Ellram et al. 2013) even after the recent growing costs in emerging markets.

3. Model

The model considers a firm that sells a product in its home country at price p, and outsources its manufacturing to two suppliers, one in the home country (denoted with subscript H) and the other in a foreign country (denoted with subscript F). The random exchange rate is denoted $\tilde{e}$, and its realization is e following a probability density function (pdf) $g(e)$ and a cumulative distribution function (cdf) $G(e)$ on a support $[e_l, e_h]$ where $e_h > e_l > 0$ with a mean $E[\tilde{e}] = \bar{e}$. Random demand is denoted $\tilde{x}$, and its realization x follows a pdf $f(x)$ and a cdf $F(x)$ on a support $[x_l, x_h]$ where $x_h > x_l > 0$ with $E[\tilde{x}] = D$. We assume that the random exchange rate and the demand for the product are independent (e.g., as in Li and Chen 2010). This assumption can be reasonable for a single product sold in a domestic market. Importantly, the assumption of independence allows some tractability that is critical for exposing insight into the character of optimal sourcing strategies. See Table 1 for a list of notation.

The formulation uses a two-stage stochastic program with recourse (i.e., select production quantities after the exchange rate is realized). In the first stage, the firm makes two decisions in the presence of exchange-rate and demand uncertainty: The amount of capacity to reserve from the home-country and foreign-country suppliers, denoted Q_H and Q_F, respectively. A per-unit capacity reservation cost, denoted k_i where $i = H$, F, is incurred in order to reserve capacity at each supplier in advance.

Q_i	capacity reservation quantity from domestic source ($i = H$) and foreign source ($i = F$)
q_i	production quantity from domestic source ($i = H$) and foreign source ($i = F$)
p	selling price per unit
k_i	capacity reservation cost per unit from source $i = H, F$
o_i	operational cost per at time of capacity reservation (stage 1) from source $i = H, F$
e	realized exchange rate at time of production decision (stage 2), i.e., $o_F e$ = realized operational cost from foreign source
$\tilde{e}$	random exchange rate at stage 1 with pdf g, cdf G, support $[e_l, e_h]$, and mean $\bar{e}$
t_i	transportation cost per unit from source $i = H, F$; in domestic currency
c_i	total stage 2 cost per unit from source $i = H, F$, i.e., $c_H = o_H + t_H$, $c_F(e) = o_F e + t_F$
$\tilde{x}$	random demand with pdf f, cdf F, and support $[x_l, x_h]$, x = realized demand
C_i	total expected cost per unit from source $i = H, F$, i.e., $C_H = c_H + k_H = o_H + t_H + k_H$, $C_F = c_F(\bar{e}) + k_F = o_F\bar{e} + t_F + k_F$
C_F^-	total expected cost per unit from foreign source accounting for decision to set $q_F = 0$ when variable cost is higher than the price, i.e., $C_F^- = o_F E\left[\tilde{e} \middle| \tilde{e} < \left(p - t_F\right)/o_F\right] + t_F + k_F$

Table 1. Notation

The first-stage objective function maximizes the expected profit $E\left[\Pi\left(Q_H, Q_F\right)\right]$ while accounting for the second-stage optimal decision. We express the first-stage formulation as follows:

$$\max_{Q_H, Q_F \geq 0} \left\{ E\left[\Pi\left(Q_H, Q_F\right)\right] = -k_H Q_H - k_F Q_F + \int_{e_l}^{e_h} \pi^*\left(Q_H, Q_F, e\right) g\left(e\right) de \right\} \tag{1}$$

where $\pi^*\left(Q_H, Q_F, e\right)$ is the optimal second-stage expected profit from the first-stage decisions (Q_H, Q_F) at the realized exchange rate e.

We note that the introduction of a fixed capacity reservation cost does not affect the basic structure of the optimization problem or our main results (i.e., the set of possible optimal sourcing policies without fixed costs remain in place if fixed costs are included). In effect, the optimization problem is initially solved without consideration of fixed cost. Then profits net of fixed cost are considered in order to identify the optimal policy.

In stage 2, the firm makes two decisions in advance of the selling season and before the demand is realized: q_H and q_F represent the amount of production ordered from the domestic and foreign suppliers, respectively. Figure 1 describes the sequence of decisions. In stage 2, the ordering cost from each supplier is denoted by c_i where $i = H, F$ which can be considered as the sum of two costs: An operational cost denoted o_i where $i = H, F$, denominated in the local currency of supplier and a transshipment cost denoted t_i where $i = H, F$ that is inclusive of duties and localization costs in the home-country currency. The operational cost o_F is paid in the foreign currency for each unit produced by the foreign supplier. At the time capacity reservation decisions are made (in stage 1), the operational cost from the foreign supplier in terms of the home-country currency is random due to the randomness in the exchange rate; it can be expressed as $o_F \tilde{e}$. The firm then determines the production amount q_F to be ordered from the foreign source which is limited by the first-stage capacity reservation decision, i.e., $q_F \leq Q_F$. Moreover, the production decision q_F will change based on the observed exchange rate. As a result, we describe the second-stage costs $c_H = o_H + t_H$ as denominated in the home-country currency and $c_F(e) = o_F e + t_F$ as changing with the realized value of the exchange rate.[1] It is worth noting that this payment timing is advantageous for the firm; Proposition A7 shows that changing the timing of production does not improve the firm's expected profit, and Corollary A1 shows that dual sourcing does not arise in a model where exchange-rate and demand random variables are both realized at the end of the second stage.

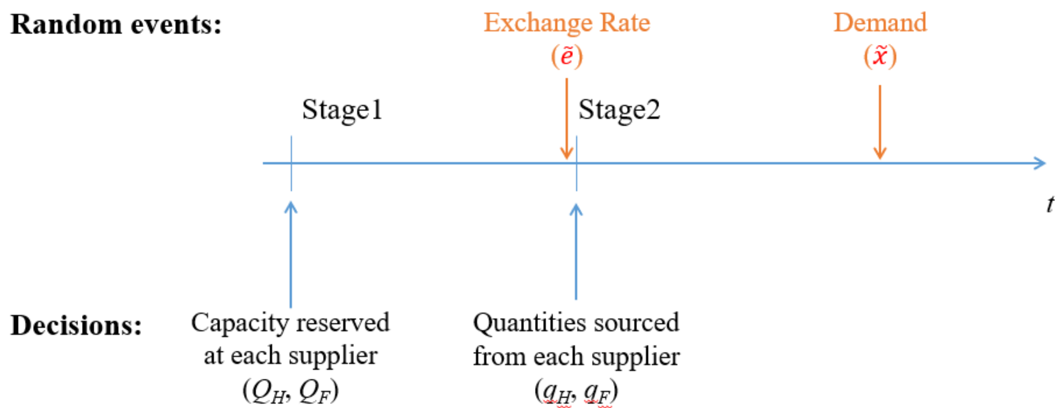

Figure 1. The natural sequence of events for a firm that reserves capacity at two suppliers.

The timing of the production order and delivery (beginning of stage 2) and demand (end of stage 2) in our model reflects settings where production/delivery lead times are long relative to

[1] The model can be supplemented with a fourth cost term penalizing the unused capacity reserved in stage 1. The inclusion of a penalty cost for unused capacity does not alter our main findings.

the selling season; the supply system operates as make-to-stock (i.e., production is ordered / delivered before demand in the selling season is known). This setting is distinct from an alternative setting where production/delivery lead times are short relative to the selling season (Li and Chen 2010, Chen et al. 2014); the supply system operates as make-to-order (i.e., production is ordered/delivered after demand in the selling season is known). Later, in Section 7.4, we analyze an alternative model where the production/leadtime is long for the foreign supplier and short for the domestic supplier.

The second-stage objective function maximizes the expected profit over random demand for a given set of first-stage decisions (Q_H, Q_F) and realized exchange rate e:

$$\pi^*\left(Q_H, Q_F, e, \tilde{x}\right) = \max_{(q_H, q_F)\geq 0}\left\{E\left[\pi_2\left(q_H, q_F, \tilde{x} \mid Q_H, Q_F, e\right)\right] \text{ s.t. } q_i \leq Q_i, i = H, F\right\} \quad (2)$$

where $E\left[\pi_2\left(q_H, q_F, \tilde{x} \mid Q_H, Q_F, e\right)\right] = -c_H q_H - c_F\left(e\right) q_F + \int_{x_l}^{x_h} p \min\left\{x, q_H + q_F\right\} f\left(x\right) dx$. The constraint in (2) ensures that the second-stage order quantities cannot exceed the first-stage capacity reservation decisions.

We next define the total landed cost, the sum of all costs from stages 1 and 2. The total landed cost at the home supplier is $C_H = c_H + k_H = o_H + t_H + k_H$. The expected total landed cost when the product is sourced from the foreign supplier is $C_F = E[c_F(\tilde{e}) + k_F] = E[o_F \tilde{e} + t_F + k_F] = o_F\bar{e} + t_F + k_F = c_F(\bar{e}) + k_F$. Note that the definition of C_F considers that the foreign supplier is utilized in every random exchange-rate outcome, and therefore, it relies on the expected value of the exchange rate $\bar{e}$. Note also that the domestic capacity reservation order cannot be replicated by a forward contract on the exchange rate with the foreign supplier, as such a contract does not provide the option to set production to zero.

Under a capacity reservation contract, it is neither necessary nor sufficient to have a cost advantage to guarantee reserving capacity at the low-cost supplier. As will be shown later, the firm might prefer to reserve capacity only at the offshore supplier (i.e., $Q_H^* = 0$ and $Q_F^* > 0$) even if the foreign supplier is expected to be more expensive (i.e., when $C_F > C_H$). Under a capacity reservation contract, the firm has the flexibility to not order from the foreign supplier when the realized exchange rate is high enough to make the second-stage margin negative. Therefore, the expected first-stage cost from the foreign supplier under a capacity reservation contract can be described as

$$C_F^- = o_F E\left[\tilde{e} \middle| \tilde{e} < \left(p - t_F\right)/o_F\right] + t_F + k_F < o_F E\left[\tilde{e}\right] + t_F + k_F = C_F.$$

The value of C_F^- is the total expected landed cost that is adjusted to account for the fact that production will not be sourced in the second stage if the variable cost is higher than selling price. Consequently, the adjusted value is less than the total expected landed cost from the foreign supplier (C_F). To avoid the trivial case of not reserving capacity, we assume that sourcing from both suppliers is economically viable, i.e.,

(A1) $C_H < p$ and $C_F^- < p$.

It is important to highlight that assumption (A1) does not require C_F to be small. Rather, C_F can take a value greater than the selling price (i.e., $C_F > p$) yielding an expected loss in the first stage. As will be shown later, the firm can still reserve capacity at the foreign supplier even if $C_F > p$ in its optimal decision and capitalize on the exchange-rate depreciation scenarios that make the foreign source less expensive.

4. Analysis

In this section, we examine the firm’s optimal capacity reservation decisions. We describe the optimal second-stage decisions before developing the optimal capacity reservation policies.

Before proceeding with the analysis, it is important to highlight that the firm does not engage in dual sourcing in the absence of exchange-rate uncertainty. This can be seen by replacing the random exchange rate variable $\tilde{e}$ with its mean $\bar{e}$ in the formulation. When uncertainty is only associated with demand, the firm has no recourse flexibility in sourcing decisions even if it has reserved capacity in the first stage. The problem becomes a single-stage Newsvendor Problem with two suppliers with the (expected) first-stage costs equal to C_H and C_F. It is easy to verify that the firm would choose the supplier with the lower first-stage cost.

Remark 1. *When demand is the only source of uncertainty, if $C_H \leq C_F$, then*

$$Q_H^* = Q_H^0 = F^{-1}((p - C_H)/p) > 0 \text{ and } Q_F^* = 0; \tag{3}$$

otherwise if $C_H > C_F$, then

$$Q_H^* = 0 \text{ and } Q_F^* = F^{-1}((p - C_F)/p) > 0. \tag{4}$$

From Remark 1, we see that dual sourcing is eliminated from the optimal solution in the absence of exchange-rate uncertainty; the firm uses a single supplier and orders only from the supplier with the lower total landed cost. Our paper shows that dual sourcing arises because of the combination of exchange-rate uncertainty and the capacity reservation flexibility.

Let us present a short summary of the optimal quantity decisions in a single-source setting in order to shed light into the analysis. The optimal order quantities from the home-country supplier are denoted $q_H^0 = F^{-1}((p - c_H)/p)$ in stage 2 (in the absence of a first-stage capacity decision limitation) and $Q_H^0 = F^{-1}((p - C_H)/p)$ in stage 1 as defined in (5). Note that $q_H^0 > Q_H^0$ because $c_H < C_H < p$. Similarly, let Q_F^0 denote the optimal capacity reservation quantity under sole-sourcing from the foreign supplier. Note that the optimal order quantity from the foreign supplier in stage 2 is $q_F^0(e) = F^{-1}((p - c_F(e))^+/p) = F^{-1}((p - o_F e - t_F)^+/p)$; its value decreases with the higher values of the realized exchange-rate e. Let $\tau(Q_F)$ denote the exchange-rate value that equates $q_F^0(e)$ to the optimal first-stage capacity reservation decision Q_F, i.e., solving $q_F^0(e = \tau(Q_F)) = Q_F$ for $\tau(Q_F)$ yields

$$\tau(Q_F) = \frac{p\left(1 - F\left(Q_F\right)\right) - t_F}{o_F}.$$

As shown in Proposition A1, Q_F^0 is the unique solution to the following:

$$\int_{e_l}^{\tau(Q_F^0)} \left(\tau\left(Q_F^0\right) - e\right) g\left(e\right) de = \frac{k_F}{o_F}. \tag{5}$$

In the presence of two suppliers, the optimal second-stage production decisions can be classified in three regions of exchange-rate realizations as depicted in Figure 2. In the first region where $e_l \le e \le \tau_1 = (c_H - t_F)/o_F$, the realized exchange rate is low and the foreign supplier is less costly than the home supplier, i.e., $c_F(e) \le c_H$. In the second region where $\tau_1 \le e \le \tau_2$, sourcing from the home supplier is more economical than the foreign supplier, i.e., $c_H < c_F(e)$. In the third region where $\tau_2 \le e \le e_h$, sourcing from the foreign supplier is no more economical because $c_F(e) \ge p$; thus, the firm does not utilize the foreign supplier even if it has reserved capacity in the first stage. In order to eliminate the trivial cases in the analysis, we assume that the support for the exchange-rate random variable is wide enough that $e_l \le \tau_1$ and $\tau_2 \le e_h$. The following proposition provides the optimal second-stage production decisions at each realization of exchange rate.

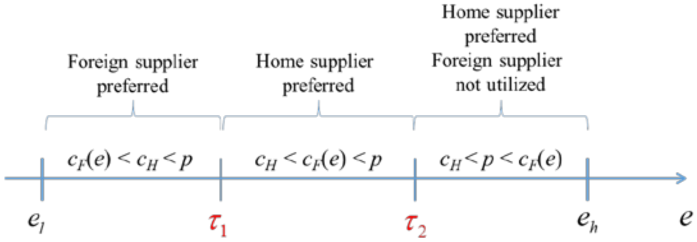

Figure 2. Exchange-rate realization in the second stage.

Proposition 1. *The optimal second-stage production decisions are*:

$$\left(q_H^*(e), q_F^*(e)\right) = \begin{cases} \left(\min\left\{Q_H, \left(q_H^0 - Q_F\right)^+\right\}, \min\left\{Q_F, q_F^0(e)\right\}\right) & \text{if } e_l \le e < \tau_1 \\ \left(\min\left\{Q_H, q_H^0\right\}, \min\left\{Q_F, \left(q_F^0(e) - Q_H\right)^+\right\}\right) & \text{if } \tau_1 \le e < \tau_2 \\ \left(\min\left\{Q_H, q_H^0\right\}, 0\right) & \text{if } \tau_2 \le e \le e_h \end{cases}. \tag{6}$$

In the remainder of the paper, we suppress the exchange rate parameter in the optimal second-stage production functions unless necessary for clarity. From Proposition 1, it can be seen that the optimal amount of capacity reserved from the domestic supplier cannot exceed q_H^0. The optimal capacity decisions in the first stage can be classified into the following three sets as depicted in Figure 3(a): Region $R1 = \{Q_H, Q_F \mid Q_H + Q_F \le q_H^0\}$, region $R2 = \{Q_H, Q_F \mid Q_H \le q_H^0, Q_F \le q_H^0 \text{ and } Q_H + Q_F > q_H^0\}$, and region $R3 = \{Q_H, Q_F \mid Q_H \le q_H^0 \text{ and } Q_F > q_H^0\}$.

Proposition 2. *The objective function in* (1) *is jointly concave in Q_H and Q_F.*

The proof of the above joint concavity proposition is not trivial and it relies on a series of intermediate results presented in the online supplement: Lemmas A1 through A5 and propositions A1 through A4. Moreover, Proposition A5 of the online supplement shows that the optimal solution does not lie in region *R*2. Together, they provide the foundation for identifying optimal solutions.

4.1. Optimal Policies

We next show that there are five potentially optimal capacity reservation policies. One of these policies utilizes onshore sourcing, two of them rely on offshore sourcing, and two policies feature dual sourcing.

(1) Policy H: Onshore sourcing with $Q_H^* = Q_H^0$, $Q_F^* = 0$,

(2) Policy F_L: Offshore sourcing with a smaller capacity reservation $Q_F^* = Q_F^0 \le q_H^0$, $Q_H^* = 0$,

(3) Policy F_H: Offshore sourcing with a higher capacity reservation $Q_F^* = Q_F^0 > q_H^0$, $Q_H^* = 0$,

(4) Policy D_R: Dual sourcing featuring a capacity rationing perspective with $Q_H^* + Q_F^* = Q_F^0$,

(5) Policy D_E: Dual sourcing featuring excess capacity with $Q_F^* = Q_F^0$, $Q_H^* < Q_H^0$.

Figure 3(b) illustrates these five potentially optimal policies along with the three regions. Policy H is the onshore policy where the firm reserves capacity only at the domestic supplier in the amount of $Q_H^* = Q_H^0 = F^{-1}((p - C_H)/p)$. Proposition A5 in the online supplement establishes Q_H^0 as the minimum total capacity reservation amount for the general problem.

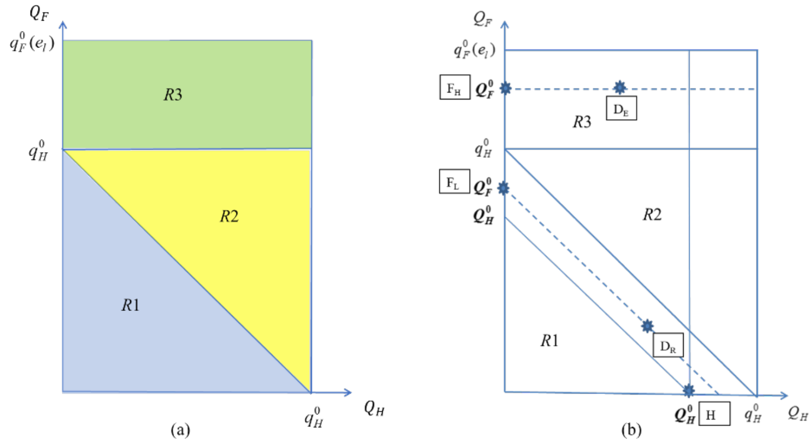

Figure 3. (a) The three regions for the capacity reservation decisions; (b) the set of potentially optimal policies (designated with rectangles).

There are two offshore policies, F_L and F_H, where the optimal capacity reservation decisions are $Q_F^* = Q_F^0$ where Q_F^0 is determined through (7). Recall that Q_F^0 can be less than or greater than q_H^0. We denote the offshore sourcing policy that leads to limited capacity investment $Q_F^* = Q_F^0 \le q_H^0$ as F_L, and the offshore sourcing policy with a higher capacity commitment $Q_F^* = Q_F^0 > q_H^0$ as F_H.

There are two potentially optimal dual sourcing policies that differ characteristically. Policy D_R is coined as the *rationing* dual sourcing policy where the firm reserves capacity from both suppliers but the total amount of capacity reserved is fixed, i.e., $Q_H^* + Q_F^* = Q_F^0$. The firm's allocation of capacity between the two suppliers can be perceived as rationing capacity in order to mitigate cost uncertainty stemming from currency fluctuations. Specifically, the firm diversifies its supply base between a cost-uncertain and a cost-certain supplier in order to mitigate the negative consequences of exchange-rate uncertainty. In this policy, the firm does not necessarily benefit much from currency swings. Thus, it is a defensive and a conservative policy that can be perceived as mitigating the negative consequences of currency fluctuations. Under D_R, the firm always utilizes the home supplier to its maximum, i.e., $q_H^* = Q_H^*$ at every realization of the random exchange rate. However, it utilizes the foreign supplier up to its limit only when the realized exchange rate is desirable, i.e., $e \le \tau_2$.

The second dual sourcing policy D_E is aggressive and opportunistic as it features *excess* capacity in order to enjoy the benefits of fluctuating exchange rates. Two observations can be made about this policy. First, the firm considers the foreign supplier as its primary source and reserves the exact amount of capacity it would have reserved under the offshore sourcing policies, i.e., $Q_F^* = Q_F^0$. Second, the firm reserves an additional capacity from the domestic source. However, this amount is strictly less than what it would have reserved under the onshore sourcing policy, i.e., $Q_H^* < Q_H^0$. The domestic supplier serves as a backup source in this policy; it is utilized only when the realized exchange rate makes the foreign source an expensive supplier, i.e., when $\tau_1 \le e \le e_h$. Thus, the foreign source is not always utilized at its maximum reserved capacity. By reserving a total capacity that exceeds the optimal amount that would be reserved from the offshore source, the firm often ends up wasting some reserved capacity, but in turn, takes advantage of the swings in the exchange rate. In sum, additional capacity reserved at the domestic source leads to an opportunistic behavior and provides the flexibility to enjoy the benefits of cost fluctuations.

These potentially optimal five policies can be obtained by reviewing four optimality conditions that explain the marginal benefits from each supplier. These four conditions provide the necessary and sufficient conditions for each policy to be the optimal decision:

(OC1): $E\left[\left(\left(p-c_F(\tilde{e})\right)^+ - \left(p-C_H\right)\right)^+\right] - k_F > 0$

(OC2): $E\left[\left(c_H - c_F(\tilde{e})\right)^+\right] - k_F > 0$

(OC3): $\left(p-C_H\right) - \left(E\left[\left(p-c_F(\tilde{e})\right)^+\right] - k_F\right) > 0$

(OC4): $E\left[\left(\left(p-c_H\right) - \left(p-c_F(\tilde{e})\right)^+\right)^+\right] - k_H > 0$

Optimality condition (OC1) describes whether the marginal benefit from using the foreign supplier over the domestic supplier is sufficient to pay for the reservation fee at the foreign source. Condition (OC2) enforces a stronger condition than (OC1) because it requires that the expected benefits in the second stage exceed k_F in order to justify reserving capacity in the foreign source. Condition (OC3) compares expected total landed costs between the two sources; when satisfied, it indicates that the marginal benefit from the domestic supplier is higher than the expected profit from the foreign source. Condition (OC4) reverses the requirement in (OC2) for the domestic supplier; it requires the expected benefits in the second stage needs to exceed the capacity reservation cost k_H in order to justify using the domestic source.

When the foreign supplier is less costly on average than the domestic source (i.e., when $C_F < C_H$) conditions (OC2) and (OC4) are sufficient to assure that the optimal decision is limited to offshoring and dual sourcing policies. However, when the domestic supplier's total landed cost is less than the expected total landed cost of the foreign source (i.e., when $C_H < C_F$), all four optimal conditions might be necessary to determine the optimal solution. Proposition A6 of the online supplement provides four technical results about the relationships between these optimality conditions: Satisfying (OC2) implies satisfying (OC1) (Proposition A6(a)); not satisfying (OC1) implies not satisfying (OC2) (Proposition A6(b)); not satisfying (OC1) implies satisfying (OC3) (Proposition A6(c)); and, not satisfying (OC3) implies satisfying (OC1) (Proposition A6(d)). These relationships help establish the following necessary and sufficient conditions that lead to the optimal policy choice.

Proposition 3.

(a) *Policy* H *is optimal iff* (OC1) *does not hold;*

(b) *Policy* $\mathrm{F_L}$ *is optimal iff* (OC2) *and* (OC3) *do not hold;*

(c) *Policy* $\mathrm{F_H}$ *is optimal iff* (OC2) *holds and* (OC4) *does not hold;*

(d) *Policy* $\mathrm{D_R}$ *is optimal iff* (OC1) *and* (OC3) *hold and* (OC2) *does not hold;*

(e) *Policy* $\mathrm{D_E}$ *is optimal iff* (OC2) *and* (OC4) *hold.*

Table A1 in the online supplement tabulates the necessary and sufficient conditions for each policy to be optimal. Note that the left-hand-side of the above four optimality conditions can be computed for any given selling price, cost structure, and the probability density function of the exchange rate; it is important to highlight that these conditions do not depend on the demand distribution. An algorithm that sourcing managers can use to locate the optimal sourcing policy appears in the online supplement.

Dual sourcing is the prevailing policy under certain conditions. The following proposition shows a simple comparison that reveals which one of the dual sourcing policies, $\mathrm{D_R}$ or $\mathrm{D_E}$, can be optimal.

Proposition 4. (a) *If* $Q_F^0 < q_H^0$, *then policy* $\mathrm{D_E}$ *cannot be optimal, leaving policy* $\mathrm{D_R}$ *as the only viable dual sourcing policy;* (b) *If* $Q_F^0 > q_H^0$, *then policy* $\mathrm{D_R}$ *cannot be optimal, leaving policy* $\mathrm{D_E}$ *as the only viable dual sourcing policy.*

5. Impact of Exchange-Rate Uncertainty

In this section, we compare the optimal sourcing decisions under exchange-rate and demand uncertainty with those obtained under deterministic exchange rate (by replacing the random exchange rate with its mean $\bar{e}$) and stochastic demand. The comparison provides insights regarding the impact of exchange-rate uncertainty on capacity reservation decisions. It is shown earlier that, under a deterministic exchange rate in our model, the firm engages in only single sourcing utilizing either the onshore source or the offshore source depending on which supplier has the lower total cost of sourcing. In other words, under a deterministic exchange rate and stochastic demand, dual sourcing does not exist in our setting. We examine the capacity choices under the cases with one source featuring the lower total sourcing cost; recall that the total landed cost for the two sources are $C_H = o_H + t_H + k_H$ and $C_F = o_F\bar{e} + t_F + k_F$.

Case 1: Lower expected cost at the foreign supplier: $C_F < C_H$. When the foreign supplier has the lower expected total landed cost, the firm always chooses offshore sourcing under the deterministic exchange rate. However, this is not necessarily the case if the exchange rate is uncertain.

Case 2: Lower cost at the domestic supplier: $C_H \leq C_F$. When sourcing from the domestic supplier is less costly, the firm always chooses onshore sourcing under a deterministic exchange rate. The next proposition shows that the offshore sourcing policy can be optimal despite featuring a more expensive foreign supplier.

Proposition 5. (a) *When $C_F < C_H$, the firm utilizes either an offshore sourcing policy* (F_L *or* F_H) *or the dual sourcing policy* D_E *under exchange-rate and demand uncertainty;* (b) *when $C_H \leq C_F$, all five policies can be optimal, and consequently, offshore sourcing policies* (F_L *or* F_H) *can be optimal under exchange-rate and demand uncertainty.*

Proposition 5(a) implies that a lower expected sourcing cost is not a sufficient condition for offshore sourcing. When the exchange-rate uncertainty is incorporated into the model, dual sourcing can be preferred over offshore sourcing even if the expected total landed cost is lower for the foreign supplier. In this case, the firm reserves capacity at the more expensive domestic source in addition to the capacity reserved at the foreign supplier.

Proposition 5(b) shows that it is not necessary to have the lowest total sourcing cost in order to reserve capacity at a single source. It shows that a lower cost at the domestic supplier does not eliminate the possibility of offshore sourcing. Alternatively said, offshore sourcing does not need to feature the lower expected sourcing cost to be the optimal policy. This result contrasts the conclusions of earlier publications that indicate that the supplier with the lowest total cost always receives a positive order quantity (e.g., Dada et al. 2007). We find that, in the presence of exchange-rate uncertainty, offshore sourcing can be the optimal policy even if the expected total landed cost is greater than that of the domestic source. The implication of this finding is that the domestic source with the lower sourcing cost does not get any orders – thus, lower cost is *not* an order qualifier. This result relies on the recourse capability of the capacity reservation contracts. Specifically, the firm does not have to utilize the offshore source when the realized exchange rate is unfavorable (e.g., cost of sourcing is expensive). Thus, the effective cost of utilizing the foreign source is lower than its expected total landed cost.

The finding in Proposition 5(b), which is illustrated numerically in Section 6, has important practical implications. It may be natural for a firm to exclude a high-cost foreign supplier from consideration when reserving capacity. Our findings may help protect against this inclination by highlighting that a firm should consider the volatility of exchange rate – in addition to cost at the current exchange rate – when making sourcing decisions.

<table>
<tr><th>Cost Structure</th><th>Demand Uncertainty</th><th colspan="3">Optimality Conditions</th><th>Exchange-Rate and Demand Uncertainty</th></tr>
<tr><td rowspan="3">Case 1
$C_F < C_H$</td><td rowspan="3">Offshore Sourcing</td><td rowspan="2">OC2</td><td colspan="2">OC4</td><td>Excess Dual Sourcing (D_E)</td></tr>
<tr><td colspan="2">$\overline{\text{OC4}}$</td><td>Offshore Sourcing (F_H)</td></tr>
<tr><td colspan="3">$\overline{\text{OC2}}$</td><td>Offshore Sourcing (F_L)</td></tr>
<tr><td rowspan="5">Case 2
$C_H \leq C_F$</td><td rowspan="5">Onshore Sourcing</td><td rowspan="2">OC2</td><td colspan="2">OC4</td><td>Excess Dual Sourcing (D_E)</td></tr>
<tr><td colspan="2">$\overline{\text{OC4}}$</td><td>Offshore Sourcing (F_H)</td></tr>
<tr><td rowspan="3">$\overline{\text{OC2}}$</td><td rowspan="2">OC1</td><td>OC3</td><td>Rationing Dual Sourcing (D_R)</td></tr>
<tr><td>$\overline{\text{OC3}}$</td><td>Offshore Sourcing (F_L)</td></tr>
<tr><td colspan="2">$\overline{\text{OC1}}$</td><td>Onshore Sourcing (H)</td></tr>
</table>

Table 2. The impact of exchange-rate uncertainty on sourcing decisions. An overscore implies the reversed condition.

Table 2 illustrates how introducing exchange-rate uncertainty can significantly influence the optimal sourcing policy when compared to the case where uncertainty is associated only with demand. In other words, ignoring the exchange-rate uncertainty may result in decisions that are far from optimal. Table 2 shows that when $C_H \leq C_F$ under a deterministic exchange-rate setting, the optimal sourcing policy is onshore sourcing (H). However, when exchange-rate uncertainty is incorporated into the same decision-making process, the optimal choice can be offshoring policies F_L or F_H (without using the less expensive domestic source) or dual sourcing policies D_R

and D_E. The table also provides the optimality conditions that are necessary for these policies to be optimal.

The next proposition shows that offshore sourcing can be the optimal choice even when it has a negative expected profit margin in the second stage.

Proposition 6. *The optimal sourcing policy may be dual sourcing* (D_R *or* D_E) *or offshore sourcing* (F_L *or* F_H) *when* $p - c_F(\bar{e}) < 0$.

The consequence of the above proposition is that the firm can choose offshore sourcing even if it is expected to lose money in the second stage with $p - c_F(\bar{e}) < 0$. Note that this is a stronger condition than a negative expected margin in the first stage with $p - C_F < 0$. Under a deterministic exchange rate, the firm may not even consider the foreign supplier when the second-stage return is negative. Proposition 6 implies that the firm may benefit from giving up a deterministic positive profit margin from the domestic supplier ($p - C_H$), and instead engage with a single foreign source even if the expected unit cost from the offshore source (in stage 2) is higher than the unit price, i.e., $p - c_F(\bar{e}) < 0$. In this case, postponing the sourcing decision until the revelation of the exchange rate provides the benefit of potentially high profit margin caused by lower exchange-rate realizations. It also comes at the expense of incurring a potential loss, e.g., capacity reservation cost, at higher exchange rate realizations. This flexibility can increase the desirability of the foreign supplier so much that the firm prefers to utilize offshore sourcing without a domestic supplier even if the foreign supplier has an expected stage 2 cost higher than the selling price. The result shows how a foreign supplier becomes attractive based on exchange-rate fluctuations even if it is not an economically desirable one at the expected exchange rate.

We next investigate how exchange-rate volatility influences the optimal sourcing policy. The next proposition shows that the optimal domestic capacity does not behave monotonically in exchange-rate volatility as it shows both an increasing and decreasing behavior.

Proposition 7. (a) *The optimal foreign capacity* Q_F^* *always increases in exchange-rate volatility. But the optimal domestic capacity* Q_H^* *may increase or decrease;* (b) *The expected profit increases in exchange-rate volatility.*

The above proposition formally establishes that exchange-rate uncertainty helps the firm improve its expected profit. The result stems from the recourse flexibility enabled by the capacity reservation contracts. It implies that the firm benefits from paying for production in the foreign currency when there is a higher degree of exchange-rate uncertainty.

5.1. Exchange-Rate Volatility

One might intuit that higher degrees of exchange-rate volatility creates the incentive to invest in additional flexibility through domestic capacity. However, it is possible that the reverse holds – that domestic capacity may decrease as exchange rate volatility increases. We use a uniform exchange-rate distribution in the next two propositions only to provide technical proofs for the findings associated with the impact of volatility.

Proposition 8. *When the exchange rate is uniformly distributed, under policy* $\mathrm{D_E}$*, the optimal domestic capacity* Q_H^* *decreases in exchange-rate volatility iff* $C_H < c_F(\bar{e})$*.*

The condition in Proposition 8 may appear counter-intuitive at first because it suggests that when total landed cost from the domestic source is lower than the expected cost of sourcing from the foreign source, the domestic capacity decreases in exchange-rate uncertainty. It is worth noting that this proposition does not imply that reducing the cost of onshore sourcing leads to lower optimal capacity at the domestic source. In fact, when $C_H < c_F(\bar{e})$, the firm reserves higher levels of domestic capacity at any degree of exchange-rate volatility compared to the opposite case (i.e., $C_H > c_F(\bar{e})$). The condition in Proposition 8 requires that the cost of sourcing from the domestic source is low, and thus, the firm reserves a sufficiently high level of domestic capacity under policy $\mathrm{D_E}$. Consequently, the domestic capacity becomes strictly a substitute for the foreign capacity which increases in exchange-rate volatility (according to Proposition 7). As a result, the firm reduces its capacity commitment at the domestic source in order to capitalize more on the prospects of sourcing from the foreign supplier.

Proposition 9. *When the exchange rate is uniformly distributed, as the exchange-rate volatility increases*:

(a) *if* $C_F < C_H$*, then the optimal sourcing policy changes according to the following path* (*or a continuous portion thereof*): $\mathrm{F_L} \rightarrow \mathrm{F_H} \rightarrow \mathrm{D_E}$;

(b) *if* $C_F \geq C_H$*, then the optimal sourcing policy changes according to the following paths* (*or a continuous portion thereof*): $\mathrm{H} \rightarrow \mathrm{D_R} \rightarrow \mathrm{F_L} \rightarrow \mathrm{F_H} \rightarrow \mathrm{D_E}$, *or* $\mathrm{H} \rightarrow \mathrm{D_R} \rightarrow \mathrm{D_E}$.

Proposition 9 establishes the optimal policy paths as the exchange-rate volatility increases. Specifically, when the foreign supplier has the lower expected sourcing cost, the firm either keeps offshore sourcing or switches to policy $\mathrm{D_E}$ utilizing excess capacity. On the other hand, when the total landed cost from the domestic supplier is lower, the firm switches from onshore sourcing to the dual sourcing policy exhibiting rationing behavior (policy $\mathrm{D_R}$). With increasing exchange-rate volatility, the firm either directly switches to the excess dual sourcing policy (policy D_E) or first adopts offshore sourcing policies before implementing the excess dual sourcing policy. We observe that as the degree of exchange-rate variation increases, the foreign supplier becomes a more desirable source. This is because there are higher chances of a large savings due to the lower exchange rate realizations; in the meantime, the possible loss due to the appreciation of the exchange rate—which is as much as the capacity reservation cost plus the penalty cost of unused capacity—remains unchanged. Consequently, a higher volatility in the exchange rate results in choosing the foreign supplier as the primary source (corresponding to policies F_L, F_H, and D_E). Figure 4 visually depicts the path of policy switches described in Proposition 9(b) by increasing the exchange-rate volatility while preserving the mean.

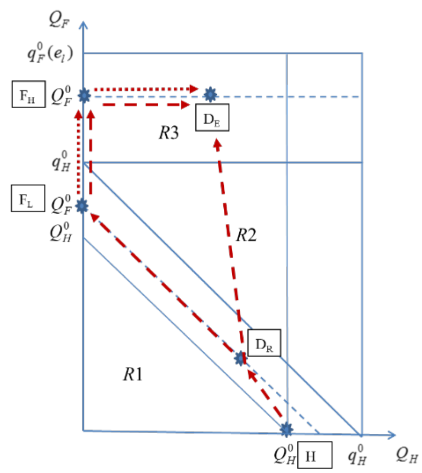

Figure 4. The impact of increasing volatility in the exchange-rate distribution (while preserving the mean) on the choice of the optimal policy. The dashed (dotted) arrows illustrate how the optimal policy switches as the exchange rate becomes more volatile when $C_F > C_H$.

6. Numerical Illustration

We illustrate our theoretical findings using data from the furniture manufacturer that motivated our study. The firm has three selling seasons in a year, each spanning a four-month interval, following the traditional school semesters. The firm reserves capacity at its suppliers one selling season in advance, corresponding to four months prior to the season. Unit reservation costs (k_H, k_F) for the firm's products range from 1% (e.g., book carts, the firm's best-known products) to 5% (e.g., office desks and chairs) of the selling price p. From another perspective, these unit reservation costs (k_H, k_F) range between 1.25% (for book carts) to 6.25% (for office desks and chairs) of the total landed costs C_H and C_F. It is valuable to point out that, the furniture firm estimates that these unit reservation costs (k_H, k_F) correspond to a significant proportion of the supplier's profit margin. The managers estimate that the unit capacity reservations fees are in the range of 15% to 80% of the profit margins the suppliers would have earned if they were to manufacture the whole product and sell it to the specialty furniture company. These estimations clearly point to a benefit on the part of the suppliers, creating inertia against a strategic behavior. In the remaining part of this section, we highlight only the results associated with surprising insights using the data provided by the firm (e.g., a book cart) and ignore the results from products that lead to trivial outcomes.

The capacity reservation payment is made at the spot exchange rate one season before the selling season approaches (equivalent to four months); it is paid by converting the foreign currency to the domestic currency using the spot exchange rate at the time of the initial payment. Shortly before the beginning of the selling season, corresponding to stage 2 of our model, the furniture maker specifies the exact quantity of products to be manufactured at each supplier and makes the appropriate payment at the exchange-rate of the day. It is important to remind that the firm continues to operate under demand uncertainty during the selling season; schools and libraries continue to place orders during the selling season.

In order to provide a meaningful and comparative analysis, while keeping costs and prices confidential, we normalize the selling price to \$100 and scale the capacity reservation and transportation costs accordingly. In the analysis presented here, we use conservative numbers where the unit capacity reservation costs are k_H = \$1 and k_F = \$1; the unit transportation costs are t_H = \$2, t_F = \$4. We illustrate the impact of operational costs through a range of values: \$75 $\leq o_H \leq$ \$85 and \$75 $\leq o_F \leq$ \$85. (We report the value of o_F denominated in \$ in Figure 5, which corresponds to €56.18 $\leq o_F \leq$ €63.67 at the mean exchange rate of 1.335 \$/€.) Demand is uniformly distributed between 0 and 200.

We construct the exchange-rate distribution from the fluctuations in daily exchange-rates between 2010 and 2012 (three years of data) – this is the period corresponding to the planning period of the data provided by the motivating firm. The Euro-to-Dollar exchange rate has a mean of 1.335 and a standard deviation of 0.065 during this period; we use the changes in the daily rates from four months prior in order to construct the distribution describing the exchange-rate fluctuations; we provide the details of this process in the online supplement. As shown earlier, the demand distribution has no impact on the choice of the optimal policy.

Figure 5 demonstrates the optimal policy under these cost terms using the same policy indicators H, F_L, F_H, D_R and D_E. For the parameter combinations in Figure 5 where dual sourcing is optimal, the average percentage improvement in profit relative to the best single-sourcing alternative is 6.3%.[2] The percentage differences range from 0% (at boundary points between single and dual sourcing) to 21.8%. It is easy to see that when the operational cost of the home (foreign) source is significantly less expensive than that of the foreign (domestic) source, then the optimal policy is a single source policy H (F_H). Dual sourcing policies are optimal in a region where the second-stage operational costs are relatively similar. Although not shown here, the region of dual sourcing shrinks with increasing values of k_H and k_F.

o_F \ o_H	75.0	75.5	76.0	76.5	77.0	77.5	78.0	78.5	79.0	79.5	80.0	80.5	81.0	81.5	82.0	82.5	83.0	83.5	84.0	84.5	85.0
85.0	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	D_R	D_R	D_E
84.5	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	D_R	D_R	D_E	D_E
84.0	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	D_R	D_R	D_E	D_E	D_E
83.5	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	D_R	D_R	D_E	D_E	D_E	D_E
83.0	H	H	H	H	H	H	H	H	H	H	H	H	H	H	D_R	D_R	D_E	D_E	D_E	D_E	D_E
82.5	H	H	H	H	H	H	H	H	H	H	H	H	H	D_R	D_R	D_E	D_E	D_E	D_E	D_E	D_E
82.0	H	H	H	H	H	H	H	H	H	H	H	H	D_R	D_R	D_E	D_E	D_E	D_E	D_E	D_E	D_E
81.5	H	H	H	H	H	H	H	H	H	H	H	D_R	D_R	D_E	D_E	D_E	D_E	D_E	D_E	D_E	D_E
81.0	H	H	H	H	H	H	H	H	H	H	D_R	D_R	D_E	D_E	D_E	D_E	D_E	D_E	D_E	D_E	D_E
80.5	H	H	H	H	H	H	H	H	H	D_R	D_R	D_E	D_E	D_E	D_E	D_E	D_E	D_E	D_E	D_E	D_E
80.0	H	H	H	H	H	H	H	H	D_R	D_R	D_E	D_E	D_E	D_E	D_E	D_E	D_E	D_E	D_E	F_H	F_H
79.5	H	H	H	H	H	H	H	D_R	D_R	D_E	D_E	D_E	D_E	D_E	D_E	D_E	D_E	D_E	F_H	F_H	F_H
79.0	H	H	H	H	H	H	D_R	D_R	D_E	D_E	D_E	D_E	D_E	D_E	D_E	D_E	D_E	F_H	F_H	F_H	F_H
78.5	H	H	H	H	H	D_R	D_R	D_E	D_E	D_E	D_E	D_E	D_E	D_E	D_E	D_E	F_H	F_H	F_H	F_H	F_H
78.0	H	H	H	H	D_R	D_R	D_E	D_E	D_E	D_E	D_E	D_E	D_E	D_E	D_E	F_H	F_H	F_H	F_H	F_H	F_H
77.5	H	H	H	D_R	D_R	D_E	D_E	D_E	D_E	D_E	D_E	D_E	D_E	D_E	F_H	F_H	F_H	F_H	F_H	F_H	F_H
77.0	H	H	H	D_R	D_R	D_E	D_E	D_E	D_E	D_E	D_E	D_E	D_E	F_H	F_H	F_H	F_H	F_H	F_H	F_H	F_H
76.5	H	H	D_R	D_R	D_E	D_E	D_E	D_E	D_E	D_E	D_E	D_E	F_H	F_H	F_H	F_H	F_H	F_H	F_H	F_H	F_H
76.0	H	D_R	D_R	D_E	D_E	D_E	D_E	D_E	D_E	D_E	D_E	F_H	F_H	F_H	F_H	F_H	F_H	F_H	F_H	F_H	F_H
75.5	D_R	D_R	D_E	D_E	D_E	D_E	D_E	D_E	D_E	D_E	F_H	F_H	F_H	F_H	F_H	F_H	F_H	F_H	F_H	F_H	F_H
75.0	D_R	D_E	D_E	D_E	D_E	D_E	D_E	D_E	D_E	F_H	F_H	F_H	F_H	F_H	F_H	F_H	F_H	F_H	F_H	F_H	F_H

Figure 5. Optimal policies under varying stage 2 costs (o_H, o_F); t_H = \$2, t_F = \$4, $k_H = k_F$ = \$1.

The region for dual sourcing in Figure 5 expands and shrinks with increasing values of the second-stage production costs ($c_H = o_H + t_H$ and $c_F(e) = o_F e + t_F$). There are two opposing drivers for this phenomenon. First, higher operational costs in stage 2 diminish the relative magnitude of the unit capacity reservation costs of stage 1. Therefore, the firm perceives reserving capacity to

[2] Based on the most profitable alternative between the domestic source and the foreign source. For the foreign source, we compute profit using the average exchange rate, which is consistent with historical practice at the firm motivating the research.

be less expensive in sourcing decisions; this leads to a wider range of second stage operational costs that feature dual sourcing policies as the optimal choice. On the other hand, higher operational costs reduce the value that can be gained from dual sourcing policies. Specifically, as the operational costs increase, sourcing from both suppliers become less profitable, and thus the potential gain from dual sourcing policies declines with higher second-stage operational costs. As a consequence, the region where dual sourcing is preferred shrinks as the operational costs continue to increase in relative to the selling price.

Policy D_E is often the desired dual sourcing policy over policy D_R when the unit reservation fees are lower as in Figure 5. This is because the firm has a higher degree of benefit from exchange rate fluctuations under policy D_E in stage 2, without having to make a significant payout in stage 1. However, policy D_R is more desirable than policy D_E at higher values of k_H and k_F. This is because there is not a sufficient degree of exchange-rate volatility in order to benefit from having excess capacity in place.

7. Extensions

7.1. Risk Aversion

We extend the model of Section 3 to account for risk aversion. The first-stage problem is to maximize expected profit subject to value-at-risk (VaR) constraint that limits the risk associated with the realized returns under exchange-rate and demand uncertainty. In VaR, two parameters describe the firm's risk preference: β represents the loss (VaR) that the firm is willing to tolerate at probability α, where $0 \leq \alpha \leq 1$.[3] In addition to capacity reservation quantities, the firm determines amount of financial hedging contracts to acquire, denoted Q_C, in order to mitigate the currency risk. As is common in the literature, we assume that the exchange rate spot market is efficient (i.e., no arbitrage opportunities). The unit cost of financial hedging is denoted $k_C(e_s)$ where e_s is the exercise price of the financial hedging instrument, i.e., the rate at which the foreign currency will be converted to the domestic currency. The first-stage decision triplet (Q_H, Q_F, Q_C) is an infeasible solution when the probability of realized loss (VaR) exceeding the tolerable loss β is greater than the tolerated probability α. We express the first-stage problem as follows:

[3] VaR probabilities of 1% and 5% are relatively common in practice (Pearson 2002).

$$\max_{Q_H, Q_F, Q_C \ge 0} \left\{ E\left[\Pi\left(Q_H, Q_F, Q_C\right)\right] = -k_H Q_H - k_F Q_F - k_C\left(e_s\right) Q_C + \int_{e_l}^{e_h} \pi^*\left(Q_H, Q_F, Q_C, e\right) g\left(e\right) de \right\}$$

$$\text{s.t. } P_{(\tilde{e},\tilde{x})}\left[-k_H Q_H - k_F Q_F - k_C\left(e_s\right) Q_C + \pi_2\left(q_H^*, q_F^*, q_C^*, \tilde{x} \middle| Q_H, Q_F, Q_C, \tilde{e}\right) < -\beta\right] \le \alpha \qquad (7)$$

where $P_{(\tilde{e},\tilde{x})}[\cdot]$ represents the probability over the exchange-rate and demand random variables and q_C^* is the optimal number of currency options exercised in the second stage. Inequality (7) enforces the VaR restriction in which the probability that the firm's realized loss exceeding the tolerable loss of β is less than or equal to the tolerable loss probability α. This VaR constraint combines the realized losses in stage 2 (from the realizations of exchange-rate and demand random variables, $\tilde{e}$ and $\tilde{x}$, respectively) with the reservation costs of stage 1, accounting for the overall risks in sourcing decisions. For a given α, if VaR is more than the tolerable loss β, then first-stage decisions (Q_H, Q_F, Q_C) correspond to an infeasible solution. VaR is the most widely used risk measure in practice as it is the recommended approach in Basel II and Basel III Accords (2013). It is important to note that our main findings continue to hold under a Conditional Value at Risk (CVaR) measure.

We let $r_C(e_s)$ denote the return from exercising one unit of the financial hedging instrument. The second-stage objective function maximizes the expected profit over random demand for a given set of first-stage decisions and realized exchange rate e:

$$\pi^*\left(Q_H, Q_F, Q_C, e, \tilde{x}\right) = \max_{(q_H, q_F, q_C) \ge 0} \left\{ E\left[\pi_2\left(q_H, q_F, q_C, \tilde{x} \mid Q_H, Q_F, Q_C, e\right)\right] \text{ s.t. } q_i \le Q_i, i = H, F, C \right\} \qquad (8)$$

where $E\left[\pi_2\left(q_H, q_F, q_C, \tilde{x} \mid Q_H, Q_F, Q_C, e\right)\right] = -c_H q_H - c_F\left(e\right) q_F + r_C\left(e_s\right) q_C + \int_{x_l}^{x_h} p \min\left\{x, q_H + q_F\right\} f\left(x\right) dx$

To highlight the impact of risk aversion, we first examine the problem in the absence of financial hedging and set $Q_C = q_C = 0$ (we present the influence of financial hedging in Section 7.2). Before proceeding with the analysis of risk aversion, it is important to make several observations. In the absence of exchange-rate uncertainty, the introduction of risk aversion through a VaR constraint does not lead to dual sourcing. Remark 1 has already pointed out that when the random exchange rate is replaced with its mean $\bar{e}$ in the risk-neutral setting, the firm works with only one supplier and reserves capacity at the source with the lower cost. When the optimal capacity reserved at the low-cost source (let us denote it with Q^N) violates the VaR constraint due to the stochastic demand, then the firm would reduce its initial capacity reservation amount in order to satisfy the constraint. Specifically, let x_α denote the value of the demand random variable that corresponds to α probability in its cdf. It is sufficient to check the value of the realized profit at x_α from reserving Q^N units of capacity at the low cost supplier j: If $p\,x_\alpha - (k_j + c_j + t_j)Q^N < -\beta$, then the firm reduces its initial capacity investment from Q^N to

$$Q^A = (p\,x_\alpha - \beta)/(k_j + c_j + t_j) < Q^N$$

where Q^A describes the amount of capacity reserved in stage 1 due to risk aversion. In conclusion, in the absence of exchange-rate uncertainty, the firm reduces its initial capacity reservation commitment as a result of risk aversion, but it does not switch to dual sourcing.

We next examine the impact of risk aversion in the presence of exchange-rate uncertainty. Let e_α denote the exchange rate realization at fractile $1 - \alpha$, i.e., $[1 - G(e_\alpha)] = \alpha$. Because demand uncertainty in isolation does not lead to any policy change in our model, but rather a reduction in reserved capacity, we focus on problem settings where exchange-rate is a source of uncertainty in violating the VaR requirement. This setting occurs when $e_\alpha \geq \tau_2$; exchange-rate realizations that are greater than τ_2 are most detrimental to the firm because it would waste the entire capacity reserved at the foreign source due to the fact that $q_F^*(e) = 0$ for $e \geq \tau_2$.

In the presence of exchange-rate uncertainty, incorporating risk aversion encourages the firm to engage in dual sourcing. This can be seen when the optimal decision in the risk-neutral setting is an offshore sourcing policy as in the case of policies F_H and F_L. When risk aversion is included and the VaR constraint is violated, the firm can decrease the level of capacity investment in the foreign source in order to comply with the VaR constraint. Let us define Q_F^A as the level of capacity reserved at the foreign source that yields a realized profit equal to $-\beta$ at the exchange-rate realization $e_\alpha \geq \tau_2$, i.e.,

$$Q_F^A = \beta/k_F. \tag{9}$$

The following two conditions help characterize when the firm switches from single sourcing at the foreign source (i.e., F_H and F_L) to dual sourcing:

$$\text{(RA1):}\ \left(p - C_H\right)\left[\frac{o_F}{k_F}\int_{e_l}^{\tau\left(Q_F^A\right)} G(e)\,de - 1\right] > k_H - E\left[\left(p - C_H\right) - \left(p - c_F\left(\tilde{e}\right)^+\right)^+\right],$$

$$\text{(RA2):}\ \left(\frac{\left(p - C_H\right)o_F}{k_F} + o_F\right)\int_{e_l}^{\tau\left(Q_F^A\right)} G(e)\,de > E\left[\left(p - c_F\left(\tilde{e}\right)\right)\right].$$

Proposition 10. *Suppose the risk-neutral optimal solution violates the VaR constraint, and* $e_\alpha \geq \tau_2$.

(a) *When the optimal sourcing policy in the risk-neutral setting is* $\mathrm{F_H}$ *and* $Q_F^A > q_H^0$, *the firm switches to dual sourcing under risk aversion if condition* (RA1) *holds;*

(b) *When the optimal sourcing policy in the risk-neutral setting is either* $\mathrm{F_L}$ *or* $\mathrm{F_H}$ *with* $Q_F^A \leq q_H^0$, *the firm switches to dual sourcing under risk aversion if condition* (RA2) *holds.*

Inequalities (RA1) and (RA2) in the above proposition correspond to conditions under which the marginal value of home-country capacity is positive under risk aversion, given that sourcing from the foreign supplier is optimal for a risk-neutral firm. As shown in the proposition, (RA1) is relevant when q_H^0 is low (i.e., $Q_F^A > q_H^0$) and (RA2) is relevant when q_H^0 is high (i.e., $Q_F^A \leq q_H^0$). In other words, (RA1) applies when the stage-2 costs of the home-country source are relatively high compared to (RA2). This characteristic underlies the intuition that explains why (RA1) is a stronger condition than (RA2) (i.e., dual sourcing is less likely because of high-cost in the home country); further analysis shows that the right-hand side (RHS) of condition (RA1) is greater than that of (RA2) when $Q_F^A > q_H^0$. In this case, when (RA1) holds, condition (RA2) also holds. Thus, (RA1) can be perceived as a sufficient condition that warrants switching from the risk-neutral optimal offshoring policy to dual sourcing under risk aversion. It can also be shown that when dual sourcing policies are optimal in the risk-neutral setting, they continue to be optimal under risk aversion in our model.

In conclusion, our analysis shows that the introduction of risk aversion through a VaR constraint leads to a higher likelihood of dual sourcing. Our analysis is conducted using VaR as the underlying risk measure in describing risk aversion. It is also important to note that the same analysis can be replicated using a Conditional Value at Risk (CVaR) as the risk measure. While the two conditions in (RA1) and (RA2) would be revised, the conclusions from the analysis would remain unchanged, i.e., risk aversion encourages dual sourcing.

7.2. Financial Hedging

We next examine the impact of financial hedging on the firm's optimal policy choices. The buying firm is concerned about the future value of $o_F \tilde{e}$ that it needs to pay to the foreign supplier in stage 2. This payment increases with higher values of the realized exchange rate e and can cause the loss to be higher than β. The VaR requirement ensures that the firm's realized losses exceeding β is less than or equal to the tolerated loss probability α. For any given capacity reservation decisions Q_H and Q_F, if the VaR constraint is violated, then firm can engage in financial hedging contracts in order to mitigate the potential losses stemming from $o_F e$. Thus, the firm's financial hedging initiative is intended to avoid realized losses to be less than β at $e = e_\alpha$ and at $x = x_l$. When this is accomplished, the firm's realized losses is assured to be less than β at all $e \leq e_\alpha$ and at all $x \in [x_l, x_h]$.

How do the firm's cash flows change with financial hedging? Each unit of financial hedging contract is purchased at a unit cost of $k_C(e_s)$ (also referred to as the premium) from the financial institution. This contract has a strike (or, exercise) price of e_s. The firm brings e_s to the financial institution and receives a payment of e. The return from the financial hedging instrument purchased at the unit cost of $k_C(e_s)$ is then $r_C(e_s) = e - e_s$. This contract then protects the buying firm by limiting the losses at exchange-rate realizations $e > e_s$.

We consider two different financial hedging instruments in our analysis. The first alternative is a currency futures contract. In the futures alternative, the firm buys Q_C units of contracts each at $k_C(e_s)$ and pays a total of $k_C(e_s)Q_C$ upfront to the financial institution. After observing e in stage 2, the firm brings the money e_sQ_C to the financial institution, and in return, receives a payment of eQ_C. Thus, the return from each futures contract can be written as $r_C(e_s) = (e - e_s)$, yielding a total payback of $(e - e_s)Q_C$. Note that the return value of $r_C(e_s)$ can be both positive and negative under a futures contract. The firm will get a positive return when $e \geq e_s$ but will have to pay more when $e < e_s$.

The second alternative for financial hedging is a currency option contract. In this alternative, the firm buys the right to exchange its currency at the exercise price e_s; thus, buying Q_C units of option contracts each at $k_C(e_s)$ has a total cost of $k_C(e_s)Q_C$. In stage 2, the firm only exercises this option when the realized exchange rate e is greater than e_s, i.e., when the cost of the foreign supplier becomes expensive with $e \geq e_s$. Exercising the option means paying e_sQ_C to the financial institution and obtaining eQ_C. Thus, the return from each contract in stage 2 is equal to $r_C(e_s) = (e - e_s)^+$ and its value is always positive.

Let us next examine the premium, i.e., the unit purchasing cost, of these financial hedging instruments. The financial institution is going to cover its own expected losses, and therefore, will sell a futures contract at the following cost:

$$k_C(e_s) = \int_{e_s}^{e_h} (e - e_s) g(e) de - \int_{e_l}^{e_s} (e_s - e) g(e) de + \Delta \tag{10}$$

where $\Delta \geq 0$ represents the friction cost including the buying firm's transactions costs and the financial institution's profit. In practice, it is possible that Δ exceeds the selling firm's friction cost. In such cases (i.e., contango), the additional premium may reflect the market's willingness to pay for the lower risk of a certain (versus uncertain) price in the future. Conceivably, the value of Δ could also be less than the selling firm's friction costs (i.e., backwardation), though this can create arbitrage opportunities and is likely to be short-lived. Our results below are linked to certain ranges or values of Δ, and thus help to show how results change when Δ is low or high.

If the financial instrument is a currency option, then the financial institution will sell it at the following unit cost:

$$k_C(e_s) = \int_{e_s}^{e_h} (e - e_s) g(e) de + \Delta . \tag{11}$$

If the firm purchased futures contracts, all contracts purchased in stage 1 must be exercised in stage 2 regardless of the realization of the exchange rate; this implies that $q_C = Q_C$. However, if the firm purchased a currency option contract, it is only exercised when the realized exchange rate is greater than or equal to the strike price, i.e., when $e \geq e_s$. Thus, q_C can be less than Q_C under the currency option contracts. It is easy to see that a currency option contract is valuable when it has a strike price $e_s \leq e_\alpha$. Then, at realizations of $e > e_s$, the firm would exercise all of its currency option contracts, i.e., $q_C = Q_C$, and protect the realized profit in the VaR constraint in (2). We can then express the VaR constraint as follows:

$$P_{(\tilde{e},\tilde{x})}\left[\left\{\begin{array}{l} -k_H Q_H - k_F Q_F - k_C(e_s) Q_C - (o_H + t_H) q_H^*(\tilde{e}) - (o_F \tilde{e} + t_F) q_F^*(\tilde{e}) \\ +r_C(e_s) Q_C + p \min\left\{\tilde{x}, q_H^*(\tilde{e}) + q_F^*(\tilde{e})\right\} \end{array}\right\} < -\beta\right] \leq \alpha . \tag{12}$$

We consider realistic risk tolerance probabilities, and therefore, examine the case with $e_\alpha > \bar{e}$. The friction cost should be limited for the buying firm to make financial hedging attractive; we limit the friction cost to $\Delta \leq e_\alpha - \bar{e}$ for the futures contracts and $\Delta \leq e_\alpha - e_s - \int_{e_s}^{e_h} (e - e_s) g(e) de$ for the options contract.

The next proposition establishes the number of financial hedging contracts the firm should buy in order to satisfy the VaR constraint in (12).

Proposition 11. *If the capacity reservation decisions Q_H and Q_F do not satisfy the VaR constraint in* (12),

(a) *the firm can purchase the following number of futures contracts that has a strike price e_s and a friction cost $\Delta \le e_\alpha - \bar{e}$*

$$Q_C^*(e_s) = \frac{\left\{k_H Q_H + k_F Q_F + (o_H + t_H) q_H^*(e_\alpha) + (o_F e_\alpha + t_F) q_F^*(e_\alpha) - p \min\left\{x_l, q_H^*(e_\alpha) + q_F^*(e_\alpha)\right\} - \beta\right\}}{(e_\alpha - \bar{e} - \Delta)} \quad (13)$$

and satisfy (12)*; and*

(b) *the firm can purchase the following number of currency option contracts that has a strike price e_s and a friction cost $\Delta \le e_\alpha - e_s - \int_{e_l}^{e_s} (e_s - e) g(e) de$*

$$Q_C^*(e_s) = \frac{\left\{k_H Q_H + k_F Q_F + (o_H + t_H) q_H^*(e_\alpha) + (o_F e_\alpha + t_F) q_F^*(e_\alpha) - p \min\left\{x_l, q_H^*(e_\alpha) + q_F^*(e_\alpha)\right\} - \beta\right\}}{\left(e_\alpha - \bar{e} - \int_{e_l}^{e_S} G(e) de - \Delta\right)} \quad (14)$$

and satisfy (12)*; and*

(c) *when* $\Delta = 0$, *the expected profit* $E[\Pi(Q_H, Q_F, Q_C^*(e_s))]$ *under risk aversion and financial hedging is equivalent to* $E[\Pi(Q_H, Q_F, 0)]$ *of the risk-neutral setting in the absence of the VaR constraint* (12).

Proposition 11(a) and (b) show that financial hedging enables the firm to satisfy the VaR constraint. The number of hedging contracts specified in (13) and (14) account for the losses that can incur at all of the undesirable exchange-rate realizations in the range of $e_\alpha \le e \le e_h$ corresponding to α percent in the cdf. Thus, the number of financial hedging contracts in (13) and (14) guarantee that the firm's losses exceeding β is less than or equal to the firm's tolerated loss probability α.

The comparison of the hedging contracts recommended by (13) and (14) shows that the firm would be buying a greater number of currency option contracts than futures contracts in order to satisfy the VaR constraint in (12). The denominator of (14) is smaller in value than that of (13) while the numerators are identical. Thus, the number of necessary options contracts described by (14) is greater than the minimum number of futures contracts described by (13).

Proposition 11(c) shows that in the absence of friction costs, i.e., when $\Delta = 0$, financial hedging is beneficial in terms of protecting the expected profit. For any capacity reservation decisions Q_H and Q_F made in stage 1, financial hedging enables the firm to obtain the same expected profit it earned in the risk-neutral setting without having to sacrifice initial capacity reservation in order to satisfy the VaR constraint. However, when the friction cost is positive, i.e., $\Delta > 0$, then financial hedging cannot replicate the expected profits obtained in the risk-neutral setting even if it retains the same optimal policy. In the presence of friction costs, the reduction in profits from switching to dual sourcing can be less detrimental than the additional cost of financial hedging. Proposition 10 has shown that the introduction of risk aversion encourages dual sourcing in the absence of financial hedging. Under significant friction costs, it is easy to see that dual sourcing can be preferred over financial hedging. Thus, our findings developed under risk aversion continue to be prevailing even under financial hedging.

The consequence of Proposition 11(c) is that, in the absence of friction costs, the firm's set of potential optimal policies under risk aversion and financial hedging is identical to the set of policies developed in Section 4 for the risk-neutral setting: H, $\mathrm{F_L}$, $\mathrm{H_F}$, $\mathrm{D_R}$ and $\mathrm{D_E}$. Thus, it is concluded that financial hedging not only eliminates the negative consequences of risk aversion, but it can also make our five potentially optimal policies to hold under more general settings.

In sum, we find that our insights into the role of exchange rate uncertainty on optimal sourcing decisions are robust. While the introduction of risk aversion can make dual sourcing more likely, our earlier conclusions are unaffected through the use of financial hedging in risk mitigation.

7.3. Ordering More Quantity Than Reserved Capacity

Capacity reservation contracts designate a maximum amount that can be ordered as the season approaches. Inequalities $q_i \leq Q_i$ i = H, F restrict the amount that can be ordered in stage 2 by the amount of capacity reserved in stage 1. What if the supplier allows for a larger order quantity in stage 2 than the amount of capacity reserved in stage 1? Let $q_H{}^a$ and $q_F{}^a$ denote the additional amount of goods ordered in stage 2 from the domestic and foreign suppliers, respectively. Like q_H and q_F, $q_H{}^a$ and $q_F{}^a$ decisions are made before realizing the demand random variable, making the second-stage problem consist of four sourcing decisions. Let Δ_H and Δ_F (> 0) describe the additional cost from ordering without a reservation contract from the two suppliers. The unit cost of acquiring goods without a reservation contract is described as:

$$C_H{}^a = k_H + o_H + \Delta_H + t_H = c_H + \Delta_H < p$$

$$C_F{}^a(e) = k_F\bar{e} + (o_F + \Delta_F)e + t_F = C_F(e) + \Delta_F e$$

where $C_F(\bar{e}) + \Delta_F\bar{e} < p$ (otherwise the firm would not order additional goods in stage 2). Note that when $\Delta_H = \Delta_F = 0$ the firm does not have sufficient incentive to reserve capacity. The second-stage objective function becomes:

$$\pi^*\left(Q_H, Q_F, Q_C, e, \tilde{x}\right) = \max_{\left(q_H, q_F, q_C, q_H^a, q_F^a\right) \geq 0} \begin{Bmatrix} E\left[\pi_2\left(q_H, q_{F,} q_H^a, q_F^a, q_C, \tilde{x} \mid Q_H, Q_F, Q_C, e\right)\right] \\ \text{s.t. } q_i \leq Q_i, i = H, F, C \end{Bmatrix} \tag{15}$$

$$E\left[\pi_2\left(q_H, q_{F,} q_H^a, q_F^a, q_C, \tilde{x} \mid Q_H, Q_F, Q_C, e\right)\right] = -c_H q_H - c_F\left(e\right) q_F - c_H^a q_H^a - c_F^a q_F^a + r_C\left(e_s\right) q_C + \int_{x_l}^{x_h} p \min\left\{x, q_H + q_F + q_H^a + q_F^a\right\} f\left(x\right) dx$$

The maximum values for q_H^a and q_F^a can be determined by analyzing the sourcing alternatives in isolation: $q_H^{a*} \leq Q_H^a = F^{-1}((p - C_H^a)/p) < Q_H^0$ and $q_F^{a*}(e) \leq q_F^{a0}(e) = F^{-1}((p - C_F^a(e))/p) < q_F^0(e)$. Recall that τ_1 and τ_2 represent the thresholds which equate $c_F(e) = c_H$ and $c_F(e) = p$, respectively. We next define τ_1^a, $\tau_1^{a=}$ and τ_2^a as the threshold points that equate $C_F^a(e) = C_H$, $C_F^a(e) = C_H^a$ and $C_F^a(e) = p$, respectively. From comparison, $\tau_1^a < \tau_1$ and $\tau_2^a < \tau_2$. We present the analysis for the setting when $\tau_1^{a=} < \tau_1$, the opposite setting is analogous. Proposition A8 of the online supplement shows that either foreign or domestic source is utilized for additional supplies depending on the exchange rate realization. This finding implies that the firm does not buy additional units from both sources simultaneously. The next proposition extends the findings of Proposition 1 by incorporating the flexibility to source more than what is reserved in stage 1.

Proposition 12. *The optimal second-stage decisions that maximize* (15) *are as follows:*

$$\left(q_H^*(e), q_F^*(e), q_H^{a*}(e), q_F^{a*}(e)\right) = \begin{cases} \Gamma_1 & \text{if } e_l \leq e < \tau_1^a \\ \Gamma_2 & \text{if } \tau_1^a \leq e < \tau_1^{a=} \\ \Gamma_3 & \text{if } \tau_1^{a=} \leq e \leq \tau_1 \\ \Gamma_4 & \text{if } \tau_1 \leq e \leq \tau_2^a \\ \Gamma_5 & \text{if } \tau_2^a \leq e \leq \tau_2 \\ \Gamma_6 & \text{if } \tau_2 \leq e \leq e_h \end{cases} \tag{16}$$

$$\Gamma_1 = \left(0, \min\left\{Q_F, q_F^0(e)\right\}, 0, \left(q_F^{a0}(e) - Q_F\right)^+\right)$$

$$\Gamma_2 = \left(\min\left\{Q_H, \left(q_H^0 - Q_F\right)^+\right\}, \min\left\{Q_F, q_F^0(e)\right\}, 0, \left(q_F^{a0}(e) - Q_H + Q_F\right)^+\right)$$

$$\Gamma_3 = \left(\min\left\{Q_H, \left(q_H^0 - Q_F\right)^+\right\}, \min\left\{Q_F, q_F^0(e)\right\}, \left(Q_H^{a0} - Q_H - Q_F\right)^+, 0 \right)$$

$$\Gamma_4 = \left(\min\left\{Q_H, q_H^0\right\}, \min\left\{Q_F, \left(q_F^0(e) - Q_H\right)^+\right\}, \left(Q_H^{a0} - Q_H + Q_F\right)^+, 0 \right)$$

$$\Gamma_5 = \left(\min\left\{Q_H, q_H^0\right\}, 0, \left(Q_H^{a0} - Q_H\right)^+, 0 \right)$$

$$\Gamma_6 = \left(\min\left\{Q_H, q_H^0\right\}, 0, \left(Q_H^{a0} - Q_H\right)^+, 0 \right).$$

Proposition 12 establishes when to purchase additional supplies without reserving capacity in stage 1. Let Q_H^{a*} and Q_F^{a*} denote the optimal capacity reservation decisions from the domestic and foreign supplier in the presence of the flexibility to purchase additional units. While the optimal policy structure in stage 1 continues to be the same five policies established in Section 4.1, the following proposition shows that the optimal amount of capacity to be reserved is smaller in the presence of the flexibility to purchase additional units.

Proposition 13. *$Q_H^{a*} \leq Q_H^*$ and $Q_F^{a*} \leq Q_F^*$ in all policies.*

In conclusion, the flexibility to purchase additional units decreases the initial commitment to capacity but it does eliminate it.

7.4. Non-Homogenous Leadtimes

This section examines the impact of non-homogenous leadtimes between the two suppliers. Specifically, we consider the event that the domestic supplier is in close proximity where the firm can postpone the ordering decision q_H until after the random demand is observed. The sequence of events is as follows: In stage 1, the firm reserves capacity Q_H and Q_F from both suppliers. After realizing the random exchange rate e, the firm places an order quantity q_F from the foreign supplier. This decision continues to be made under demand uncertainty and is restricted by the reservation amount from stage 1, i.e., $q_F \leq Q_F$. After observing the random demand x, the firm places the order quantity q_H from the domestic supplier; this decision is constrained by Q_H. Rapid replenishment from a domestic supplier is consistent with quick response manufacturing strategies (Iyer and Bergen 1997).

Proposition 14. *The optimal amount of production from the two suppliers are:*

$$\left(q_H^*(e,x), q_F^*(e)\right) = \begin{cases} \left(\min\left\{Q_H, \left(x - Q_F\right)^+\right\}, \min\left\{Q_F, q_F^0(e)\right\}\right) & \text{if } e_l \le e < \tau_1 \\ \left(\min\left\{Q_H, x\right\}, \min\left\{Q_F, \left(q_F^0(e) - Q_H\right)^+\right\}\right) & \text{if } \tau_1 \le e < \tau_2 \\ \left(\min\left\{Q_H, x\right\}, 0\right) & \text{if } \tau_2 \le e \le e_h \end{cases}. \tag{17}$$

Under the non-homogenous leadtime setting, the returns from the foreign supplier from a unit reserved in stage 1 is identical; thus, the optimal decisions involving the foreign source continue to be the identical to earlier findings. As a result, policies F_L, F_H and D_R (which has a total capacity reservation equal to the amount reserved in the foreign supplier) do not get impacted by this adjustment. The only difference in the five policies established in Section 4.1 is that the optimal reservation amount in policies H and DE increase due to more efficient use of the capacity reserved in the domestic supplier. Let Q_H^p denote the amount of capacity reserved in the domestic supplier under non-homogenous leadtimes.

Proposition 15. $Q_H^{p*} \geq Q_H^*$ *in policies* H *and* DE.

In conclusion, the firm reserves a larger amount of capacity from the domestic supplier in the presence of non-homogenous leadtimes.

8. Conclusions and Managerial Insights

This paper examines the impact of exchange-rate uncertainty on capacity reservation decisions for a global firm. We develop an analytical model for a firm that sources from two suppliers, one domestic and one foreign, and sells in a single market. While demand uncertainty in isolation (i.e., ignoring the impact of exchange-rate uncertainty) does not lead to dual sourcing in the context of capacity reservation contracts, exchange-rate uncertainty creates the incentive for the firm to engage in dual sourcing.

The paper makes four contributions. First, we identify the set of potentially optimal policies and the conditions that lead to these policies. Five potentially optimal sourcing policies emerge: One onshore sourcing, two kinds of offshore souring policies, and two characteristically different dual sourcing policies. One dual sourcing policy commits to a total capacity in an amount it would have reserved under the offshore sourcing policy; but this dual sourcing policy rations the total capacity investment between the domestic and foreign suppliers according to the exchange-rate uncertainty. This dual-sourcing policy can be perceived as a defensive and a conservative approach as it is motivated to negate the detrimental consequences of an appreciating exchange rate making the foreign supplier an expensive source. The same policy foregoes the benefits of a lower cost foreign supplier under devaluing exchange-rate realizations. The second dual sourcing policy features excess capacity. The same amount of capacity is reserved at the foreign supplier as with the offshore sourcing policy, and there is additional capacity reserved at the domestic supplier. However, the amount of capacity reserved at the domestic supplier is less than the amount that would have been reserved under the onshore sourcing policy. Thus, the capacity reserved at the domestic supplier is perceived as a backup capability, and it is intended to be utilized in order to benefit from exchange-rate fluctuations. We show that these five policies can be located by checking four optimality conditions. The conditions clarify how the firm switches its optimal policy choice from one sourcing policy to another with increasing degrees of exchange-rate volatility.

Second, our analysis shows that a lower sourcing cost is not a qualifier to reserve capacity at a supplier under exchange-rate uncertainty. It can be optimal to source only from a foreign supplier that has a higher expected total landed cost. This finding (i) departs from the conclusions of earlier publications; and (ii) goes against the common practice of low-cost sourcing and suggests that managers should think more deeply about their sourcing policies under exchange-rate uncertainty. Moreover, our results show that the firm may reserve capacity only at the foreign supplier even if the expected operational cost (inclusive of production, transportation, duties and localization expenses) is higher than the selling price. Together, these findings exemplify the potential gains from currency fluctuations in the context of capacity reservation contracts in sourcing.

Third, we show that risk aversion makes dual sourcing more desirable. When the firm's optimal policy is offshore sourcing in the risk-neutral setting, the introduction of risk aversion with a VaR requirement can trigger a policy switch to dual sourcing. Thus, under risk aversion, dual sourcing becomes more attractive than single sourcing.

Fourth, financial hedging can help eliminate the negative consequences of risk aversion and enable the firm to replicate the expected profit of the risk-neutral policies. When financial hedging instruments are sold at cost without premiums, the firm obtains the same set of five potentially optimal global sourcing policies in the presence of financial hedging. When there are premiums, the behavior observed under risk aversion continues to be prevalent. We conclude that our policy findings are robust as they continue to hold under risk aversion and financial hedging.

In addition to the above four contributions, our study provides two thought-provoking insights regarding the impact of exchange-rate volatility on capacity reservation decisions. First, the firm may not reserve capacity at a domestic supplier even if the sourcing cost from the domestic supplier is less than the expected cost of a foreign supplier. Fluctuations in exchange rates can create the opportunity to source strategically and improve profitability. Second, dual sourcing arises in two different forms and benefits. Policy D_R is recommended for a firm that is worried about controlling the variations in profits. If the exchange rates are substantially more volatile, then the firm is encouraged to follow policy D_E, to be more aggressive and opportunistic to benefit from fluctuations through capacity reserved beyond needs. While the reserved capacity of the foreign supplier increases with volatility, capacity reservation decisions at the domestic supplier can exhibit both an increasing and decreasing behavior. The conditions for the increasing and decreasing behavior are described under increasing exchange-rate uncertainty. Greater degrees of exchange-rate volatility do not always increase domestic capacity and do not regularly lead to higher flexibility. The firm may prefer to give up some of its allocation flexibility under policy D_E, for example, by reducing its domestic capacity and capitalizing more on the foreign capacity.

Extensions to our study with the flexibility to order more than the initial capacity reserved and non-homogenous leadtimes prove that the five potentially optimal policies are robust. When the firm can order more than its initial reservation amount, the amount of capacity reserved decreases, however, this flexibility does not eliminate any of the five policies. When the two suppliers have non-homogenous leadtimes, even if the supplier is in close proximity to postpone the final order quantity until after the random demand is realized, (i) the same five policies continue to be optimal, (ii) single sourcing policies that feature the foreign supplier is not impacted, and (iii) the initial capacity commitment in domestic supplier increases.

We have stated earlier that our work is motivated from the challenges faced by a specialty furniture maker. Our paper demonstrates numerically how our model applies to the products of this firm. The firm’s prior sourcing decisions have ignored exchange-rate uncertainty and relied on utilizing a single supplier which is selected based on the lower cost. Incorporating exchange-rate uncertainty, our analysis shows that dual sourcing policies are better than single sourcing policies especially when the unit capacity reservation costs are smaller relative to the selling price (e.g., book carts). The firm now engages in dual sourcing for a range of products; both the rationing dual sourcing policy and the dual sourcing policy with excess capacity are utilized among the firm's product portfolio. When the unit capacity reservation costs are relatively high (e.g., office desks and chairs), a single-sourcing policy is desired.

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Online Supplement

Capacity Reservation and Sourcing under Exchange-Rate Uncertainty

Proposition A1. (a) *The first-stage expected profit function for offshore sourcing is concave in Q_F*; (b) $Q_F^0 > 0$.

Proof. (a) The objective function in (1) can be written as follows:

$$
\begin{aligned}
E\left[\Pi\left(Q_H=0,Q_F,Q_C\right)\right] &= -k_F Q_F - k_C\left(e_s\right)Q_C \\
&+\int_{e_l}^{\tau(Q_F)}\left[pD-\left(o_F e+t_F\right)Q_F-\int_{Q_F}^{\infty}p\left(x-Q_F\right)f\left(x\right)dx+r_C\left(e_s\right)Q_C\right]g\left(e\right)de \\
&+\int_{\tau(Q_F)}^{\tau_2}\left[pD-\left(o_F e+t_F\right)q_F^0\left(e\right)+\int_{q_F^0(e)}^{\infty}p\left(x-q_F^0\left(e\right)\right)f\left(x\right)dx+r_C\left(e_s\right)Q_C\right]g\left(e\right)de.
\end{aligned}
$$

The first-order derivative of the profit function with respect to Q_F is:

$$
\frac{\partial E\left[\Pi\left(Q_H=0,Q_F,Q_C\right)\right]}{\partial Q_F}=-k_F+\int_{e_l}^{\tau(Q_F)}\left[-c_F\left(e\right)+p\left(1-F(Q_F)\right)\right]g\left(e\right)de\,. \tag{18}
$$

Let us consider the stage 2 problem. The objective function is

$$
\begin{aligned}
\pi^*\left(q_H,q_F,q_C,e,\tilde{x}\right) &= -\left(o_H+t_H\right)q_H-\left(o_F e+t_F\right)q_F \\
&+\int_{x_l}^{(q_H+q_F)}pxf\left(x\right)dx+\int_{(q_H+q_F)}^{x_h}p\left(q_H+q_F\right)f\left(x\right)dx+r_C\left(e_s\right)q_C
\end{aligned}
$$

and its first- and second-order derivatives are:

$$
\frac{\partial \pi^*\left(q_H,q_F,q_C,e,\tilde{x}\right)}{\partial q_F}=-\left(o_F e+t_F\right)+p\left[1-F\left(q_F\right)\right] \tag{19}
$$

$$
\frac{\partial^2 \pi^*\left(q_H,q_F,q_C,e,\tilde{x}\right)}{\partial {q_F}^2}=-pf\left(q_F\right)\le 0
$$

which proves concavity in q_F.

Recall that we define $\tau(Q_F)$ as the exchange rate value that equates q_F to Q_F. From (16), we have $\tau(Q_F) = (p[1 - F(Q_F)] - t_F)/o_F$ with $\partial\tau(Q_F)/\partial Q_F = - pf(Q_F)/o_F \le 0$. Substituting $\tau(Q_F)$ into (15) and replacing $p[1 - F(Q_F)]$ with $o_F\tau(Q_F) + t_F$, we can rewrite the first-order derivative of the first-stage objective function as:

$$
\frac{\partial E\left[\Pi\left(Q_H=0,Q_F,Q_C\right)\right]}{\partial Q_F}=-k_F+\int_{e_l}^{\tau(Q_F)}\left(o_F\tau\left(Q_F\right)-o_F e\right)g\left(e\right)de\,. \tag{20}
$$

Note that the second-order derivative is non-positive because $\partial\tau(Q_F)/\partial Q_F \le 0$

$$\frac{\partial^2 E\left[\Pi\left(Q_H = 0, Q_F, Q_C\right)\right]}{\partial {Q_F}^2} = o_F \int_{e_l}^{\tau(Q_F)} \frac{d\tau(Q_F)}{\partial Q_F} g\left(e\right) de \le 0;$$

therefore, the first-stage objective function is concave in Q_F. Finally, it should be observed that (17) is the expression that leads to (7).

(b) Assumption (A1) states that $C_F^- = E[c_F(\tilde{e}|\tilde{e} < (p - t_F)/o_F)] + k_F < p$. Observe that at $(Q_H, Q_F) = 0$, we have $F(Q_F) = 1$ and $\tau(Q_F) = \tau_2 = (p - t_F)/o_F$. We can see that the first-order derivative of the first-stage objective function is positive at $Q_F = 0$ due to Assumption (A1):

$$-k_F + \int_{e_l}^{\tau_2} \left(p - o_F e - t_F\right) g\left(e\right) de = -k_F - E[c_F(\tilde{e}|\tilde{e} < (p - t_F)/o_F)] + p > 0;$$

therefore, $Q_F^0 > 0$. ❑

Lemma A1. *In a Capacitated Newsvendor problem, the optimal solution is min* (q^*, Q) *where* q^* *is the optimal quantity for the typical newsvendor problem* (*without capacity constraints*) *and* Q *is the capacity level.*

Lemma A2. *For a newsvendor who is initially granted an initial inventory of* q_I, *it is optimal to order* $(q^* - q_I)^+$ *where* q^* *is the optimal quantity for the typical newsvendor problem* (*no initial inventory*).

Proof of Proposition 1. When there are two suppliers each associated with a different set of second-stage costs, and given capacity level (i.e., Q_H and Q_F), the firm prioritizes sourcing from the less costly supplier. Therefore, by Lemma A1, $q_H^* = \min\{Q_H, q_H^0\}$ when $c_H \le c_F$ and $q_F^* = \min\{Q_F, q_F^0(e)\}$ when $c_H > c_F$.

If the desired level of second-stage production is not completed, then the firm continues to source from the expensive supplier. We present the proof for q_H^*. The proof for q_F^* is analogous.

When $c_H > c_F$, if the foreign supplier is in short of capacity (i.e., $\min\{Q_F, q_F^0(e)\} = Q_F$)), we know from Lemma A2 that the domestic source can still be utilized as long as $q_H^0 > Q_F = q_F^*$. In this case, the optimal production quantity would be $(q_H^0 - Q_F)^+$ if there were no capacity constraints. It follows that $q_H^* = \min\{Q_H, (q_H^0 - Q_F)^+\}$ when Q_H restricts the production quantity. ❑

Proposition A2. *The first-stage expected profit function in region* $R1$ *is jointly concave in* (Q_H, Q_F).

Proof. The profit function in region $R1$ can be expressed as follows:

$$
\begin{aligned}
&E\left[\Pi\left(Q_H,Q_F,Q_C)\middle|R1\right)\right]=-k_HQ_H-k_FQ_F-k_C\left(e_s\right)Q_C\\
&+\int_{e_l}^{\tau_1}\left[pD-\left(o_H+t_H\right)Q_H-\left(o_Fe+t_F\right)Q_F-\int_{Q_H+Q_F}^{\infty}p\left(x-Q_H-Q_F\right)f\left(x\right)dx\right]g\left(e\right)de\\
&+\int_{\tau_1}^{\tau(Q_H+Q_F)}\left[pD-\left(o_H+t_H\right)Q_H-\left(o_Fe+t_F\right)Q_F-\int_{Q_H+Q_F}^{\infty}p\left(x-Q_H-Q_F\right)f\left(x\right)dx\right]g\left(e\right)de \qquad (21)\\
&+\int_{\tau(Q_H+Q_F)}^{\tau(Q_H)}\left[pD-\left(o_H+t_H\right)Q_H-\left(o_Feo+t_F\right)\left(q_F^0(e)-Q_H\right)-\int_{q_F^0(e)}^{\infty}p\left(x-q_F^0(e)\right)f\left(x\right)dx\right]g\left(e\right)de\\
&+\int_{\tau(Q_H)}^{e_h}\left[pD-\left(o_H+t_H\right)Q_H-\int_{Q_H}^{\infty}p\left(x-Q_H\right)f\left(x\right)dx\right]g\left(e\right)de+\int_{e_l}^{e_h}r_C\left(e_s\right)Q_Cg\left(e\right)de
\end{aligned}
$$

$$
\frac{\partial E\left[\Pi\left(Q_H,Q_F,Q_C\middle|R1\right)\right]}{\partial Q_F}=-k_F+\int_{e_l}^{\tau(Q_H+Q_F)}\left[-\left(o_Fe+t_F\right)-p\left[1-F\left(Q_H+Q_F\right)\right]\right]g\left(e\right)de
$$

We know from stage 2 optimization that at the exchange rate $e = \tau(Q_H + Q_F)$, we have $p\left[1-F\left(Q_H+Q_F\right)\right]=o_F\tau\left(Q_H+Q_F\right)+t_F$. Substituting this expression in the first-order derivative, we get

$$
\frac{\partial E\left[\Pi\left(Q_H,Q_F,Q_C\middle|R1\right)\right]}{\partial Q_F}=-k_F+o_F\int_{e_l}^{\tau(Q_H+Q_F)}\left(\tau(Q_H+Q_F)-e\right)g\left(e\right)de\,.
$$

$$
\begin{aligned}
\frac{\partial E\left[\Pi\left(Q_H,Q_F,Q_C\middle|R1\right)\right]}{\partial Q_H}=-k_H&+\int_{e_l}^{\tau(Q_H+Q_F)}\left[-\left(o_H+t_H\right)+p\left[1-F\left(Q_H+Q_F\right)\right]\right]g\left(e\right)de\\
&+\int_{\tau(Q_H+Q_F)}^{\tau(Q_H)}\left[-\left(o_H+t_H\right)+\left(o_Fe+t_F\right)\right]g\left(e\right)de\\
&+\int_{\tau(Q_H)}^{e_h}\left[-\left(o_H+t_H\right)+p\left[1-F\left(Q_H\right)\right]\right]g\left(e\right)de
\end{aligned}
$$

We know from stage 2 optimization that, at the exchange rate $e = \tau(Q)$, we have $p\left[1-F\left(Q\right)\right]=o_F\tau\left(Q\right)+t_F$. We substitute this transformation in the above derivative. Moreover, we add and subtract the following terms: $\int_{e_l}^{\tau(Q_H+Q_F)}\left(o_Fe+t_F\right)g\left(e\right)de$ and $\int_{\tau(Q_H)}^{e_h}\left(o_Fe+t_F\right)g\left(e\right)de$, and obtain

$$
\frac{\partial E\left[\Pi\left(Q_H,Q_F,Q_C\middle|R1\right)\right]}{\partial Q_H}=-C_H+E\left[c_F\left(\tilde{e}\right)\right]
$$

$$+o_F \int_{e_l}^{\tau(Q_H+Q_F)} \left(\tau(Q_H+Q_F)-e\right) g\left(e\right) de - o_F \int_{\tau(Q_H)}^{e_h} \left(e-\tau(Q_H)\right) g\left(e\right) de$$

Because $\frac{\partial \tau\left(Q\right)}{\partial Q} = \frac{\partial\left[\frac{\left(p\left[1-F\left(Q\right)-t_F\right]\right)}{o_F}\right]}{\partial Q} = \frac{-pf\left(Q\right)}{o_F}$, we have

$$\frac{\partial^2 E\left[\Pi\left(Q_H, Q_F, Q_C \middle| R1\right)\right]}{\partial Q_F^2} = \frac{\partial^2 E\left[\Pi\left(Q_H, Q_F, Q_C \middle| R1\right)\right]}{\partial Q_H \partial Q_F} = -pf\left(Q_H+Q_F\right) G\left(\tau(Q_H+Q_F)\right) < 0,$$

$$\frac{\partial^2 E\left[\Pi\left(Q_H, Q_F, Q_C \middle| R1\right)\right]}{\partial Q_H^2} = -pf\left(Q_H+Q_F\right) G\left(\tau(Q_H+Q_F)\right) - pf\left(Q_H\right)\left[1-G\left(\tau(Q_H)\right)\right] < 0,$$

$$\begin{aligned}\frac{\partial^2 E\left[\Pi\left(Q_H, Q_F, Q_C \middle| R1\right)\right]}{\partial Q_H^2} &= \frac{\partial^2 E\left[\Pi\left(Q_H, Q_F, Q_C \middle| R1\right)\right]}{\partial Q_F^2} - pf\left(Q_H\right)\left[1-G\left(\tau(Q_H)\right)\right] \\ &< \frac{\partial^2 E\left[\Pi\left(Q_H, Q_F, Q_C \middle| R1\right)\right]}{\partial Q_F^2}.\end{aligned}$$

The determinant is positive, and the objective function jointly concave in Q_H and Q_F in region $R1$ because

$$\left[\begin{array}{l}\frac{\partial^2 E\left[\Pi\left(Q_H, Q_F, Q_C \middle| R1\right)\right]}{\partial Q_H^2} \\ \times \frac{\partial^2 E\left[\Pi\left(Q_H, Q_F, Q_C \middle| R1\right)\right]}{\partial Q_F^2}\end{array}\right] > \left(\frac{\partial^2 E\left[\Pi\left(Q_H, Q_F, Q_C \middle| R1\right)\right]}{\partial Q_H \partial Q_F}\right)^2 = \left(\frac{\partial^2 E\left[\Pi\left(Q_H, Q_F, Q_C \middle| R1\right)\right]}{\partial Q_H^2}\right)^2. \ \square$$

Proposition A3. *The first-stage expected profit function in region* $R2$ *is jointly concave in* (Q_H, Q_F).

Proof. The objective function in region $R2$ can be expressed as follows:

$$\begin{aligned}E\left[\Pi\left(Q_H, Q_F, Q_C \middle| R2\right)\right] &= -k_H Q_H - k_F Q_F - k_C\left(e_s\right) Q_C \\ &+ \int_{e_l}^{\tau_1}\left[pD - \left(o_H+t_H\right)\left(q_H^0 - Q_F\right) - \left(o_F e + t_F\right) Q_F - \int_{q_H^o}^{\infty} p\left(x - q_H^0\right) f\left(x\right) dx\right] g\left(e\right) de \\ &+ \int_{\tau_1}^{\tau(Q_H)}\left[pD - \left(o_H+t_H\right) Q_H - \left(o_F e + t_F\right)\left(q_F^0(e) - Q_H\right) - \int_{q_F^0(e)}^{\infty} p\left(x - q_F^0(e)\right) f\left(x\right) dx\right] g\left(e\right) de \\ &+ \int_{\tau(Q_H)}^{e_h}\left[pD - \left(o_H+t_H\right) Q_H - \int_{Q_H}^{\infty} p\left(x - Q_H\right) f\left(x\right) dx\right] g\left(e\right) de + \int_{e_l}^{e_h} r_C\left(e_s\right) Q_C g\left(e\right) de\end{aligned} \quad (22)$$

$$\frac{\partial E\left[\Pi\left(Q_H, Q_F, Q_C \middle| R2\right)\right]}{\partial Q_H} = -k_H + \int_{\tau_1}^{\tau(Q_H)} \left[-\left(o_H + t_H\right) + \left(o_F e + t_F\right)\right] g\left(e\right) de$$
$$+ \int_{\tau(Q_H)}^{e_h} \left[-\left(o_H + t_H\right) + p\left[1 - F\left(Q_H\right)\right]\right] g\left(e\right) de$$

We know from stage 2 optimization that, at the exchange rate $e = \tau(Q)$, we have $p\left[1 - F\left(Q\right)\right] = o_F \tau\left(Q\right) + t_F$. We substitute this transformation in the above derivative.

$$\frac{\partial E\left[\Pi\left(Q_H, Q_F, Q_C \middle| R2\right)\right]}{\partial Q_H} = -k_H + \int_{\tau_1}^{\tau(Q_H)} \left[-\left(o_H + t_H\right) + \left(o_F e + t_F\right)\right] g\left(e\right) de$$
$$+ \int_{\tau(Q_H)}^{e_h} \left[-\left(o_H + t_H\right) + o_F \tau\left(Q_H\right) + t_F\right] g\left(e\right) de$$

$$\frac{\partial^2 E\left[\Pi\left(Q_H, Q_F, Q_C \middle| R2\right)\right]}{\partial Q_H^2} = \int_{\tau(Q_H)}^{e_h} o_F \left[\frac{-pf\left(Q_H\right)}{o_F}\right] g\left(e\right) de = -pf\left(Q_H\right)\left[1 - G\left(\tau(Q_H)\right)\right] < 0.$$

Therefore, the objective function is concave in Q_H.

We next show that the objective function is linear in Q_F.

$$\begin{aligned}\frac{\partial E\left[\Pi\left(Q_H, Q_F, Q_C \middle| R2\right)\right]}{\partial Q_F} &= -k_F + \int_{e_l}^{\tau_1} \left[\left(o_H + t_H\right) - \left(o_F e + t_F\right)\right] g\left(e\right) de = -k_F + \int_{e_l}^{\tau_1} \left[c_H - c_F\left(e\right)\right] g\left(e\right) de \\ &= -k_F + E\left[\left(c_H - c_F\left(\tilde{e}\right)\right)^+\right]\end{aligned} \tag{23}$$

Note that equation (23) becomes optimality condition (OC2) in later derivations. Equation (23) is constant and independent of Q_F and Q_H implying that

$$\frac{\partial^2 E\left[\Pi\left(Q_H, Q_F, Q_C \middle| R2\right)\right]}{\partial Q_F^2} = \frac{\partial^2 E\left[\Pi\left(Q_H, Q_F, Q_C \middle| R2\right)\right]}{\partial Q_H \partial Q_F} = 0.$$

The Hessian for the objective function in $R2$ is 0, and the objective function is jointly concave in Q_H and Q_F in region $R2$.❑

Proposition A4. *The first-stage expected profit function in region R3 is jointly concave in* (Q_H, Q_F).

Proof. The objective function in region $R3$ can be expressed as follows:

$$E\left[\Pi\left(Q_H, Q_F, Q_C \middle| R3\right)\right] = -k_H Q_H - k_F Q_F - k_C\left(e_s\right) Q_C$$
$$+ \int_{e_l}^{\tau(Q_F)} \left[pD - \left(o_F e + t_F\right) Q_F - \int_{Q_F}^{\infty} p\left(x - Q_F\right) f\left(x\right) dx\right] g\left(e\right) de$$

$$
\begin{aligned}
&+\int_{\tau(Q_F)}^{\tau_1}\left[pD-\left(o_F e+t_F\right)q_F^o(e)-\int_{q_F^0(e)}^{\infty}p\left(x-q_F^0(e)\right)f\left(x\right)dx\right]g\left(e\right)de\\
&+\int_{\tau_1}^{\tau(Q_H)}\left[pD-\left(o_H+t_H\right)Q_H-\left(o_F e+t_F\right)\left(q_F^0(e)-Q_H\right)-\int_{q_F^0(e)}^{\infty}p\left(x-q_F^0(e)\right)f\left(x\right)dx\right]g\left(e\right)de \quad (24)\\
&+\int_{\tau(Q_H)}^{e_h}\left[pD-\left(o_H+t_H\right)Q_H-\int_{Q_H}^{\infty}p\left(x-Q_H\right)f\left(x\right)dx\right]g\left(e\right)de+\int_{e_l}^{e_h}r_C\left(e_s\right)Q_C g\left(e\right)de
\end{aligned}
$$

$$
\frac{\partial E\left[\Pi\left(Q_H,Q_F,Q_C\,|R3\right)\right]}{\partial Q_F}=-k_F+\int_{e_l}^{\tau(Q_F)}\left(p\left[1-F\left(Q_F\right)\right]-\left(o_F e+t_F\right)\right)g\left(e\right)de
$$

We know from stage 2 optimization that at the exchange rate $e = \tau(Q_F)$, we have $p\left[1-F\left(Q_F\right)\right]=o_F\tau\left(Q_F\right)+t_F$. Substituting this expression in the first-order derivative, we get

$$
\frac{\partial E\left[\Pi\left(Q_H,Q_F,Q_C\,|R3\right)\right]}{\partial Q_F}=-k_F+o_F\int_{e_l}^{\tau(Q_F)}\left(\tau(Q_F)-e\right)g\left(e\right)de
$$

which is identical to (5).

$$
\begin{aligned}
\frac{\partial E\left[\Pi\left(Q_H,Q_F,Q_C\,|R3\right)\right]}{\partial Q_H}=&-k_H-\int_{\tau_1}^{\tau(Q_H)}\left[\left(o_H+t_H\right)-\left(o_F e+t_F\right)\right]g\left(e\right)de\\
&-\int_{\tau(Q_H)}^{e_h}\left(\left(o_H+t_H\right)-p\left[1-F\left(Q_H\right)\right]\right)g\left(e\right)de
\end{aligned}
$$

Note that

$$
\begin{aligned}
E\left[\left(\left(p-c_H\right)-\left(p-c_F\left(e\right)\right)^+\right)^+\right]=&\int_{\tau_1}^{\tau(Q)}\left[\left(\left(p-o_H-t_H\right)-\left(p-o_F e-t_F\right)\right)\right]g\left(e\right)de\\
&+\int_{\tau(Q)}^{e_h}\left[\left(\left(p-o_H-t_H\right)-\left(p-o_F\tau\left(Q\right)-t_F\right)\right)\right]g\left(e\right)de
\end{aligned}
$$

And we know from stage 2 optimization that, at $\tau(Q_H)$, we have $p\left[1-F\left(Q_H\right)\right]=o_F\tau\left(Q_H\right)+t_F$.

We substitute both expressions in the first-order derivative and obtain the following:

$$
\frac{\partial E\left[\Pi\left(Q_H,Q_F,Q_C\,|R3\right)\right]}{\partial Q_H}=-k_H+E\left[\left(\left(p-c_H\right)-\left(p-c_F\left(e\right)\right)^+\right)^+\right]. \quad (25)
$$

Note that in later derivations, (25) becomes the optimality condition (OC4).

Because $\frac{\partial \tau(Q)}{\partial Q} = \frac{\partial\left[\frac{\left(p\left[1-F(Q)-t_F\right]\right)}{o_F}\right]}{\partial Q} = \frac{-pf(Q)}{o_F}$, we have

$$\frac{\partial^2 E\left[\Pi\left(Q_H, Q_F, Q_C \middle| R3\right)\right]}{\partial Q_H^2} = o_F \int_{\tau(Q_H)}^{e_h} \frac{-pf(Q_H)}{o_F} g(e)\,de = -pf(Q_H)\left[1-G\left(\tau(Q_H)\right)\right] < 0$$

$$\frac{\partial^2 E\left[\Pi\left(Q_H, Q_F, Q_C \middle| R3\right)\right]}{\partial Q_F^2} = o_F \int_{e_l}^{\tau(Q_F)} \frac{-pf(Q_F)}{o_F} g(e)\,de = -pf(Q_F)\left[1-G\left(\tau(Q_F)\right)\right] < 0$$

$$\frac{\partial^2 E\left[\Pi\left(Q_H, Q_F, Q_C \middle| R3\right)\right]}{\partial Q_H \partial Q_F} = 0\,.$$

The determinant of the Hessian is positive, leading to joint concavity in region $R3$. ❑

Lemma A3. *The objective function in* (1) *is continuous.*

Proof. Note that

$$\begin{aligned}
\lim_{(Q_H+Q_F)\to q_H^{0-}} E\left[\Pi\left(Q_H, Q_F, Q_C\right)\right] &= \lim_{(Q_H+Q_F)\to q_H^0} E\left[\Pi\left(Q_H, Q_F, Q_C \middle| R1\right)\right] = -k_H Q_H - k_F Q_F - k_C(e_s) Q_C \\
&+ \int_{e_l}^{\tau_1}\left[pD - (o_H + t_H)\left(q_H^0 - Q_F\right) - (o_F e + t_F) Q_F - \int_{q_H^0}^{\infty} p\left(x - q_H^0\right) f(x)\,dx\right] g(e)\,de \\
&+ \int_{\tau_1}^{\tau(Q_H)}\left[\begin{array}{l} pD - (o_H + t_H) Q_H - (o_F e + t_F)\left(q_F^0(e) - Q_H\right) \\ - \int_{q_F^0(e)}^{\infty} p\left(x - q_F^0(e)\right) f(x)\,dx \end{array}\right] g(e)\,de \\
&+ \int_{\tau(Q_H)}^{e_h}\left[pD - (o_H + t_H) Q_H - \int_{Q_H}^{\infty} p\left(x - Q_H\right) f(x)\,dx\right] g(e)\,de \\
&+ \int_{e_l}^{e_h} r_C(e_s) Q_C\, g(e)\,de
\end{aligned}$$

where by definition, $\tau(Q_H + Q_F = q_H^0) = \tau_1$. This expression is equal to

$$E\left[\Pi\left(Q_H, Q_F, Q_C \middle| R2\right)\right]\Big|_{Q_H+Q_F=q_H^0} = \lim_{(Q_H+Q_F)\to q_H^{0+}} E\left[\Pi\left(Q_H, Q_F, Q_C\right)\right] = E\left[\Pi\left(Q_H, Q_F, Q_C\right)\right]\Big|_{Q_H+Q_F=q_H^0}$$

where the first equality holds by definition. Hence, the objective function is continuous along the boundary line of $Q_H + Q_F = q_H^0$.

Similarly, along the boundary $Q_F = q_H^0$,

$$\lim_{Q_F\to q_H^{0+}} E\left[\Pi\left(Q_H, Q_F, Q_C\right)\right] = \lim_{Q_F\to q_H^0} E\left[\Pi\left(Q_H, Q_F, Q_C \middle| R3\right)\right] = -k_H Q_H - k_F q_H^0 - k_C(e_s) Q_C$$

$$
\begin{aligned}
&+\int_{e_l}^{\tau_1}\left[pD-\left(o_F e+t_F\right)q_H^0-\int_{q_H^0}^{\infty}p\left(x-q_H^0\right)f\left(x\right)dx\right]g\left(e\right)de\\
&+\int_{\tau_1}^{\tau(Q_H)}\left[\begin{array}{l} pD-\left(o_H+t_H\right)Q_H-\left(o_F e+t_F\right)\left(q_F^0(e)-Q_H\right)\\ -\int_{q_F^0(e)}^{\infty}p\left(x-q_F^0(e)\right)f\left(x\right)dx \end{array}\right]g\left(e\right)de\\
&+\int_{\tau(Q_H)}^{e_h}\left[pD-\left(o_H+t_H\right)Q_H-\int_{Q_H}^{\infty}p\left(x-Q_H\right)f\left(x\right)dx\right]g\left(e\right)de\\
&+\int_{e_l}^{e_h}r_C\left(e_s\right)Q_C g\left(e\right)de
\end{aligned}
$$

where by definition, $\tau(Q_F = q_H^0) = \tau_1$. This expression is equal to

$$
E\left[\Pi\left(Q_H,Q_F,Q_C\left|R2\right.\right)\right]|_{Q_F=q_H^0}=\lim_{Q_F\to q_H^{0+}}E\left[\Pi\left(Q_H,Q_F,Q_C\right)\right]=E\left[\Pi\left(Q_H,Q_F,Q_C\right)\right]|_{Q_F=q_H^0}
$$

which implies that the objective function is continuous along this boundary line as well. As a result, the objective function is continuous everywhere.❑

Lemma A4. *The profit function in* (1) *is differentiable.*

Proof. Along the boundary line of $Q_H + Q_F = q_H^0$, the left derivative of the objective function with respect to Q_F is

$$
\begin{aligned}
\lim_{(Q_H+Q_F)\to q_H^{0-}}\frac{\partial E\left[\Pi\left(Q_H,Q_F,Q_C\left|R1\right.\right)\right]}{\partial Q_F}&=-k_F+o_F\int_{e_l}^{\tau_1}\left(\tau_1-e\right)g\left(e\right)de\\
&=-k_F+\int_{e_l}^{\tau_1}\left[\left(o_H+t_H\right)-\left(o_F e+t_F\right)\right]g\left(e\right)de\\
&=\frac{\partial E\left[\Pi\left(Q_H,Q_F,Q_C\left|R2\right.\right)\right]}{\partial Q_F}=\frac{\partial E\left[\Pi\left(Q_H,Q_F,Q_C\left|R2\right.\right)\right]}{\partial Q_F}|_{Q_H+Q_F=q_H^0}
\end{aligned}
$$

where the last expression is the right derivative of objective function along the same boundary line.

The left derivative of the objective function with respect to Q_H is

$$
\begin{aligned}
\lim_{(Q_H+Q_F)\to q_H^{0-}}\frac{\partial E\left[\Pi\left(Q_H,Q_F,Q_C\left|R1\right.\right)\right]}{\partial Q_H}&=-\left(k_H+o_H+t_H\right)+o_F\overline{e}+t_F-o_F\int_{\tau(Q_H)}^{e_h}\left(e-\tau(Q_H)\right)g\left(e\right)de\\
&+o_F\int_{e_l}^{\tau_1}\left(\tau_1-e\right)g\left(e\right)de
\end{aligned}
.
$$

By rearranging the terms, we can rewrite the above derivative as follows:

$$\frac{\partial E\left[\Pi\left(Q_H,Q_F,Q_C\left|R1\right.\right)\right]}{\partial Q_H}\Big|_{Q_H+Q_F=q_H^0}=-k_H+\int_{\tau_1}^{\tau(Q_H)}\left[-\left(o_H+t_H\right)+\left(o_Fe+t_F\right)\right]g\left(e\right)de$$

$$+\int_{\tau(Q_H)}^{e_h}\left[-\left(o_H+t_H\right)+o_Ft(Q_H)+t_F\right]g\left(e\right)de$$

$$=\frac{\partial E\left[\Pi\left(Q_H,Q_F,Q_C\left|R2\right.\right)\right]}{\partial Q_H}\Big|_{Q_H+Q_F=q_H^0}$$

By definition, the above derivative is equal to the right derivative of the objective function along the same boundary line. Thus, the objective function is differentiable along the boundary line of $Q_H + Q_F = q_H^0$. Along the boundary line of $Q_F = q_H^0$, the right derivative of the objective function with respect to Q_F is

$$\lim_{Q_F\to q_H^{0+}}\frac{\partial E\left[\Pi\left(Q_H,Q_F,Q_C\right)\right]}{\partial Q_F}=\frac{\partial E\left[\Pi\left(Q_H,Q_F,Q_C\left|R3\right.\right)\right]}{\partial Q_F}\Big|_{Q_F=q_H^0}$$

$$=-k_F+o_F\int_{e_l}^{\tau_1}\left(\tau_1-e\right)g\left(e\right)de=-k_F+\int_{e_l}^{\tau_1}\left[\left(o_H+t_H\right)-\left(o_Fe+t_F\right)\right]g\left(e\right)de$$

$$=\frac{\partial E\left[\Pi\left(Q_H,Q_F,Q_C\left|R2\right.\right)\right]}{\partial Q_F}\Big|_{Q_F=q_H^0}$$

where the last term is the left derivative of objective function along the same boundary line.

The right derivative of the profit function with respect to Q_H is

$$\lim_{Q_F\to q_H^{0+}}\frac{\partial E\left[\Pi\left(Q_H,Q_F,Q_C\left|R3\right.\right)\right]}{\partial Q_H}=-k_H+o_F\int_{\tau_1}^{\tau(Q_H)}\left(e-\tau_1\right)g\left(e\right)de+o_F\int_{\tau(Q_H)}\left(\tau(Q_H)-\tau_1\right)g\left(e\right)de$$

$$=-k_H+\int_{\tau_1}^{\tau(Q_H)}\left[-\left(o_H+t_H\right)+\left(o_Fe+t_F\right)\right]g\left(e\right)de$$

$$+\int_{\tau(Q_H)}^{e_h}\left[-\left(o_H+t_H\right)+o_F\tau(Q_H)+t_F\right]g\left(e\right)de$$

$$=\frac{\partial E\left[\Pi\left(Q_H,Q_F,Q_C\left|R2\right.\right)\right]}{\partial Q_H}\Big|_{Q_F=q_H^0}$$

By definition, the last expression is the left derivative of the objective function along that same boundary line. Therefore, the objective function is differentiable along the boundary line of $Q_F = q_H^0$ as well. In sum, the objective function is differentiable everywhere. ❑

Lemma A5. *Onshore sourcing* (H) *is the optimal policy if and only if* $Q_F^0 < Q_H^0$.

Proof. From Proposition A2, we observe that in $R1$ the shadow price of Q_H monotonically decreases in Q_H as long as the total capacity is fixed. Note that this shadow price is positive at solution ($Q_H = Q_F^0$, Q_F

= 0) due to concavity of the profit function in Q_H. Moreover, the shadow price of Q_F in $R1$ remains the same as long as the total capacity is unchanged. Therefore, the solution ($Q_H = Q_F^0$, $Q_F = 0$) dominates all solutions along the line $Q_H + Q_F = Q_F^0$. However, this dominant solution corresponds to onshore sourcing which itself is dominated by the optimal onshore sourcing policy H ($Q_H^* = Q_H^0$, $Q_F^* = Q_F^0$). Consequently, below the line $Q_H + Q_F = Q_H^0$, the optimal solution is the onshore sourcing policy H. ❑

Proof of Proposition 2. From differentiability (lemmas A3 and A4) and piecewise concavity of the objective function (propositions A2, A3, and A4), it follows that the objective function is jointly concave in Q_H and Q_F everywhere.❑

Proposition A5. *The optimal solution does not lie in region R2.*

Proof of Proposition A5. The first derivative of the objective function in $R2$ with respect to Q_F is given in (20) as follows:

$$\frac{\partial E\left[\Pi\left(Q_H, Q_F, Q_C \middle| R2\right)\right]}{\partial Q_F} = -k_F + \int_{e_l}^{\tau_1}\left[\left(o_H + t_H\right) - \left(o_F e + t_F\right)\right] g(e)\,de = -k_F + \int_{e_l}^{\tau_1}\left[c_H - c_F(e)\right] g(e)\,de\,.$$

The above derivative is constant and is independent of Q_F and Q_H. Therefore, the objective function in region $R2$ is linear in Q_F, which implies that there is no interior solution in this region. ❑

Expressions for Optimality Conditions:

(OC1): $E\left[\left(\left(p - c_F(\tilde{e})\right)^+ - \left(p - C_H\right)\right)^+\right] - k_F > 0$ implies that

$$\int_{e_l}^{\tau_1 + (k_H/o_F)} \left(\left(p - o_F e - t_F\right) - \left(p - o_H - t_H - k_H\right)\right) g(e)\,de - k_F > 0\,. \tag{26}$$

(OC2): $E\left[\left(c_H - c_F(\tilde{e})\right)^+\right] - k_F > 0$ implies that

$$\int_{e_l}^{\tau_1} \left(\left(p - o_F e - t_F\right) - \left(p - o_H - t_H\right)\right) g(e)\,de - k_F > 0\,. \tag{27}$$

(OC3): $\left(p - C_H\right) - \left(E\left[\left(p - c_F(\tilde{e})\right)^+\right] - k_F\right) > 0$ implies that

$$\left(p - o_H - t_H - k_H\right) - \left(\int_{e_l}^{\tau_2} \left(p - o_F e - t_F\right) g(e)\,de - k_F\right) > 0\,. \tag{28}$$

(OC4): $E\left[\left(\left(p - c_H\right) - \left(p - c_F(\tilde{e})\right)^+\right)^+\right] - k_H > 0$ implies that

$$\int_{\tau_1}^{\tau_2}\left(\left(p-o_H-t_H\right)-\left(p-o_Fe-t_F\right)\right)g\left(e\right)de+\int_{\tau_2}^{e_h}\left(p-o_H-t_H\right)g\left(e\right)de-k_H>0\,. \tag{29}$$

Proposition A6. (a) *When* (OC2) *holds*, (OC1) *also holds;* (b) *when* (OC1) *does not hold*, (OC2) *does not hold either*; (c) *when* (OC1) *does not hold*, (OC3) *holds*; (d) *when* (OC3) *does not hold*, (OC1) *holds*.

Proof of Proposition A6. Part (a) requires the comparison of (23) and (27). Let us define the left hand side of (27) as $LOC2$, i.e.,

$$LOC2=\int_{e_l}^{\tau_1}\left(\left(p-o_Fe-t_F\right)-\left(p-o_H-t_H\right)\right)g\left(e\right)de-k_F\,.$$

The LHS of (23) is greater than the LHS of (27). This can be seen from the LHS of (23):

$$\begin{aligned}\int_{e_l}^{\tau_1+(k_H/o_F)}\begin{pmatrix}\left(p-o_Fe-t_F\right)\\-\left(p-o_H-t_H-k_H\right)\end{pmatrix}g\left(e\right)de-k_F&=\int_{e_l}^{\tau_1}\begin{pmatrix}\left(p-o_Fe-t_F\right)\\-\left(p-o_H-t_H\right)\\+k_H\end{pmatrix}g\left(e\right)de-k_F\\&\quad+\int_{e_l}^{\tau_1}k_Hg\left(e\right)de+\int_{\tau_1}^{\tau_1+(k_H/o_F)}\begin{pmatrix}\left(p-o_Fe-t_F\right)\\-\left(p-o_H-t_H\right)\\+k_H\end{pmatrix}g\left(e\right)de\\&=LOC2+\int_{e_l}^{\tau_1}k_Hg\left(e\right)de+\int_{\tau_1}^{\tau_1+(k_H/o_F)}\begin{pmatrix}\left(p-o_Fe-t_F\right)\\-\left(p-o_H-t_H\right)\\+k_H\end{pmatrix}g\left(e\right)de\\&>LOC2\end{aligned}$$

because

$$\int_{e_l}^{\tau_1}k_Hg\left(e\right)de+\int_{\tau_1}^{\tau_1+(k_H/o_F)}\left(\left(p-o_Fe-t_F\right)-\left(p-o_H-t_H\right)+k_H\right)g\left(e\right)de>0$$

in the region $\tau_1 \le e \le \tau_1 + (k_H/o_F)$. The LHS of (23) is greater than the LHS of (27).

Part (a): When (OC2) holds, the LHS of (27) is positive. Because the LHS of (23) is greater than the LHS of (23), the LHS of(27) is also positive. This implies that when (OC2) holds, (OC1) also holds.

Part (b): The relationship holds because it is the contrapositive of the relation given in Part (a).

Part (c): The proof requires the comparison of (23) and (28). Let us define the LHS of (23) as $LOC1$, i.e.,

$$LOC1=\int_{e_l}^{\tau_1+(k_H/o_F)}\left(\left(p-o_Fe-t_F\right)-\left(p-o_H-t_H-k_H\right)\right)g\left(e\right)de-k_F\,.$$

When (OC1) does not hold, $LOC1$ is negative. Let us rewrite (OC3) as follows:

$$\left(\int_{e_l}^{\tau}\left(p-o_F e-t_F\right)g\left(e\right)de-k_F\right)-\left(p-o_H-t_H-k_H\right)<0. \tag{30}$$

The LHS of (30) can be rewritten as follows:

$$\begin{aligned}\left(\int_{e_l}^{\tau_2}\left(p-o_F e-t_F\right)g\left(e\right)de-k_F\right)-\left(p-o_H-t_H-k_H\right)=&\int_{e_l}^{\tau_1+\left(k_H/o_F\right)}\left[\left(p-o_F e-t_F\right)-\left(p-o_H-t_H-k_H\right)\right]g\left(e\right)de\\&-k_F\\&-\int_{\tau_1+\left(k_H/o_F\right)}^{\tau_2}\left[\left(p-o_H-t_H-k_H\right)-\left(p-o_F e-t_F\right)\right]g\left(e\right)de\\&-\int_{\tau_1}^{\tau_2}\left[\left(p-o_H-t_H-k_H\right)\right]g\left(e\right)de\\=&\,LOC1\\&-\int_{\tau_1+\left(k_H/o_F\right)}^{\tau_2}\left[\left(p-o_H-t_H-k_H\right)-\left(p-o_F e-t_F\right)\right]g\left(e\right)de\\&-\int_{\tau_1}^{\tau_2}\left[\left(p-o_H-t_H-k_H\right)\right]g\left(e\right)de\\<&\,LOC1\end{aligned}$$

If $LOC1$ is negative, then the LHS of (30) is also negative because the remaining two terms are both negative. i.e.,

$$-\int_{\tau_1+\left(k_H/o_F\right)}^{\tau_2}\left[\left(p-o_H-t_H-k_H\right)-\left(p-o_F e-t_F\right)\right]g\left(e\right)de-\int_{\tau_1}^{\tau_2}\left[\left(p-o_H-t_H-k_H\right)\right]g\left(e\right)de<0.$$

Thus, when (OC1) is not satisfied, the LHS of (23) takes a negative value, and this means that the LHS of (30) is also negative, and therefore, (OC3) holds.

Part (d): The relationship holds because it is the contrapositive of the relation given in Part (c). ❑

Optimal Sourcing Policy / Optimality Condition	Onshore Sourcing	Offshore Sourcing		Dual Sourcing	
	H	F_L	F_H	D_R	D_E
(OC1)	×			✓	
(OC2)		×	✓	×	✓
(OC3)		×		✓	
(OC4)			×		✓

Table A1. Necessary and sufficient conditions for the first-stage optimal decisions. A check mark ("✓") indicates that the corresponding inequality (i.e., optimality condition) holds when a particular sourcing policy is optimal, and a cross mark ("×") indicates that the opposite inequality holds.

Proof of Proposition 3. Proposition 2 has shown that the objective function in (1) is jointly concave in Q_H and Q_F, and Proposition A5 has shown that there is no interior solution in region $R2$ where the shadow price of Q_F is constant (independent of Q_H and Q_F). Therefore, the sign of this shadow price in (20), equivalent to (OC2), leads us to the region where the optimal solution is located:

$$\frac{\partial E\left[\Pi\left(Q_H, Q_F, Q_C \middle| R2\right)\right]}{\partial Q_F} = -k_F + \int_{e_l}^{\tau_1}\left[c_H - c_F\left(e\right)\right] g\left(e\right) de \, .$$

If it is positive (negative), the optimal solution lies in region $R3$ (region $R1$) and that the optimal solution is an interior solution.

Suppose the shadow price in (20) is positive, i.e., (OC2) holds corresponding to

$$\int_{e_l}^{\tau_1}\left[c_H - c_F\left(e\right)\right] g\left(e\right) de > k_F \tag{31}$$

The proof for parts (c) and (e) of the proposition requires analyzing region $R3$.

Region $R3$: In this region, the first-order condition for Q_F corresponds to the expression in (7), and it results in the same optimal solution Q_F^0.

In order for Q_H^* to be positive, we must have $\frac{\partial E\left[\Pi\left(Q_H, Q_F, Q_C \middle| R3\right)\right]}{\partial Q_H}\Big|_{Q_H=0}$ as positive. It is shown in the proof of Proposition A4 that this derivative does not depend on the value of Q_F. Hence,

$$\frac{\partial E\left[\Pi\left(Q_H, Q_F, Q_C \middle| R3\right)\right]}{\partial Q_H}\Big|_{Q_H=0} = -k_H + \int_{\tau_1}^{\tau_2}\left(\left(p - o_H - t_H\right) - \left(p - o_F e - t_F\right)\right) g\left(e\right) de$$
$$+\int_{\tau_2}^{e_h}\left(p - o_H - t_H\right) g\left(e\right) de - k_H$$

must be positive, which is equivalent to saying (OC4) holds. Therefore, in order for policy D_E to be optimal, conditions (OC4) and (OC2) must hold. This completes the proof for part (e). Otherwise, i.e., when the condition (OC4) does not hold, the first-order derivative with respect to Q_H is negative and $Q_H^* = 0$; then the offshore sourcing policy F_H becomes optimal. This completes part (c) of the proof.

Moreover, defining $\tau' = \tau\,(Q_H + Q_F = Q_H^0) = (o_H + t_H + k_H - t_F)/o_F$

$$\frac{\partial E\left[\Pi\left(Q_H, Q_F, Q_C \middle| R3\right)\right]}{\partial Q_H}\Big|_{Q_H=Q_H^0} = \begin{bmatrix} E\left[\left(\left(p - c_F\left(\tilde{e}\right)\right)^+ - \left(p - c_H\right)\right)^+\right] \\ -E\left[\left(\left(p - c_F\left(\tilde{e}\right)\right)^+ - \left(p - C_H\right)\right)^+\right] \end{bmatrix} < 0 \qquad (32)$$

As a result, Q_H^* is lower than Q_H^0 in this region.

For parts (a), (b) and (d) of this proposition, we investigate Region $R1$.

Note that (OC2) is identical to inequality (31). Therefore, if OC2 does not hold (i.e., the first-order derivative of the first-stage objective function with respect to Q_F in region $R2$ is negative), the optimal solution must lie in region $R1$.

Region $R1$: In this region, we know that

$$Q_H^* + Q_F^* = Q_F^0\,. \qquad (33)$$

From (33), we have $\frac{\partial E\left[\Pi\left(Q_H, Q_F, Q_C \middle| R1\right)\right]}{\partial Q_F}\Big|_{\left(Q_H+Q_F=Q_F^0\right)} = 0$ at the optimal solution. In order for Q_H^* to be positive, we need $\frac{\partial E\left[\Pi\left(Q_H, Q_F, Q_C \middle| R1\right)\right]}{\partial Q_H}\Big|_{Q_H=0,\,Q_F=Q_F^0}$ to be positive. We have

$$\frac{\partial E\left[\Pi\left(Q_H, Q_F, Q_C \middle| R1\right)\right]}{\partial Q_H}\Big|_{Q_H=0,\,Q_F=Q_F^0} = \left(\frac{\partial E\left[\Pi\left(Q_H, Q_F, Q_C \middle| R1\right)\right]}{\partial Q_H} - \frac{\partial E\left[\Pi\left(Q_H, Q_F, Q_C \middle| R1\right)\right]}{\partial Q_F}\right)\Big|_{Q_H=0,\,Q_F=Q_F^0}$$

$$\frac{\partial E\left[\Pi\left(Q_H, Q_F, Q_C \middle| R1\right)\right]}{\partial Q_H}\Big|_{Q_H=0,\,Q_F=Q_F^0} = \left(p - o_H - t_H - k_H\right) - \left(\int_{e_l}^{\tau_2}\left(p - o_F e - t_F\right) g\left(e\right) de - k_F\right). \qquad (34)$$

This is equivalent to the LHS of (OC3). Thus, the expression in (30) must be positive for D_R to be optimal. Otherwise, $Q_H^* = 0$ corresponding to the offshore sourcing policy F_L – this completes part (b) of the proof.

On the other hand, from Lemma A5, in order for Q_F^* to be positive, $Q_F^0 > Q_H^0$ must hold. Equivalently, the derivative of the first-stage objective function with respect to Q_F along the line $Q_H + Q_F = Q_H^0$ (i.e., $\frac{\partial E\left[\Pi\left(Q_H, Q_F, Q_C \middle| R1\right)\right]}{\partial Q_F}\Big|_{Q_H+Q_F=Q_H^0}$) must be positive. We have

$$\frac{\partial E\left[\Pi\left(Q_H, Q_F, Q_C \middle| R1\right)\right]}{\partial Q_F}\Big|_{Q_H+Q_F=Q_H^0} = E\left[\left(\left(p - c_F\left(\tilde{e}\right)\right)^+ - \left(p - C_H\right)\right)^+\right] - k_F . \tag{35}$$

Hence, $E\left[\left(\left(p - c_F\left(\tilde{e}\right)\right)^+ - \left(p - C_H\right)\right)^+\right] - k_F$, corresponding to the LHS of (OC1) must also be positive for D_R to be optimal. This completes part (d) of the proof. Otherwise, when the LHS of (OC1), equivalently (35) is negative, we have $Q_F^* = 0$ corresponding to the onshore sourcing policy H. This completes part (a) of the proof.

Furthermore, as $\frac{\partial E\left[\Pi\left(Q_H, Q_F, Q_C\right)\right]}{\partial Q_H}\Big|_{Q_H=Q_H^0, Q_F=0} = 0$ by definition, and $\frac{\partial^2 E\left[\Pi\left(Q_H, Q_F, Q_C\right)\right]}{\partial Q_H \partial Q_F} < 0$, we have $\frac{\partial E\left[\Pi\left(Q_H, Q_F, Q_C\right)\right]}{\partial Q_H}\Big|_{Q_H=Q_H^0, Q_F>0} < 0$, which implies that Q_H^* must be lower than Q_H^0 in region $R1$. ❑

<u>The Special Case of Low Volatility in Exchange Rate and/or High Profit Margin:</u>

This is the special case where the volatility in exchange rate is so low, or the profit margin is so high, that $p - c_F(e) > 0$ for all exchange-rate realizations in the second stage (i.e., $\tau_2 > e_h$ in Figure 3). First, the set of potentially optimal solutions remains the same. This is because the expected profit function in the first stage does not depend on τ_2 in any of the regions. Moreover, the optimality conditions remain the same. The reason is as follows: First, inequality (31) is independent of τ_2. Second, in the derivations of equations, (28), (32), (31) and (35), τ_2 is replaced with e_h, which causes $E[(p - c_F(\tilde{e}))^+]$ to be replaced with $E[(p - c_F(\tilde{e}))]$. Therefore, if m_F is always positive, then we can simplify all of the optimality conditions by replacing $E[(p - c_F(\tilde{e}))^+]$ with $E[(p - c_F(\tilde{e}))]$. However, keeping $E[(p - c_F(\tilde{e}))^+]$ leads to the general conditions.

Therefore, the set of potentially optimal solutions and the optimality conditions are robust to the magnitude of exchange-rate volatility and to the variation in profit margin. ❑

Proof of Proposition 4. Note that the first-order derivative of the objective function in an offshore sourcing policy with respect to Q_F evaluated at q_H^0 is equal to the derivative of the global sourcing objective function with respect to Q_F evaluated at the same point. i.e.,

$$\frac{\partial E\left[\Pi\left(Q_H = 0, Q_F, Q_C\right)\right]}{\partial Q_F}\Big|_{Q_F=q_H^0} = \frac{\partial E\left[\Pi\left(Q_H, Q_F, Q_C\right)\right]}{\partial Q_F}\Big|_{Q_F=q_H^0} = \int_{e_l}^{\tau_1}\left(\left(p - o_F e - t_F\right) - \left(p - o_H - t_H\right)\right)g\left(e\right)de - k_F$$

which is equivalent to the LHS of (OC1).

Hence, $Q_F^0 > q_H^0$ is equivalent to the optimal global sourcing policy being in region $R3$, and $Q_F^0 < q_H^0$ is equivalent to the optimal global sourcing policy being in region $R1$. Moreover, for part (b), $Q_F^0 > Q_H^0$ ensures that the optimal policy is not H. ❑

Proof of Proposition 5. Part (a): If $C_F < C_H$, we have $k_F + o_F\,\bar{e} + t_F < k_H + o_H + t_H$; then $p - o_F\,\bar{e} - t_F - k_F > p - o_H - t_H - k_H$, which means $E[(p - c_F(\tilde{e}))] - - k_F > p - o_H - t_H - k_H$. This has two implications: First, the opposite of condition (OC3) holds (i.e., $E[(p - c_F(\tilde{e}))^+] > p - o_H - t_H - k_H$), which in turn implies D_R is never optimal in this case. Second, condition (OC1) holds because

$$E[((p - c_F(\tilde{e}))^+ - (p - C_H))^+] \geq E[(p - c_F(\tilde{e}))^+ - (p - C_H)]$$
$$= E[(p - c_F(\tilde{e}))^+] - (p - C_H) > E[(p - c_F(\tilde{e}))] - (p - C_H) > k_F.$$

Therefore, from Table A1, if condition (OC2) does not hold, then policy F_L is optimal. Otherwise, if condition (OC2) holds but condition (OC4) does not hold, then policy F_H is optimal. Finally, if both conditions (OC2) and (OC4) hold, then policy D_E is optimal.

Part (b): When $C_F \geq C_H$, the inequality $k_F + c_F\bar{e} + t_F \geq k_H + c_H + t_H$ by itself does not imply that the optimality conditions hold or not. If condition (OC2) holds and condition (OC4) does not hold, then the optimal policy is F_H. Moreover, if conditions (OC2) and (OC3) do not hold and (OC1) holds, the optimal policy is F_L. ❑

Proof of Proposition 6. The inequality $p - o_F\bar{e} - t_F < 0$ does not eliminate the possibility of the optimality conditions associated with dual sourcing and offshore sourcing to hold. Thus, they may still be optimal sourcing policies. ❑

Proof of Proposition 7. (a) In region $R3$, the shadow price of Q_F is

$$\lambda_{Q_F}^{R3} = \frac{\partial E\left[\Pi\left(Q_H, Q_F, Q_C \,|R3\right)\right]}{\partial Q_F} = -k_F + o_F \int_{e_l}^{e_h} \left(\tau(Q_F) - e\right)^+ g\left(e\right) de$$

Let us define $n(e) = (\tau(Q_F) - e)^+$. Then, the shadow price can be expressed as $-k_F + o_F E\left[n\left(\tilde{e}\right)\right]$. Note that $n(e)$ is piecewise linear and convex in e. Therefore, by definition of second-degree stochastic dominance, $E\left[n\left(\ddot{e}\right)\right] \geq E\left[n\left(\dot{e}\right)\right]$ if $\ddot{e}$ is a mean-preserving spread of $\dot{e}$. Consequently, the higher exchange-rate volatility, the higher the shadow price, and the higher the value of Q_F^* in this region.

In region $R1$, the shadow price of Q_F is

$$\lambda_{Q_F}^{R1} = \frac{\partial E\left[\Pi\left(Q_H, Q_F, Q_C \,|R1\right)\right]}{\partial Q_F} = -k_F + o_F \int_{e_l}^{e_h} \left(\tau(Q_H + Q_F) - e\right)^+ g\left(e\right) de$$

By similar argument Q_F^* increases in exchange-rate volatility in this region as well.

Note that in region $R2$, the shadow price is constant, and its sign, positive or negative, determines that the optimal solution is whether in region $R3$ or region $R1$, respectively. This shadow price is

$$E\left[\left(\left(p-c_F\left(\tilde{e}\right)\right)^+-\left(p-c_H\right)\right)^+\right]-k_F=\int_{e_l}^{\tau_1}\left(\left(p-o_Fe-t_F\right)-\left(p-o_H-t_H\right)\right)g\left(e\right)de-k_F$$
$$=\int_{e_l}^{\tau_1}\left(o_H+t_H-o_Fe-t_F\right)g\left(e\right)de-k_F$$

where $(o_H + t_H - o_F e - t_F)$ is convex in e. Therefore, this shadow price increases in exchange-rate volatility, which implies that as volatility increases the location of the optimal solution may switch from region $R1$ to region $R3$ but not in an opposite way. Since Q_F^* is always higher in region $R3$ than in region $R1$, it follows that Q_F^* always increases in exchange-rate volatility.

For Q_H^*, the shadow price in region $R3$ is

$$\lambda_{Q_H}^{R3}=\frac{\partial E\left[\Pi\left(Q_H,Q_F,Q_C\left|R3\right.\right)\right]}{\partial Q_H}=-\left(k_H+o_H+t_H\right)+o_F\overline{e}+t_F+o_FE\left[n\left(\tilde{e}\right)\right]$$

where $n(e)=[(\tau_1-e)^+-(e-\tau(Q_H))^+]$ is piecewise linear but neither convex nor concave in e, which leads to an inconclusive result regarding the behavior of the domestic capacity in exchange-rate volatility.

(b) Let us consider the second-stage expected profit expression in (5). Using the Envelope Theorem, we have

$$\frac{\partial\pi^*\left(Q_H,Q_F,Q_C,e\right)}{\partial e}=\frac{\partial E\left[\pi_2\left(q_H^*,q_F^*,q_C,x\mid Q_H,Q_F,Q_C,e\right)\right]}{\partial e}$$
$$=\left(\frac{\partial E\left[\pi_2\left(q_H,q_F,q_C,x\mid Q_H,Q_F,Q_C,e\right)\right]}{\partial e}\right)\Bigg|_{q_H=q_H^*,q_F=q_F^*}\tag{36}$$

where

$$E\left[\pi_2\left(q_H,q_F,x\mid Q_H,Q_F,e\right)\right]=-\left(o_H+t_H\right)q_H-\left(o_Fe+t_F\right)q_F+\int_{x_l}^{x_h}p\min\left\{x,q_H+q_F\right\}f\left(x\right)dx$$

We have $\frac{\partial E\left[\pi_2\left(q_H,q_F,q_C,x\mid Q_H,Q_F,Q_C,e\right)\right]}{\partial e}=-o_Fq_F$. Thus, from (32), $\frac{\partial\pi^*\left(Q_H,Q_F,Q_C,e\right)}{\partial e}=-o_Fq_F^*$ which implies $\frac{\partial^2\pi^*\left(Q_H,Q_F,Q_C,e\right)}{\partial e^2}=-o_F\frac{\partial q_F^*(e)}{\partial e}$ which is positive because q_F^* decreases in e.

Consequently, $\pi^*\left(Q_H,Q_F,Q_C,e\right)$ is convex in e, which by the definition of second-order stochastic dominance implies $E\left[\pi^*\left(Q_H,Q_F,Q_C,\ddot{e}\right)\right]>E\left[\pi^*\left(Q_H,Q_F,Q_C,\dot{e}\right)\right]$ if $\ddot{e}$ is a mean-preserving spread of $\dot{e}$. As a result, the first-stage expected profit increases in exchange-rate volatility.❑

Proof of Proposition 8. It is sufficient to examine the first-order derivative with respect to Q_H in Region $R3$. We employ a uniformly distributed exchange rate ($\tilde{e} \sim U\left[\bar{e}-d, \bar{e}+d\right]$), we have

$$o_F \int_{\tau(Q_H)}^{\bar{e}+d} \left(e-\tau(Q_H)\right)\frac{1}{2d}de - \int_{\tau_1}^{\bar{e}+d}\left(\left(o_F e + t_F\right)-\left(o_H + t_H\right)\right)\frac{1}{2d}de + k_H = 0$$

$$\frac{o_F}{4d}\left(\bar{e}+d-\tau(Q_H)\right)^2 - \frac{\left[o_F\left(\bar{e}+d\right)+t_F-\left(o_H+t_H\right)\right]^2}{4do_F} + k_H = 0$$

$$\left(\bar{e}+d-\tau(Q_H)\right)^2 = \frac{\left[o_F\left(\bar{e}+d\right)+t_F-\left(o_H+t_H\right)\right]^2 - 4do_F k_H}{{o_F}^2}.$$

The feasible solution to this quadratic equation is

$$\tau_H^* = \bar{e}+d-\frac{1}{o_F}\sqrt{\left[o_F\left(\bar{e}+d\right)+t_F-\left(o_H+t_H\right)\right]^2 - 4do_F k_H}$$

The square root term is guaranteed to be positive by OC4 so that it holds under policy D_E. It follows that

$$\frac{\partial \tau_H^*}{\partial d} = 1 - \frac{\left[o_F\left(\bar{e}+d\right)+t_F-\left(o_H+t_H\right)\right]-2k_H}{\sqrt{\left[o_F\left(\bar{e}+d\right)+t_F-\left(o_H+t_H\right)\right]^2 - 4do_F k_H}}.$$

The root of the above partial derivative occurs when

$$\left[o_F\left(\bar{e}+d\right)+t_F-\left(o_H+t_H\right)\right]^2 - 4do_F k_H - \left(\left[o_F\left(\bar{e}+d\right)+t_F-\left(o_H+t_H\right)\right]-2k_H\right)^2 = 0$$

which reduces to $o_F\bar{e} + t_F - (k_H + o_H + t_H) = 0$. As a result, when $k_H + o_H + t_H < o_F\bar{e} + t_F$, we have $\frac{\partial \tau_H^*}{\partial d} > 0$ which by definition ($\tau_H{}^* = \tau(Q_H{}^*) = (p\,[(1 - F(Q_H{}^*)] - t_F)/o_F$) implies $\frac{\partial Q_H^*}{\partial d} < 0$. ❑

Proof of Proposition 9. In case 1, Proposition 5(a) established the fact that the potentially optimal policies are F_L, F_H, and D_E. From Proposition 8, we know that the LHS of (OC2) and $Q_F{}^*$ increase in exchange-rate volatility. As a result, the optimal policy can switch from F_L in region $R1$ to F_H in region $R3$, and not in the opposite way. Depending on whether (OC4) holds in region $R3$, the optimal policy may change to D_E.

In case 2, all policies can potentially be optimal. At sufficiently low degrees of exchange-rate volatility, the firm adopts policy H due to the lower cost of onshore sourcing. Note that under policy H, when (OC1) does not hold, (OC2) does not hold either. Moreover, Proposition A6(c) established that if (OC1) does not hold, then (OC3) holds.

Higher levels of exchange-rate volatility cause the LHS of (OC1) to increase (by a similar argument presented in the proof of proposition 8) until (OC1) holds. At this switching point, (OC3) still holds. This is where D_R becomes the optimal policy. As exchange-rate volatility increases, the LHS of (OC2) increases and the LHS of (OC3) decreases. The latter is because $E[(p - c_F(\tilde{e}))^+]$ is convex in e, thus $E[(p$

$- c_F(\tilde{e}))^+] - k_F - C_H$ increases in exchange-rate volatility by a similar argument as made in the proof of Proposition 8. Depending on whether (OC3) does not hold or (OC2) holds first, the optimal policy path is different. If (OC3) does not hold first, F_L becomes the optimal policy first and similar to case 1, the next optimal policies are F_H (when (OC2) holds) and D_E if (OC4) holds as well. Otherwise, if (OC2) holds before (OC3) is reversed, the optimal solution moves to *R*3. But this also causes (OC4) to hold because the LHS of (OC4) is the summation of the left-hand sides of (OC2) and (OC3). Therefore, if both (OC2) and (OC3) hold, then (OC4) holds as well. That implies that the optimal solution may directly switch from D_R to D_E.❑

Proof of Proposition 10. (a) For $e_\alpha \geq \tau_2$, we have $q_F^*(e_\alpha) = 0$. By supposition, we have

$$P_{(\tilde{e},\tilde{x})}\left[\Pi\left(0, Q_F^*, 0\right) < -\beta\right] > \alpha .$$

Furthermore,

$$\Pi\left(0, Q_F^A, 0 \mid e_\alpha\right) = \Pi\left(0, Q_F^A, 0 \mid e\right) = -\beta \text{ for all } e \geq \tau_2,$$

which implies $P_{(\tilde{e},\tilde{x})}\left[\Pi\left(0, Q_F^A, Q_C = 0\right) < -\beta\right] = 0$. Note that for all $e \geq \tau_2$,

$$\Pi\left(0, Q_F, 0 \mid e_\alpha\right) = \Pi\left(0, Q_F, 0 \mid e\right) < -\beta \text{ for all } Q_F > Q_F^A \text{ and } e \geq \tau_2,$$

and for all $Q_F > Q_F^A$ and for some $e < \tau_2$,

$$\Pi\left(0, Q_F, 0 \mid e\right) = -\beta \text{ for all } Q_F > Q_F^A \text{ and some } e < \tau_2,$$

which implies $P_{(\tilde{e},\tilde{x})}\left[\Pi\left(0, Q_F, Q_C = 0\right) < -\beta\right] > \alpha$ for all $Q_F > Q_F^A$. Thus, since the risk-neutral optimal solution Q_F^* is larger than Q_F^A, it follows that Q_F^A is the unique optimal reservation quantity under sole sourcing from the foreign supplier.

Next, suppose that $Q_H > 0$. Define

$$Q_F^A(Q_H) \equiv \left[\left(p - C_H\right)Q_H + \beta\right] / k_F \tag{37}$$

and note that $P_{(\tilde{e},\tilde{x})}\left[\Pi\left(Q_H, Q_F^A(Q_H), 0\right) < -\beta\right] = 0 < \alpha$ for all $0 \leq Q_H \leq x_L$, and $P_{(\tilde{e},\tilde{x})}\left[\Pi\left(Q_H, Q_F, 0\right) < -\beta\right] > \alpha$ for all $Q_F > Q_F^A(Q_H)$ and $0 \leq Q_H \leq x_L$.

Part (a): If $Q_F^A = Q_F^A(0) > q_H^0$, we can substitute the capacity investment in (34) into the first-stage objective function associated with *R*3 and take its first-order derivative with respect to Q_H. The total derivative of that function with respect to Q_H can be expressed as:

$$\frac{dE\left[\Pi\left(Q_H, Q_F^A(Q_H), 0 \middle| R3\right)\right]}{dQ_H} = \frac{\partial E\Pi\left(Q_H, Q_F, 0 \middle| R3\right)}{\partial Q_F}\Big|_{Q_F^A(Q_H)} \cdot \frac{dQ_F^A(Q_H)}{dQ_H} + \frac{\partial E\Pi\left(Q_H, Q_F, 0 \middle| R3\right)}{\partial Q_H}\Big|_{Q_F^A(Q_H)}$$

From Proposition A4,

$$\frac{\partial E\left[\Pi\left(Q_H,Q_F,0\middle|R3\right)\right]}{\partial Q_F}=-k_F+o_F\int_{e_l}^{\tau(Q_F)}\left(\tau(Q_F)-e\right)g\left(e\right)de\text{, and,}$$

$$\frac{\partial E\left[\Pi\left(Q_H,Q_F,0\middle|R3\right)\right]}{\partial Q_H}=-k_H+E\left[\left(\left(p-c_H\right)-\left(p-c_F\right)^+\right)^+\right].$$

Thus,

$$\frac{dE\left[\Pi\left(Q_H,Q_F^A\left(Q_H\right),0\middle|R3\right)\right]}{dQ_H}\Big|_{Q_H=0}=\frac{\left(p-C_H\right)}{k_F}\left[-k_F+o_F\int_{e_l}^{\tau\left(Q_F^A\left(Q_H\right)\right)}\left(\tau\left(Q_F^A\left(Q_H\right)\right)-e\right)g(e)de\right].$$
$$-k_H+E\left[\left(\left(p-c_H\right)-\left(p-c_F\right)^+\right)^+\right]$$

Note that the first term is positive since $Q_F^A < Q_F^0$, and the last two terms form the LHS of (OC4) is negative because of the definition of policy F_H. Because $\int_{e_l}^{\tau}\left(\tau-e\right)g\left(e\right)de=\int_{e_l}^{\tau}G\left(e\right)de$, the above condition is equivalent to RA1. Therefore, when $Q_F^A > q_H^0$, if condition (RA1) holds, the firm satisfies the VaR constraint and increases expected profit by increasing Q_H from 0 to any $Q_H \in (0, x_L]$ while simultaneously increasing Q_F from $Q_F^A\left(0\right)$ to $Q_F^A\left(Q_H\right)$, and thus engages in dual sourcing under risk aversion.

Part (b): If $Q_F^A=Q_F^A\left(0\right)\leq q_H^0$, regardless of whether F_H or F_L is the optimal policy in the risk-neutral setting, we can substitute the capacity investment in (37) into the first-stage objective function associated with region $R1$ and take its first-order derivative with respect to Q_H.

$$\frac{dE\left[\Pi\left(Q_H,Q_F^A\left(Q_H\right),0\middle|R1\right)\right]}{dQ_H}=\frac{\partial E\Pi\left(Q_H,Q_F,0\middle|R1\right)}{\partial Q_F}\Big|_{Q_F^A\left(Q_H\right)}\cdot\frac{dQ_F^A\left(Q_H\right)}{dQ_H}+\frac{\partial E\Pi\left(Q_H,Q_F,0\middle|R1\right)}{\partial Q_H}\Big|_{Q_F^A\left(Q_H\right)}$$

From Proposition A2,

$$\frac{\partial E\left[\Pi\left(Q_H,Q_F,0\middle|R1\right)\right]}{\partial Q_F}=-k_F+o_F\int_{e_l}^{\tau(Q_H+Q_F)}\left(\tau(Q_H+Q_F)-e\right)g\left(e\right)de\text{, and}$$

$$\frac{\partial E\left[\Pi\left(Q_H, Q_F, 0 \middle| R1\right)\right]}{\partial Q_H} = -C_H + E\left[c_F\left(\tilde{e}\right)\right] - o_F \int_{\tau(Q_H)}^{e_h} \left(e - \tau(Q_H)\right) g\left(e\right) de$$

$$+ o_F \int_{e_l}^{\tau(Q_H + Q_F)} \left(\tau(Q_H + Q_F) - e\right) g\left(e\right) de$$

$$= \left(p - C_H\right) - \left(p - E\left[c_F\left(\tilde{e}\right)\right]\right) - o_F \int_{\tau(Q_H)}^{e_h} \left(e - \tau(Q_H)\right) g\left(e\right) de$$

$$+ o_F \int_{e_l}^{\tau(Q_H + Q_F)} \left(\tau(Q_H + Q_F) - e\right) g\left(e\right) de$$

Thus,

$$\frac{dE\left[\Pi\left(Q_H, Q_F^A\left(Q_H\right), 0 \middle| R1\right)\right]}{dQ_H}\Big|_{Q_H = 0} = \left[\frac{\left(p - C_H\right) o_F}{k_F} \int_{e_l}^{\tau\left(Q_F^A\right)} \left(\tau\left(Q_F^A\right) - e\right) g(e) \left(e\right) de\right]$$

$$- E\left[\left(p - c_F\left(\tilde{e}\right)\right)\right] + o_F \int_{e_l}^{\tau\left(Q_F^A\right)} \left[\tau\left(Q_F^A\right) - e\right] g\left(e\right) de$$

Therefore, when $Q_F^A \leq q_H^0$ and dual sourcing is the optimal sourcing policy under risk aversion if

$$\left(\frac{\left(p - C_H\right) o_F}{k_F} + o_F\right) \int_{e_l}^{\tau\left(Q_F^A\right)} G\left(e\right) de > E\left[\left(p - c_F\left(\tilde{e}\right)\right)\right].$$

The above condition provides the condition (RA2). Due to submodularity of the expected profit function, the LHS of (RA2) is larger than that of (RA1). Thus, under risk aversion, it is more likely for dual sourcing to be adopted when $Q_F^A \leq q_H^0$ than when $Q_F^A > q_H^0$. ❑

Proof of Proposition 11. Part (a): Recall that e_α denotes the exchange rate realization at fractile $1 - \alpha$ (i.e., $[1 - G(e_\alpha)] = \alpha$). The realized profit is greater than or equal to $-\beta$ when the exchange-rate random variable takes values in the range $e_l \leq e \leq e_\alpha$, and the probability of loss greater than or equal to β is less than or equal to α; this is typically a smaller probability percentage than 50%, so we assume $e_\alpha > \bar{e}$. In stage 2, the buying firm brings e_s to the financial institution for each contract; in return, the firm receives e from the financial institution. Then, the return from the financial hedging contract is $r_C(e_s) = e - e_s$. For the futures contract, it is sufficient to equate the profit at $e = e_\alpha$ inside the probability expression in (12) to $-\beta$ in order to determine the number of hedging contracts necessary to warrant profitability that satisfies the VaR requirement:

$$\left\{\begin{matrix} -k_H Q_H - k_F Q_F - k_C\left(e_s\right) Q_C - \left(o_H + t_H\right) q_H^*\left(e_\alpha\right) - \left(o_F e_\alpha + t_F\right) q_F^*\left(e_\alpha\right) \\ + \left(e_\alpha - e_s\right) Q_C + p \min\left\{x, q_H^*\left(e_\alpha\right) + q_F^*\left(e_\alpha\right)\right\} \end{matrix}\right\} = -\beta .$$

Solving for Q_C at the lowest demand realization $x = x_l$ provides the minimum (optimal) number of hedging contracts satisfying VaR at $e = e_\alpha$:

$$Q_C^*(e_s) = \frac{\left\{k_H Q_H + k_F Q_F + (o_H + t_H) q_H^*(e_\alpha) + (o_F e_\alpha + t_F) q_F^*(e_\alpha) - p \min\left\{x_l, q_H^*(e_\alpha) + q_F^*(e_\alpha)\right\} - \beta\right\}}{\left(e_\alpha - e_s - k_C(e_s)\right)}.$$

Taking the expectation of the term $(e - e_s)\, Q_C$ in the first-stage objective over all exchange-rate realizations provides the expected return of $r_C(e_s)Q_C = Q_C\left(\int_{e_l}^{e_h}(e - e_s)\, f(e)\, de\right)$ in stage 2.

Note that the cost of engaging in financial hedging is $k_C(e_s)Q_C$. If the firm engages in currency futures contracts, the expense from engaging in financial hedging $k_C(e_s)$ is defined as in (10). Then the cost of financial hedging is:

$$k_C(e_s)Q_C = Q_C\left(\int_{e_s}^{e_h}(e - e_s)\, g(e) de - \int_{e_l}^{e_s}(e_s - e)\, g(e) de + \Delta\right) = Q_C\left(\int_{e_l}^{e_h}(e - e_s)\, g(e) de + \Delta\right) = Q_C((\bar{e} - e_s) + \Delta)$$

for Q_C units of hedging contracts. The buying firm is worried about the exchange rate realizations that are greater than or equal to e_α; thus, the concern is for exchange rate realizations in the range $e_\alpha \leq e \leq e_h$. Note that $r_C(e_s) = (e - e_s)$ increases for exchange rate realizations that exceed e_α; thus, it is sufficient to examine the VaR constraint at $e = e_\alpha$. Substituting $k_C(e_s) = ((\bar{e} - e_s) + \Delta)$ and $r_C(e_s) = (e_\alpha - e_s)$ into $Q_C^*(e_s)$ expression above, we can express the minimum (optimal) number of currency hedging contracts as:

$$Q_C^*(e_s) = \frac{\left\{k_H Q_H + k_F Q_F + (o_H + t_H) q_H^*(e_\alpha) + (o_F e_\alpha + t_F) q_F^*(e_\alpha) - p \min\left\{x_l, q_H^*(e_\alpha) + q_F^*(e_\alpha)\right\} - \beta\right\}}{\left(e_\alpha - \bar{e} - \Delta\right)}.$$

Part (b): If the firm engages in options contracts, then $k_C(e_s)$ is defined as in (11) where $k_C(e_s) = \int_{e_s}^{e_h}(e - e_s)\, g(e)\, de + \Delta = \int_{e_s}^{e_h} e g(e)\, de - e_s\left[1 - G(e_s)\right] + \Delta = \bar{e} - e_s - \int_{e_l}^{e_s} G(e)\, de + \Delta$. In this case, the buying firm brings $e_s Q_C$ to the financial institution in stage 2, and receives a return of eQ_C whenever the realized exchange rate takes a high value. i.e., when $e_s \leq e \leq e_h$. Thus, return from each contract is $r_C(e_s) = (e - e_s)$ when $e_s \leq e \leq e_h$. The cost of financial hedging is

$$k_C(e_s)Q_C = Q_C\left(\int_{e_s}^{e_h}(e - e_s)\, g(e) de + \Delta\right) = Q_C\left(\bar{e} - e_s - \int_{e_l}^{e_s} G(e)\, de + \Delta\right).$$

Substituting $k_C(e_s) = \left(\int_{e_s}^{e_h} e g(e)\, de - e_s\left[1 - G(e_s)\right] + \Delta\right)$ and $r_C(e_s) = (e_\alpha - e_s)$ into $Q_C^*(e_s)$ expression above, we can determine the minimum (optimal) number of currency options contracts as:

$$Q_C^*(e_s) = \frac{\left\{k_H Q_H + k_F Q_F + (o_H + t_H) q_H^*(e_\alpha) + (o_F e_\alpha + t_F) q_F^*(e_\alpha) - p\min\left\{x_l, q_H^*(e_\alpha) + q_F^*(e_\alpha)\right\} - \beta\right\}}{\left(e_\alpha - \bar{e} - \int_{e_l}^{e_S} G(e)\,de - \Delta\right)}.$$

Part (c): It is easy to see that when $\Delta = 0$, the cost of financial hedging $k_C(e_s) = E[r_C(e_s)]$. Thus, the expected profit for any given (Q_H, Q_F, Q_C) decision made in stage 1 in the risk-neutral setting can be replicated through purchasing $Q_C^*(e_s)$ units of contracts under risk aversion to satisfy VaR. ❑

Proposition A7. (a) *It is optimal* (*most profitable*) *for the firm to pay for production based on the exchange rate observed at the time the firm orders production to the foreign supplier;* (b) *Early payment and postponed payment lead to single sourcing policies.*

Proof. Proposition 7(b) has shown that the firm benefits from paying in foreign currency. We consider two points in the timeline: (1) time zero, when the firm signs the capacity reservation contracts and makes payments for the reserved capacity, and (2) the point in time at which demand is realized. These two points represent early payment and postponed payment, respectively, and help examine the impact of the payment timing on the sourcing policies and the expected profit of the firm. It is worth noting that the production decision is made prior to making payments to the supplier(s). In other words, the firm pays for production only after it first determines the production quantity to order. We assume that the exchange rate spot market is efficient (as noted in Section 3).

In the case of early payment, all cost components are known and deterministic at time zero. Therefore, the operational and penalty costs can be considered $c_F \bar{e}$ and $u_F \bar{e}$, respectively, all in the home-country currency. The firm's first-stage objective function can then be expressed as:

$$\max_{Q_H, Q_F, q_H, q_F \geq 0} E\left[\Pi(Q_H, Q_F, Q_C, q_H, q_F)\right] = -k_H Q_H - k_F Q_F - k_C(e_s) Q_C - (o_H + t_H) q_H - (o_F \bar{e} + t_F) q_F$$
$$+ r_C(e_s) Q_C + \int_{x_l}^{x_h} p \min\{x, q_H + q_F\} f(x)\,dx$$

s.t. $q_i \leq Q_i$ for $i = H, F$

or

$$\max_{Q_H, Q_F, q_H, q_F \geq 0} E\left[\Pi(Q_H, Q_F, Q_C, q_H, q_F)\right] = -k_H Q_H - k_F Q_F - k_C(e_s) Q_C - (o_H + t_H) q_H - (o_F \bar{e} + t_F) q_F$$
$$+ r_C(e_s) Q_C + \int_{x_l}^{x_h} p \min\{x, q_H + q_F\} f(x)\,dx \qquad (38)$$

s.t. $q_i \leq Q_i$ for $i = H, F$

Note that $q_i < Q_i$ leads to additional cost in the objective function, and thus, it is suboptimal to have $q_i < Q_i$. Therefore, $q_i = Q_i$ at the optimal solution, and thus, (35) can be re-written as:

$$\max_{Q_H,Q_F,q_H,q_F\geq 0} E\left[\Pi\left(Q_H,Q_F,Q_C,q_H,q_F\right)\right] = -k_H Q_H - k_F Q_F - k_C\left(e_s\right)Q_C + \int_{e_l}^{e_h}\begin{bmatrix} -(o_H+t_H)q_H - \left(o_F e + t_F\right)q_F + r_C\left(e_s\right)Q_C \\ +\int_{x_l}^{x_h} p\min\left\{x, q_H+q_F\right\} f\left(x\right)dx \end{bmatrix} g(e)de \tag{39}$$

s.t. $q_i = Q_i$ for $i = H, F$

or equivalently,

$$\max_{Q_H,Q_F\geq 0} E\left[\Pi\left(Q_H,Q_F,Q_C\right)\right] = -k_H Q_H - k_F Q_F - k_C\left(e_s\right)Q_C + \int_{e_l}^{e_h}\pi_E^*\left(Q_H,Q_F,Q_C e\right)g(e)de$$

where

$$\pi_E^*\left(Q_H,Q_F,Q_C,e\right) = \max_{q_H,q_F\geq 0}\left\{-(o_H+t_H)q_H - \left(o_F e + t_F\right)q_F + r_C\left(e_s\right)Q_C + \int_{x_l}^{x_h} p\min\left\{x, q_H+q_F\right\} f\left(x\right)dx\right\}$$

s.t. $q_i = Q_i$ for $i = H, F$

Note that $\pi_E^*\left(Q_H,Q_F,Q_C,e\right) \leq \pi^*\left(Q_H,Q_F,Q_C,e\right)$ due to the binding constraint. Therefore, it follows that early payment does not improve the expected profit.

In the case of the postponed payment scheme, the payment for the production order can be postponed to a later time (when demand is realized) and made based on the exchange-rate at that time. Therefore, both capacity reservation and production decisions are made under exchange-rate and demand uncertainty. The first-stage objective function can be expressed as:

$$\max_{Q_H,Q_F,q_H,q_F\geq 0} E\left[\Pi\left(Q_H,Q_F,Q_C,q_H,q_F\right)\right] = \int_{e_l}^{e_h}\int_{x_l}^{x_h}\begin{bmatrix} -k_H Q_H - k_F Q_F - k_C\left(e_s\right)Q_C \\ -(o_H+t_H)q_H - \left(o_F e + t_F\right)q_F \\ +r_C\left(e_s\right)Q_C + p\min\left\{x, q_H+q_F\right\} \end{bmatrix} f(x)g(e)dxde$$

s.t. $q_i \leq Q_i$ for $i = H, F$

Observe that, similar to the case of early payment, we must have $q_i = Q_i$ at the optimal solution. Therefore, the first-stage objective function can be re-written as:

$$\max_{Q_H,Q_F,q_H,q_F\geq 0} E\left[\Pi\left(Q_H,Q_F,Q_C,q_H,q_F\right)\right] = -k_H Q_H - k_F Q_F - k_C\left(e_s\right)Q_C + \int_{e_l}^{e_h}\begin{bmatrix} -(o_H+t_H)q_H - \left(o_F e + t_F\right)q_F \\ +r_C\left(e_s\right)Q_C + \int_{x_l}^{x_h} p\min\left\{x, q_H+q_F\right\} f\left(x\right)dx \end{bmatrix} g(e)de$$

s.t. $q_i = Q_i$ for $i = H, F$

which is identical to the first-stage objective function developed for early payment. Therefore, with analogous arguments made for the case of early payment, it follows that postponed payment is not beneficial for the firm either.

For part (b) observe that due to $q_i = Q_i$ in both the early and postponed payment schemes, the problem becomes equivalent to a single-stage newsvendor problem with two suppliers. The firm then procures only from the less expensive supplier resulting in a single-sourcing policy. ❑

Corollary A1. *If exchange rate and demand are observed simultaneously, the firm adopts a single sourcing policy which results in an expected profit less than that obtained when exchange rate is observed at the time of production orders.*

Proof. Note that the firm can observe the exchange rate and pay for production as late as the time at which the demand is realized. This case is identical to the postponed payment case analyzed in Proposition A7. ❑

Proposition A8. *Either q_H^a or q_F^a would be utilized in the optimal solution in the second stage.*

Proof. The proof follows from the fact that when $e \leq \tau_1^{a=}$ we have $C_F^a(e) \leq C_H^a$ making the foreign source more desirable than the domestic supplier for additional units. This is because $p - C_F^a(e) \geq p - C_H^a$. Moreover, the maximum amount of units that would be purchased from the foreign source is greater than or equal to that from the domestic source; thus, $q_F^{a0}(e) = F^{-1}((p - C_F^a(e))/p) > Q_H^{a0} = F^{-1}((p - C_H^a)/p)$ eliminating the possibility of using the domestic source. Similarly, when $e > \tau_1^{a=}$ we have $C_F^a(e) > C_H^a$ making the domestic source more desirable than the foreign supplier for additional units. This is because $p - C_F^a(e) < p - C_H^a$. Moreover, the maximum amount of units that would be purchased from the domestic source is greater than that from the foreign source; thus, $Q_H^{a0} = F^{-1}((p - C_H^a)/p) > q_F^{a0}(e) = F^{-1}((p - C_F^a(e))/p)$ eliminating the possibility of using the foreign source. ❑

Proof of Proposition 12. (i) When $e_l \leq e \leq \tau_1^a$, $c_F(e) \leq C_F^a(e) < c_H < C_H^a < p \Rightarrow p - c_F(e) \geq p - C_F^a(e) > p - c_H > p - C_H^a > 0$. These marginal returns lead to the following preference order: $q_F >> q_F^a >> q_H >> q_H^a$. First, the firm would source from the reserved capacity in the foreign source up to $q_F^0(e)$; therefore, $q_F^* = \min\{Q_F, q_F^0(e)\}$. If $Q_F < q_F^0(e)$, then the firm would purchase additional units from the foreign source up to $q_F^{a0}(e) < q_F^0(e)$; thus, $q_F^{a*} = (q_F^{a0}(e) - Q_F)^+$. Because the exchange is low making the foreign supplier significantly less expensive in this case, the firm would not need to use the domestic source even if there is capacity reserved in stage 1.

(ii) When $\tau_1^a < e \le \tau_1^{a=}$, $c_F(e) < c_H \le C_F^a(e) < C_H^a < p \Rightarrow p - c_F(e) > p - c_H \ge p - C_F^a(e) > p - C_H^a > 0$. These marginal returns lead to the following preference order: $q_F \gg q_H \gg q_F^a \gg q_H^a$. First, the firm would source from the reserved capacity in the foreign source up to $q_F^0(e)$; therefore, $q_F^* = \min\{Q_F, q_F^0(e)\}$. If $Q_F < q_F^0(e)$, then the firm would source additional units from the domestic source up to q_H^0; thus, $q_H^* = \min\{Q_H, (q_H^0 - Q_F)^+\}$. If the total reservation is less than the desired amount, i.e., $Q_H + Q_F < q_F^{a0}(e)$, then the firm would purchase additional units from the foreign source (without reserved capacity), and thus $q_F^{a*} = (q_F^{a0}(e) - Q_H - Q_F)^+$. From Proposition A8, we know that the firm would not source additional units from the domestic supplier.

(iii) When $\tau_1^{a=} < e \le \tau_1$, $c_F(e) < c_H < C_H^a \le C_F^a(e) < p \Rightarrow p - c_F(e) > p - c_H > p - C_H^a > p - C_F^a(e) > 0$. These marginal returns lead to the following preference order: $q_F \gg q_H \gg q_H^a \gg q_F^a$. First, the firm would source from the reserved capacity in the foreign supplier up to $q_F^0(e)$; therefore, $q_F^* = \min\{Q_F, q_F^0(e)\}$. If $Q_F < q_F^0(e)$, then the firm would source additional units from the reserved capacity at the domestic source up to q_H^0; thus, $q_H^* = \min\{Q_H, (q_H^0 - Q_F)^+\}$. If the total reservation is less than the desired amount, i.e., $Q_H + Q_F < Q_H^{a0}$, then the firm would purchase additional units from the domestic supplier (without reserved capacity), and thus $q_H^{a*} = (Q_H^{a0} - Q_H - Q_F)^+$.

(iv) When $\tau_1 < e \le \tau_2^a$, $c_H < c_F(e) < C_H^a \le C_F^a(e) < p \Rightarrow p - c_H > p - c_F(e) > p - C_H^a > p - C_F^a(e) \ge 0$. These marginal returns lead to the following preference order: $q_H \gg q_F \gg q_H^a \gg q_F^a$. First, the firm would source from the reserved capacity in the domestic source up to q_H^0; therefore, $q_H^* = \min\{Q_H, q_H^0\}$. If $Q_H < q_H^0$, then the firm would source from the reserved capacity of the foreign supplier up to $q_F^0(e)$; thus, $q_F^* = \min\{Q_F, (q_F^0(e) - Q_H)^+\}$. If the reserved capacity is still less, i.e., $Q_H + Q_F < Q_H^{a0}$, then the firm would purchase additional units from the domestic supplier (without reserved capacity), and thus $q_H^{a*} = (Q_H^{a0} - Q_H - Q_F)^+$.

(v) When $\tau_2^a < e \le \tau_2$, $c_H < C_H^a < c_F(e) \le p < C_F^a(e) \Rightarrow p - c_H > p - C_H^a > p - c_F(e) \ge 0 > p - C_F^a(e)$. These marginal returns lead to the following preference order: $q_H \gg q_H^a \gg q_F$. Note that sourcing additional units without capacity reservation from the foreign source is no more economical, i.e., $q_F^{a*} = 0$. First, the firm would get as much as from the reserved capacity from the domestic source up to q_H^0; therefore, $q_H^* = \min\{Q_H, q_H^0\}$. If $Q_H < q_H^0$, then the firm would source from the domestic supplier (without the reservation from stage 1) up to q_H^{a0}; thus, $q_H^{a*} = \min\{Q_H^{a0}, (q_H^{a0} - Q_H)^+\}$. In this case, the firm would not use the foreign source even if there is capacity reserved in stage 1.

(vi) When $\tau_2 < e \le e_h$, $c_H < C_H^a \le p \le c_F(e) < C_F^a(e) \Rightarrow p - c_H > p - C_H^a > 0 > p - c_F(e) > p - C_F^a(e)$. These marginal returns lead to the following preference order: $q_H >> q_F$. Note that sourcing additional units with or without capacity reservation from the foreign source is no more economical, i.e., $q_F = q_F^{a*} = 0$. First, the firm would get as much as from the reserved capacity from the domestic source up to q_H^0; therefore, $q_H^* = \min\{Q_H, q_H^0\}$. If $Q_H < q_H^0$, then the firm would source from the domestic supplier (without the reservation from stage 1) up to q_H^{a0}; thus, $q_H^{a*} = \min q_H^{a0}, (q_H^{a0} - Q_H)^+\}$. In this case, the firm would not use the foreign source even if it reserved capacity in stage 1. ❑

Proof of Proposition 13. The proof follows by comparing the optimal second-stage decisions involving q_H and q_F in propositions 2 and 12. Let $E[\Pi^a(Q_H, Q_F, Q_C)]$ represent the objective function under non-homogenous leadtimes. Let δ_H and δ_F describe the difference in the first-order derivatives w.r.t. Q_H and Q_F, respectively, of $E[\Pi^a(Q_H, Q_F, Q_C)]$ and the original objective function in (1). In all three regions,

$$\delta_H = -\int_{e_l}^{\tau_1^a} \left[\left(p - c_H - k_H\right)\right] g\left(e\right) de < 0, \text{ and}$$

$$\delta_F = -\int_{\tau_2^a}^{\tau_2} \left[\left(p - C_F\left(e\right) - k_F \bar{e}\right)\right] g\left(e\right) de < 0.$$

The negative values of δ_H and δ_F imply that the returns from a unit investment in Q_H and Q_F in stage 1 has smaller returns in stage 2 when the flexibility to purchase additional supplies is incorporated into the model. Thus, the optimal values of Q_H and Q_F are smaller as a result of the flexibility to purchase additional supplies. ❑

Proof of Proposition 14. The proof follows from the proof of Proposition 1. In the original model, the decision variable q_H was determined in the presence of demand uncertainty, and therefore, the stocking level was determined as q_H^0. In the model with non-homogeneous leadtimes, the decision variable q_H is determined after observing the realization of demand, denoted x (as well as exchange rate e); thus, we can indicate it as $q_H(e, x)$. Rather than ordering q_H^0 upfront, the firm waits to observe x in order to place the order. Thus, q_H^0 in the results of Proposition 1 can be replaced with x for the optimal amount to be ordered from the domestic supplier. ❑

Proof of Proposition 15. The proof follows by comparing the optimal second-stage decisions involving q_H in propositions 1 and 14. Let $E[\Pi^p(Q_H, Q_F, Q_C)]$ represent the objective function under non-homogenous leadtimes. Let δ_H describe the difference in the first-order derivatives wr.t. Q_H the objective function of $E[\Pi^p(Q_H, Q_F, Q_C)]$ and the original objective function in (1). In all three regions involving policies H and D_E, the postponement of q_H decision leads to a positive return which can be quantified as

$$
\begin{aligned}
\delta_H = & \int_{e_l}^{\tau_1}\left[\int_{q_F^0(e)}^{Q_H+q_F^0(e)}\left(-c_H\left(x-q_F^0(e)\right)f(x)dx\right)+\int_{Q_H+q_F^0(e)}^{x_h}\left(-c_H Q_H f(x)dx\right)-\left(-c_H Q_H\right)\right]g(e)de \\
& +\int_{\tau_1}^{\tau_2}\left[\int_{x_l}^{q_F^0(e)}\left(-c_H x f(x)dx\right)+\int_{q_F^0(e)}^{x_h}\left(-c_H Q_H f(x)dx\right)-\left(-c_H Q_H\right)\right]g(e)de \qquad > 0 \\
& +\int_{\tau_2}^{e_h}\left[\int_{x_l}^{Q_H}\left(-c_H x f(x)dx\right)+\int_{Q_H}^{x_h}\left(-c_H Q_H f(x)dx\right)-\left(-c_H Q_H\right)\right]g(e)de
\end{aligned}
$$

because all integrands are positive. The positive value of δ_H implies that the returns from a unit investment in Q_H provides a greater return in stage 2 in the non-homogenous leadtime setting. Therefore, the optimal value of Q_H is greater in the non-homogenous setting. ❑

Details of the Exchange-Rate Distribution

We first analyze the rate of change in the exchange rate in four months. We accomplish this goal by examining each daily exchange rate (or, spot rate), denoted s_t, and comparing it with the daily exchange rate of four months later, denoted s_{t+120}, within our three-year data set. Specifically, for each day over the 3-year period, we calculate the proportion of the exchange-rate four months into the future relative to the current exchange rate, i.e., $e_t = s_{t+120}/s_t$. The comparison of the daily exchange rate with its counterpart in four months results in an empirical distribution which we use in our analysis. Figure A1(a) provides the histogram of the empirical distribution representing the fluctuations in the Euro-Dollar exchange rate used in our numerical illustrations. Figure A1(b) shows the frequency distribution of the change in the value of the exchange rate in four months. As can be seen from Figure A1(b), there would not be a well-fitting statistical distribution to represent the change in exchange rates in four months. Therefore, we use the entire data set of the changes during the period of 2010 – 2012 as our distribution in the analysis.

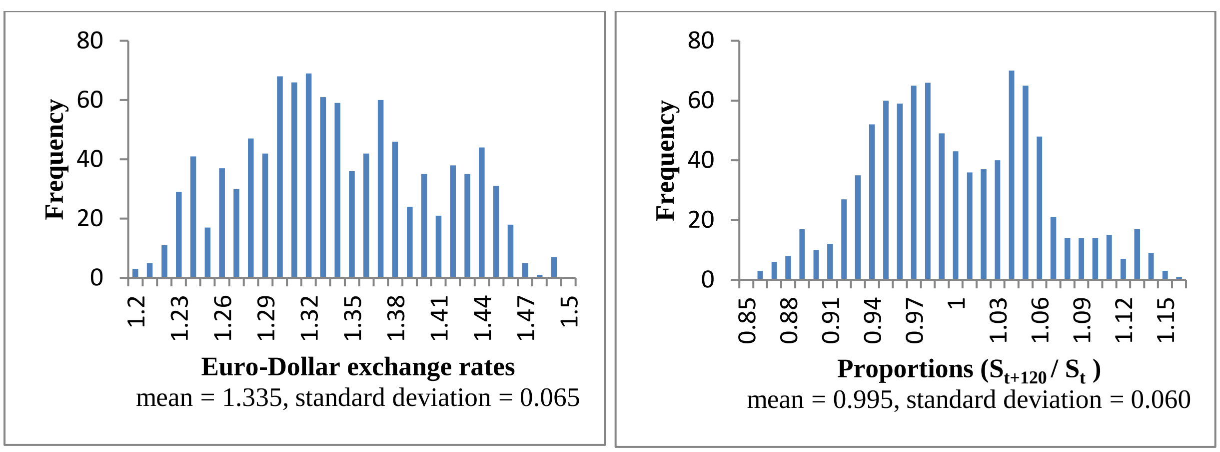

Figure A1. The Euro-Dollar exchange rate between 2010 – 2012. (a) Histogram of the actual values. (b) Frequency distribution of the proportions representing the change in the value of the exchange rate.

<u>Optimal Policy Algorithm based on Proposition 3</u>

Step 1) Check OC1.

If it holds, go to Step 2.

Otherwise (if OC1 does not hold), Policy H is optimal. Go to End.

Step 2) Check OC2.

If it holds, check OC4.

If OC4 holds, Policy D_E is optimal. Go to End.

Otherwise (if OC4 does not hold), Policy F_H is optimal. Go to End.

Otherwise (if OC2 does not hold), check OC3.

If OC3 holds, Policy D_R is optimal. Go to End.

Otherwise (if OC3 does not hold), Policy F_L is optimal. Go to End.

End.