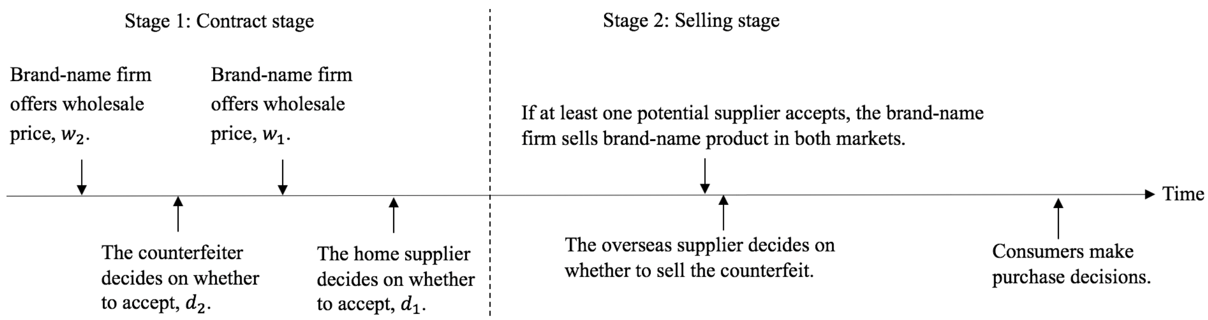
E-Companion
Converting Counterfeiters in Emerging Markets to Authorized Suppliers: A New Anti-Counterfeiting Measure

E-Companion A Extensions

A.1 Extension 1: Sequential Contract Offering

In this extension, based on the decision sequence in Figure A1, we conduct backward deduction to solve our problem. The procedures are as follows: firstly, we discuss the overseas supplier's counterfeiting decision $s(w_1, w_2, d_1, d_2)$ given $d_2 = 1$; secondly, we discuss the home supplier's acceptance decision $d_1(w_1, w_2, d_2)$; thirdly, we discuss the optimal wholesale price decision $w_1(w_2, d_2)$; fourthly, we discuss the overseas supplier's acceptance decision $d_2(w_2)$; lastly, we discuss the optimal wholesale price decision w_2.

Figure A1 Sequence of Decisions and Events (Extension 1)



A.2 Extension 2: Endogenous Counterfeit Price

In this extension, we examine the price-setting capability of the counterfeiter. We conduct the analysis by backward induction. First, for a given sourcing strategy, we derive the profit expressions and discuss the optimal counterfeiting decision of the overseas supplier, s^*.

Under each possible sourcing strategy, we obtain the profit expressions for each firm, and discuss the optimal retail price p_2^* of the counterfeit with $s = 1$. In particular, if the counterfeiter sells the counterfeits, we focus on the case when the brand-name firm has a positive market share in the overseas market, i.e., $m_{B2} > 0$. The overseas supplier decides whether to sell the counterfeit, $s^*(w_2)$ by comparing $\pi_2(w_2, s = 1)$ and $\pi_2(w_2, s = 0)$. If $\pi_2(w_2, s = 1) > \pi_2(w_2, s = 0)$, she decides to sell the counterfeit; otherwise, she does not sell the counterfeit. Recall that $e < \frac{\alpha(\beta p_B - k_2)^2}{4\beta(1-\beta)}$. Thus, under strategies H and N, the counterfeiter always sells the counterfeit products. Under strategies D and O, the overseas supplier decisions on selling the counterfeit only when w_2 is not high, which is summarized in Lemma 4.

Second, we derive the best response functions of the overseas and home suppliers, $(d_1^*(w_1, w_2), d_2^*(w_1, w_2))$. For the analysis below, it is convenient to define the following notations: $M = \frac{\alpha(\beta p_B - k_2)^2}{4\beta(1-\beta)} - e$, $M' = \frac{\alpha(\beta p_B - (1-\gamma)k_2)^2}{4(1-\gamma)\beta(1-\gamma-\beta)} - e$, and $K = \frac{\alpha(\beta - k_2)^2}{4\beta} - e$.

Given w_1 and w_2, we derive the home and overseas suppliers' optimal contract acceptance decisions. By evaluating the difference in each potential supplier's expected profit between accepting and rejecting the contract, we obtain the optimal decisions of the two suppliers:

$$(d_1^*(w_1,w_2),d_2^*(w_1,w_2)) = \begin{cases} (1,1), & \text{if } w_1 \geq k_1, \max\{k_2,\underline{w}_2\} \leq w_2 < w_2^{(0)} \text{ or } w_2 \geq \max\{w_2^{D(2)},w_2^{(0)}\}, \\ (1,0), & \text{if } w_1 \geq k_1, \underline{w}_2 < w_2 < \max\{k_2,\underline{w}_2\} \text{ or } w_2^{(0)} < w_2 < \max\{w_2^{D(2)},w_2^{(0)}\}, \\ (0,1), & \text{if } w_1 < k_1, \max\{w_2^{O(1)},\underline{w}_2\} \leq w_2 < w_2^{(0)} \text{ or } w_2 \geq \max\{w_2^{O(2)},w_2^{(0)}\}, \\ (0,0), & \text{if } w_1 < k_1, \underline{w}_2 < w_2 < \max\{w_2^{O(1)},\underline{w}_2\} \text{ or } w_2^{(0)} < w_2 < \max\{w_2^{O(2)},w_2^{(0)}\}, \end{cases}$$

where $w_2^{D(2)} = k_2 + \frac{M}{\alpha(1-\frac{p_B}{1-\gamma})}$, $w_2^{O(1)} = k_2 - \frac{2(1+\frac{1}{\alpha})(1-\gamma-p_B)(1-\gamma-\beta)-\beta p_B+(1-\gamma)k_2}{\beta} + \sqrt{\frac{4(1-\gamma)(1-\gamma-\beta)(K-M')}{\alpha\beta} + \left(\frac{2(1+\frac{1}{\alpha})(1-\gamma-p_B)(1-\gamma-\beta)-\beta p_B+(1-\gamma)k_2}{\beta}\right)^2}$, $w_2^{O(2)} = k_2 + \frac{K}{(1+\alpha)(1-\frac{p_B}{1-\gamma})}$, and $w_2^{(0)} = \max\{w_2^{(0)\prime},\underline{w}_2\}$.

Third, we discuss the optimal wholesale prices (w_1,w_2) that the brand-name firm would offer under each sourcing strategy. Substituting $(d_1^*(w_1,w_2),d_2^*(w_1,w_2))$ into the profit functions of the brand-name firm, we analyze the optimal wholesale price under each possible sourcing strategy.

$$\pi_B^H(w_1) = (p_B - w_1)(1-p_B) + \alpha(p_B - w_1 - t)\left(1 - \frac{(2-\beta)p_B - k_2}{2(1-\beta)}\right);$$

$$\pi_B^D = \begin{cases} \pi_B^{DC}(w_1,w_2) = (p_B - w_1)(1-p_B) & \\ +\alpha(p_B - w_2)\left(\frac{2(1-\gamma-\beta)(1-\gamma-p_B)-\beta p_B+(1-\gamma)k_2+\beta(w_2-k_2)}{2(1-\gamma)(1-\gamma-\beta)}\right), & \text{if } w_1 \geq k_1, \max\{k_2,\underline{w}_2\} \leq w_2 < w_2^{(0)}, \\ \pi_B^{D\dagger}(w_1,w_2) = (p_B - w_1)(1-p_B) + \alpha(p_B - w_2)(1-\frac{p_B}{1-\gamma}), & \text{if } w_1 \geq k_1, w_2 \geq \max\{w_2^{D(2)},w_2^{(0)}\}; \end{cases}$$

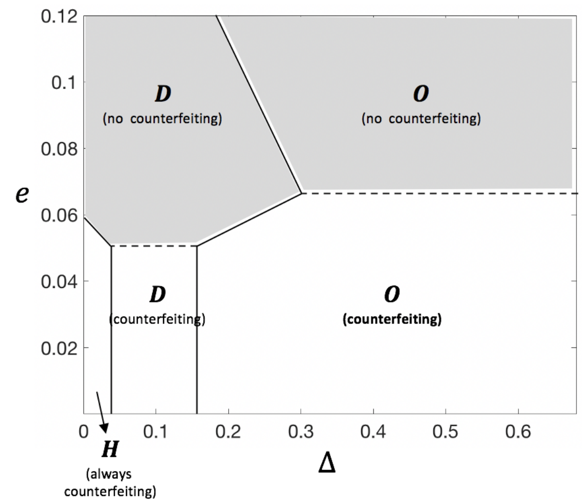
$$\pi_B^O = \begin{cases} \pi_B^{OC}(w_2) = (p_B - w_2 - t)\left(1 - \frac{p_B}{1-\gamma}\right) & \\ +\alpha(p_B - w_2)\left(\frac{2(1-\gamma-\beta)(1-\gamma-p_B)-\beta p_B+(1-\gamma)k_2+\beta(w_2-k_2)}{2(1-\gamma)(1-\gamma-\beta)}\right), & \text{if } \max\{w_2^{O(1)},\underline{w}_2\} \leq w_2 < w_2^{(0)}, \\ \pi_B^{O\dagger}(w_2) = (p_B - w_2 - t)(1-\frac{p_B}{1-\gamma}) + \alpha(p_B - w_2)(1-\frac{p_B}{1-\gamma}), & \text{if } w_2 \geq \max\{w_2^{O(2)},w_2^{(0)}\}; \end{cases}$$

$$\pi_B^N(w_1,w_2) = 0.$$

By analyzing the brand-name firm's profit under each sourcing strategy, we obtain the optimal wholesale prices in Lemma 5.

Finally, we obtain the equilibrium by comparing the brand-name firm's optimal profits among different sourcing strategies. We provide the numerical analysis about the equilibrium under the setting with the price-setting flexibility. Figure A2 illustrates how the equilibrium sourcing strategy varies with respect to the cost differential between two suppliers (Δ) and the penalty from law enforcement in the overseas market (e). We observe that in this extension, the equilibrium is similar to that developed under the base model which has been depicted in Figure 5.

Figure A2 Equilibrium Sourcing Strategy Relative to the Cost Differential Between Two Suppliers (Δ) and Penalty from Law Enforcement in the Overseas Market (e). (Shadow areas indicate that counterfeiting is prevented. In this example, $p_B = 0.7$, $k_2 = 0.02$, $\alpha = 5$, $\beta = 0.3$, $\gamma = 0.01$, $t = 0.01$.)



A.3 Extension 3: Endogenous Brand-Name Product and Counterfeit Prices

Our base model assumes retail prices p_B and p_2 are exogenously determined. This extension explores the implications of endogenizing retail prices. Solving the game with endogenous retail prices alongside endogenous sourcing decisions introduces analytical challenges. For tractability, we focus on optimizing retail pricing decisions for given wholesale prices w_1 and w_2 under strategies D and O, respectively. Specifically, we examine scenarios where the wholesale price contracts have already been structured to convert the counterfeiter through either dual sourcing or single sourcing from the overseas supplier, and it is possible for the authorized overseas supplier to sell counterfeits. The subsequent analysis investigates the conditions that the overseas supplier is prevented from selling counterfeits, considering the dynamics of endogenized retail pricing decisions.

Under Strategy D or Strategy O, the sequence of events unfolds as follows: First, the brand-name firm sets the retail price p_B of the brand-name product. Subsequently, the overseas supplier decides whether to sell counterfeits, s. If she opts to sell counterfeits in the overseas market, i.e., $s = 1$, she then determines the retail price of the counterfeit p_2. We employ backward induction to solve the game, with details provided in E-Companion B.

Endogenously setting their retail prices under competition in the overseas market introduces more interactions among players. Specifically, the endogenous retail price p_B provides the brand-name firm an additional lever to prevent counterfeiting through price competition. At the same time, it allows the overseas supplier the opportunity to adjust her retail price p_2. When retail price p_B is low enough, counterfeiting can be prevented as competition leads to zero market share for the counterfeit product. In the following lemma, we outline the conditions under which the overseas supplier does not sell counterfeits. We define $\hat{p}_B^D$ and $\hat{p}_B^O$ in Equation (17) in E-Companion B.

LEMMA A1. *Given* (w_1, w_2)*, (i) under Strategy D,* $s^* = 0$ *if* $\hat{p}_B^D \leq \frac{\sqrt{\frac{4\beta(1-\gamma)(1-\gamma-\beta)e}{\alpha}}+((1-\gamma)k_2+\beta(w_2-k_2))}{\beta}$*; (ii) under Strategy O,* $s^* = 0$ *if* $\hat{p}_B^O \leq \frac{\sqrt{\frac{4\beta(1-\gamma)(1-\gamma-\beta)e}{\alpha}}+((1-\gamma)k_2+\beta(w_2-k_2))}{\beta}$.

Recall that in our base model, the overseas supplier's profit from selling counterfeits does not depend on the wholesale price w_2. However, Lemma A1 implies that wholesale price w_2 may affect the optimal retail price when $\hat{p}_B^D$ or $\hat{p}_B^O$ is adopted to prevent counterfeit sales, which in turn affects the overseas supplier's profit from counterfeiting.

In the following, we compare the conditions with respect to (w_1, w_2) under which the overseas supplier is prevented from counterfeiting under endogenous retail prices with those from our base model under exogenous retail prices. Recall from Lemma 1 that, when retail prices are exogenous, the brand-name firm is able to prevent counterfeiting by setting a sufficiently high wholesale price $w_2 \geq w_2^{(0)}$ under strategies D and O. When retail prices are endogenously determined, counterfeiting is prevented if $w_2 \geq w_2^{D,endog}$ under Strategy D or if $w_2 \geq w_2^{O,endog}$ under Strategy O. The following proposition provides the sufficient conditions about the comparison between $w_2^{D,endog}$ and$w_2^{O,endog}$ with $w_2^{(0)}$, respectively. We define the thresholds $e_1^{D,endog}$, $e_1^{O,endog}$, $e_2^{D,endog}$ and $e_2^{O,endog}$ in Equation (18) of E-Companion B.

PROPOSITION EC.1. *For given* (w_1, w_2)*,*

(a) under Strategy D, $w_2^{D,endog} < w_2^{(0)}$ *if* $(e_1^{D,endog})^+ < e < (e_2^{D,endog})^+$*;*

(b) under Strategy O, $w_2^{O,endog} < w_2^{(0)}$ *if* $(e_1^{O,endog})^+ < e < (e_2^{O,endog})^+$.

Proposition EC.1 indicates that if the penalty from law enforcement e is not high, it becomes easier for the brand-name firm to prevent the overseas supplier from counterfeiting if he can choose the retail price optimally. Specifically, in this case, a wholesale price w_2, which satisfies $w_2^{D,endog} \leq w_2 < w_2^{(0)}$ under Strategy D or $w_2^{O,endog} \leq w_2 < w_2^{(0)}$ under Strategy O, can prevent counterfeit sales under the optimal retail prices, whereas it cannot prevent counterfeiting under fixed retail prices. This occurs because the optimal retail price of the brand-name firm increases with w_2. When the wholesale price w_2 is lower, the brand-name firm chooses a lower retail price. Consequently, the potentially intense price competition discourages the overseas supplier from selling counterfeits. This result confirms that the flexibility to adjust retail prices is a valuable leverage for the brand-name firm to prevent counterfeit sales.

A.4 Extension 4: Revenue-Dependent Penalty for Counterfeiting

In this section, our model is extended to consider a different law enforcement penalty, which depends on the revenue from selling counterfeits.

Denote the probability of a counterfeiter getting caught as ϕ, where $\phi \in (0,1)$, we examine the effect of the revenue related penalty for counterfeiting: after getting caught, the counterfeiter pays the penalty from law enforcement e and gets her investment of counterfeiting confiscated, which means she cannot sell and produce the counterfeit in the market. Thus, the overseas supplier's expected profit π_2 is given as

$$\begin{aligned}\pi_2\left(w_2,d_1,d_2,s\right) =& d_2\left(\left(1-d_1\right)\left(w_2-k_2\right)m_{B1}(d_1,d_2)+\left(w_2-k_2\right)m_{B2}(d_1,d_2,s)\right)\\ &+s\left(\left(1-\phi\right)\left(p_2-k_2\right)m_2(d_1,d_2,s)-\phi e\right),\end{aligned} \tag{5}$$

where $m_{B1}(d_1,d_2)$, $m_{B2}(d_1,d_2,s)$ and $m_2(d_1,d_2,s)$ are given in equations (1)-(3), respectively. For the second line of Equation (5), the first term represents the expected profit of selling the counterfeit, and the second term represents the expected penalty from law enforcement. In this extension, to avoid the uninteresting case where the counterfeiter never sell counterfeits if she rejects the contract, we assume the penalty is not too high, that is, $e < \frac{\alpha(\beta p_B - p_2)(p_2-k_2)(1-\phi)}{(1-\beta)\beta\phi}$. For the analysis below, it is convenient to define the following notations:

$$\begin{aligned}
&M_p = \alpha(1-\phi)\left(p_2-k_2\right)\left(\tfrac{p_B-p_2}{1-\beta}-\tfrac{p_2}{\beta}\right)-\phi e,\ w_2^{D(2)} = k_2+\tfrac{M_p}{\alpha(1-\frac{p_B}{1-\gamma})},\\
&M'_p = \alpha(1-\phi)\left(p_2-k_2\right)\left(\tfrac{p_B-p_2}{1-\gamma-\beta}-\tfrac{p_2}{\beta}\right)-\phi e,\ w_2^{(0)} = k_2+\tfrac{M'_p}{\alpha\left(\frac{p_B-p_2}{1-\gamma-\beta}-\frac{p_B}{1-\gamma}\right)},\\
&K_p = \alpha(1-\phi)\left(p_2-k_2\right)\left(1-\tfrac{p_2}{\beta}\right)-\phi e, \qquad w_2^{O(2)} = k_2+\tfrac{K_p}{(1+\alpha)(1-\frac{p_B}{1-\gamma})},\\
&K_p-M'_p = \alpha(1-\phi)\left(p_2-k_2\right)\left(1-\tfrac{p_B-p_2}{1-\gamma-\beta}\right),\ \ w_2^{O(1)} = k_2+\tfrac{K_p-M'_p}{(1-\frac{p_B}{1-\gamma})+\alpha(1-\frac{p_B-p_2}{1-\gamma-\beta})}.
\end{aligned}$$

Similar to the analysis in Section 4, in this extension, if $w_2 < w_2^{(0)}$, after being converted, she will choose to sell counterfeits in the overseas market. Further, by evaluating the difference in each potential supplier's expected profit between accepting and rejecting the contract, we obtain the best response function of two potential suppliers. As a result, the optimal decisions of two suppliers are

$$(d_1^*,d_2^*) = \begin{cases}(1,1), & \text{if } w_1\ge k_1,\ k_2\le w_2< w_2^{(0)} \text{ or } w_2\ge w_2^{(0)},\\ (1,0), & \text{if } w_1\ge k_1,\ w_2<k_2,\\ (0,1), & \text{if } w_1<k_1,\ \min\{w_2^{(0)},w_2^{O(1)}\}\le w_2<w_2^{(0)} \text{ or } w_2\ge \max\{w_2^{(0)},w_2^{O(2)}\},\\ (0,0), & \text{if } w_1<k_1,\ w_2<\min\{w_2^{(0)},w_2^{O(1)}\} \text{ or } w_2^{(0)}\le w_2<\max\{w_2^{(0)},w_2^{O(2)}\}.\end{cases}$$

Thus, for each possible sourcing strategy, the optimal wholesale price(s) of the brand-name firm, which will be accepted by the home or overseas suppliers, satisfies the following:

(a) under Strategy H, $w_1^H = k_1$;

(b) under Strategy D, $w_1^D = k_1$ and

(i) $w_2^D = k_2$ and $s^* = 1$, if $e < e_{D1}$;

(ii) $w_2^D = \max\{w_2^{(0)}, w_2^{D(2)}\}$ and $s^* = 0$, if $e \ge e_{D1}$;

(c) under Strategy O,

(i) $w_2^O = w_2^{O(1)}$ and $s^* = 1$, if $e < e_{O1}$;

(ii) $w_2^O = \max\{w_2^{(0)}, w_2^{O(2)}\}$ and $s^* = 0$, if $e \ge e_{O1}$;

where e_{D1} and e_{O1} are defined as

$$\begin{aligned}
e_{D1} &= \left((1-\phi)(p_2-k_2)-\left(\frac{(p_B-k_2)(\frac{p_B-p_2}{1-\gamma-\beta}-\frac{p_B}{1-\gamma})}{1-\frac{p_B}{1-\gamma}}\right)\frac{\beta}{1-\gamma}\right)\frac{\alpha(\beta p_B-(1-\gamma)p_2)}{(1-\gamma-\beta)\beta\phi},\\
e_{O1} &= \left((1-\phi)(p_2-k_2)-\left(\frac{\alpha(p_B-w_2^{OC*})(\frac{p_B-p_2}{1-\gamma-\beta}-\frac{p_B}{1-\gamma})}{(1+\alpha)(1-\frac{p_B}{1-\gamma})}+w_2^{OC*}-k_2\right)\frac{\beta}{1-\gamma}\right)\frac{\alpha(\beta p_B-(1-\gamma)p_2)}{(1-\gamma-\beta)\beta\phi},
\end{aligned}$$

and $w_2^{OC*} = k_2 + \frac{\alpha(1-\phi)(p_2-k_2)(1-\frac{p_B-p_2}{1-\gamma-\beta})}{(1-\frac{p_B}{1-\gamma})+\alpha(1-\frac{p_B-p_2}{1-\gamma-\beta})}$.

By further making comparisons among different scenarios and using the approach in Lemma B2, we have the following equilibrium results. We define thresholds in the below Equation (6): $R = \frac{\alpha(\beta p_B-(1-\gamma)p_2)}{(1-\gamma-\beta)\beta}$, and

$$
\begin{aligned}
&\Delta_{DH} = p_B - k_2 - \frac{(p_B-k_2)(1-\frac{p_B-p_2}{1-\gamma-\beta})}{(1-\frac{p_B-p_2}{1-\beta})} - t;\\
&\Delta_{DO} = p_B - k_2 - \frac{\left(p_B-w_2^{OC*}-t\right)(1-\frac{p_B}{1-\gamma})-\alpha\left(w_2^{OC*}-k_2\right)(1-\frac{p_B-p_2}{1-\gamma-\beta})}{(1-p_B)};\\
&\Delta_{HO} = p_B - k_2 - \frac{\left(p_B-w_2^{OC*}-t\right)(1-\frac{p_B}{1-\gamma})+\alpha\left(p_B-w_2^{OC*}\right)(1-\frac{p_B-p_2}{1-\gamma-\beta})+\alpha t(1-\frac{p_B-p_2}{1-\beta})}{(1-p_B)+\alpha(1-\frac{p_B-p_2}{1-\beta})};\\
&f_{DH} = ((1-\phi)(p_2-k_2) - (x_{DH}(\Delta)-k_2)\frac{\beta}{1-\gamma})\frac{R}{\phi}, \text{ where } x_{DH}(\Delta) = p_B - \frac{(p_B-k_2-\Delta-t)(1-\frac{p_B-p_2}{1-\beta})}{(1-\frac{p_B}{1-\gamma})};\\
&f_{DO1} = ((1-\phi)(p_2-k_2) - (x_{DO1}(\Delta)-k_2)\frac{\beta}{1-\gamma})\frac{R}{\phi}, \text{ where } x_{DO1}(\Delta) = p_B - \frac{(p_B-k_2-\Delta)(1-p_B)+\alpha(p_B-k_2)(1-\frac{p_B-p_2}{1-\gamma-\beta})+t(1-\frac{p_B}{1-\gamma})}{(1+\alpha)(1-\frac{p_B}{1-\gamma})};\\
&f_{DO2} = ((1-\phi)(p_2-k_2) - (x_{DO2}(\Delta)-k_2)\frac{\beta}{1-\gamma})\frac{R}{\phi}, \text{ where } x_{DO2}(\Delta) = p_B - \frac{\left(p_B-w_2^{OC*}-t\right)((1-\frac{p_B}{1-\gamma})+\alpha(1-\frac{p_B-p_2}{1-\gamma-\beta}))-(p_B-k_2-\Delta)(1-p_B)}{\alpha(1-\frac{p_B}{1-\gamma})};\\
&f_{DO3} = ((1-\phi)(p_2-k_2) - (x_{DO3}(\Delta)-k_2)\frac{\beta}{1-\gamma})\frac{R}{\phi}, \text{ where } x_{DO3}(\Delta) = p_B - \frac{(p_B-k_2-\Delta)(1-p_B)}{(1-\frac{p_B}{1-\gamma})} - t;\\
&f_{DO4} = \frac{(p_B-\frac{(p_B-k_2-\Delta)(1-p_B)}{1-\frac{p_B}{1-\gamma}}-k_2-t(1-\frac{p_B}{1-\gamma}))\beta(1-\frac{p_B}{1-\gamma})-(1-\phi)\alpha(p_2-k_2)(1-\frac{p_2}{\beta})\beta+(1-\phi)\alpha(1-\gamma)(1-\frac{p_B}{1-\gamma})(p_2-k_2)}{\left(\frac{(1-\gamma)(1-\gamma-\beta)(1-\frac{p_B}{1-\gamma})}{(\beta p_B-(1-\gamma)p_2)}-1\right)\beta\phi};
\end{aligned}
\tag{6}
$$

where $w_2^{OC*} = k_2 + \frac{\alpha(1-\phi)(p_2-k_2)(1-\frac{p_B-p_2}{1-\gamma-\beta})}{(1-\frac{p_B}{1-\gamma})+\alpha(1-\frac{p_B-p_2}{1-\gamma-\beta})}$.

The equilibrium sourcing strategy of the brand-name firm is as follows:

(a) Strategy H with $w_1^* = k_1$ if $e < f_{DH}$ and $\Delta < \min\{\Delta_{DH}, \Delta_{HO}\}$;

(b) Strategy D with $w_1^* = k_1$, and

$$
w_2^* = \begin{cases} k_2, & \text{if } e \le \min\{e_{D1}, f_{DO1}\} \text{ and } \min\{\Delta_{DH}, \Delta_{DO}\} \le \Delta < \Delta_{DO};\\ w_2^{(0)}, & \text{if } \max\{e_{D1}, f_{DH}, f_{DO2}\} \le e \le \min\{\frac{(1-\phi)e_3}{\phi}, f_{DO3}\}, \text{ or if } e > \max\{\frac{(1-\phi)e_3}{\phi}, f_{DO4}\};\end{cases}
$$

(c) Strategy O with

$$
w_2^* = \begin{cases} w_2^{O(1)}, & \text{if } e < \min\{e_{O1}, f_{DO2}\} \text{ and } \Delta > \max\{\Delta_{HO}, \Delta_{DO}\};\\ \max\{w_2^{(0)}, w_2^{O(2)}\}, & \text{if } \max\{e_{O1}, f_{DO1}, f_{DO3}\} \le e \le \frac{(1-\phi)e_3}{\phi}, \text{ or if } \frac{(1-\phi)e_3}{\phi} < e < f_{DO4},\end{cases}
$$

where e_3 is defined in Equation (7).

In this extension, the equilibrium is similar to that in the base model. We find that the consumer surplus under each optimal strategy is the same as that in the base model, while the social surplus can be lower or higher than that in the base model.

E-Companion B Proofs of Analytical Results

B.1 Proof of Lemma 1.

This proof has two steps: (1) we derive the profit expressions under each possible strategy; (2) we focus on the discussion about the counterfeiter or the authorised overseas supplier about whether to sell the counterfeit.

Step 1: Under each possible sourcing strategy, we obtain the profit expression of each firm as below.

Strategy H: Given wholesale prices w_1 and w_2, the home supplier accepts the contract and the counterfeiter rejects the contract, i.e., $d_1 = 1$ and $d_2 = 0$. Thus, the brand-name firm only sources from the home supplier.

(1) If the counterfeiter sells the counterfeit in the overseas market, i.e., $s = 1$, the expected profits of the brand-name firm, the home and overseas suppliers are given below:

$$\begin{aligned}
&\pi_B^H(w_1) = (p_B - w_1)(1-p_B) + \alpha(p_B - w_1 - t)\left(1 - \tfrac{p_B - p_2}{1-\beta}\right),\\
&\pi_1^H(w_1) = (w_1 - k_1)\left((1-p_B) + \alpha\left(1 - \tfrac{p_B - p_2}{1-\beta}\right)\right),\\
&\pi_2^H = \alpha(p_2 - k_2)\left(\tfrac{p_B - p_2}{1-\beta} - \tfrac{p_2}{\beta}\right) - e.
\end{aligned}$$

(2) If the counterfeiter does not sell the counterfeit, i.e., $s = 0$, the brand-name firm is the monopoly in the overseas market. Thus, their profits expressions are:

$$\pi_B^H(w_1) = (p_B - w_1)(1-p_B) + \alpha(p_B - w_1 - t)(1-p_B), \quad \pi_1^H(w_1) = (1+\alpha)(w_1 - k_1)(1-p_B), \quad \pi_2^H = 0.$$

Strategy D: Given wholesale prices w_1 and w_2, the home supplier and the counterfeiter accept their contracts, respectively, i.e., $d_1 = 1$ and $d_2 = 1$. Then, the counterfeiter is converted to an authorized overseas supplier. Thus, their profit expressions are as follows.

(1) If the overseas supplier sells the counterfeit in the overseas market, i.e., $s = 1$:

$$\begin{aligned}
&\pi_B^D(w_1, w_2) = (p_B - w_1)(1-p_B) + \alpha(p_B - w_2)\left(1 - \tfrac{p_B - p_2}{1-\gamma-\beta}\right),\\
&\pi_1^D(w_1) = (w_1 - k_1)(1-p_B),\\
&\pi_2^D(w_2) = \alpha(w_2 - k_2)\left(1 - \tfrac{p_B - p_2}{1-\gamma-\beta}\right) + \alpha(p_2 - k_2)\left(\tfrac{p_B - p_2}{1-\gamma-\beta} - \tfrac{p_2}{\beta}\right) - e.
\end{aligned}$$

(2) If the overseas supplier does not sell the counterfeit, i.e., $s = 0$:

$$\begin{aligned}
&\pi_B^D(w_1, w_2) = (p_B - w_1)(1-p_B) + \alpha(p_B - w_2)\left(1 - \tfrac{p_B}{1-\gamma}\right),\\
&\pi_1^D(w_1) = (w_1 - k_1)(1-p_B),\\
&\pi_2^D(w_2) = \alpha(w_2 - k_2)\left(1 - \tfrac{p_B}{1-\gamma}\right).
\end{aligned}$$

Strategy O: Given wholesale prices w_1 and w_2, the home supplier rejects the contract and the counterfeiter accepts the contract, i.e., $d_1 = 0$ and $d_2 = 1$. Then, the counterfeiter is converted to an authorized overseas supplier. Thus, their profit expressions are as follows.

(1) If the overseas supplier sells the counterfeit in the overseas market, i.e., $s = 1$:

$$\begin{aligned}
&\pi_B^O(w_2) = (p_B - w_2 - t)\left(1 - \tfrac{p_B}{1-\gamma}\right) + \alpha(p_B - w_2)\left(1 - \tfrac{p_B - p_2}{1-\gamma-\beta}\right),\\
&\pi_1^O = 0, \quad \pi_2^O(w_2) = (w_2 - k_2)\left(1 - \tfrac{p_B}{1-\gamma}\right) + \alpha(w_2 - k_2)\left(1 - \tfrac{p_B - p_2}{1-\gamma-\beta}\right) + \alpha(p_2 - k_2)\left(\tfrac{p_B - p_2}{1-\gamma-\beta} - \tfrac{p_2}{\beta}\right) - e.
\end{aligned}$$

(2) If the overseas supplier does not sell the counterfeit, i.e., $s = 0$:

$$\begin{aligned}
&\pi_B^O(w_2) = (p_B - w_2 - t)\left(1 - \tfrac{p_B}{1-\gamma}\right) + \alpha(p_B - w_2)\left(1 - \tfrac{p_B}{1-\gamma}\right),\\
&\pi_1^O = 0, \quad \pi_2^O(w_2) = (1+\alpha)(w_2 - k_2)\left(1 - \tfrac{p_B}{1-\gamma}\right).
\end{aligned}$$

Strategy N: Given wholesale prices w_1 and w_2, the home supplier and the counterfeiter reject their contracts, respectively, i.e., $d_1 = 0$ and $d_2 = 0$. Under this strategy, the brand-name firm does not have suppliers, and later we will show that it is not an equilibrium strategy.

(1) If the counterfeiter sells the counterfeit in the overseas market, i.e., $s = 1$:

$$\pi_B^N = 0, \quad \pi_1^N = 0, \quad \pi_2^N = \alpha\,(p_2 - k_2)\left(1 - \frac{p_2}{\beta}\right) - e.$$

(2) If the counterfeiter does not sell the counterfeit, i.e., $s = 0$:

$$\pi_B^N = 0, \quad \pi_1^N = 0, \quad \pi_2^N = 0.$$

For the analysis below, it is convenient to define the following notations:

$$\begin{aligned}
&M = \alpha\,(p_2 - k_2)\,(\tfrac{p_B - p_2}{1-\beta} - \tfrac{p_2}{\beta}) - e, \;\; w_2^{D(2)} = k_2 + \tfrac{M}{\alpha(1-\frac{p_B}{1-\gamma})},\\
&M' = \alpha\,(p_2 - k_2)\,(\tfrac{p_B - p_2}{1-\gamma-\beta} - \tfrac{p_2}{\beta}) - e, \;\; w_2^{(0)} = k_2 + \tfrac{M'}{\alpha\left(\frac{p_B-p_2}{1-\gamma-\beta} - \frac{p_B}{1-\gamma}\right)},\\
&K = \alpha\,(p_2 - k_2)\,(1 - \tfrac{p_2}{\beta}) - e, \qquad w_2^{O(2)} = k_2 + \tfrac{K}{(1+\alpha)(1-\frac{p_B}{1-\gamma})},\\
&K - M' = \alpha\,(p_2 - k_2)\,(1 - \tfrac{p_B - p_2}{1-\gamma-\beta}), \;\; w_2^{O(1)} = k_2 + \tfrac{K - M'}{(1-\frac{p_B}{1-\gamma}) + \alpha(1-\frac{p_B-p_2}{1-\gamma-\beta})}.
\end{aligned}$$

With the assumption $0 \le e < \alpha(p_2 - k_2)(\frac{p_B - p_2}{1-\beta} - \frac{p_2}{\beta})$, we know, $M > 0$, $M' > 0$ and $K > 0$. Note that $M < M'$.

Step 2: We discuss whether the counterfeiting exists.

In the following, we make a comparison between $\pi_2^H\,(s = 1)$ and $\pi_2^H\,(s = 0)$. There are two scenarios depending on d_2.

1. When the counterfeiter does not accept the contract, i.e., $d_2 = 0$, which means she is not converted to an authorized overseas supplier, we have the below discussion.

(1) Under Strategy H, if the counterfeiter sells the counterfeit in the overseas market, her profit is $\pi_2^H\,(w_2, s = 1) = \alpha\,(p_2 - k_2)\,(\frac{p_B - p_2}{1-\beta} - \frac{p_2}{\beta}) - e$.

(2) Under Strategy N, if the counterfeiter sells the counterfeit in the overseas market, her profit is $\pi_2^N\,(w_2, s = 1) = \alpha\,(p_2 - k_2)\,(1 - \frac{p_2}{\beta}) - e$.

Note that we assume $0 \le e < \alpha(p_2 - k_2)(\frac{p_B - p_2}{1-\beta} - \frac{p_2}{\beta})$. Thus, when the counterfeiter does not accept the contract, she will sell the counterfeit in the overseas market.

2. When the counterfeiter accepts the contract, i.e., $d_2 = 1$, which means she becomes an authorized overseas supplier, we have the below discussion.

(1) Under Strategy D, if the overseas supplier does not sell the counterfeit in the overseas market, her profit is $\pi_2^D\,(w_2, s = 0) = \alpha\,(w_2 - k_2)\,(1 - \frac{p_B}{1-\gamma})$. If the overseas supplier sells the counterfeit in the overseas market, her profit is $\pi_2^D\,(w_2, s = 1) = \alpha\,(w_2 - k_2)\,(1 - \frac{p_B - p_2}{1-\gamma-\beta}) + (\alpha\,(p_2 - k_2)\,(\frac{p_B - p_2}{1-\gamma-\beta} - \frac{p_2}{\beta}) - e)$.

Then, from $\pi_2^D\,(w_2, s = 0) \ge \pi_2^D\,(w_2, s = 1)$, we obtain, $w_2 \ge w_2^{(0)}$, where $w_2^{(0)} = k_2 + \frac{M'}{\alpha(\frac{p_B - p_2}{1-\gamma-\beta} - \frac{p_B}{1-\gamma})}$.

(2) Under Strategy O, if the overseas supplier does not sell the counterfeit in the overseas market, her profit is $\pi_2^O(w_2, s=0) = (w_2-k_2)(1-\frac{p_B}{1-\gamma}) + \alpha(w_2-k_2)(1-\frac{p_B}{1-\gamma})$. If the overseas supplier sells the counterfeit in the overseas market, her profit is $\pi_2^O(w_2, s=1) = (w_2-k_2)(1-\frac{p_B}{1-\gamma}) + \alpha(w_2-k_2)(1-\frac{p_B-p_2}{1-\gamma-\beta}) + (\alpha(p_2-k_2)(\frac{p_B-p_2}{1-\gamma-\beta}-\frac{p_2}{\beta})-e)$.

Then, from $\pi_2^O(w_2, s=0) \geq \pi_2^O(w_2, s=1)$, we obtain, $w_2 \geq w_2^{(0)}$.

Thus, if the wholesale price of w_2 satisfies $w_2 < w_2^{(0)}$, then even the counterfeiter is converted to an authorized overseas supplier, she would still sell counterfeits in the overseas market, i.e., $s(w_1, w_2, d_1) = 1$ with $d_2 = 1$. ■

B.2 Proof of Lemma 2.

This proof has two steps: (1) we derive the best response of two suppliers; (2) we discuss the possible optimal wholesale prices offered by the brand-name firm under each sourcing strategy. In order to differentiate the cases that the overseas supplier sells counterfeits, we use the superscripts "D†", "O†" to denote the Strategy D without counterfeiting, Strategy O without counterfeiting, respectively; and use the superscripts "DC", "OC" to denote the Strategy D with counterfeiting, Strategy O with counterfeiting, respectively.

Step 1: We derive the best responses of the overseas and home suppliers.

With each sourcing strategy, the overseas supplier's profit function is as follows:

$$\pi_2^H(w_2) = \alpha(p_2-k_2)\left(\frac{p_B-p_2}{1-\beta}-\frac{p_2}{\beta}\right)-e,$$

$$\pi_2^D = \begin{cases} \pi_2^{DC}(w_2) = \alpha(w_2-k_2)\left(1-\frac{p_B-p_2}{1-\gamma-\beta}\right) + \left(\alpha(p_2-k_2)\left(\frac{p_B-p_2}{1-\gamma-\beta}-\frac{p_2}{\beta}\right)-e\right), & \text{if } w_2 < w_2^{(0)}, \\ \pi_2^{D\dagger}(w_2) = \alpha(w_2-k_2)(1-\frac{p_B}{1-\gamma}), & \text{if } w_2 \geq w_2^{(0)}, \end{cases}$$

$$\pi_2^O = \begin{cases} \pi_2^{OC}(w_2) = (w_2-k_2)\left(1-\frac{p_B}{1-\gamma}\right) \\ \qquad +\alpha(w_2-k_2)\left(1-\frac{p_B-p_2}{1-\gamma-\beta}\right) + \left(\alpha(p_2-k_2)\left(\frac{p_B-p_2}{1-\gamma-\beta}-\frac{p_2}{\beta}\right)-e\right), & \text{if } w_2 < w_2^{(0)}, \\ \pi_2^{O\dagger}(w_2) = (w_2-k_2)\left(1-\frac{p_B}{1-\gamma}\right) + \alpha(w_2-k_2)\left(1-\frac{p_B}{1-\gamma}\right), & \text{if } w_2 \geq w_2^{(0)}, \end{cases}$$

$$\pi_2^N = \alpha(p_2-k_2)(1-\frac{p_2}{\beta})-e.$$

1.1 Below, we discuss the conditions for overseas supplier's accepting.

(1) Under $w_2 < w_2^{(0)}$, where $w_2^{(0)} = k_2 + \frac{M'}{\alpha(\frac{p_B-p_2}{1-\gamma-\beta}-\frac{p_B}{1-\gamma})}$, we discuss for a given belief on the home supplier's contact decision $\widetilde{d_1} = 1$ and $\widetilde{d_1} = 0$, respectively.

(i) If $\widetilde{d_1} = 1$, then, we compare the overseas supplier's profits between Strategy D with counterfeiting and Strategy H, i.e., $\pi_2^{DC}(w_2)$ and π_2^H. If the overseas supplier decides to accept, then it should satisfy

$$\begin{aligned} &\pi_2^{DC}(w_2) \geq \pi_2^H, \\ \Rightarrow\ &\alpha(w_2-k_2)\left(1-\frac{p_B-p_2}{1-\gamma-\beta}\right) + M' \geq M, \\ \Rightarrow\ &w_2 \geq k_2 + \frac{M-M'}{\alpha\left(1-\frac{p_B-p_2}{1-\gamma-\beta}\right)}. \end{aligned}$$

Note that $w_2 \geq k_2$. As $M < M'$, then, we have, $w_2 \geq k_2$.

(ii) If $\widetilde{d_1} = 0$, then, we compare the overseas supplier's profits between Strategy O with counterfeiting and Strategy N, i.e., $\pi_2^{OC}(w_2)$ and π_2^N. If the overseas supplier decides to accept, then it should satisfy

$$\begin{aligned}
&\pi_2^{OC}(w_2) \geq \pi_2^N, \\
\Rightarrow\ &(w_2 - k_2)\left(1 - \tfrac{p_B}{1-\gamma}\right) + \alpha (w_2 - k_2)\left(1 - \tfrac{p_B - p_2}{1-\gamma-\beta}\right) + M' \geq K, \\
\Rightarrow\ &w_2 \geq w_2^{O(1)}\text{, where } w_2^{O(1)} = k_2 + \tfrac{K - M'}{\left(1-\frac{p_B}{1-\gamma}\right) + \alpha\left(1 - \frac{p_B-p_2}{1-\gamma-\beta}\right)} = k_2 + \tfrac{\alpha(p_2-k_2)\left(1-\frac{p_B-p_2}{1-\gamma-\beta}\right)}{\left(1-\frac{p_B}{1-\gamma}\right) + \alpha\left(1 - \frac{p_B-p_2}{1-\gamma-\beta}\right)}.
\end{aligned}$$

Thus, in the case of $w_2 < w_2^{(0)}$, we obtain

$$d_2(\widetilde{d_1}) = \begin{cases} d_2\left(\widetilde{d_1} = 1\right) = 1, & \text{if } k_2 \leq w_2 < w_2^{(0)}, \\ d_2\left(\widetilde{d_1} = 1\right) = 0, & \text{if } w_2 < k_2, \\ d_2\left(\widetilde{d_1} = 0\right) = 1, & \text{if } \min\{w_2^{O(1)}, w_2^{(0)}\} \leq w_2 < w_2^{(0)}, \\ d_2\left(\widetilde{d_1} = 0\right) = 0, & \text{if } w_2 < \min\{w_2^{O(1)}, w_2^{(0)}\}. \end{cases}$$

(2) Under $w_2 \geq w_2^{(0)}$, we discuss for given $\widetilde{d_1} = 1$ and $\widetilde{d_1} = 0$, respectively.

(i) If $\widetilde{d_1} = 1$, then, we compare the overseas supplier's profits between Strategy D without counterfeiting and Strategy H, i.e., $\pi_2^{D\dagger}(w_2)$ and π_2^H. If the overseas supplier decides to accept, then it should satisfy

$$\begin{aligned}
&\pi_2^{D\dagger}(w_2) \geq \pi_2^H, \\
\Rightarrow\ &\alpha (w_2 - k_2)\left(1 - \tfrac{p_B}{1-\gamma}\right) \geq M, \\
\Rightarrow\ &w_2 \geq w_2^{D(2)}\text{, where } w_2^{D(2)} = k_2 + \tfrac{M}{\alpha\left(1-\frac{p_B}{1-\gamma}\right)}.
\end{aligned}$$

(ii) If $\widetilde{d_1} = 0$, then, we compare the overseas supplier's profits between Strategy O without counterfeiting and Strategy N, i.e., $\pi_2^{O\dagger}(w_2)$ and π_2^N. If the overseas supplier decides to accept, then it should satisfy

$$\begin{aligned}
&\pi_2^{O\dagger}(w_2) \geq \pi_2^N, \\
\Rightarrow\ &(w_2 - k_2)\left(1 - \tfrac{p_B}{1-\gamma}\right) + \alpha (w_2 - k_2)\left(1 - \tfrac{p_B}{1-\gamma}\right) \geq K, \\
\Rightarrow\ &w_2 \geq w_2^{O(2)}\text{, where } w_2^{O(2)} = k_2 + \tfrac{K}{(1+\alpha)\left(1-\frac{p_B}{1-\gamma}\right)}.
\end{aligned}$$

Thus, in the case of $w_2 \geq w_2^{(0)}$, we obtain

$$d_2(\widetilde{d_1}) = \begin{cases} d_2\left(\widetilde{d_1} = 1\right) = 1, & \text{if } w_2 \geq \max\{w_2^{D(2)}, w_2^{(0)}\}, \\ d_2\left(\widetilde{d_1} = 1\right) = 0, & \text{if } w_2^{(0)} < w_2 < \max\{w_2^{D(2)}, w_2^{(0)}\}, \\ d_2\left(\widetilde{d_1} = 0\right) = 1, & \text{if } w_2 \geq \max\{w_2^{O(2)}, w_2^{(0)}\}, \\ d_2\left(\widetilde{d_1} = 0\right) = 0, & \text{if } w_2^{(0)} < w_2 < \max\{w_2^{O(2)}, w_2^{(0)}\}. \end{cases}$$

1.2 Similarly, we derive the best response function of the home supplier $d_1(\widetilde{d_2})$ to the overseas supplier's action $\widetilde{d_2} \in \{0, 1\}$ as follows:

$$d_1(\widetilde{d_2}) = \begin{cases} d_1\left(\widetilde{d_2} = 1\right) = 1, & \text{if } w_1 \geq k_1, \\ d_1\left(\widetilde{d_2} = 0\right) = 1, & \text{if } w_1 \geq k_1, \\ d_1\left(\widetilde{d_2} = 1\right) = 0, & \text{if } w_1 < k_1, \\ d_1\left(\widetilde{d_2} = 0\right) = 0, & \text{if } w_1 < k_1. \end{cases}$$

1.3 Given best response functions $d_1(\widetilde{d_2})$ and $d_2(\widetilde{d_1})$, we obtain the following fixed point (d_1^*, d_2^*) that satisfies $(d_1(\widetilde{d_2}), \widetilde{d_2}) = (\widetilde{d_1}, d_2(\widetilde{d_1}))$. Thus, the optimal decisions of two suppliers are

$$(d_1^*, d_2^*) = \begin{cases} (1,1), & \text{if } w_1 \geq k_1,\, k_2 \leq w_2 < w_2^{(0)} \text{ or } w_2 \geq \max\{w_2^{D(2)}, w_2^{(0)}\}, \\ (1,0), & \text{if } w_1 \geq k_1,\, w_2 < k_2 \text{ or } w_2^{(0)} \leq w_2 < \max\{w_2^{D(2)}, w_2^{(0)}\}, \\ (0,1), & \text{if } w_1 < k_1,\, \min\{w_2^{O(1)}, w_2^{(0)}\} \leq w_2 < w_2^{(0)} \text{ or } w_2 \geq \max\{w_2^{O(2)}, w_2^{(0)}\}, \\ (0,0), & \text{if } w_1 < k_1,\, w_2 < \min\{w_2^{O(1)}, w_2^{(0)}\} \text{ or } w_2^{(0)} \leq w_2 < \max\{w_2^{O(2)}, w_2^{(0)}\}, \end{cases}$$

where

$$\begin{aligned} &M = \alpha\,(p_2 - k_2)\,(\tfrac{p_B - p_2}{1-\beta} - \tfrac{p_2}{\beta}) - e, \;\; w_2^{D(2)} = k_2 + \tfrac{M}{\alpha(1 - \frac{p_B}{1-\gamma})}; \\ &M' = \alpha\,(p_2 - k_2)\,(\tfrac{p_B - p_2}{1-\gamma-\beta} - \tfrac{p_2}{\beta}) - e, \; w_2^{(0)} = k_2 + \tfrac{M'}{\alpha\left(\frac{p_B - p_2}{1-\gamma-\beta} - \frac{p_B}{1-\gamma}\right)}; \\ &K = \alpha\,(p_2 - k_2)\,(1 - \tfrac{p_2}{\beta}) - e, \qquad w_2^{O(2)} = k_2 + \tfrac{K}{(1+\alpha)(1 - \frac{p_B}{1-\gamma})}; \\ &K - M' = \alpha\,(p_2 - k_2)\,(1 - \tfrac{p_B - p_2}{1-\gamma-\beta}), \;\; w_2^{O(1)} = k_2 + \tfrac{K - M'}{(1 - \frac{p_B}{1-\gamma}) + \alpha(1 - \frac{p_B - p_2}{1-\gamma-\beta})}. \end{aligned}$$

Note that $w_2^{O(1)}$ is independent on e; and $w_2^{(0)}$, $w_2^{O(2)}$ and $w_2^{D(2)}$ are dependent on e.

Step 2: We derive the optimal wholesale price(s) with each case.

Substituting (d_1^*, d_2^*) into the profit functions of the brand-name firm, we analyze the optimal wholesale price under each possible sourcing strategy.

$$\pi_B^H\,(w_1) = (p_B - w_1)\,(1 - p_B) + \alpha\,(p_B - w_1 - t)\,(1 - \tfrac{p_B - p_2}{1-\beta}), \text{ if } w_1 \geq k_1,$$

$$\pi_B^D = \begin{cases} \pi_B^{DC}\,(w_1, w_2) = (p_B - w_1)\,(1 - p_B) + \alpha\,(p_B - w_2)\,(1 - \frac{p_B - p_2}{1-\gamma-\beta}), \text{ if } w_1 \geq k_1,\, k_2 \leq w_2 < w_2^{(0)}, \\ \pi_B^{D\dagger}\,(w_1, w_2) = (p_B - w_1)\,(1 - p_B) + \alpha\,(p_B - w_2)\,(1 - \frac{p_B}{1-\gamma}), \quad \text{if } w_1 \geq k_1,\, w_2 \geq \max\{w_2^{D(2)}, w_2^{(0)}\}, \end{cases}$$

$$\pi_B^O = \begin{cases} \pi_B^{OC}\,(w_2) = (p_B - w_2 - t)\,(1 - \frac{p_B}{1-\gamma}) + \alpha\,(p_B - w_2)\,(1 - \frac{p_B - p_2}{1-\gamma-\beta}), \text{ if } \min\{w_2^{O(1)}, w_2^{(0)}\} \leq w_2 < w_2^{(0)}, \\ \pi_B^{O\dagger}\,(w_2) = (p_B - w_2 - t)\,(1 - \frac{p_B}{1-\gamma}) + \alpha\,(p_B - w_2)\,(1 - \frac{p_B}{1-\gamma}), \quad \text{if } w_2 \geq \max\{w_2^{O(2)}, w_2^{(0)}\}, \end{cases}$$

$$\pi_B^N = 0.$$

In the following, we have two steps: (1) firstly check the feasible region of π_B under each case; (2) then make a comparison between $\pi_B^{D\dagger}$ and π_B^{DC}, $\pi_B^{O\dagger}$ and π_B^{OC}, respectively.

2.1 We first check the feasibility of π_B under our assumption of $e < \bar{e}$, where $\bar{e} = \alpha(p_2 - k_2)(\frac{p_B - p_2}{1-\beta} - \frac{p_2}{\beta})$.

Recall that $w_2^{O(1)}$ is independent on e; and $w_2^{(0)}$, $w_2^{O(2)}$ and $w_2^{D(2)}$ are dependent on e. From the conditions of the brand-name firm's profit expression under each possible strategy, we know:

$$\begin{aligned} &w_2^{O(1)} < w_2^{(0)}(e) \quad\;\; \Rightarrow e < e_1, \text{ where } e_1 = \left(p_2 - k_2 - \frac{\alpha(p_2 - k_2)(1 - \frac{p_B - p_2}{1-\gamma-\beta})}{(1 - \frac{p_B}{1-\gamma}) + \alpha(1 - \frac{p_B - p_2}{1-\gamma-\beta})}\frac{\beta}{1-\gamma}\right)\frac{\alpha(\beta p_B - (1-\gamma)p_2)}{(1-\gamma-\beta)\beta}; \\ &w_2^{D(2)}(e) < w_2^{(0)}(e) \Rightarrow e < e_2, \text{ where } e_2 = \frac{(1-\gamma)(p_2 - k_2)\alpha(1 - \frac{p_B}{1-\gamma}) - \alpha(p_2 - k_2)(\frac{p_B - p_2}{1-\beta} - \frac{p_2}{\beta})\beta}{\left(\frac{(1-\gamma)(1-\gamma-\beta)(1 - \frac{p_B}{1-\gamma})}{(\beta p_B - (1-\gamma)p_2)} - 1\right)\beta}; \\ &w_2^{O(2)}(e) < w_2^{(0)}(e) \Rightarrow e < e_3, \text{ where } e_3 = \frac{(1-\gamma)(p_2 - k_2)(1+\alpha)(1 - \frac{p_B}{1-\gamma}) - \alpha(p_2 - k_2)(1 - \frac{p_2}{\beta})\beta}{\left(\frac{(1-\gamma)(1-\gamma-\beta)(1+\alpha)(1 - \frac{p_B}{1-\gamma})}{\alpha(\beta p_B - (1-\gamma)p_2)} - 1\right)\beta}. \end{aligned} \tag{7}$$

Note that $e_2 \geq \bar{e} > e_3 \geq e_1$. When $\gamma = 0$, we have, $e_2 = \bar{e} > e_3 = e_1$. Thus, with the assumption of $e < \bar{e}$ in our base model, if $w_2^{D(2)} > w_2^{(0)}$, then $e > e_2$ and $w_2 > w_2^{D(2)}$. As $e > e_2$ is out of the feasible region, it implies that under Strategy D with $e < \bar{e}$, the feasible condition is $w_2 > w_2^{(0)}$.

Note that for Strategy O with counterfeiting, i.e., $\pi_B^{OC}(w_2)$, the condition $\min\{w_2^{O(1)}, w_2^{(0)}\} \leq w_2 < w_2^{(0)}$ is non-empty if $e < e_1$.

As $\pi_B(w_1, w_2)$ decreases in w_1, then, the optimal wholesale price of the home supplier that the brand-name firm is willing to offer is equal to the production cost, that is, $w_1^H = k_1$ with Strategy H, and $w_1^D = k_1$ with Strategy D.

As $\pi_B(w_1, w_2)$ decreases in w_2, then, the optimal wholesale price of the overseas supplier that the brand-name firm is willing to offer is the lower bound of the feasible regions. We use $*$ to indicate the optimal wholesale decision of these cases. Then, the optimal wholesale prices w_2 for these cases are $w_2^{DC*} = k_2$, $w_2^{D\dagger *} = w_2^{(0)}$, $w_2^{OC*} = w_2^{O(1)}$, $w_2^{O\dagger *} = \max\{w_2^{O(2)}, w_2^{(0)}\}$, respectively.

Thus, we have following profit expression under each case:

$$\pi_B^H = (p_B - k_1)(1 - p_B) + \alpha(p_B - k_1 - t)(1 - \tfrac{p_B - p_2}{1-\beta}),$$

$$\pi_B^D = \begin{cases} \pi_B^{DC}(w_2^{DC*}) = (p_B - k_1)(1 - p_B) + \alpha(p_B - w_2^{DC*})(1 - \frac{p_B - p_2}{\gamma-\beta}), \\ \pi_B^{D\dagger}\left(w_2^{D\dagger *}\right) = (p_B - k_1)(1 - p_B) + \alpha\left(p_B - w_2^{D\dagger *}\right)(1 - \frac{p_B}{\gamma}), \end{cases}$$

$$\pi_B^O = \begin{cases} \pi_B^{OC}(w_2^{OC*}) = (p_B - w_2^{OC*} - t)(1 - \frac{p_B}{\gamma}) + \alpha(p_B - w_2^{OC*})(1 - \frac{p_B - p_2}{\gamma-\beta}), \text{ if } e < e_1, \\ \pi_B^{O\dagger}\left(w_2^{O\dagger *}\right) = \left(p_B - w_2^{O\dagger *} - t\right)(1 - \frac{p_B}{\gamma}) + \alpha\left(p_B - w_2^{O\dagger *}\right)(1 - \frac{p_B}{\gamma}), \end{cases}$$

$$\pi_B^N = 0,$$

where $w_2^{DC*} = k_2$, $w_2^{D\dagger *} = w_2^{(0)}$, $w_2^{OC*} = w_2^{O(1)}$, $w_2^{O\dagger *} = \max\{w_2^{O(2)}, w_2^{(0)}\}$.

2.2 Then, we make comparisons for strategies D and O, respectively.

Under Strategy D:

$$\pi_B^D = \begin{cases} \pi_B^{DC}(w_2^{DC*}) = (p_B - k_1)(1 - p_B) + \alpha(p_B - w_2^{DC*})(1 - \frac{p_B - p_2}{1-\gamma-\beta}), \\ \pi_B^{D\dagger}\left(w_2^{D\dagger *}\right) = (p_B - k_1)(1 - p_B) + \alpha\left(p_B - w_2^{D\dagger *}\right)(1 - \frac{p_B}{1-\gamma}). \end{cases}$$

Then,

$$\begin{aligned} & \pi_B^{D\dagger}\left(w_2^{D\dagger *}\right) \geq \pi_B^{DC}(w_2^{DC*}), \\ \Rightarrow\ & (p_B - w_2^{D*})(1 - \tfrac{p_B}{1-\gamma}) \geq (p_B - w_2^{DC*})(1 - \tfrac{p_B - p_2}{1-\gamma-\beta}), \\ \Rightarrow\ & w_2^{D*} \leq \frac{p_B(\frac{p_B - p_2}{1-\gamma-\beta} - \frac{p_B}{1-\gamma}) + w_2^{DC*}(1 - \frac{p_B - p_2}{1-\gamma-\beta})}{1 - \frac{p_B}{1-\gamma}} = p_B - \frac{\left(p_B - w_2^{DC*}\right)(1 - \frac{p_B - p_2}{1-\gamma-\beta})}{1 - \frac{p_B}{1-\gamma}}, \\ \Rightarrow\ & e \geq e_{D1}\text{ , where } e_{D1} = \left(p_2 - k_2 - (\frac{(p_B - k_2)(\frac{p_B - p_2}{1-\gamma-\beta} - \frac{p_B}{1-\gamma})}{1 - \frac{p_B}{1-\gamma}})\frac{\beta}{1-\gamma}\right)\frac{\alpha(\beta p_B - (1-\gamma)p_2)}{(1-\gamma-\beta)\beta}. \end{aligned}$$

Under Strategy O:

$$\pi_B^O = \begin{cases} \pi_B^{OC}(w_2^{OC*}) = (p_B - w_2^{OC*} - t)(1 - \frac{p_B}{1-\gamma}) + \alpha(p_B - w_2^{OC*})(1 - \frac{p_B - p_2}{1-\gamma-\beta}), \text{ if } e < e_1, \\ \pi_B^{O\dagger}\left(w_2^{O\dagger *}\right) = \left(p_B - w_2^{O\dagger *} - t\right)(1 - \frac{p_B}{1-\gamma}) + \alpha\left(p_B - w_2^{O\dagger *}\right)(1 - \frac{p_B}{1-\gamma}). \end{cases}$$

Then,

$$\begin{aligned}
&\pi_B^{O\dagger}\left(w_2^{O\dagger*}\right) \geq \pi_B^{OC}\left(w_2^{OC*}\right),\\
\Rightarrow\ &\alpha(p_B - w_2^{OC*})(\tfrac{p_B-p_2}{1-\gamma-\beta} - \tfrac{p_B}{1-\gamma}) \geq (w_2^{O\dagger*} - w_2^{OC*})(1+\alpha)(1-\tfrac{p_B}{1-\gamma}),\\
\Rightarrow\ &w_2^{O\dagger*} \leq \frac{\alpha(p_B-w_2^{OC*})(\frac{p_B-p_2}{1-\gamma-\beta}-\frac{p_B}{1-\gamma})}{(1+\alpha)(1-\frac{p_B}{1-\gamma})} + w_2^{OC*},\\
\Rightarrow\ &e \geq e_{O1}\text{ , where } e_{O1} = \left(p_2 - k_2 - (\frac{\alpha(p_B-w_2^{OC*})(\frac{p_B-p_2}{1-\gamma-\beta}-\frac{p_B}{\gamma})}{(1+\alpha)(1-\frac{p_B}{1-\gamma})} + w_2^{OC*} - k_2)\frac{\beta}{1-\gamma}\right)\frac{\alpha(\beta p_B-(1-\gamma)p_2)}{(1-\gamma-\beta)\beta}.
\end{aligned}$$

Then, based on above discussion, we have the following optimal wholesale price w_2 for Strategy D and Strategy O, respectively. Note that $e_{O1} < e_1$.

(a) Under Strategy D, (i) $w_2^D = k_2$ and $s^* = 1$, if $e < e_{D1}$; (ii) $w_2^D = w_2^{(0)}$ and $s^* = 0$, if $e \geq e_{D1}$;

(b) under Strategy O, (i) $w_2^O = w_2^{O(1)}$ and $s^* = 1$, if $e < e_{O1}$; (ii) $w_2^O = \max\{w_2^{O(2)}, w_2^{(0)}\}$ and $s^* = 0$, if $e \geq e_{O1}$;

where

$$\begin{aligned}
e_{D1} &= \left(p_2 - k_2 - (\frac{(p_B-k_2)(\frac{p_B-p_2}{1-\gamma-\beta}-\frac{p_B}{1-\gamma})}{1-\frac{p_B}{1-\gamma}})\frac{\beta}{1-\gamma}\right)\frac{\alpha(\beta p_B-(1-\gamma)p_2)}{(1-\gamma-\beta)\beta},\\
e_{O1} &= \left(p_2 - k_2 - (\frac{\alpha(p_B-w_2^{OC*})(\frac{p_B-p_2}{1-\gamma-\beta}-\frac{p_B}{\gamma})}{(1+\alpha)(1-\frac{p_B}{1-\gamma})} + w_2^{OC*} - k_2)\frac{\beta}{1-\gamma}\right)\frac{\alpha(\beta p_B-(1-\gamma)p_2)}{(1-\gamma-\beta)\beta};
\end{aligned}\tag{8}$$

and $w_2^{OC*} = k_2 + \frac{\alpha(p_2-k_2)(1-\frac{p_B-p_2}{1-\gamma-\beta})}{(1-\frac{p_B}{1-\gamma})+\alpha(1-\frac{p_B-p_2}{1-\gamma-\beta})}$.

Thus, we have the results. ■

B.3 Proof of Proposition 1.

Recall that the brand-name firm's optimal profit under each case is as follows:

$$\pi_B^H = (p_B - k_1)(1-p_B) + \alpha(p_B - k_1 - t)(1 - \tfrac{p_B-p_2}{1-\beta}),$$

$$\pi_B^D = \begin{cases} \pi_B^{D\dagger}\left(w_2^{D\dagger*}\right) = (p_B-k_1)(1-p_B) + \alpha\left(p_B - w_2^{D\dagger*}\right)(1-\frac{p_B}{1-\gamma}), & \text{if } e \geq e_{D1},\\ \pi_B^{DC}\left(w_2^{DC*}\right) = (p_B-k_1)(1-p_B) + \alpha\left(p_B - w_2^{DC*}\right)(1-\frac{p_B-p_2}{1-\gamma-\beta}), & \text{if } e < e_{D1}, \end{cases}$$

$$\pi_B^O = \begin{cases} \pi_B^{O\dagger}\left(w_2^{O\dagger*}\right) = \left(p_B - w_2^{O\dagger*} - t\right)(1-\frac{p_B}{1-\gamma}) + \alpha\left(p_B - w_2^{O\dagger*}\right)(1-\frac{p_B}{1-\gamma}), & \text{if } e \geq e_{O1},\\ \pi_B^{OC}\left(w_2^{OC*}\right) = \left(p_B - w_2^{OC*} - t\right)(1-\frac{p_B}{1-\gamma}) + \alpha\left(p_B - w_2^{OC*}\right)(1-\frac{p_B-p_2}{1-\gamma-\beta}), & \text{if } e < e_{O1}, \end{cases}$$

$$\pi_B^N = 0,$$

where $w_2^{DC*} = k_2$, $w_2^{D\dagger*} = w_2^{(0)}$, $w_2^{OC*} = w_2^{O(1)}$, $w_2^{O\dagger*} = \max\{w_2^{O(2)}, w_2^{(0)}\}$.

Note that $w_2^{D\dagger*}$ and $w_2^{O\dagger*}$ are dependent on e, and w_2^{DC*} and w_2^{OC*} are independent on e.

In the following, before we analyze the comparison results in our equilibrium, we have below lemma for the general comparison results.

LEMMA B2. *The equilibrium sourcing strategy of the brand-name firm is as follows:*

(a) Strategy H with $w_1^* = k_1$ *if* $e < \min\{f_{DH}, f_{HO}\}$ *and* $\Delta < \min\{\Delta_{DH}, \Delta_{HO}\}$*;*

(b) Strategy D with $w_1^* = k_1$*, and*

$$w_2^* = \begin{cases} w_2^{DC*}, & \text{if } e < \min\{e_{D1}, f_{DO1}\} \text{ and } \min\{\Delta_{DH}, \Delta_{DO}\} \leq \Delta < \Delta_{DO},\\ w_2^{D\dagger*}, & \text{if } \max\{e_{D1}, f_{DH}, f_{DO2}\} \leq e \leq \min\{e_3, f_{DO3}\}, \text{ or if } e > \max\{e_3, f_{DO4}\}; \end{cases}$$

(c) Strategy O with

$$w_2^* = \begin{cases} w_2^{OC*}, & \textit{if } e < \min\{e_{O1}, f_{DO2}\} \textit{ and } \Delta > \max\{\Delta_{HO}, \Delta_{DO}\}, \\ w_2^{O\dagger *}, & \textit{if } \max\{e_{O1}, f_{DO1}, f_{DO3}, f_{HO}\} \le e < e_3, \textit{ or if } \max\{e_3, f_{HO}\} < e < f_{DO4}. \end{cases}$$

where the thresholds are derived by

$$\begin{array}{lll}
\pi_B^H > \pi_B^{O\dagger} & \Rightarrow w_2^{O*} > p_B - \frac{(p_B-k_2-\Delta)((1-p_B)+\alpha(1-\frac{p_B-p_2}{1-\beta}))-\alpha t(1-\frac{p_B-p_2}{1-\beta})+t(1-\frac{p_B}{\gamma})}{(1+\alpha)(1-\frac{p_B}{\gamma})}, & \Rightarrow e < f_{HO}, \\
\pi_B^H > \pi_B^{OC} & \Rightarrow \Delta < p_B - k_2 - \frac{\left(p_B-w_2^{OC*}-t\right)(1-\frac{p_B}{\gamma})+\alpha\left(p_B-w_2^{OC*}\right)(1-\frac{p_B-p_2}{\gamma-\beta})+\alpha t(1-\frac{p_B-p_2}{1-\beta})}{(1-p_B)+\alpha(1-\frac{p_B-p_2}{1-\beta})}, & \Rightarrow \Delta < \Delta_{HO}, \\
\pi_B^H > \pi_B^{DC} & \Rightarrow \Delta < p_B - k_2 - \frac{\left(p_B-w_2^{DC*}\right)(1-\frac{p_B-p_2}{1-\gamma-\beta})}{1-\frac{p_B-p_2}{1-\beta}} - t, & \Rightarrow \Delta < \Delta_{DH}, \\
\pi_B^H > \pi_B^{D\dagger} & \Rightarrow w_2^{D\dagger *} > p_B - \frac{(p_B-k_2-\Delta-t)(1-\frac{p_B-p_2}{1-\beta})}{1-\frac{p_B}{1-\gamma}}, & \Rightarrow e < f_{DH}, \\
\pi_B^{D\dagger} > \pi_B^{O\dagger} & \Rightarrow w_2^{O\dagger *} > p_B - \frac{\alpha(w_2^{O*}-w_2^{D*})(1-\frac{p_B}{1-\gamma})+(p_B-k_2-\Delta)(1-p_B)}{(1-\frac{p_B}{1-\gamma})} - t, & \Rightarrow e < f_{DO3}, or, e > f_{DO4}, \\
\pi_B^{D\dagger} > \pi_B^{OC} & \Rightarrow w_2^{D\dagger *} < p_B - \frac{\left(p_B-w_2^{OC*}-t\right)(1-\frac{p_B}{1-\gamma})+\alpha\left(p_B-w_2^{OC*}\right)(1-\frac{p_B-p_2}{1-\gamma-\beta})-(p_B-k_2-\Delta)(1-p_B)}{\alpha(1-\frac{p_B}{1-\gamma})}, & \Rightarrow e > f_{DO2}, \\
\pi_B^{D\dagger} > \pi_B^{DC} & \Rightarrow w_2^{D\dagger *} < \frac{p_B(\frac{p_B-p_2}{1-\gamma-\beta}-\frac{p_B}{1-\gamma})+w_2^{DC*}(1-\frac{p_B-p_2}{1-\gamma-\beta})}{1-\frac{p_B}{1-\gamma}} = p_B - \frac{\left(p_B-w_2^{DC*}\right)(1-\frac{p_B-p_2}{1-\gamma-\beta})}{1-\frac{p_B}{1-\gamma}}, & \Rightarrow e > e_{D1}, \\
\pi_B^{DC} > \pi_B^{O\dagger} & \Rightarrow w_2^{O\dagger *} > p_B - \frac{(p_B-k_2-\Delta)(1-p_B)+\alpha\left(p_B-w_2^{DC*}\right)(1-\frac{p_B-p_2}{1-\gamma-\beta})+t(1-\frac{p_B}{1-\gamma})}{(1+\alpha)(1-\frac{p_B}{1-\gamma})}, & \Rightarrow e < f_{DO1}, \\
\pi_B^{DC} > \pi_B^{OC} & \Rightarrow \Delta < p_B - k_2 - \frac{\left(p_B-w_2^{OC*}-t\right)(1-\frac{p_B}{1-\gamma})-\alpha\left(w_2^{OC*}-w_2^{DC*}\right)(1-\frac{p_B-p_2}{1-\gamma-\beta})}{1-p_B}, & \Rightarrow \Delta < \Delta_{DO}, \\
\pi_B^{O\dagger} > \pi_B^{OC} & \Rightarrow w_2^{O\dagger *} < \frac{\alpha(p_B-w_2^{OC*})(\frac{p_B-p_2}{1-\gamma-\beta}-\frac{p_B}{1-\gamma})}{(1+\alpha)(1-\frac{p_B}{1-\gamma})} + w_2^{OC*}, & \Rightarrow e > e_{O1}.
\end{array}$$

Proof of Lemma B2: There are three steps to make comparisons about the brand-name firm's profits: (1) we compare Strategy D without counterfeiting, Strategy D with counterfeiting, and Strategy O without counterfeiting, Strategy O with counterfeiting; (2) we compare every strategy with Strategy H; (3) we summarize the whole conditions for each Strategy.

1. We make comparisons between strategies D and O. Then, we have four cases to compare:

(1.1) $\pi_B^{D\dagger}$ and $\pi_B^{O\dagger}$:

$$\begin{cases} \pi_B^{D\dagger} = (p_B - k_1)(1-p_B) + \alpha\left(p_B - w_2^{D\dagger *}\right)(1 - \frac{p_B}{1-\gamma}), & \text{if } e \ge e_{D1}, \\ \pi_B^{O\dagger} = \left(p_B - w_2^{O\dagger *} - t\right)(1 - \frac{p_B}{1-\gamma}) + \alpha\left(p_B - w_2^{O\dagger *}\right)(1 - \frac{p_B}{1-\gamma}), & \text{if } e \ge e_{O1}. \end{cases}$$

Thus,

$$\begin{aligned} & \pi_B^{O\dagger} > \pi_B^{D\dagger}, \\ \Rightarrow & \left(p_B - w_2^{O\dagger *} - t\right)(1 - \tfrac{p_B}{1-\gamma}) > \alpha(w_2^{O\dagger *} - w_2^{D\dagger *})(1 - \tfrac{p_B}{1-\gamma}) + (p_B - k_2 - \Delta)(1-p_B), \\ \Rightarrow & w_2^{O\dagger *} < p_B - \frac{\alpha(w_2^{O\dagger *}-w_2^{D*})(1-\frac{p_B}{1-\gamma})+(p_B-k_2-\Delta)(1-p_B)}{(1-\frac{p_B}{1-\gamma})} - t. \end{aligned}$$

(1.2) $\pi_B^{D\dagger}$ and π_B^{OC}:

$$\begin{cases} \pi_B^{D\dagger} = (p_B - k_1)(1-p_B) + \alpha\left(p_B - w_2^{D\dagger *}\right)(1 - \frac{p_B}{1-\gamma}), & \text{if } e \ge e_{D1}, \\ \pi_B^{OC} = (p_B - w_2^{OC*} - t)(1 - \frac{p_B}{1-\gamma}) + \alpha\left(p_B - w_2^{OC*}\right)(1 - \frac{p_B-p_2}{1-\gamma-\beta}), & \text{if } e < e_{O1}. \end{cases}$$

Thus,

$$\begin{aligned} & \pi_B^{OC} > \pi_B^{D\dagger}, \\ \Rightarrow & (p_B - w_2^{OC*} - t)(1 - \tfrac{p_B}{1-\gamma}) + \alpha\left(p_B - w_2^{OC*}\right)(1 - \tfrac{p_B-p_2}{1-\gamma-\beta}) > (p_B - k_2 - \Delta)(1-p_B) + \alpha\left(p_B - w_2^{D\dagger *}\right)(1 - \tfrac{p_B}{\gamma}), \\ \Rightarrow & w_2^{D\dagger *} > p_B - \frac{\left(p_B-w_2^{OC*}-t\right)(1-\frac{p_B}{1-\gamma})+\alpha\left(p_B-w_2^{OC*}\right)(1-\frac{p_B-p_2}{1-\gamma-\beta})-(p_B-k_2-\Delta)(1-p_B)}{\alpha(1-\frac{p_B}{1-\gamma})}. \end{aligned}$$

(1.3) π_B^{DC} and $\pi_B^{O\dagger}$:

$$\begin{cases} \pi_B^{DC} = (p_B - k_1)(1-p_B) + \alpha(p_B - w_2^{DC*})(1 - \frac{p_B - p_2}{1-\gamma-\beta}), & \text{if } e < e_{D1}, \\ \pi_B^{O\dagger} = \left(p_B - w_2^{O\dagger *} - t\right)(1 - \frac{p_B}{1-\gamma}) + \alpha\left(p_B - w_2^{O\dagger *}\right)(1 - \frac{p_B}{1-\gamma}), & \text{if } e \geq e_{O1}. \end{cases}$$

Thus,

$$\begin{aligned} &\pi_B^{O\dagger} > \pi_B^{DC}, \\ \Rightarrow\ &(1+\alpha)\left(p_B - w_2^{O\dagger *}\right)(1 - \frac{p_B}{1-\gamma}) - t(1 - \frac{p_B}{1-\gamma}) > (p_B - k_1)(1-p_B) + \alpha(p_B - w_2^{DC*})(1 - \frac{p_B - p_2}{1-\gamma-\beta}), \\ \Rightarrow\ &w_2^{O\dagger *} < p_B - \frac{(p_B - k_2 - \Delta)(1-p_B) + \alpha\left(p_B - w_2^{DC*}\right)(1 - \frac{p_B - p_2}{1-\gamma-\beta}) + t(1 - \frac{p_B}{1-\gamma})}{(1+\alpha)(1 - \frac{p_B}{1-\gamma})}. \end{aligned}$$

(1.4) π_B^{DC} and π_B^{OC}:

$$\begin{cases} \pi_B^{DC} = (p_B - k_1)(1-p_B) + \alpha(p_B - w_2^{DC*})(1 - \frac{p_B - p_2}{1-\gamma-\beta}), & \text{if } e < e_{D1}, \\ \pi_B^{OC} = (p_B - w_2^{OC*} - t)(1 - \frac{p_B}{1-\gamma}) + \alpha(p_B - w_2^{OC*})(1 - \frac{p_B - p_2}{1-\gamma-\beta}), & \text{if } e < e_{O1}. \end{cases}$$

Thus,

$$\begin{aligned} &\pi_B^{OC} > \pi_B^{DC}, \\ \Rightarrow\ &(p_B - w_2^{OC*} - t)(1 - \frac{p_B}{1-\gamma}) > (p_B - k_2 - \Delta)(1-p_B) + \alpha(w_2^{OC*} - w_2^{DC*})(1 - \frac{p_B - p_2}{1-\gamma-\beta}), \\ \Rightarrow\ &\Delta > p_B - k_2 - \frac{\left(p_B - w_2^{OC*} - t\right)(1 - \frac{p_B}{1-\gamma}) - \alpha\left(w_2^{OC*} - w_2^{DC*}\right)(1 - \frac{p_B - p_2}{1-\gamma-\beta})}{1-p_B}. \end{aligned}$$

2. We make comparisons for Strategy D and Strategy O, and Strategy H:

(2.1) $\pi_B^{D\dagger}$ and π_B^{H}:

$$\begin{cases} \pi_B^{H} = (p_B - k_1)(1-p_B) + \alpha(p_B - k_1 - t)(1 - \frac{p_B - p_2}{1-\beta}), \\ \pi_B^{D\dagger} = (p_B - k_1)(1-p_B) + \alpha\left(p_B - w_2^{D\dagger *}\right)(1 - \frac{p_B}{1-\gamma}), & \text{if } e \geq e_{D1}. \end{cases}$$

Thus,

$$\begin{aligned} &\pi_B^{D\dagger} > \pi_B^{H}, \\ \Rightarrow\ &\left(p_B - w_2^{D\dagger *}\right)(1 - \frac{p_B}{1-\gamma}) > (p_B - k_2 - \Delta - t)(1 - \frac{p_B - p_2}{1-\beta}), \\ \Rightarrow\ &w_2^{D\dagger *} < p_B - \frac{(p_B - k_2 - \Delta - t)(1 - \frac{p_B - p_2}{1-\beta})}{1 - \frac{p_B}{1-\gamma}}. \end{aligned}$$

(2.2) π_B^{DC} and π_B^{H}:

$$\begin{cases} \pi_B^{H} = (p_B - k_1)(1-p_B) + \alpha(p_B - k_1 - t)(1 - \frac{p_B - p_2}{1-\beta}), \\ \pi_B^{DC} = (p_B - k_1)(1-p_B) + \alpha(p_B - w_2^{DC*})(1 - \frac{p_B - p_2}{1-\gamma-\beta}), & \text{if } e < e_{D1}. \end{cases}$$

Thus,

$$\begin{aligned} &\pi_B^{DC} > \pi_B^{H}, \\ \Rightarrow\ &(p_B - w_2^{DC*})(1 - \frac{p_B - p_2}{1-\gamma-\beta}) > (p_B - k_2 - \Delta - t)(1 - \frac{p_B - p_2}{1-\beta}), \\ \Rightarrow\ &\Delta > p_B - k_2 - \frac{\left(p_B - w_2^{DC*}\right)(1 - \frac{p_B - p_2}{1-\gamma-\beta})}{1 - \frac{p_B - p_2}{1-\beta}} - t. \end{aligned}$$

(2.3) $\pi_B^{O\dagger}$ and π_B^{H}:

$$\begin{cases} \pi_B^{H} = (p_B - k_1)(1-p_B) + \alpha(p_B - k_1 - t)(1 - \frac{p_B - p_2}{1-\beta}), \\ \pi_B^{O\dagger} = \left(p_B - w_2^{O\dagger *} - t\right)(1 - \frac{p_B}{1-\gamma}) + \alpha\left(p_B - w_2^{O\dagger *}\right)(1 - \frac{p_B}{1-\gamma}), & \text{if } e \geq e_{O1}. \end{cases}$$

Thus,

$$\begin{aligned}&\pi_B^{O\dagger} > \pi_B^H,\\ \Rightarrow\ & (1+\alpha)\left(p_B - w_2^{O\dagger *}\right)(1-\tfrac{p_B}{1-\gamma}) - t(1-\tfrac{p_B}{1-\gamma}) > (p_B - k_2 - \Delta)\left((1-p_B) + \alpha(1-\tfrac{p_B-p_2}{1-\beta})\right) - \alpha t(1-\tfrac{p_B-p_2}{1-\beta}),\\ \Rightarrow\ & w_2^{O\dagger *} < p_B - \frac{(p_B-k_2-\Delta)((1-p_B)+\alpha(1-\frac{p_B-p_2}{1-\beta})) - \alpha t(1-\frac{p_B-p_2}{1-\beta}) + t(1-\frac{p_B}{1-\gamma})}{(1+\alpha)(1-\frac{p_B}{1-\gamma})}.\end{aligned}$$

(2.4) π_B^{OC} and π_B^H:

$$\begin{cases} \pi_B^H = (p_B - k_1)(1-p_B) + \alpha(p_B - k_1 - t)(1-\frac{p_B-p_2}{1-\beta}),\\ \pi_B^{OC} = (p_B - w_2^{OC*} - t)(1-\frac{p_B}{1-\gamma}) + \alpha(p_B - w_2^{OC*})(1-\frac{p_B-p_2}{1-\gamma-\beta}), \text{ if } e < e_{O1}. \end{cases}$$

Thus,

$$\begin{aligned}&\pi_B^{OC} > \pi_B^H,\\ \Rightarrow\ & (p_B - w_2^{OC*} - t)(1-\tfrac{p_B}{1-\gamma}) + \alpha(p_B - w_2^{OC*})(1-\tfrac{p_B-p_2}{1-\gamma-\beta}) > (p_B-k_2-\Delta)\left((1-p_B)+\alpha(1-\tfrac{p_B-p_2}{1-\beta})\right) - \alpha t(1-\tfrac{p_B-p_2}{1-\beta}),\\ \Rightarrow\ & \Delta > p_B - k_2 - \frac{\left(p_B-w_2^{OC*}-t\right)(1-\frac{p_B}{1-\gamma}) + \alpha\left(p_B - w_2^{OC*}\right)(1-\frac{p_B-p_2}{1-\gamma-\beta}) + \alpha t(1-\frac{p_B-p_2}{1-\beta})}{(1-p_B)+\alpha(1-\frac{p_B-p_2}{1-\beta})}.\end{aligned}$$

3. Therefore, we obtain the results as follows.

(3.1) The conditions for $\pi_B^* = \pi_B^{D\dagger}$ are $e \geq e_{D1}$, and

$$\begin{aligned}
&\pi_B^{D\dagger} > \pi_B^{O\dagger} \Rightarrow w_2^{O\dagger *} > p_B - \frac{\alpha(w_2^{O\dagger *} - w_2^{D\dagger *})(1-\frac{p_B}{1-\gamma}) + (p_B-k_2-\Delta)(1-p_B)}{(1-\frac{p_B}{1-\gamma})} - t, && \Rightarrow e < f_{DO3}, or, e > f_{DO4},\\
&\pi_B^{D\dagger} > \pi_B^{OC} \Rightarrow w_2^{D\dagger *} < p_B - \frac{\left(p_B - w_2^{OC*} - t\right)(1-\frac{p_B}{1-\gamma}) + \alpha\left(p_B - w_2^{OC*}\right)(1-\frac{p_B-p_2}{1-\gamma-\beta}) - (p_B-k_2-\Delta)(1-p_B)}{\alpha(1-\frac{p_B}{1-\gamma})}, && \Rightarrow e > f_{DO2},\\
&\pi_B^{D\dagger} > \pi_B^{H} \Rightarrow w_2^{D\dagger *} < p_B - \frac{(p_B-k_2-\Delta-t)(1-\frac{p_B-p_2}{1-\beta})}{1-\frac{p_B}{1-\gamma}}, && \Rightarrow e > f_{DH};
\end{aligned}$$

(3.2) the conditions for $\pi_B^* = \pi_B^{DC}$ are $e < e_{D1}$, and

$$\begin{aligned}
&\pi_B^{DC} > \pi_B^{O\dagger} \Rightarrow w_2^{O\dagger *} > p_B - \frac{(p_B-k_2-\Delta)(1-p_B) + \alpha\left(p_B - w_2^{DC*}\right)(1-\frac{p_B-p_2}{1-\gamma-\beta}) + t(1-\frac{p_B}{1-\gamma})}{(1+\alpha)(1-\frac{p_B}{1-\gamma})}, && \Rightarrow e < f_{DO1},\\
&\pi_B^{DC} > \pi_B^{OC} \Rightarrow \Delta < p_B - k_2 - \frac{\left(p_B - w_2^{OC*} - t\right)(1-\frac{p_B}{1-\gamma}) - \alpha\left(w_2^{OC*} - w_2^{DC*}\right)(1-\frac{p_B-p_2}{1-\gamma-\beta})}{1-p_B}, && \Rightarrow \Delta < \Delta_{DO},\\
&\pi_B^{DC} > \pi_B^{H} \Rightarrow \Delta > p_B - k_2 - \frac{\left(p_B - w_2^{DC*}\right)(1-\frac{p_B-p_2}{1-\gamma-\beta})}{1-\frac{p_B-p_2}{1-\beta}} - t, && \Rightarrow \Delta > \Delta_{DH};
\end{aligned}$$

(3.3) the conditions for $\pi_B^* = \pi_B^O$ are $e \geq e_{O1}$, and

$$\begin{aligned}
&\pi_B^{O\dagger} > \pi_B^{D\dagger} \Rightarrow w_2^{O\dagger *} < p_B - \frac{\alpha(w_2^{O\dagger *} - w_2^{D\dagger *})(1-\frac{p_B}{1-\gamma}) + (p_B-k_2-\Delta)(1-p_B)}{(1-\frac{p_B}{1-\gamma})} - t, && \Rightarrow e > f_{DO3}, or, e < f_{DO4},\\
&\pi_B^{O\dagger} > \pi_B^{DC} \Rightarrow w_2^{O\dagger *} < p_B - \frac{(p_B-k_2-\Delta)(1-p_B) + \alpha\left(p_B - w_2^{DC*}\right)(1-\frac{p_B-p_2}{1-\gamma-\beta}) + t(1-\frac{p_B}{1-\gamma})}{(1+\alpha)(1-\frac{p_B}{1-\gamma})}, && \Rightarrow e > f_{DO1},\\
&\pi_B^{O\dagger} > \pi_B^{H} \Rightarrow w_2^{O\dagger *} < p_B - \frac{(p_B-k_2-\Delta)((1-p_B)+\alpha(1-\frac{p_B-p_2}{1-\beta})) - \alpha t(1-\frac{p_B-p_2}{1-\beta}) + t(1-\frac{p_B}{1-\gamma})}{(1+\alpha)(1-\frac{p_B}{1-\gamma})}, && \Rightarrow e > f_{HO};
\end{aligned}$$

(3.4) the conditions for $\pi_B^* = \pi_B^{OC}$ are $e < e_{O1}$, and

$$\begin{aligned}
&\pi_B^{OC} > \pi_B^{D\dagger} \Rightarrow w_2^{D*} > p_B - \frac{\left(p_B - w_2^{OC*} - t\right)(1-\frac{p_B}{1-\gamma}) + \alpha\left(p_B - w_2^{OC*}\right)(1-\frac{p_B-p_2}{1-\gamma-\beta}) - (p_B-k_2-\Delta)(1-p_B)}{\alpha(1-\frac{p_B}{1-\gamma})}, && \Rightarrow e < f_{DO2},\\
&\pi_B^{OC} > \pi_B^{DC} \Rightarrow \Delta > p_B - k_2 - \frac{\left(p_B - w_2^{OC*} - t\right)(1-\frac{p_B}{1-\gamma}) - \alpha\left(w_2^{OC*} - w_2^{DC*}\right)(1-\frac{p_B-p_2}{1-\gamma-\beta})}{1-p_B}, && \Rightarrow \Delta > \Delta_{DO},\\
&\pi_B^{OC} > \pi_B^{H} \Rightarrow \Delta > p_B - k_2 - \frac{\left(p_B - w_2^{OC*} - t\right)(1-\frac{p_B}{1-\gamma}) + \alpha\left(p_B - w_2^{OC*}\right)(1-\frac{p_B-p_2}{1-\gamma-\beta}) + \alpha t(1-\frac{p_B-p_2}{1-\beta})}{(1-p_B)+\alpha(1-\frac{p_B-p_2}{1-\beta})}, && \Rightarrow \Delta > \Delta_{HO};
\end{aligned}$$

(3.5) the conditions for $\pi_B^* = \pi_B^H$ are

$$
\begin{aligned}
&\pi_B^H > \pi_B^{O\dagger} \Rightarrow w_2^{O\dagger *} > p_B - \frac{(p_B-k_2-\Delta)((1-p_B)+\alpha(1-\frac{p_B-p_2}{1-\beta}))-\alpha t(1-\frac{p_B-p_2}{1-\beta})+t(1-\frac{p_B}{1-\gamma})}{(1+\alpha)(1-\frac{p_B}{\gamma})}, && \Rightarrow e < f_{HO},\\
&\pi_B^H > \pi_B^{OC} \Rightarrow \Delta < p_B - k_2 - \frac{\left(p_B-w_2^{OC*}-t\right)(1-\frac{p_B}{1-\gamma})+\alpha\left(p_B-w_2^{OC*}\right)(1-\frac{p_B-p_2}{1-\gamma-\beta})+\alpha t(1-\frac{p_B-p_2}{1-\beta})}{(1-p_B)+\alpha(1-\frac{p_B-p_2}{1-\beta})}, && \Rightarrow \Delta < \Delta_{HO},\\
&\pi_B^H > \pi_B^{DC} \Rightarrow \Delta < p_B - k_2 - \frac{\left(p_B-w_2^{DC*}\right)(1-\frac{p_B-p_2}{1-\gamma-\beta})}{1-\frac{p_B-p_2}{1-\beta}} - t, && \Rightarrow \Delta < \Delta_{DH},\\
&\pi_B^H > \pi_B^{D\dagger} \Rightarrow w_2^{D\dagger *} > p_B - \frac{(p_B-k_2-\Delta-t)(1-\frac{p_B-p_2}{1-\beta})}{1-\frac{p_B}{1-\gamma}}, && \Rightarrow e < f_{DH}.
\end{aligned}
$$

By combining the conditions for each strategy, we have the results. ■

Following the general result in Lemma B2, we further derive the conditions of (Δ, e) for different wholesale price w_2^*. We define below thresholds: $w_2^{OC*} = k_2 + \frac{\alpha(p_2-k_2)(1-\frac{p_B-p_2}{1-\gamma-\beta})}{(1-\frac{p_B}{1-\gamma})+\alpha(1-\frac{p_B-p_2}{1-\gamma-\beta})}$, and $R = \frac{\alpha(\beta p_B-(1-\gamma)p_2)}{(1-\gamma-\beta)\beta}$,

$$
\begin{aligned}
&\Delta_{DH} = p_B - k_2 - \frac{(p_B-k_2)(1-\frac{p_B-p_2}{1-\gamma-\beta})}{(1-\frac{p_B-p_2}{1-\beta})} - t;\\
&\Delta_{DO} = p_B - k_2 - \frac{\left(p_B-w_2^{OC*}-t\right)(1-\frac{p_B}{1-\gamma})-\alpha\left(w_2^{OC*}-k_2\right)(1-\frac{p_B-p_2}{1-\gamma-\beta})}{(1-p_B)};\\
&\Delta_{HO} = p_B - k_2 - \frac{\left(p_B-w_2^{OC*}-t\right)(1-\frac{p_B}{1-\gamma})+\alpha\left(p_B-w_2^{OC*}\right)(1-\frac{p_B-p_2}{1-\gamma-\beta})+\alpha t(1-\frac{p_B-p_2}{1-\beta})}{(1-p_B)+\alpha(1-\frac{p_B-p_2}{1-\beta})};\\
&f_{DH} = (p_2 - k_2 - (x_{DH}(\Delta) - k_2)\frac{\beta}{1-\gamma})R, \text{ where } x_{DH}(\Delta) = p_B - \frac{(p_B-k_2-\Delta-t)(1-\frac{p_B-p_2}{1-\beta})}{(1-\frac{p_B}{1-\gamma})};\\
&f_{DO1} = (p_2 - k_2 - (x_{DO1}(\Delta) - k_2)\frac{\beta}{1-\gamma})R, \text{ where } x_{DO1}(\Delta) = p_B - \frac{(p_B-k_2-\Delta)(1-p_B)+\alpha(p_B-k_2)(1-\frac{p_B-p_2}{1-\gamma-\beta})+t(1-\frac{p_B}{1-\gamma})}{(1+\alpha)(1-\frac{p_B}{1-\gamma})};\\
&f_{DO2} = (p_2 - k_2 - (x_{DO2}(\Delta) - k_2)\frac{\beta}{1-\gamma})R, \text{ where } x_{DO2}(\Delta) = p_B - \frac{\left(p_B-w_2^{OC*}-t\right)((1-\frac{p_B}{1-\gamma})+\alpha(1-\frac{p_B-p_2}{1-\gamma-\beta}))-(p_B-k_2-\Delta)(1-p_B)}{\alpha(1-\frac{p_B}{1-\gamma})};\\
&f_{DO3} = (p_2 - k_2 - (x_{OD3}(\Delta) - k_2)\frac{\beta}{1-\gamma})R, \text{ where } x_{OD3}(\Delta) = p_B - \frac{(p_B-k_2-\Delta)(1-p_B)}{(1-\frac{p_B}{1-\gamma})} - t;\\
&f_{DO4} = \frac{(p_B-\frac{(p_B-k_2-\Delta)(1-p_B)}{1-\frac{p_B}{1-\gamma}}-k_2-t(1-\frac{p_B}{1-\gamma}))\beta(1-\frac{p_B}{1-\gamma})-\alpha(p_2-k_2)(1-\frac{p_2}{\beta})\beta+\alpha\gamma(1-\frac{p_B}{1-\gamma})(p_2-k_2)}{\left(\frac{(1-\gamma)(1-\gamma-\beta)(1-\frac{p_B}{1-\gamma})}{(\beta p_B-(1-\gamma)p_2)}-1\right)\beta};\\
&f_{HO1} = (p_2 - k_2 - (x_{HO1}(\Delta) - k_2)\frac{\beta}{1-\gamma})R, \text{ where } x_{HO1}(\Delta) = p_B - \frac{(p_B-k_2-\Delta)((1-p_B)+\alpha(1-\frac{p_B-p_2}{1-\beta}))-\alpha t(1-\frac{p_B-p_2}{1-\beta})+t(1-\frac{p_B}{1-\gamma})}{(1+\alpha)(1-\frac{p_B}{\gamma})};\\
&f_{HO2} = \alpha\,(p_2-k_2)\,(1-\tfrac{p_2}{\beta}) - (x_{HO2}(\Delta)-k_2)(1+\alpha)(1-\tfrac{p_B}{\gamma}), \text{ where } x_{HO2}(\Delta) = p_B - \frac{(p_B-k_2-\Delta)\left((1-p_B)+\alpha\left(1-\frac{p_B-p_2}{1-\beta}\right)\right)}{(1+\alpha)\left(1-\frac{p_B}{1-\gamma}\right)}\\
&-t(1-\tfrac{p_B}{1-\gamma}) + t\alpha(1-\tfrac{p_B-p_2}{1-\beta}).
\end{aligned}
\tag{9}
$$

Note that for the condition of $\pi_B^* = \pi_B^H$, $f_{DH} < f_{HO1}$, $f_{DH} < f_{HO2}$; for the condition of $\pi_B^* = \pi_B^{O\dagger}$, $f_{DO3} > f_{HO1}$, $f_{DO3} > f_{HO2}$. Thus, in our base case, the equilibrium sourcing strategy of the brand-name firm is as follows:

(a) Strategy H with $w_1^* = k_1$ if $e < f_{DH}$ and $\Delta < \min\{\Delta_{DH}, \Delta_{HO}\}$;

(b) Strategy D with $w_1^* = k_1$, and

$$
w_2^* = \begin{cases} k_2, & \text{if } e \le \min\{e_{D1}, f_{DO1}\} \text{ and } \min\{\Delta_{DH}, \Delta_{DO}\} \le \Delta < \Delta_{DO},\\ w_2^{(0)}, & \text{if } \max\{e_{D1}, f_{DH}, f_{DO2}\} \le e \le \min\{e_3, f_{DO3}\}, \text{ or if } e \ge \max\{e_3, f_{DO4}\}; \end{cases}
$$

(c) Strategy O with

$$
w_2^* = \begin{cases} w_2^{O(1)}, & \text{if } e \le \min\{e_{O1}, f_{DO2}\} \text{ and } \Delta > \max\{\Delta_{HO}, \Delta_{DO}\},\\ \max\{w_2^{O(2)}, w_2^{(0)}\}, & \text{if } \max\{e_{O1}, f_{DO1}, f_{DO3}\} < e \le e_3, \text{ or if } e_3 < e < f_{DO4}; \end{cases}
$$

where e_3 is defined in Equation (7). ■

B.4 Proof of Proposition 2.

Recall that $\pi_B^{D\dagger} \geq \pi_B^{DC}$ if $e \geq e_{D1}$, and $\pi_B^{O\dagger} \geq \pi_B^{OC}$ if $e \geq e_{O1}$, where

$$e_{D1} = \left(p_2 - k_2 - \left(\frac{(p_B-k_2)(\frac{p_B-p_2}{1-\gamma-\beta} - \frac{p_B}{1-\gamma})}{1-\frac{p_B}{1-\gamma}}\right)\frac{\beta}{1-\gamma} \right) \frac{\alpha(\beta p_B-(1-\gamma)p_2)}{(1-\gamma-\beta)\beta};$$
$$e_{O1} = \left(p_2 - k_2 - \left(\frac{\alpha(p_B-w_2^{OC*})(\frac{p_B-p_2}{1-\gamma-\beta} - \frac{p_B}{1-\gamma})}{(1+\alpha)(1-\frac{p_B}{1-\gamma})} + w_2^{OC*} - k_2\right)\frac{\beta}{1-\gamma} \right) \frac{\alpha(\beta p_B-(1-\gamma)p_2)}{(1-\gamma-\beta)\beta},$$

and $w_2^{OC*} = k_2 + \frac{\alpha(p_2-k_2)\left(1-\frac{p_B-p_2}{1-\gamma-\beta}\right)}{\left(1-\frac{p_B}{1-\gamma}\right)+\alpha\left(1-\frac{p_B-p_2}{1-\gamma-\beta}\right)}$.

Under Strategy D, if $e \geq e_{D1}$, the counterfeiter does not sell the counterfeit; under Strategy O, if $e \geq e_{O1}$, the counterfeiter does not sell the counterfeit. Thus, we compare thresholds e_{D1} and e_{O1}, to analyze which sourcing strategy helps prevent counterfeiting at a lower e. If $e_{D1} > e_{O1}$, it means that Strategy O is easier to prevent counterfeiting. Otherwise, Strategy D is easier to prevent counterfeiting.

Below we derive the condition of $e_{D1} > e_{O1}$.

$$\begin{aligned}
& e_{D1} > e_{O1}, \\
\Rightarrow\ & p_2 - k_2 - \left(\frac{(p_B-k_2)(\frac{p_B-p_2}{1-\gamma-\beta} - \frac{p_B}{1-\gamma})}{1-\frac{p_B}{1-\gamma}}\right)\frac{\beta}{1-\gamma} > (p_2-k_2) - \left(\frac{\alpha(p_B-w_2^{OC*})(\frac{p_B-p_2}{1-\gamma-\beta} - \frac{p_B}{1-\gamma})}{(1+\alpha)(1-\frac{p_B}{1-\gamma})} + w_2^{OC*} - k_2\right)\frac{\beta}{1-\gamma}, \\
\Rightarrow\ & \alpha > \frac{(p_B-k_2)\left(\frac{p_B-p_2}{1-\gamma-\beta} - \frac{p_B}{1-\gamma}\right)}{(p_2-k_2)\left(1-\frac{p_B-p_2}{1-\gamma-\beta}\right)}, \\
\Rightarrow\ & ((1-\gamma)(1-\gamma-\beta) - (1-\gamma)(p_B-p_2)) > \frac{(p_B-k_2)}{\alpha(p_2-k_2)}(\beta p_B - (1-\gamma)p_2), \\
\Rightarrow\ & ((1-\gamma)(1-\gamma-\beta) - (1-\gamma)(p_B-p_2)) > x(\beta p_B - (1-\gamma)p_2), \text{where } x = \frac{(p_B-k_2)}{\alpha(p_2-k_2)}, \\
\Rightarrow\ & (1-\gamma - \tfrac{\beta+p_B-(1+x)p_2}{2})^2 > x\beta p_B + (\tfrac{\beta+p_B-(1+x)p_2}{2})^2, \\
\Rightarrow\ & 1-\gamma > \tfrac{\beta+p_B-(1+x)p_2}{2} + \sqrt{x\beta p_B + (\tfrac{\beta+p_B-(1+x)p_2}{2})^2}, \\
& 1-\gamma < \tfrac{\beta+p_B-(1+x)p_2}{2} - \sqrt{x\beta p_B + (\tfrac{\beta+p_B-(1+x)p_2}{2})^2}, \text{invalid.}
\end{aligned}$$

Define

$$\widehat{\gamma} = 1 - \min\left\{\frac{\beta+p_B-\left(1+\frac{p_B-k_2}{\alpha(p_2-k_2)}\right)p_2}{2} + \sqrt{\frac{\beta p_B(p_B-k_2)}{\alpha(p_2-k_2)} + \frac{\left(\beta+p_B-\left(1+\frac{p_B-k_2}{\alpha(p_2-k_2)}\right)p_2\right)^2}{4}}, 1\right\}. \tag{10}$$

Thus, we obtain the result. ■

B.5 Proof of Corollary 1.

Note that in equilibrium of the base model, under Strategy H, the profit of each firm is the same as that under the benchmark, i.e., $\pi_1^H = \bar{\pi}_1^*$, $\pi_2^H = \bar{\pi}_2^*$, $\pi_B^H = \bar{\pi}_B^*$. In equilibrium, under Strategy D or Strategy O, the home supplier obtains zero profit, that is, $\pi_1^D = \pi_1^O = \bar{\pi}_1^* = 0$. Thus, in the following, we focus on comparing the profits of the brand-name firm, the overseas supplier, between the benchmark and Strategy D as well as Strategy O in equilibrium from Proposition 1, respectively. Note that we assume $0 \leq e < \alpha(p_2-k_2)(\frac{p_B-p_2}{1-\beta} - \frac{p_2}{\beta})$. Recall that

$$\begin{aligned}
M' &= \alpha\,(p_2-k_2)\,(\tfrac{p_B-p_2}{1-\gamma-\beta} - \tfrac{p_2}{\beta}) - e, & w_2^{(0)} &= k_2 + \frac{M'}{\alpha\left(\frac{p_B-p_2}{1-\gamma-\beta} - \frac{p_B}{1-\gamma}\right)}, \\
K &= \alpha\,(p_2-k_2)\,(1 - \tfrac{p_2}{\beta}) - e, & w_2^{O(2)} &= k_2 + \frac{K}{(1+\alpha)(1-\frac{p_B}{1-\gamma})}, \\
K - M' &= \alpha\,(p_2-k_2)\,(1 - \tfrac{p_B-p_2}{1-\gamma-\beta}), & w_2^{O(1)} &= k_2 + \frac{K-M'}{(1-\frac{p_B}{1-\gamma})+\alpha(1-\frac{p_B-p_2}{1-\gamma-\beta})}.
\end{aligned}$$

1.If there is no counterfeiting after conversion, i.e., $s^* = 0$, then, we have the comparison of profits as follows.

For the brand-name firm:

$$\begin{aligned}
\pi_B^H &= (p_B - k_1)(1 - p_B) + \alpha (p_B - k_1 - t)\left(1 - \tfrac{p_B - p_2}{1-\beta}\right), \\
\pi_B^D\left(w_2^* = w_2^{(0)}\right) &= (p_B - k_1)(1 - p_B) + \alpha (p_B - w_2^*)\left(1 - \tfrac{p_B}{1-\gamma}\right), \\
\pi_B^O\left(w_2^* = \max\{w_2^{O(2)}, w_2^{(0)}\}\right) &= (p_B - w_2^* - t)\left(1 - \tfrac{p_B}{1-\gamma}\right) + \alpha (p_B - w_2^*)\left(1 - \tfrac{p_B}{1-\gamma}\right).
\end{aligned}$$

For the overseas supplier:

$$\begin{aligned}
\pi_2^H &= \alpha (p_2 - k_2)\left(\tfrac{p_B - p_2}{1-\beta} - \tfrac{p_2}{\beta}\right) - e = M, \\
\pi_2^D\left(w_2^* = w_2^{(0)}\right) &= \alpha (w_2^* - k_2)\left(1 - \tfrac{p_B}{1-\gamma}\right), \\
\pi_2^O\left(w_2^* = \max\{w_2^{O(1)}, w_2^{(0)}\}\right) &= (w_2^* - k_2)\left(1 - \tfrac{p_B}{1-\gamma}\right) + \alpha (w_2^* - k_2)\left(1 - \tfrac{p_B}{1-\gamma}\right).
\end{aligned}$$

(1) When Strategy D is optimal,

for the brand-name firm, $\pi_B^D - \bar{\pi}_B^* \geq 0$;

for the overseas supplier, $\pi_2^D - \bar{\pi}_2^* = \alpha (w_2^* - k_2)\left(1 - \dfrac{p_B}{1-\gamma}\right) - M \geq 0.$

(2) When Strategy O is optimal,

for the brand-name firm, $\pi_B^O - \bar{\pi}_B^* \geq 0$;

for the overseas supplier, $\pi_2^O - \bar{\pi}_2^* = (w_2^* - k_2)\left(1 - \dfrac{p_B}{1-\gamma}\right) + \alpha (w_2^* - k_2)\left(1 - \dfrac{p_B}{1-\gamma}\right) - M \geq 0.$

2. If there is counterfeiting after conversion, i.e., $s^* = 0$, then, we have the comparison as follows.

For the brand-name firm:

$$\begin{aligned}
\pi_B^H &= (p_B - k_1)(1 - p_B) + \alpha (p_B - k_1 - t)\left(1 - \tfrac{p_B - p_2}{1-\beta}\right), \\
\pi_B^D(w_2^* = k_2) &= (p_B - k_1)(1 - p_B) + \alpha (p_B - w_2^*)\left(1 - \tfrac{p_B - p_2}{1-\gamma-\beta}\right), \\
\pi_B^O\left(w_2^* = w_2^{O(1)}\right) &= (p_B - w_2^* - t)\left(1 - \tfrac{p_B}{1-\gamma}\right) + \alpha (p_B - w_2^*)\left(1 - \tfrac{p_B - p_2}{1-\gamma-\beta}\right).
\end{aligned}$$

For the overseas supplier:

$$\begin{aligned}
\pi_2^H &= \alpha (p_2 - k_2)\left(\tfrac{p_B - p_2}{1-\beta} - \tfrac{p_2}{\beta}\right) - e = M, \\
\pi_2^D(w_2^* = k_2) &= \alpha (w_2^* - k_2)\left(1 - \tfrac{p_B - p_2}{1-\gamma-\beta}\right) + \left(\alpha (p_2 - k_2)\left(\tfrac{p_B - p_2}{1-\gamma-\beta} - \tfrac{p_2}{\beta}\right) - e\right), \\
\pi_2^O\left(w_2^* = w_2^{O(1)}\right) &= (w_2^* - k_2)\left(1 - \tfrac{p_B}{1-\gamma}\right) + \alpha (w_2^* - k_2)\left(1 - \tfrac{p_B - p_2}{1-\gamma-\beta}\right) + \left(\alpha (p_2 - k_2)\left(\tfrac{p_B - p_2}{1-\gamma-\beta} - \tfrac{p_2}{\beta}\right) - e\right).
\end{aligned}$$

(1) When Strategy D is optimal,

for the brand-name firm, $\pi_B^D - \bar{\pi}_B^* \geq 0$;

for the overseas supplier, $\pi_2^D - \bar{\pi}_2^* = M' - M > 0.$

(2) When Strategy O is optimal,

for the brand-name firm, $\pi_B^O - \bar{\pi}_B^* \geq 0$;
for the overseas supplier, $\pi_2^O - \bar{\pi}_2^* = (w_2^* - k_2)\left(1 - \dfrac{p_B}{1-\gamma}\right) + \alpha\,(w_2^* - k_2)\left(1 - \dfrac{p_B - p_2}{1-\gamma-\beta}\right) + M' - M > 0.$

Thus, based on the equilibrium in Proposition 1, under strategies D and O, for the brand-name firm, $\pi_B^D \geq \bar{\pi}_B^*$, $\pi_B^O \geq \bar{\pi}_B^*$, respectively; for the overseas supplier, $\pi_2^D \geq \bar{\pi}_2^*$, $\pi_2^O \geq \bar{\pi}_2^*$, respectively. ■

B.6 Proof of Proposition 3.

Recall that we have below thresholds of θ: $\widetilde{\theta} = \frac{p_B - p_2}{1-\beta}$, $\widetilde{\theta}' = \frac{p_B - p_2}{1-\gamma-\beta}$, $\hat{\theta}_B = \frac{p_B}{1-\gamma}$, and $\hat{\theta}_2 = \frac{p_2}{\beta}$.

Firstly, under the benchmark: in the equilibrium,

(1) consumer surplus in the home market is $\overline{CS}_1 = \frac{1-(p_B)^2}{2} - p_B(1-p_B) = \frac{(1-p_B)^2}{2}$;

(2) consumer surplus in the overseas market is

$$\overline{CS}_2 = \alpha(\frac{\beta(\widetilde{\theta}^2 - (\hat{\theta}_2)^2)}{2} - p_2(\widetilde{\theta} - \hat{\theta}_2) + \frac{1-\widetilde{\theta}^2}{2} - p_B(1-\widetilde{\theta})) = \alpha(\frac{\beta\left(\widetilde{\theta} - \hat{\theta}_2\right)^2}{2} + \frac{1-\widetilde{\theta}^2}{2} - p_B\left(1-\widetilde{\theta}\right)).$$

Secondly, under the base model: in the equilibrium,

(i) when Strategy H is optimal, consumer surplus in the home and overseas markets are as follows, respectively: $CS_1^H = \overline{CS}_1$, $CS_2^H = \overline{CS}_2$;

(ii) when Strategy D is optimal, consumer surplus in the home and overseas markets are as follows, respectively:

$$CS_1^D = \tfrac{1-(p_B)^2}{2} - p_B(1-p_B) = \tfrac{(1-p_B)^2}{2},$$
$$CS_2^{D\dagger} = \alpha((1-\gamma)(\tfrac{1-(\hat{\theta}_B)^2}{2}) - p_B(1-\hat{\theta}_B)) = \alpha(1-\gamma)(\tfrac{(1-\hat{\theta}_B)^2}{2}),$$
$$CS_2^{DC} = \alpha(\tfrac{\beta((\widetilde{\theta}')^2-(\hat{\theta}_2)^2)}{2} - p_2(\widetilde{\theta}' - \hat{\theta}_2) + \tfrac{1-(\widetilde{\theta}')^2}{2} - p_B(1-\widetilde{\theta}')) = \alpha(\tfrac{\beta(\widetilde{\theta}'-\hat{\theta}_2)^2}{2} + \tfrac{1-(\widetilde{\theta}')^2}{2} - p_B(1-\widetilde{\theta}'));$$

(iii) when Strategy O is optimal, consumer surplus in the home and overseas markets are as follows, respectively:

$$CS_1^O = (1-\gamma)(\tfrac{1-(\hat{\theta}_B)^2}{2}) - p_B(1-\hat{\theta}_B) = (1-\gamma)(\tfrac{(1-\hat{\theta}_B)^2}{2}),$$
$$CS_2^{O\dagger} = \alpha((1-\gamma)(\tfrac{1-(\hat{\theta}_B)^2}{2}) - p_B(1-\hat{\theta}_B)) = \alpha(1-\gamma)(\tfrac{(1-\hat{\theta}_B)^2}{2}),$$
$$CS_2^{OC} = \alpha(\tfrac{\beta((\widetilde{\theta}')^2-(\hat{\theta}_2)^2)}{2} - p_2(\widetilde{\theta}' - \hat{\theta}_2) + \tfrac{1-(\widetilde{\theta}')^2}{2} - p_B(1-\widetilde{\theta}')) = \alpha(\tfrac{\beta(\widetilde{\theta}'-\hat{\theta}_2)^2}{2} + \tfrac{1-(\widetilde{\theta}')^2}{2} - p_B(1-\widetilde{\theta}')).$$

Lastly, by comparing consumer surplus between the benchmark and Strategy D as well as Strategy O in equilibrium, respectively, we have the following results.

(1) In the home market, $CS_1^O \leq CS_1^D = \overline{CS}_1$. Because

$$CS_1^O - \overline{CS}_1 = (1-\gamma)(\tfrac{1-(\hat{\theta}_B)^2}{2}) - p_B(1-\hat{\theta}_B) - (\tfrac{1-(p_B)^2}{2} - p_B(1-p_B)) = \tfrac{(1-\gamma)(1-\hat{\theta}_B)^2}{2} - \tfrac{(1-p_B)^2}{2} \leq 0,$$

where the equality is achieved if $\gamma = 0$.

(2) In the overseas market, $CS_2^D = CS_2^O$. By comparing CS_2^D with $\overline{CS}_2$, we get the following results.

$$\begin{aligned} CS_2^{D\dagger} - \overline{CS}_2 =& \ \alpha(\tfrac{(1-\gamma)(1-\hat{\theta}_B)^2}{2} - (\tfrac{\beta(\widetilde{\theta}-\hat{\theta}_2)^2}{2} + \tfrac{1-\widetilde{\theta}^2}{2} - p_B(1-\widetilde{\theta})) \\ =& -\tfrac{\alpha(\beta p_B - p_2)^2}{2\beta(1-\beta)} + \alpha(\tfrac{(1-\gamma)(1-\hat{\theta}_B)^2}{2} - \tfrac{(1-p_B)^2}{2}) < 0; \\ CS_2^{DC} - \overline{CS}_2 =& \ \alpha((\tfrac{\beta(\widetilde{\theta}'-\hat{\theta}_2)^2}{2} + \tfrac{1-(\widetilde{\theta}')^2}{2} - p_B(1-\widetilde{\theta}')) - (\tfrac{\beta(\widetilde{\theta}-\hat{\theta}_2)^2}{2} + \tfrac{1-\widetilde{\theta}^2}{2} - p_B(1-\widetilde{\theta})) \\ =& \ \tfrac{-\alpha(1-\beta)(\widetilde{\theta}' - \frac{p_B-p_2}{1-\beta})^2}{2} \leq 0. \end{aligned}$$

The sign of $(CS_2^{DC} - \overline{CS}_2)$ is analyzed as follows. Note that the function $f(x) = \frac{\beta(x-\hat{\theta}_2)^2}{2} + \frac{1-(x)^2}{2} - p_B(1-x) = \frac{-(1-\beta)x^2 + 2(p_B-p_2)x + 1 - 2p_B + \beta(\hat{\theta}_2)^2}{2} = \frac{-(1-\beta)(x - \frac{p_B-p_2}{1-\beta})^2 + \frac{(p_B-p_2)^2}{1-\beta} + 1 - 2p_B + \frac{p_2^2}{\beta}}{2}$ increases for $x < \frac{p_B-p_2}{1-\beta}$, and decreases for $x > \frac{p_B-p_2}{1-\beta}$. Recall $\widetilde{\theta}' = \frac{p_B-p_2}{1-\gamma-\beta} \geq \frac{p_B-p_2}{1-\beta}$. Thus, $CS_2^{DC} - \overline{CS}_2 \leq 0$.

Thus, in the overseas market, $CS_2^D = CS_2^O \leq \overline{CS}_2$.

For the total consumer surplus, $CS = CS_1 + CS_2$, thus, we know, $CS^O \leq CS^D \leq \overline{CS}$.

From above comparisons, we can obtain that in both the home and overseas markets, the consumer surplus loss increases in γ. When $\gamma = 0$, we have $CS_1^O = \overline{CS}_1$ and $CS_2^{DC} = \overline{CS}_2$. ■

B.7 Proof of Proposition 4.

Firstly, under the benchmark: in the equilibrium,

$\overline{SS} = \overline{CS}_1 + \overline{CS}_2 + \bar{\pi}_B^* + \bar{\pi}_1^* + \bar{\pi}_2^* = \frac{(1-p_B)^2}{2} + \alpha(\frac{\beta(\widetilde{\theta}-\hat{\theta}_2)^2}{2} + \frac{1-\widetilde{\theta}^2}{2} - p_B(1-\widetilde{\theta})) + (p_B - k_1)(1-p_B) + \alpha(p_B - k_1 - t)(1 - \frac{p_B-p_2}{1-\beta}) + (\alpha(p_2-k_2)(\frac{p_B-p_2}{1-\beta} - \frac{p_2}{\beta}) - e)$.

Secondly, under the base model: in the equilibrium,

(i) when Strategy D is optimal, the social surplus is

$SS^{D\dagger} = CS_1^D + CS_2^D + \pi_B^D + \pi_1^D + \pi_2^D = \frac{(1-p_B)^2}{2} + \alpha\gamma(\frac{(1-\hat{\theta}_B)^2}{2}) + (p_B-k_1)(1-p_B) + \alpha(p_B-k_2)\left(1 - \frac{p_B}{1-\gamma}\right)$,

$SS^{DC} = CS_1^{DC} + CS_2^{DC} + \pi_B^{DC} + \pi_1^{DC} + \pi_2^{DC} = \frac{(1-p_B)^2}{2} + \alpha(\frac{\beta(\widetilde{\theta}'-\hat{\theta}_2)^2}{2} + \frac{1-(\widetilde{\theta}')^2}{2} - p_B(1-\widetilde{\theta}')) + (p_B-k_1)(1-p_B) + \alpha(p_B-k_2)(1 - \frac{p_B-p_2}{1-\gamma-\beta}) + (\alpha(p_2-k_2)(\frac{p_B-p_2}{1-\gamma-\beta} - \frac{p_2}{\beta}) - e)$;

(ii) when Strategy O is optimal, the social surplus is

$SS^{O\dagger} = CS_1^O + CS_2^O + \pi_B^O + \pi_1^O + \pi_2^O = (1+\alpha)(1-\gamma)(\frac{(1-\hat{\theta}_B)^2}{2}) + (p_B - k_2 - t)\left(1 - \frac{p_B}{1-\gamma}\right) + \alpha(p_B-k_2)\left(1 - \frac{p_B}{1-\gamma}\right)$,

$SS^{OC} = CS_1^{OC} + CS_2^{OC} + \pi_B^{OC} + \pi_1^{OC} + \pi_2^{OC} = (1-\gamma)(\frac{(1-\hat{\theta}_B)^2}{2}) + \alpha(\frac{\beta(\widetilde{\theta}'-\hat{\theta}_2)^2}{2} + \frac{1-(\widetilde{\theta}')^2}{2} - p_B(1-\widetilde{\theta}')) + (p_B-k_2-t)\left(1 - \frac{p_B}{1-\gamma}\right) + \alpha(p_B-k_2)(1 - \frac{p_B-p_2}{1-\gamma-\beta}) + (\alpha(p_2-k_2)(\frac{p_B-p_2}{1-\gamma-\beta} - \frac{p_2}{\beta}) - e)$.

Lastly, by comparing the social surplus between the benchmark and Strategy D as well as Strategy O in equilibrium, respectively, we have the following discussions. We define

$$\begin{aligned} \bar{\Delta}_D &= \frac{\alpha(p_B-p_2)(\frac{p_B-p_2}{1-\gamma-\beta} - \frac{p_B-p_2}{1-\beta}) - g_1}{\alpha(1 - \frac{p_B-p_2}{1-\beta})} - t, \\ \bar{\Delta}_O &= \frac{(p_B-k_2)(\frac{p_B}{1-\gamma} - p_B) + \alpha(p_B-p_2)(\frac{p_B-p_2}{1-\gamma-\beta} - \frac{p_B-p_2}{1-\beta}) - t(\alpha(1-\frac{p_B-p_2}{1-\beta}) - (1-\frac{p_B}{1-\gamma})) - g_2}{(1-p_B) + \alpha(1 - \frac{p_B-p_2}{1-\beta})}; \\ e_1' &= \bar{e}_{D1} - g_1, \\ e_2' &= \bar{e}_{O1} - g_2, \end{aligned} \tag{11}$$

where

$$\bar{e}_{D1} = \alpha(p_2-k_2)(\frac{p_B-p_2}{1-\beta}-\frac{p_2}{\beta}) - \alpha(\Delta+t)(1-\frac{p_B-p_2}{1-\beta}) - \alpha(p_B-k_2)(\frac{p_B-p_2}{1-\beta}-\frac{p_B}{1-\gamma}),$$

$$\bar{e}_{O1} = \alpha(p_2-k_2)(\frac{p_B-p_2}{1-\beta}-\frac{p_2}{\beta}) - \Delta(1-p_B) - (\Delta+t)\alpha(1-\frac{p_B-p_2}{1-\beta}) + (p_B-k_2)(\frac{p_B}{1-\gamma}-p_B)$$
$$-\alpha(p_B-k_2)(\frac{p_B-p_2}{1-\beta}-\frac{p_B}{1-\gamma}),$$

and

$$g_1 = \begin{cases} -\frac{\alpha(\beta p_B-p_2)^2}{2\beta(1-\beta)} + \alpha(\frac{(1-\gamma)(1-\hat{\theta}_B)^2}{2} - \frac{(1-p_B)^2}{2}), & \text{if } s^*=0, \\ \frac{-\alpha(1-\beta)(\widetilde{\theta}'-\frac{p_B-p_2}{1-\beta})^2}{2}, & \text{if } s^*=1, \end{cases}$$

$g_2 = g_1 + (\frac{(1-\gamma)(1-\frac{p_B}{1-\gamma})^2}{2} - \frac{(1-p_B)^2}{2})$. Note that $g_1 \le 0$ and $g_2 \le 0$ represent the loss of consumer surplus under strategies D and O, respectively.

Then, we have the following comparisons about social welfare.

(1) If there is no counterfeiting after conversion, i.e., $s^*=0$, then: recall that $\widetilde{\theta} = \frac{p_B-p_2}{1-\beta}$, and $\hat{\theta}_B = \frac{p_B}{1-\gamma}$,

(i) when Strategy D is optimal,

$$\begin{aligned} SS^{D\dagger} - \overline{SS} &= (CS_1^D - \overline{CS}_1) + (CS_2^D - \overline{CS}_2) + (\pi_B^D + \pi_1^D + \pi_2^D) - (\bar{\pi}_B^* + \bar{\pi}_1^* + \bar{\pi}_2^*) \\ &= -\frac{\alpha(\beta p_B-p_2)^2}{2\beta(1-\beta)} + \alpha(\frac{(1-\gamma)(1-\hat{\theta}_B)^2}{2} - \frac{(1-p_B)^2}{2}) \\ &\quad +\alpha(1-\frac{p_B-p_2}{1-\beta})(\Delta+t) + \alpha(p_B-k_2)(\frac{p_B-p_2}{1-\beta}-\frac{p_B}{1-\gamma}) - (\alpha(p_2-k_2)(\frac{p_B-p_2}{1-\beta}-\frac{p_2}{\beta}) - e); \end{aligned}$$

(ii) when Strategy O is optimal,

$$\begin{aligned} SS^{O\dagger} - \overline{SS} &= (CS_1^O - \overline{CS}_1) + (CS_2^O - \overline{CS}_2) + (\pi_B^O + \pi_1^O + \pi_2^O) - (\bar{\pi}_B^* + \bar{\pi}_1^* + \bar{\pi}_2^*) \\ &= -\frac{\alpha(\beta p_B-p_2)^2}{2\beta(1-\beta)} + (1+\alpha)(\frac{(1-\gamma)(1-\hat{\theta}_B)^2}{2} - \frac{(1-p_B)^2}{2}) \\ &\quad +\Delta(1-p_B) + (\Delta+t)\alpha(1-\frac{p_B-p_2}{1-\beta}) - (p_B-k_2)(\frac{p_B}{1-\gamma}-p_B) + \alpha(p_B-k_2)(\frac{p_B-p_2}{1-\beta}-\frac{p_B}{1-\gamma}) \\ &\quad -(\alpha(p_2-k_2)(\frac{p_B-p_2}{1-\beta}-\frac{p_2}{\beta}) - e). \end{aligned}$$

Since $e < \alpha(p_2-k_2)(\frac{p_B-p_2}{1-\beta}-\frac{p_2}{\beta})$, then, we derive below conditions: $SS^{D\dagger} > \overline{SS}$ when $e > (e_1')^+$; $SS^{O\dagger} > \overline{SS}$ when $e > (e_2')^+$.

(2) If there is counterfeiting after conversion, i.e., $s^*=1$, then: recall that $\widetilde{\theta}' = \frac{p_B-p_2}{1-\gamma-\beta}$, and $\hat{\theta}_B = \frac{p_B}{1-\gamma}$,

(i) when Strategy *DC* is optimal,

$$\begin{aligned} SS^{DC} - \overline{SS} &= (CS_1^D - \overline{CS}_1) + (CS_2^D - \overline{CS}_2) + (\pi_B^D + \pi_1^D + \pi_2^D) - (\bar{\pi}_B^* + \bar{\pi}_1^* + \bar{\pi}_2^*) \\ &= \frac{-\alpha(1-\beta)(\widetilde{\theta}'-\frac{p_B-p_2}{1-\beta})^2}{2} \\ &\quad +\alpha(1-\frac{p_B-p_2}{1-\beta})(\Delta+t) - \alpha(p_B-p_2)(\frac{p_B-p_2}{1-\gamma-\beta}-\frac{p_B-p_2}{1-\beta}); \end{aligned}$$

(ii) when Strategy *OC* is optimal,

$$\begin{aligned} SS^{OC} - \overline{SS} &= (CS_1^O - \overline{CS}_1) + (CS_2^O - \overline{CS}_2) + (\pi_B^O + \pi_1^O + \pi_2^O) - (\bar{\pi}_B^* + \bar{\pi}_1^* + \bar{\pi}_2^*) \\ &= \frac{-\alpha(1-\beta)(\widetilde{\theta}'-\frac{p_B-p_2}{1-\beta})^2}{2} + (\frac{(1-\gamma)(1-\hat{\theta}_B)^2}{2} - \frac{(1-p_B)^2}{2}) \\ &\quad +\Delta((1-p_B) + \alpha(1-\frac{p_B-p_2}{1-\beta})) + t\alpha(1-\frac{p_B-p_2}{1-\beta}) - t(1-\frac{p_B}{1-\gamma}) - (p_B-k_2)(\frac{p_B}{1-\gamma}-p_B) \\ &\quad -\alpha(p_B-p_2)(\frac{p_B-p_2}{1-\gamma-\beta}-\frac{p_B-p_2}{1-\beta}). \end{aligned}$$

Thus, we obtain the conditions: $SS^{DC} > \overline{SS}$ when $\Delta > \bar{\Delta}_D$; $SS^{OC} > \overline{SS}$ when $\Delta > \bar{\Delta}_O$. ■

B.8 Proofs For Extension 1: Sequential Contract Offering

B.8.1 Proof of Lemma 3.

In order to differentiate the cases that the overseas supplier sells counterfeits, we call Strategy D without counterfeiting as Strategy D†, Strategy O without counterfeiting as Strategy O†; and call Strategy D with counterfeiting as Strategy D^{C}, Strategy O with counterfeiting as Strategy O^{C}.

Recall that

$$\begin{aligned}
&M' = \alpha\left(p_2 - k_2\right)\left(\tfrac{p_B - p_2}{1-\gamma-\beta} - \tfrac{p_2}{\beta}\right) - e, && w_2^{(0)} = k_2 + \frac{M'}{\alpha\left(\frac{p_B-p_2}{1-\gamma-\beta} - \frac{p_B}{1-\gamma}\right)},\\
&K = \alpha\left(p_2 - k_2\right)\left(1 - \tfrac{p_2}{\beta}\right) - e, && w_2^{O(2)} = k_2 + \frac{K}{(1+\alpha)(1-\frac{p_B}{1-\gamma})},\\
&K - M' = \alpha\left(p_2 - k_2\right)\left(1 - \tfrac{p_B-p_2}{1-\gamma-\beta}\right), && w_2^{O(1)} = k_2 + \frac{K-M'}{(1-\frac{p_B}{1-\gamma})+\alpha(1-\frac{p_B-p_2}{1-\gamma-\beta})},\\
& && \widehat{w}_2 = p_B - \frac{(p_B-k_2-\Delta)(1-p_B)}{1-\frac{p_B}{1-\gamma}} - t.
\end{aligned}$$

We observe that $w_2^{O(1)}$ is independent on e and Δ; $w_2^{(0)}$ and $w_2^{O(2)}$ are dependent on e; and $\widehat{w}_2$ is dependent on Δ.

Step 1: We derive the overseas supplier's counterfeiting decision $s(w_1, w_2, d_1, d_2)$. If the overseas supplier decides to sell counterfeits, then, it should satisfy: $\pi_2\left(s=1\right) \geq \pi_2\left(s=0\right)$ for $d_2 = 1$. That is,

$$\begin{aligned}
&\max\{\pi_2\left(s=1; w_1, w_2, d_1=0, d_2=1\right), \pi_2\left(s=1; w_1, w_2, d_1=1, d_2=1\right)\}\\
\geq &\max\{\pi_2\left(s=0; w_1, w_2, d_1=0, d_2=1\right), \pi_2\left(s=0; w_1, w_2, d_1=1, d_2=1\right)\}.
\end{aligned}$$

Note that with the assumption $0 \leq e < \alpha(p_2 - k_2)(\frac{p_B-p_2}{1-\beta} - \frac{p_2}{\beta})$, $\pi_2\left(s=1\right) \geq \pi_2\left(s=0\right)$ holds for $d_2 = 0$. Thus, we obtain,

$$s^*\left(w_1, w_2, d_1, d_2\right) = \begin{cases} 0, & \text{if } d_2 = 1 \text{ and } w_2 \geq w_2^{(0)},\\ 1, & \text{if } d_2 = 1 \text{ and } k_2 \leq w_2 < w_2^{(0)}, \text{ or if } d_2 = 0.\end{cases}$$

Step 2: We derive the home supplier's acceptance decision $d_1(w_1, w_2, d_2)$. If the home supplier decides to accept the contract, i.e., $d_1 = 1$, then, it should satisfy: $\pi_1\left(d_1=1\right) \geq \pi_1\left(d_1=0\right)$. That is,

$$\begin{aligned}
&\max\{\pi_1\left(d_1=1; w_1, w_2, d_2=0\right), \pi_1\left(d_1=1; w_1, w_2, d_2=1, s=1\right), \pi_1\left(d_1=1; w_1, w_2, d_2=1, s=0\right)\}\\
\geq &\max\{\pi_1\left(d_1=0; w_1, w_2, d_2=0\right), \pi_1\left(d_1=0; w_1, w_2, d_2=1, s=1\right), \pi_1\left(d_1=0; w_1, w_2, d_2=1, s=0\right)\}.
\end{aligned}$$

Thus, we obtain,

$$d_1(w_1, w_2, d_2) = \begin{cases} 1, & \text{if } w_1 \geq k_1,\\ 0, & \text{otherwise.}\end{cases}$$

Step 3: We derive the brand-name firm's optimal wholesale price $w_1(w_2, d_2)$.

$$\pi_B\left(w_1; w_2, d_1=1, d_2\right) \geq \pi_B\left(w_1; w_2, d_1=0, d_2\right).$$

Note that the brand-name firm's profit decreases in w_1.

Given $d_2 = 0$, we know, it should satisfy: $\pi_B(w_1; w_2, d_1 = 1, d_2 = 0) \geq \pi_B(w_1; w_2, d_1 = 0, d_2 = 0)$. That is, $k_1 \leq w_1 \leq p_B$. Thus, $w_1(w_2, d_2 = 0) = k_1$.

Given $d_2 = 1$, we know, it should satisfy:

$$\begin{aligned} &\max\{\pi_B(w_1; w_2, d_1 = 1, d_2 = 1, s = 1), \pi_B(w_1; w_2, d_1 = 1, d_2 = 1, s = 0)\} \\ \geq &\max\{\pi_B(w_1; w_2, d_1 = 0, d_2 = 1, s = 1), \pi_B(w_1; w_2, d_1 = 0, d_2 = 1, s = 0)\}. \end{aligned}$$

Then, from $\pi_B^D \geq \pi_B^O$, which means $(p_B - w_1)(1 - p_B) \geq (p_B - w_2 - t)(1 - \frac{p_B}{1-\gamma})$, then, we obtain: $w_1 \leq p_B - \frac{(p_B - w_2 - t)(1 - \frac{p_B}{1-\gamma})}{1 - p_B}$. Note that $w_1 \geq k_1$. From $p_B - \frac{(p_B - w_2 - t)(1 - \frac{p_B}{1-\gamma})}{1 - p_B} \geq k_1$, we obtain, $w_2 \geq \widehat{w}_2$, where $\widehat{w}_2 = p_B - \frac{(p_B - k_2 - \Delta)(1 - p_B)}{1 - \frac{p_B}{1-\gamma}} - t$, and $\widehat{w}_2 < k_1$.

Thus, we have:

$$w_1(w_2, d_2) = \begin{cases} k_1, & \text{if } d_2 = 0, \\ & \text{or, if } d_2 = 1 \text{ and } w_2 \geq \widehat{w}_2, \\ 0, & \text{otherwise.} \end{cases}$$

Step 4: We derive the overseas supplier's acceptance decision $d_2(w_2)$:

If the overseas supplier decides to accept the contract, i.e., $d_2 = 1$, then, it should satisfy:

$$\pi_2(d_2 = 1) \geq \pi_2(d_2 = 0).$$

That is,

$$\begin{aligned} &\max\{\pi_2(d_2 = 1; w_2, d_1 = 1, s = 1), \pi_2(d_2 = 1; w_2, d_1 = 1, s = 0) \\ &\quad \pi_2(d_2 = 1; w_2, d_1 = 0, s = 1), \pi_2(d_2 = 1; w_2, d_1 = 0, s = 0)\} \\ \geq &\max\{\pi_2(d_2 = 0; w_2, d_1 = 1, s = 1), \pi_2(d_2 = 0; w_2, d_1 = 1, s = 0) \\ &\quad \pi_2(d_2 = 0; w_2, d_1 = 0, s = 1), \pi_2(d_2 = 0; w_2, d_1 = 0, s = 0)\}. \end{aligned}$$

(1) For the case of $w_2 < w_2^{(0)}$, we obtain

$$d_2(w_2; d_1) = \begin{cases} d_2(w_2; d_1 = 1) = 1, & \text{if } \min\{w_2^{O(1)}, w_2^{(0)}\} \leq w_2 < w_{20}, \text{ and } w_2 \geq \widehat{w}_2, \\ d_2(w_2; d_1 = 1) = 0, & \text{if } w_2 < \min\{w_2^{O(1)}, w_2^{(0)}\}, \\ d_2(w_2; d_1 = 0) = 1, & \text{if } \min\{w_2^{O(1)}, w_2^{(0)}\} \leq w_2 < w_{20}, \text{ and } w_2 < \widehat{w}_2, \\ d_2(w_2; d_1 = 0) = 0, & \text{if } w_2 < \min\{w_2^{O(1)}, w_2^{(0)}\}. \end{cases}$$

(2) For the case of $w_2 \geq w_2^{(0)}$, we obtain

$$d_2(w_2; d_1) = \begin{cases} d_2(w_2; d_1 = 1) = 1, & \text{if } w_2 \geq \max\{w_2^{O(2)}, w_2^{(0)}\}, \text{ and } w_2 \geq \widehat{w}_2, \\ d_2(w_2; d_1 = 1) = 0, & \text{if } w_{20} < w_2 < \max\{w_2^{O(2)}, w_2^{(0)}\}, \\ d_2(w_2; d_1 = 0) = 1, & \text{if } w_2 \geq \max\{w_2^{O(2)}, w_2^{(0)}\}, \text{ and } w_2 < \widehat{w}_2, \\ d_2(w_2; d_1 = 0) = 0, & \text{if } w_{20} < w_2 < \max\{w_2^{O(2)}, w_2^{(0)}\}. \end{cases}$$

Thus, combined above discussions, the overseas supplier's optimal decision is

$$d_2(w_2; d_1) = \begin{cases} d_2(w_2; d_1 = 1) = 1, & \text{if } w_2 \geq \widehat{w}_2, \min\{w_2^{O(1)}, w_2^{(0)}\} \leq w_2 < w_{20} \text{ or } w_2 \geq \max\{w_2^{O(2)}, w_2^{(0)}\}, \\ d_2(w_2; d_1 = 1) = 0, & \text{if } w_2 < \min\{w_2^{O(1)}, w_2^{(0)}\} \text{ or } w_2^{(0)} < w_2 < \max\{w_2^{O(2)}, w_2^{(0)}\}, \\ d_2(w_2; d_1 = 0) = 1, & \text{if } w_2 \geq \widehat{w}_2, \min\{w_2^{O(1)}, w_2^{(0)}\} \leq w_2 < w_2^{(0)} \text{ or } w_2 \geq \max\{w_2^{O(2)}, w_2^{(0)}\}, \\ d_2(w_2; d_1 = 0) = 0, & \text{if } w_2 < \min\{w_2^{O(1)}, w_2^{(0)}\} \text{ or } w_2^{(0)} < w_2 < \max\{w_2^{O(2)}, w_2^{(0)}\}. \end{cases}$$

Step 5: We derive the brand-name firm's optimal wholesale price w_2.

By substituting $d_2(w_2; d_1)$ into the brand-name firm's profit function, we obtain

$$\pi_B^H = (p_B - k_1)(1 - p_B) + \alpha(p_B - k_1 - t)(1 - \tfrac{p_B - p_2}{1-\beta}),$$

$$\pi_B^D = \begin{cases} \pi_B^{D\dagger}(w_2) = (p_B - k_1)(1-p_B) + \alpha(p_B - w_2)(1 - \frac{p_B}{1-\gamma}), & \text{if } w_2 \geq \max\{w_2^{O(2)}, w_2^{(0)}\}, \text{ and } w_2 \geq \widehat{w}_2, \\ \pi_B^{DC}(w_2) = (p_B - k_1)(1 - \frac{p_B}{1-\gamma}) + \alpha(p_B - w_2)(1 - \frac{p_B - p_2}{1-\gamma-\beta}), & \text{if } \min\{w_2^{O(1)}, w_2^{(0)}\} \leq w_2 < w_2^{(0)}, \text{ and } w_2 \geq \widehat{w}_2, \end{cases}$$

$$\pi_B^O = \begin{cases} \pi_B^{O\dagger}(w_2) = (p_B - w_2 - t)(1 - \frac{p_B}{1-\gamma}) + \alpha(p_B - w_2)(1 - \frac{p_B}{1-\gamma}), & \text{if } w_2 \geq \max\{w_2^{O(2)}, w_2^{(0)}\}, \text{ and } w_2 < \widehat{w}_2, \\ \pi_B^{OC}(w_2) = (p_B - w_2 - t)(1 - \frac{p_B}{1-\gamma}) + \alpha(p_B - w_2)(1 - \frac{p_B - p_2}{1-\gamma-\beta}), & \text{if } \min\{w_2^{O(1)}, w_2^{(0)}\} \leq w_2 < w_2^{(0)}, \text{ and } w_2 < \widehat{w}_2, \end{cases}$$

Note that the brand-name firm's profit decreases in w_2. Then, the optimal wholesale price(s) of the brand-name firm, which will be accepted by the counterfeiter, satisfies the following:

(a) under Strategy D,

$$w_2^D = \begin{cases} w_2^{D\dagger *} = \max\{w_2^{O(2)}, w_2^{(0)}, \widehat{w}_2\}, & \text{if } s = 0, \\ w_2^{DC*} = \max\{w_2^{O(1)}, \widehat{w}_2\}, & \text{if } s = 1 \text{ and } \max\{w_2^{O(1)}, \widehat{w}_2\} < w_2^{(0)}, \end{cases}$$

(b) under Strategy O,

$$w_2^O = \begin{cases} w_2^{O\dagger *} = \max\{w_2^{O(2)}, w_2^{(0)}\}, & \text{if } s = 0 \text{ and } \max\{w_2^{O(2)}, w_2^{(0)}\} < \widehat{w}_2, \\ w_2^{OC*} = w_2^{O(1)}, & \text{if } s = 1 \text{ and } w_2^{O(1)} < \min\{w_2^{(0)}, \widehat{w}_2\}. \end{cases}$$

Recall that $w_2^{O(1)}$ is independent on e and Δ; $w_2^{(0)}$ and $w_2^{O(2)}$ are dependent on e; and $\widehat{w}_2$ is dependent on Δ. Then, we know the wholesale price $w_2^{D\dagger *}$ could be dependent on Δ and e, w_2^{DC*} could be dependent on Δ; $w_2^{O\dagger *}$ is dependent on e, w_2^{OC*} is independent on both e and Δ.

For Strategy D and Strategy O, the brand-name firm may offer different wholesale prices w_2 which helps prevent counterfeiting, below, we further check the feasible region of π_B under Strategy D and Strategy O. Then, there are four cases for the existence of possible strategies:

Case 1: $w_2^{O(1)} < \widehat{w}_2 < w_2^{(0)}$, in which both Strategy D^C and Strategy O^C are possible;

Case 2: $\widehat{w}_2 < w_2^{O(1)} < w_2^{(0)}$, in which only Strategy D^C is possible;

Case 3: $w_2^{O(1)} < w_2^{(0)} < \widehat{w}_2$, in which both Strategy $O^\dagger$ and Strategy O^C are possible. In particular, only if $\max\{w_2^{O(2)}, w_2^{(0)}\} < \widehat{w}_2$, Strategy $O^\dagger$ exists;

Case 4: $w_2^{O(1)} > w_2^{(0)}$, in which only Strategy $O^\dagger$ is possible.

Note that

$$\begin{aligned} & w_2^{O(1)} < w_2^{(0)}, \Rightarrow e < e_1; \\ & w_2^{O(1)} < \widehat{w}_2, \quad \Rightarrow \Delta > \Delta_0, \text{ where } \Delta_0 = (w_2^{OC*} - (p_B - \tfrac{(p_B-k_2)(1-p_B)}{1-\frac{p_B}{1-\gamma}}))\tfrac{1-\frac{p_B}{1-\gamma}}{1-p_B}; \\ & \widehat{w}_2 < w_2^{(0)}, \quad \Rightarrow e < \hat{e}_2, \text{ where } \hat{e}_2 = (p_2 - k_2 - (\widehat{w}_2 - k_2)\tfrac{\beta}{1-\gamma})\tfrac{\alpha(\beta p_B - (1-\gamma)p_2)}{(1-\gamma-\beta)\beta}; \\ & w_2^{O(2)} < \widehat{w}_2, \quad \Rightarrow e > \hat{e}_3, \text{ where } \hat{e}_3 = \alpha(p_2 - k_2)(1 - \tfrac{p_2}{\beta}) - (\widehat{w}_2 - k_2)(1+\alpha)(1 - \tfrac{p_B}{1-\gamma}). \end{aligned} \tag{12}$$

Thus, the feasible regions of each possible case are as follows:

Strategy $D^\dagger$: exists for all cases;

Strategy D^C: exists for case 1 and case 2, which means $e < \min\{e_1, \hat{e}_2\}$;

Strategy $O^\dagger$: exists for case 3 and case 4, which means $e > \max\{\hat{e}_2, \hat{e}_3\}$;

Strategy O^C: exists for case 1 and case 3, which means $e < e_1$ and $\Delta > \Delta_0$.

Note that it is easy to know that Strategy D^C and Strategy $O^\dagger$ do not exist in the same feasible region.

Next, we make a comparison between Strategy D† and Strategy D^{C}, Strategy O† and Strategy O^{C}, respectively.

5.1 With Strategy D, we have

$$\pi_B^D = \begin{cases} \pi_B^{D\dagger}\left(w_2^{D\dagger *}\right) = (p_B - k_1)(1-p_B) + \alpha\left(p_B - w_2^{D\dagger *}\right)(1-\frac{p_B}{1-\gamma}), \\ \pi_B^{DC}\left(w_2^{DC*}\right) = (p_B - k_1)(1-p_B) + \alpha\left(p_B - w_2^{DC*}\right)(1-\frac{p_B-p_2}{1-\gamma-\beta}), \text{ if } e < \min\{e_1, \hat{e}_2\}, \end{cases}$$

and

$$\begin{aligned} &\pi_B^{D\dagger}\left(w_2^{D\dagger *}\right) \geq \pi_B^{DC}\left(w_2^{DC*}\right), \\ \Rightarrow &\left(p_B - w_2^{D\dagger *}\right)(1-\tfrac{p_B}{1-\gamma}) \geq \left(p_B - w_2^{DC*}\right)(1-\tfrac{p_B-p_2}{1-\gamma-\beta}), \\ \Rightarrow &w_2^{D\dagger *} \leq \frac{p_B(\frac{p_B-p_2}{1-\gamma-\beta}-\frac{p_B}{1-\gamma})+w_2^{DC*}(1-\frac{p_B-p_2}{1-\gamma-\beta})}{1-\frac{p_B}{1-\gamma}} = p_B - \frac{\left(p_B-w_2^{DC*}\right)(1-\frac{p_B-p_2}{1-\gamma-\beta})}{1-\frac{p_B}{1-\gamma}}. \end{aligned}$$

Recall that $w_2^{DC*} = \max\{w_2^{O(1)}, \widehat{w}_2\}$, $w_2^{D\dagger *} = \max\{w_2^{O(2)}, w_2^{(0)}, \widehat{w}_2\}$. Note that when $\widehat{w}_2 > \max\{w_2^{O(2)}, w_2^{(0)}\}$ with Strategy D†, Strategy D^{C} does not exist. Thus, when Strategy D^{C} exists, that is, $e < \min\{e_1, \hat{e}_2\}$, the optimal wholesale price of Strategy D† is $w_2^{D\dagger *} = \max\{w_2^{O(2)}, w_2^{(0)}\}$, which is independent on Δ, and dependent on e.

If $w_2^{O(1)} > \widehat{w}_2$, then $w_2^{DC*} = w_2^{O(1)}$, which is independent on both e and Δ. Then,

$$\pi_B^{DC} > \pi_B^{D\dagger} \Rightarrow w_2^{D\dagger *} > \frac{p_B(\frac{p_B-p_2}{1-\gamma-\beta}-\frac{p_B}{1-\gamma})+w_2^{O(1)}(1-\frac{p_B-p_2}{1-\gamma-\beta})}{1-\frac{p_B}{1-\gamma}} = p_B - \frac{\left(p_B-w_2^{O(1)}\right)(1-\frac{p_B-p_2}{1-\gamma-\beta})}{1-\frac{p_B}{1-\gamma}} \Rightarrow e < e_{D2}, \text{ [case 1, case 2]}$$

If $w_2^{O(1)} < \widehat{w}_2$, then $w_2^{DC*} = \widehat{w}_2$, which is dependent on Δ. Then,

$$\pi_B^{DC} > \pi_B^{D\dagger} \Rightarrow w_2^{D\dagger *} > \frac{p_B(\frac{p_B-p_2}{1-\gamma-\beta}-\frac{p_B}{1-\gamma})+\widehat{w}_2(1-\frac{p_B-p_2}{1-\gamma-\beta})}{1-\frac{p_B}{1-\gamma}} = p_B - \frac{(p_B-\widehat{w}_2)(1-\frac{p_B-p_2}{1-\gamma-\beta})}{1-\frac{p_B}{1-\gamma}} \Rightarrow e < e_{D3}, \text{ [case 1, case 2]}$$

5.2 With Strategy O, similarly, for the comparison between Strategy O† and Strategy O^{C}, we know,

$$\pi_B^O = \begin{cases} \pi_B^{O\dagger}\left(w_2^{O\dagger *}\right) = (p_B - w_2^{O*} - t)(1-\frac{p_B}{1-\gamma}) + \alpha\left(p_B - w_2^{O\dagger *}\right)(1-\frac{p_B}{1-\gamma}), \\ \pi_B^{OC}\left(w_2^{OC*}\right) = (p_B - w_2^{OC*} - t)(1-\frac{p_B}{1-\gamma}) + \alpha\left(p_B - w_2^{OC*}\right)(1-\frac{p_B-p_2}{1-\gamma-\beta}), \end{cases}$$

and

$$\pi_B^{OC}\left(w_2^{OC*}\right) > \pi_B^{O\dagger}\left(w_2^{O\dagger *}\right) \Rightarrow w_2^{O\dagger *} > \frac{\alpha(p_B-w_2^{OC*})(\frac{p_B-p_2}{1-\gamma-\beta}-\frac{p_B}{1-\gamma})}{(1+\alpha)(1-\frac{p_B}{1-\gamma})} + w_2^{OC*} \Rightarrow e < e_{O1}. \text{ [case 3]}$$

Then, based on above discussion, we have the following optimal wholesale price w_2 for Strategy D and Strategy O, respectively. Note that $e'_{D1} < \min\{e_1, \hat{e}_2\}$, $e_{O1} < e_1$.

(a) Under Strategy D, (i) $w_2^D = \max\{w_2^{O(1)}, \widehat{w}_2\}$ and $s^* = 1$, if $e < e'_{D1}$; (ii) $w_2^D = \max\{w_2^{(0)}, w_2^{O(2)}, \widehat{w}_2\}$ and $s^* = 0$, if $e \geq e'_{D1}$;

(b) under Strategy O, (i) $w_2^O = w_2^{O(1)}$ and $s^* = 1$, if $\Delta > \Delta_0$ and $e < \max\{e_{O1}, \hat{e}_2\}$; (ii) $w_2^O = \max\{w_2^{(0)}, w_2^{O(2)}\}$ and $s^* = 0$, if $e \geq \max\{e_{O1}, \hat{e}_2, \hat{e}_3\}$;

where

$$\begin{aligned} &e'_{D1} = \min\{e_{D2}, e_{D3}\}, \\ &e_{D2} \text{ is the threshold value of } e \text{ satisfying} \max\{w_2^{O(2)}, w_2^{(0)}\} = p_B - \frac{\left(p_B-w_2^{O(1)}\right)(1-\frac{p_B-p_2}{1-\gamma-\beta})}{1-\frac{p_B}{1-\gamma}}, \\ &e_{D3} \text{ is the threshold value of } e \text{ satisfying} \max\{w_2^{O(2)}, w_2^{(0)}\} = p_B - \frac{(p_B-\widehat{w}_2)(1-\frac{p_B-p_2}{1-\gamma-\beta})}{1-\frac{p_B}{1-\gamma}}, \\ &e_{O1} = \left(p_2 - k_2 - (\frac{\alpha(p_B-w_2^{OC*})(\frac{p_B-p_2}{1-\gamma-\beta}-\frac{p_B}{\gamma})}{(1+\alpha)(1-\frac{p_B}{1-\gamma})} + w_2^{OC*} - k_2)\frac{\beta}{1-\gamma}\right)\frac{\alpha(\beta p_B-(1-\gamma)p_2)}{(1-\gamma-\beta)\beta}, \end{aligned} \tag{13}$$

and $w_2^{OC*} = k_2 + \frac{\alpha(p_2-k_2)(1-\frac{p_B-p_2}{1-\gamma-\beta})}{(1-\frac{p_B}{1-\gamma})+\alpha(1-\frac{p_B-p_2}{1-\gamma-\beta})}$.

Thus, we have the results. ■

B.8.2 Proof of Proposition 5.

The brand-name firm makes comparisons among different strategies. Recall that the profits are as follows:

$$\pi_B^H = (p_B - k_1)(1-p_B) + \alpha (p_B - k_1 - t)(1 - \tfrac{p_B - p_2}{1-\beta}),$$

$$\pi_B^D = \begin{cases} \pi_B^{D\dagger}\left(w_2^{D\dagger *}\right) = (p_B - k_1)(1-p_B) + \alpha\left(p_B - w_2^{D\dagger *}\right)(1 - \tfrac{p_B}{1-\gamma}), & \text{if } e \geq e'_{D1}, \\ \pi_B^{DC}\left(w_2^{DC*}\right) = (p_B - k_1)(1-p_B) + \alpha\left(p_B - w_2^{DC*}\right)(1 - \tfrac{p_B - p_2}{1-\gamma-\beta}), & \text{if } e < e'_{D1}, \end{cases}$$

$$\pi_B^O = \begin{cases} \pi_B^{O\dagger}\left(w_2^{O\dagger *}\right) = \left(p_B - w_2^{O\dagger *} - t\right)(1 - \tfrac{p_B}{1-\gamma}) + \alpha\left(p_B - w_2^{O\dagger *}\right)(1 - \tfrac{p_B}{1-\gamma}), & \text{if } e \geq \max\{e_{O1}, \hat{e}_2, \hat{e}_3\}, \\ \pi_B^{OC}\left(w_2^{OC*}\right) = \left(p_B - w_2^{OC*} - t\right)(1 - \tfrac{p_B}{1-\gamma}) + \alpha\left(p_B - w_2^{OC*}\right)(1 - \tfrac{p_B - p_2}{1-\gamma-\beta}), & \text{if } \Delta > \Delta_0,\ e < \max\{e_{O1}, \hat{e}_2\}, \end{cases}$$

$$\pi_B^N = 0,$$

where $w_2^{DC*} = \max\{w_2^{O(1)}, \widehat{w}_2\}$, $w_2^{D\dagger *} = \max\{w_2^{O(2)}, w_2^{(0)}, \widehat{w}_2\}$; $w_2^{OC*} = w_2^{O(1)}$, $w_2^{O\dagger *} = \max\{w_2^{O(2)}, w_2^{(0)}\}$; and e_1, $\hat{e}_2$, $\hat{e}_3$, and Δ_0 are defined in Equation (12). Recall that Strategy D^C and Strategy $\mathrm{O}^\dagger$ do not exist at the same feasible region. Thus, there is no comparison between them.

Below, following the approach in Lemma B2, we derive the conditions for each possible strategy.

(1) The conditions for $\pi_B^* = \pi_B^{D\dagger}$ are $e \geq e'_{D1}$, and

$$\begin{aligned}
&\pi_B^{D\dagger} > \pi_B^{O\dagger} \Rightarrow w_2^{O\dagger *} > p_B - \frac{\alpha(w_2^{O\dagger *} - w_2^{D\dagger *})(1-\frac{p_B}{1-\gamma}) + (p_B - k_2 - \Delta)(1-p_B)}{(1-\frac{p_B}{1-\gamma})}, && \Rightarrow e < f'_{DO3}, \text{ [case 3, case 4]} \\
&\pi_B^{D\dagger} > \pi_B^{OC} \Rightarrow w_2^{D\dagger *} < p_B - \frac{\left(p_B - w_2^{OC*}\right)(1-\frac{p_B}{1-\gamma}) + \alpha\left(p_B - w_2^{OC*}\right)(1-\frac{p_B-p_2}{1-\gamma-\beta}) - (p_B - k_2 - \Delta)(1-p_B)}{\alpha(1-\frac{p_B}{1-\gamma})}, && \Rightarrow e > f'_{DO2}, \text{ [case 1, case 3]} \\
&\pi_B^{D\dagger} > \pi_B^{H} \Rightarrow w_2^{D\dagger *} < p_B - \frac{(p_B - k_2 - \Delta)(1-\frac{p_B-p_2}{1-\beta})}{1-\frac{p_B}{1-\gamma}}, && \Rightarrow e > f'_{DH},
\end{aligned}$$

(2) the conditions for $\pi_B^* = \pi_B^{DC}$ are $e < e'_{D1}$, and

$$\begin{aligned}
&\pi_B^{DC} > \pi_B^{OC} \Rightarrow \Delta < p_B - k_2 - \frac{\left(p_B - w_2^{OC*}\right)(1-\frac{p_B}{1-\gamma}) - \alpha\left(w_2^{OC*} - w_2^{DC*}\right)(1-\frac{p_B-p_2}{1-\gamma-\beta})}{1-p_B}, && \Rightarrow \Delta < \Delta'_{DO}, \text{ [case 1]} \\
&\pi_B^{DC} > \pi_B^{H} \Rightarrow \Delta > p_B - k_2 - \frac{\left(p_B - w_2^{DC*}\right)(1-\frac{p_B-p_2}{1-\gamma-\beta})}{1-\frac{p_B-p_2}{1-\beta}}, && \Rightarrow \Delta > \Delta'_{DH}, \text{ [case 1, case 2]}
\end{aligned}$$

(3) the conditions for $\pi_B^* = \pi_B^{O\dagger}$ are $e \geq \max\{e_{O1}, \hat{e}_2, \hat{e}_3\}$, and

$$\begin{aligned}
&\pi_B^{O\dagger} > \pi_B^{D\dagger} \Rightarrow w_2^{O\dagger *} < p_B - \frac{\alpha(w_2^{O\dagger *} - w_2^{D\dagger *})(1-\frac{p_B}{1-\gamma}) + (p_B - k_2 - \Delta)(1-p_B)}{(1-\frac{p_B}{1-\gamma})}, && \Rightarrow e > f'_{DO3}, \text{ [case 3, case 4]} \\
&\pi_B^{O\dagger} > \pi_B^{H} \Rightarrow w_2^{O\dagger *} < p_B - \frac{(p_B - k_2 - \Delta)((1-p_B) + \alpha(1-\frac{p_B-p_2}{1-\beta}))}{(1+\alpha)(1-\frac{p_B}{1-\gamma})}, && \Rightarrow e > f_{HO}, \text{ [case 3, case 4]}
\end{aligned}$$

(4) the conditions for $\pi_B^* = \pi_B^{OC}$ are $\Delta > \Delta_0$, $e < \max\{e_{O1}, \hat{e}_2\}$, and

$$\begin{aligned}
&\pi_B^{OC} > \pi_B^{D\dagger} \Rightarrow w_2^{D\dagger *} > p_B - \frac{\left(p_B - w_2^{OC*}\right)(1-\frac{p_B}{1-\gamma}) + \alpha\left(p_B - w_2^{OC*}\right)(1-\frac{p_B-p_2}{1-\gamma-\beta}) - (p_B - k_2 - \Delta)(1-p_B)}{\alpha(1-\frac{p_B}{1-\gamma})}, && \Rightarrow e < f'_{DO2}, \text{ [case 1, case 3]} \\
&\pi_B^{OC} > \pi_B^{DC} \Rightarrow \Delta > p_B - k_2 - \frac{\left(p_B - w_2^{OC*}\right)(1-\frac{p_B}{1-\gamma}) - \alpha\left(w_2^{OC*} - w_2^{DC*}\right)(1-\frac{p_B-p_2}{1-\gamma-\beta})}{1-p_B}, && \Rightarrow \Delta > \Delta'_{DO}, \text{ [case 1]} \\
&\pi_B^{OC} > \pi_B^{H} \Rightarrow \Delta > p_B - k_2 - \frac{\left(p_B - w_2^{OC*}\right)(1-\frac{p_B}{1-\gamma}) + \alpha\left(p_B - w_2^{OC*}\right)(1-\frac{p_B-p_2}{1-\gamma-\beta})}{(1-p_B) + \alpha(1-\frac{p_B-p_2}{1-\beta})}, && \Rightarrow \Delta > \Delta_{HO}, \text{ [case 1, case 3]}
\end{aligned}$$

(5) the conditions for $\pi_B^* = \pi_B^H$ are

$$\begin{aligned}
&\pi_B^H > \pi_B^{O\dagger} \Rightarrow w_2^{O\dagger *} > p_B - \frac{(p_B-k_2-\Delta)((1-p_B)+\alpha(1-\frac{p_B-p_2}{1-\beta}))}{(1+\alpha)(1-\frac{p_B}{1-\gamma})}, && \Rightarrow e < f_{HO}, \text{ [case 3, case 4]}\\
&\pi_B^H > \pi_B^{OC} \Rightarrow \Delta < p_B - k_2 - \frac{\left(p_B-w_2^{OC*}\right)(1-\frac{p_B}{1-\gamma})+\alpha\left(p_B-w_2^{OC*}\right)(1-\frac{p_B-p_2}{1-\gamma-\beta})}{(1-p_B)+\alpha(1-\frac{p_B-p_2}{1-\beta})}, && \Rightarrow \Delta < \Delta_{HO}, \text{ [case 1, case 3]}\\
&\pi_B^H > \pi_B^{DC} \Rightarrow \Delta < p_B - k_2 - \frac{\left(p_B-w_2^{DC*}\right)(1-\frac{p_B-p_2}{1-\gamma-\beta})}{1-\frac{p_B-p_2}{1-\beta}}, && \Rightarrow \Delta < \Delta'_{DH}, \text{ [case 1, case 2]}\\
&\pi_B^H > \pi_B^{D\dagger} \Rightarrow w_2^{D\dagger *} > p_B - \frac{(p_B-k_2-\Delta)(1-\frac{p_B-p_2}{1-\beta})}{1-\frac{p_B}{1-\gamma}}, && \Rightarrow e < f'_{DH}.
\end{aligned}$$

Note that $f'_{DH} < f_{HO}$, $\max\{e_{O1}, \hat{e}_2, \hat{e}_3\} > f'_{DO3}$, $\max\{e_{O1}, \hat{e}_2, \hat{e}_3\} > f_{HO}$, and $\Delta_{HO} < \Delta_0$. Thus, we summarize the thresholds for comparisons, and are derived as follows:

$$\begin{aligned}
&e_{O1} < \hat{e}_2, && \Rightarrow \Delta < \Delta'_0,\\
&\Delta < p_B - k_2 - \frac{\left(p_B-w_2^{DC*}\right)(1-\frac{p_B-p_2}{1-\gamma-\beta})}{1-\frac{p_B-p_2}{1-\beta}}, && \Rightarrow \Delta < \Delta'_{DH},\\
&w_2^{D\dagger *} > p_B - \frac{(p_B-k_2-\Delta)(1-\frac{p_B-p_2}{1-\beta})}{1-\frac{p_B}{1-\gamma}}, && \Rightarrow e < f'_{DH},\\
&w_2^{D\dagger *} < p_B - \frac{\left(p_B-w_2^{OC*}\right)(1-\frac{p_B}{1-\gamma})+\alpha\left(p_B-w_2^{OC*}\right)(1-\frac{p_B-p_2}{1-\gamma-\beta})-(p_B-k_2-\Delta)(1-p_B)}{\alpha(1-\frac{p_B}{1-\gamma})}, && \Rightarrow e > f'_{DO2},\\
&\Delta < p_B - k_2 - \frac{\left(p_B-w_2^{OC*}\right)(1-\frac{p_B}{1-\gamma})-\alpha\left(w_2^{OC*}-w_2^{DC*}\right)(1-\frac{p_B-p_2}{1-\gamma-\beta})}{1-p_B}, && \Rightarrow \Delta < \Delta'_{DO},
\end{aligned} \tag{14}$$

where $w_2^{D\dagger *} = w_2^{O\dagger *} = \max\{w_2^{O(2)}, w_2^{(0)}\}$, $w_2^{DC*} = w_2^{OC*} = w_2^{O(1)}$.

The equilibrium sourcing strategy of the brand-name firm is as follows:

(a) Strategy H with $w_1^* = k_1$ if $e < f'_{DH}$ and $\Delta < \min\{\Delta'_{DH}, \Delta_0\}$;

(b) Strategy D with $w_1^* = k_1$, and

$$w_2^* = \begin{cases} w_2^{O(1)}, & \text{if } e < e'_{D1} \text{ and } \Delta'_{DH} \le \Delta \le \Delta_0;\\ \max\{w_2^{O(2)}, w_2^{(0)}\}, & \text{if } e \ge \max\{e'_{D1}, f'_{DH}\} \text{ and } \Delta \le \Delta_0,\\ & \text{or, if } f'_{DO2} \le e \le \max\{\hat{e}_2, \hat{e}_3\} \text{ and } \Delta > \Delta_0; \end{cases}$$

(c) Strategy O with

$$w_2^* = \begin{cases} w_2^{O(1)}, & \text{if } e < \min\{\hat{e}_2, f'_{DO2}\} \text{ and } \Delta_0 < \Delta \le \Delta'_0;\\ & \text{or, if } e < e_{O1} \text{ and } \Delta > \Delta'_0;\\ \max\{w_2^{O(2)}, w_2^{(0)}\}, & \text{if } e \ge \max\{e_{O1}, \hat{e}_2, \hat{e}_3\}. \end{cases}$$

Thus, by combining the conditions for each strategy, we obtain the results. ■

B.9 Proofs of Extension 2: Endogenous Counterfeit Price

B.9.1 Proof of Lemma 4.

Under each possible sourcing strategy, we obtain the profit expressions for each firm, and discuss the overseas supplier's counterfeiting decision, s^*, and the corresponding retail price of the counterfeit product if $s^* = 1$. Note that we focus on the case in which both the brand-name firm and the counterfeiter have positive market shares in the overseas market if the counterfeiter sells counterfeits.

Strategy H: Given wholesale prices w_1 and w_2, the home supplier accepts the contract and the counterfeiter rejects the contract, i.e., $d_1 = 1$ and $d_2 = 0$. Thus, the brand-name firm only sources from the home supplier.

(1) If the counterfeiter does not sell the counterfeit, i.e., $s=0$, the brand-name firm is the monopoly in the overseas market. Thus, their profit expressions are as follows:

$$\pi_B^H(w_1)=(p_B-w_1)(1-p_B)+\alpha(p_B-w_1-t)(1-p_B), \quad \pi_1^H(w_1)=(1+\alpha)(w_1-k_1)(1-p_B), \quad \pi_2^H=0.$$

(2) If the counterfeiter sells the counterfeit in the overseas market, i.e., $s=1$, the expected profits of the brand-name firm, the home and overseas suppliers are given below:

$$\begin{aligned}
&\pi_B^H(w_1)=(p_B-w_1)(1-p_B)+\alpha(p_B-w_1-t)\left(1-\tfrac{p_B-p_2}{1-\beta}\right),\\
&\pi_1^H(w_1)=(w_1-k_1)\left((1-p_B)+\alpha\left(1-\tfrac{p_B-p_2}{1-\beta}\right)\right),\\
&\pi_2^H(p_2)=\alpha(p_2-k_2)\left(\tfrac{p_B-p_2}{1-\beta}-\tfrac{p_2}{\beta}\right)-e.
\end{aligned}$$

If both the brand-name firm and the overseas supplier get positive overseas market share, i.e., $m_{B2}=\alpha\left(1-\frac{p_B-p_2}{1-\beta}\right)>0$, and $m_2=\alpha\left(\frac{p_B-p_2}{1-\beta}-\frac{p_2}{\beta}\right)>0$, then, $p_B-(1-\beta)<p_2<\beta p_B$.

In order to discuss the most interesting cases, we focus on $\frac{k_2}{p_B}<\beta<\frac{k_2+2(1-p_B)}{2-p_B}$, in which both the brand-name firm and the counterfeiter obtain positive market shares in the overseas market. The profit of the counterfeiter is

$$\pi_2^H(p_2)=\alpha(p_2-k_2)\left(\tfrac{p_B-p_2}{1-\beta}-\tfrac{p_2}{\beta}\right)-e.$$

By taking the first order derivative of $\pi_2^H(p_2)$ with respect to p_2, the optimal retail price of the counterfeit is $p_2^H=\frac{\beta p_B+k_2}{2}$. Substituting the expression of p_2^H into the profit functions, we obtain

$$\pi_B^H(w_1)=(p_B-w_1)(1-p_B)+\alpha(p_B-w_1-t)\left(1-\frac{(2-\beta)p_B-k_2}{2(1-\beta)}\right),$$

$$\pi_1^H(w_1)=(w_1-k_1)\left((1-p_B)+\alpha\left(1-\frac{(2-\beta)p_B-k_2}{2(1-\beta)}\right)\right), \quad \pi_2^H=\frac{\alpha(\beta p_B-k_2)^2}{4\beta(1-\beta)}-e.$$

Recall that $e<\frac{\alpha(\beta p_B-k_2)^2}{4\beta(1-\beta)}$, resulting in $\pi_2^H(s=1)>\pi_2^H(s=0)$. It means that the counterfeiter always sells counterfeit products.

Strategy D: Given wholesale prices w_1 and w_2, both the home supplier and the counterfeiter accept their contracts, i.e., $d_1=1$ and $d_2=1$.

(1) If the overseas supplier does not sell the counterfeit in the market, i.e., $s=0$, the expected profits of the brand-name firm, the home and overseas suppliers are given below:

$$\begin{aligned}
&\pi_B^D(w_1,w_2)=(p_B-w_1)(1-p_B)+\alpha(p_B-w_2)\left(1-\tfrac{p_B}{1-\gamma}\right),\\
&\pi_1^D(w_1)=(w_1-k_1)(1-p_B),\\
&\pi_2^D(w_2)=\alpha(w_2-k_2)\left(1-\tfrac{p_B}{1-\gamma}\right).
\end{aligned}$$

(2) If the overseas supplier sells the counterfeit in the market, i.e., $s=1$, then, for given p_B for the brand-name product, the overseas supplier decides on the retail price p_2 for the counterfeit. Their profits are as follows:

$$\begin{aligned}
&\pi_B^D(w_1,w_2,p_2)=(p_B-w_1)(1-p_B)+\alpha(p_B-w_2)\left(1-\tfrac{p_B-p_2}{1-\gamma-\beta}\right),\\
&\pi_1^D(w_1)=(w_1-k_1)(1-p_B),\\
&\pi_2^D(w_2,p_2)=\alpha(w_2-k_2)\left(1-\tfrac{p_B-p_2}{1-\gamma-\beta}\right)+\alpha(p_2-k_2)\left(\tfrac{p_B-p_2}{1-\gamma-\beta}-\tfrac{p_2}{\beta}\right)-e.
\end{aligned}$$

If both the brand-name firm and the overseas supplier get positive overseas market share, i.e., $m_{B2} = \alpha\left(1-\frac{p_B-p_2}{1-\gamma-\beta}\right) > 0$, and $m_2 = \alpha\left(\frac{p_B-p_2}{1-\gamma-\beta}-\frac{p_2}{\beta}\right) > 0$, then, $p_B-(1-\gamma-\beta) < p_2 < \frac{\beta p_B}{1-\gamma}$. The profit of the overseas supplier is

$$\pi_2^D\left(w_2,p_2\right)=\alpha\left(w_2-k_2\right)\left(1-\frac{p_B-p_2}{1-\gamma-\beta}\right)+\alpha\left(p_2-k_2\right)\left(\frac{p_B-p_2}{1-\gamma-\beta}-\frac{p_2}{\beta}\right)-e.$$

By taking the first-order derivative of $\pi_2^D\left(w_2,p_2\right)$ with respect to p_2, we have,

$$\frac{\partial(\pi_2^D(w_2,p_2))}{\partial(p_2)}=\alpha\left(\frac{p_B+k_2-2p_2+(w_2-k_2)}{1-\gamma-\beta}-\frac{2p_2-k_2}{\beta}\right)=\alpha\left(\frac{p_B-2p_2+w_2}{1-\gamma-\beta}-\frac{2p_2-k_2}{\beta}\right).$$

From $\frac{\partial\left(\pi_2^D(w_2,p_2)\right)}{\partial(p_2)}=0$, we obtain the critical point $\hat{p}_2=\frac{\beta p_B+(1-\gamma)k_2+\beta(w_2-k_2)}{2(1-\gamma)}$. Next, we check whether $\hat{p}_2$ is in the feasible region $p_B-(1-\gamma-\beta)<p_2<\frac{\beta p_B}{1-\gamma}$. From $p_B-(1-\gamma-\beta)<\hat{p}_2<\frac{\beta p_B}{1-\gamma}$, we obtain,

$$\underline{w}_2 < w_2 < k_2+\frac{\beta p_B-(1-\gamma)k_2}{\beta},$$

where $\underline{w}_2=k_2+\frac{2(1-\gamma)[p_B-(1-\gamma-\beta)]-(\beta p_B+(1-\gamma)k_2)}{\beta}$. Note that if $w_2\le \underline{w}_2$, there is no market share for the counterfeiter in the overseas market.

We focus on the case when the brand-name firm has a positive market share in the overseas market, i.e., $m_{B2}>0$. Thus, with Strategy D, if the overseas supplier sells the counterfeit, i.e., $s=1$, the optimal retail price p_2 for the counterfeit is

$$p_2^D=\begin{cases}\frac{\beta p_B}{1-\gamma}, & \text{if } w_2\ge k_2+\frac{\beta p_B-(1-\gamma)k_2}{\beta}, \text{[note that } m_2=0]\\ \hat{p}_2, & \text{if } \underline{w}_2<w_2<k_2+\frac{\beta p_B-(1-\gamma)k_2}{\beta}, \text{[note that } m_2>0]\end{cases} \tag{15}$$

and the overseas supplier's profit is

$$\pi_2^D(w_2,s=1)=\begin{cases}\pi_2^{DC1}=\alpha\left(w_2-k_2\right)\left(1-\frac{p_B-p_2^D}{1-\gamma-\beta}\right)-e, & \text{if } w_2\ge k_2+\frac{\beta p_B-(1-\gamma)k_2}{\beta},\\ \hat{\pi}_2^{DC}=\alpha\left(w_2-k_2\right)\left(1-\frac{p_B-p_2^D}{1-\gamma-\beta}\right) & \\ +\alpha\left(p_2^D-k_2\right)\left(\frac{p_B-p_2^D}{1-\gamma-\beta}-\frac{p_2^D}{\beta}\right)-e, & \text{if } \underline{w}_2<w_2<k_2+\frac{\beta p_B-(1-\gamma)k_2}{\beta},\end{cases}$$

and the brand-name firm's profit is

$$\pi_B^D(w_1,w_2,s=1)=\begin{cases}\pi_B^{DC1}=\left(p_B-w_1\right)\left(1-p_B\right)+\alpha\left(p_B-w_2\right)\left(1-\frac{p_B-p_2^D}{1-\gamma-\beta}\right), & \text{if } w_2\ge k_2+\frac{\beta p_B-(1-\gamma)k_2}{\beta},\\ \hat{\pi}_B^{DC}=\left(p_B-w_1\right)\left(1-p_B\right) & \\ +\alpha\left(p_B-w_2\right)\left(\frac{2(1-\gamma-\beta)(1-\gamma-p_B)-\beta p_B+(1-\gamma)k_2+\beta(w_2-k_2)}{2(1-\gamma)(1-\gamma-\beta)}\right), & \text{if } \underline{w}_2<w_2<k_2+\frac{\beta p_B-(1-\gamma)k_2}{\beta}.\end{cases}$$

Next, the overseas supplier determines whether to sell the counterfeit, $s^*(w_2)$. For the overseas supplier, if $\pi_2^D(w_2,s=1)>\pi_2^D(w_2,s=0)$, she decides to sell the counterfeit; otherwise, she does not sell the counterfeit. Recall that when $s=0$, the overseas supplier's profit is

$$\pi_2^D\left(w_2,s=0\right)=\alpha\left(w_2-k_2\right)\left(1-\frac{p_B}{1-\gamma}\right).$$

Note that, given p_B, for the overseas supplier has the following two scenarios:

(i) If $w_2 \geq k_2 + \frac{\beta p_B-(1-\gamma)k_2}{\beta}$, then the overseas supplier's profit of counterfeiting is $\pi_2^D(w_2, s=1) = \pi_2^{DC1}$, which implies $p_2^D = \frac{\beta p_B}{1-\gamma}$. Then, we know that the optimal decision is $s^* = 0$ because $\pi_2^D(w_2, s=0) > \pi_2^{DC1}$ always holds.

(ii) If $\underline{w}_2 < w_2 < k_2 + \frac{\beta p_B-(1-\gamma)k_2}{\beta}$, then the overseas supplier's profit from counterfeiting is $\pi_2^D(w_2, s = 1) = \hat{\pi}_2^{DC}$, which implies $p_2^D = \hat{p}_2 = \frac{\beta p_B+(1-\gamma)k_2+\beta(w_2-k_2)}{2(1-\gamma)}$. Then, we know that the optimal decision is: $s = 0$ if $\pi_2^D(w_2, s=0) > \hat{\pi}_2^{DC}$, which means:

$$
\begin{aligned}
&\alpha(w_2-k_2)(1-\frac{p_B}{1-\gamma}) > \alpha(w_2-k_2)\left(1-\frac{p_B-\hat{p}_2}{1-\gamma-\beta}\right) + \left(\alpha(\hat{p}_2-k_2)(\frac{p_B-\hat{p}_2}{1-\gamma-\beta}-\frac{\hat{p}_2}{\beta})-e\right),\\
\Rightarrow &\alpha(w_2-k_2)(1-\frac{p_B}{1-\gamma}) > \alpha(w_2-k_2)\left(\frac{2(1-\gamma-\beta)(1-\gamma)-(2(1-\gamma)-\beta)p_B+(1-\gamma)k_2+\beta(w_2-k_2)}{2(1-\gamma)(1-\gamma-\beta)}\right)\\
&+\left(\alpha\left(\frac{\beta p_B-(1-\gamma)k_2+\beta(w_2-k_2)}{2(1-\gamma)}\right)\frac{\beta p_B-(1-\gamma)k_2-\beta(w_2-k_2)}{2\beta(1-\gamma-\beta)}-e\right),\\
\Rightarrow &w_2^{(0)\prime} < w_2 < w_2^{(0)\prime\prime},
\end{aligned}
$$

where $w_2^{(0)\prime} = k_2 + \frac{\beta p_B-(1-\gamma)k_2}{\beta} - \sqrt{\frac{4(1-\gamma)(1-\gamma-\beta)e}{\alpha\beta}}$, $w_2^{(0)\prime\prime} = k_2 + \frac{\beta p_B-(1-\gamma)k_2}{\beta} + \sqrt{\frac{4(1-\gamma)(1-\gamma-\beta)e}{\alpha\beta}}$.

Note that $w_2^{(0)\prime} < k_2 + \frac{\beta p_B-(1-\gamma)k_2}{\beta} < w_2^{(0)\prime\prime}$. Thus, combining these two scenarios, the overseas supplier's optimal decision of counterfeiting is

$$
s^*(w_2) = \begin{cases} 0, & \text{if } w_2 \geq \max\{w_2^{(0)\prime}, \underline{w}_2\}, \text{[note that } m_2 = 0] \\ 1, & \text{if } \underline{w}_2 < w_2 < \max\{w_2^{(0)\prime}, \underline{w}_2\}. \text{[note that } m_2 > 0] \end{cases}
$$

Subsequently, the brand-name firm's profit is

$$
\pi_B^D(w_2) = \begin{cases} \pi_B^D(w_2, s=0) = (p_B-w_1)(1-p_B) + \alpha(p_B-w_2)(1-\frac{p_B}{1-\gamma}), & \text{if } w_2 \geq \max\{w_2^{(0)\prime}, \underline{w}_2\}, \\ \pi_B^D(w_2, s=1) = (p_B-w_1)(1-p_B) & \\ +\alpha(p_B-w_2)\left(\frac{2(1-\gamma-\beta)(1-\gamma-p_B)-\beta p_B+(1-\gamma)k_2+\beta(w_2-k_2)}{2(1-\gamma)(1-\gamma-\beta)}\right), & \text{if } \underline{w}_2 < w_2 < w_2^{(0)\prime}, \underline{w}_2\}. \end{cases}
$$

Strategy O: Given wholesale prices w_1 and w_2, the home supplier rejects and the counterfeiter accepts their respective contracts, i.e., $d_1 = 0$ and $d_2 = 1$.

(1) If the overseas supplier does not sell the counterfeit in the market, i.e., $s = 0$, we know:

$$
\begin{aligned}
&\pi_B^O(w_2, s=0) = (p_B-w_2-t)(1-\frac{p_B}{1-\gamma}) + \alpha(p_B-w_2)(1-\frac{p_B}{1-\gamma}),\\
&\pi_1^O = 0, \quad \pi_2^O(w_2, s=0) = (w_2-k_2)(1-\frac{p_B}{1-\gamma}) + \alpha(w_2-k_2)(1-\frac{p_B}{1-\gamma}).
\end{aligned}
$$

(2) If the overseas supplier sells the counterfeit in the market, i.e., $s = 1$, then the overseas supplier determines the selling price p_2 for the counterfeit. Their profits are as follows.

$$
\begin{aligned}
&\pi_B^O(w_2, p_2, s=1) = (p_B-w_2-t)\left(1-\frac{p_B}{1-\gamma}\right) + \alpha(p_B-w_2)\left(1-\frac{p_B-p_2}{1-\gamma-\beta}\right),\\
&\pi_1^O = 0, \quad \pi_2^O(w_2, p_2, s=1) = (w_2-k_2)\left(1-\frac{p_B}{1-\gamma}\right) + \alpha(w_2-k_2)\left(1-\frac{p_B-p_2}{1-\gamma-\beta}\right) + \alpha(p_2-k_2)\left(\frac{p_B-p_2}{1-\gamma-\beta}-\frac{p_2}{\beta}\right) - e.
\end{aligned}
$$

Similar with the discussion in Strategy D, we derive the optimal retail price p_2 for the overseas supplier under Strategy O by backward deduction. Thus, with Strategy O, if the overseas supplier sells the counterfeit, i.e., $s = 1$, we have $\hat{p}_2 = \frac{\beta p_B+(1-\gamma)k_2+\beta(w_2-k_2)}{2(1-\gamma)}$, and the optimal retail price is

$$
p_2^O = \begin{cases} \frac{\beta p_B}{1-\gamma}, & \text{if } w_2 \geq k_2 + \frac{\beta p_B-(1-\gamma)k_2}{\beta}, \text{[note that } m_2 = 0] \\ \hat{p}_2, & \text{if } \underline{w}_2 < w_2 < k_2 + \frac{\beta p_B-(1-\gamma)k_2}{\beta}, \text{[note that } m_2 > 0] \end{cases}
$$

and the overseas supplier's profit is

$$\pi_2^O(w_2)=\begin{cases}\pi_2^{OC1}=(w_2-k_2)\left(1-\frac{p_B}{1-\gamma}\right)+\alpha\,(w_2-k_2)\left(1-\frac{p_B-p_2^O}{1-\gamma-\beta}\right)-e, & \text{if } w_2\geq k_2+\frac{\beta p_B-(1-\gamma)k_2}{\beta},\\ \hat{\pi}_2^{OC}=(w_2-k_2)\left(1-\frac{p_B}{1-\gamma}\right) & \\ +\alpha\,(w_2-k_2)\left(1-\frac{p_B-p_2^O}{1-\gamma-\beta}\right)+\left(\alpha\left(p_2^O-k_2\right)\left(\frac{p_B-p_2^O}{1-\gamma-\beta}-\frac{p_2^O}{\beta}\right)-e\right), & \text{if } \underline{w}_2<w_2<k_2+\frac{\beta p_B-(1-\gamma)k_2}{\beta},\end{cases}$$

and the brand-name firm's profit is

$$\pi_B^O(w_2)=\begin{cases}\pi_B^{OC1}=(p_B-w_2-t)\,(1-p_B)+\alpha\,(p_B-w_2)\left(1-\frac{p_B-p_2^O}{1-\gamma-\beta}\right), & \text{if } w_2\geq k_2+\frac{\beta p_B-(1-\gamma)k_2}{\beta},\\ \hat{\pi}_B^{OC}=(p_B-w_2-t)\,(1-p_B) & \\ +\alpha\,(p_B-w_2)\left(\frac{2(1-\gamma-\beta)(1-\gamma-p_B)-\beta p_B+(1-\gamma)k_2+\beta(w_2-k_2)}{2(1-\gamma)(1-\gamma-\beta)}\right), & \text{if } \underline{w}_2<w_2<k_2+\frac{\beta p_B-(1-\gamma)k_2}{\beta}.\end{cases}$$

Next, the overseas supplier determines whether to sell the counterfeit, $s^*(w_2)$. For the overseas supplier, if $\pi_2^O(w_2,s=1)>\pi_2^O(w_2,s=0)$, she decides to sell the counterfeit; otherwise, she does not sell the counterfeit. Recall that when $s=0$, the overseas supplier's profit is

$$\pi_2^O\,(w_2,s=0)=(w_2-k_2)\,(1-\frac{p_B}{1-\gamma})+\alpha\,(w_2-k_2)\,(1-\frac{p_B}{1-\gamma}).$$

Similarly, we obtain:

$$s^*(w_2)=\begin{cases}0, & \text{if } w_2\geq \max\{w_2^{(0)\prime},\underline{w}_2\}, \text{ [note that } m_2=0]\\ 1, & \text{if } \underline{w}_2<w_2<\max\{w_2^{(0)\prime},\underline{w}_2\}. \text{ [note that } m_2>0]\end{cases}$$

Subsequently, the brand-name firm's profit is

$$\pi_B^O(w_2)=\begin{cases}\pi_B^O\,(w_2,s=0)=(p_B-w_2-t)\,(1-\frac{p_B}{1-\gamma})+\alpha\,(p_B-w_2)\,(1-\frac{p_B}{1-\gamma}), & \text{if } w_2\geq\max\{w_2^{(0)\prime},\underline{w}_2\},\\ \pi_B^{OC}\,(w_2,s=1)=\hat{\pi}_B^{OC}=(p_B-w_2-t)\left(1-\frac{p_B}{1-\gamma}\right) & \\ +\alpha\,(p_B-w_2)\left(\frac{2(1-\gamma-\beta)(1-\gamma-p_B)-\beta p_B+(1-\gamma)k_2+\beta(w_2-k_2)}{2(1-\gamma)(1-\gamma-\beta)}\right), & \text{if } \underline{w}_2<w_2<\max\{w_2^{(0)\prime},\underline{w}_2\}.\end{cases}$$

Strategy N: Given wholesale prices w_1 and w_2, both the home supplier rejects and the counterfeiter reject their contracts, i.e., $d_1=0$ and $d_2=0$.

(1) If the counterfeiter does not enter the overseas market to sell the counterfeit, *i.e.*, $s=0$, then their profits are:

$$\pi_B^N\,(w_1,w_2)=0,\quad \pi_1^N(w_1)=0,\quad \pi_2^N=0.$$

(2) If the counterfeiter enters the overseas market to sell the counterfeit, *i.e.*, $s=1$, she is the monopoly in the overseas market and determines retail price p_2^N of the counterfeit and obtains the below profit:

$$\pi_2^N\,(p_2)=\alpha\,(p_2-k_2)\left(1-\frac{p_2}{\beta}\right)-e.$$

By taking the first-order derivative of $\pi_2^N(p_2)$ with respect to p_2, the optimal retail price of the counterfeit is $p_2^N=\frac{\beta+k_2}{2}$. Substituting the expression of p_2^N into Equation (3), we obtain $m_2=\alpha\left(1-\frac{\beta+k_2}{2\beta}\right)$. Thus, their profits are:

$$\pi_B^N\,(w_1,w_2)=0,\quad \pi_1^N=0,\quad \pi_2^N=\frac{\alpha(\beta-k_2)^2}{4\beta}-e.$$

Recall that $e < \frac{\alpha(\beta p_B - k_2)^2}{4\beta(1-\beta)}$, resulting in $\pi_2^N(s=1) > \pi_2^N(s=0)$. It means that the counterfeiter always sells the counterfeit products.

Based on above discussions, for given (w_1, w_2), under either Strategy D or Strategy O,

$$s^*(w_2) = \begin{cases} 0, & \text{if } w_2 \geq \max\{w_2^{(0)\prime}, \underline{w}_2\}, \\ 1, & \text{if } \underline{w}_2 < w_2 < \max\{w_2^{(0)\prime}, \underline{w}_2\}; \end{cases}$$

where $w_2^{(0)\prime} = k_2 + \frac{\beta p_B - (1-\gamma)k_2}{\beta} - \sqrt{\frac{4(1-\gamma)(1-\gamma-\beta)e}{\alpha\beta}}$, $\underline{w}_2 = k_2 - \frac{2(1-\gamma-p_B)(1-\gamma-\beta) - \beta p_B + (1-\gamma)k_2}{\beta}$. In particular, when $s^*(w_2) = 1$, the optimal retail price of the counterfeit product is $p_2^*(w_2) = \frac{\beta p_B + (1-\gamma)k_2 + \beta(w_2 - k_2)}{2(1-\gamma)}$. ■

B.9.2 Proof of Lemma 5.

There are two parts in this proof. In part 1, we analyze the suppliers' optimal participation decision by discussing the best response functions $(d_1^*(w_1, w_2), d_2^*(w_1, w_2))$. In part 2, we determine the optimal wholesale prices that the brand-name firm offers.

Part 1. We discuss the suppliers' best response functions $(d_1^*(w_1, w_2), d_2^*(w_1, w_2))$.

With each sourcing strategy, the overseas supplier's profit function is as follows:

$$\pi_2^H = \frac{\alpha(\beta p_B - k_2)^2}{4\beta(1-\beta)} - e = M.$$

$$\pi_2^D = \begin{cases} \pi_2^{DC}(w_2) = \alpha(w_2 - k_2)\left(\frac{2(1-\gamma-\beta)(1-\gamma-p_B) - \beta p_B + (1-\gamma)k_2 + \beta(w_2-k_2)}{2(1-\gamma)(1-\gamma-\beta)}\right) & \\ +\alpha\left(\frac{\beta p_B - (1-\gamma)k_2 + \beta(w_2-k_2)}{2(1-\gamma)}\right)\frac{\beta p_B - (1-\gamma)k_2 - \beta(w_2-k_2)}{2\beta(1-\gamma-\beta)} - e, & \text{if } \underline{w}_2 < w_2 < \max\{w_2^{(0)\prime}, \underline{w}_2\}, \\ \pi_2^{D\dagger}(w_2) = \alpha(w_2 - k_2)(1 - \frac{p_B}{1-\gamma}), & \text{if } w_2 \geq \max\{w_2^{(0)\prime}, \underline{w}_2\}; \end{cases}$$

$$\pi_2^O = \begin{cases} \pi_2^{OC}(w_2) = (w_2 - k_2)\left(1 - \frac{p_B}{1-\gamma}\right) & \\ +\alpha(w_2 - k_2)\left(\frac{2(1-\gamma-\beta)(1-\gamma-p_B) - \beta p_B + (1-\gamma)k_2 + \beta(w_2-k_2)}{2(1-\gamma)(1-\gamma-\beta)}\right) & \\ +\alpha\left(\frac{\beta p_B - (1-\gamma)k_2 + \beta(w_2-k_2)}{2(1-\gamma)}\right)\frac{\beta p_B - (1-\gamma)k_2 - \beta(w_2-k_2)}{2\beta(1-\gamma-\beta)} - e, & \text{if } \underline{w}_2 < w_2 < \max\{w_2^{(0)\prime}, \underline{w}_2\}, \\ \pi_2^{O\dagger}(w_2) = (p_B - w_2 - t)(1 - \frac{p_B}{1-\gamma}) + \alpha(p_B - w_2)(1 - \frac{p_B}{1-\gamma}), & \text{if } w_2 \geq \max\{w_2^{(0)\prime}, \underline{w}_2\}; \end{cases}$$

$$\pi_2^N = \frac{\alpha(\beta - k_2)^2}{4\beta} - e = K.$$

Recall that $M = \frac{\alpha(\beta p_B - k_2)^2}{4\beta(1-\beta)} - e$, $M' = \frac{\alpha(\beta p_B - (1-\gamma)k_2)^2}{4(1-\gamma)\beta(1-\gamma-\beta)} - e$ and $K = \frac{\alpha(\beta - k_2)^2}{4\beta} - e$. With the assumption $0 \leq e < \frac{\alpha(\beta p_B - k_2)^2}{4\beta(1-\beta)}$, we know that $0 < M < M' < K$.

Step 1: We first discuss the conditions for the overseas supplier's decision to accept the wholesale contract.

(1) Under $\underline{w}_2 < w_2 < \max\{w_2^{(0)\prime}, \underline{w}_2\}$, where $\underline{w}_2 = k_2 - \frac{2(1-\gamma-p_B)(1-\gamma-\beta) - \beta p_B + (1-\gamma)k_2}{\beta}$, $w_2^{(0)\prime} = k_2 + \frac{\beta p_B - (1-\gamma)k_2}{\beta} - \sqrt{\frac{4(1-\gamma)(1-\gamma-\beta)e}{\alpha\beta}}$, we discuss the decision d_2 for a given belief on the home supplier's contact decision $\widetilde{d_1} = 1$ and $\widetilde{d_1} = 0$, respectively.

(i) If $\widetilde{d}_1 = 1$, then we compare the overseas supplier's profits between Strategy D with counterfeiting and Strategy H, i.e., $\pi_2^{DC}(w_2)$ and π_2^H. If the overseas supplier decides to accept, then it should satisfy

$$\begin{aligned}
&\pi_2^{DC}(w_2) \geq \pi_2^H, \\
\Rightarrow\ &\alpha(w_2-k_2)\left(\tfrac{2(1-\gamma-\beta)(1-\gamma-p_B)-\beta p_B+(1-\gamma)k_2+\beta(w_2-k_2)}{2(1-\gamma)(1-\gamma-\beta)}\right)+\alpha\left(\tfrac{\beta p_B-(1-\gamma)k_2+\beta(w_2-k_2)}{2(1-\gamma)}\right)\tfrac{\beta p_B-(1-\gamma)k_2-\beta(w_2-k_2)}{2\beta(1-\gamma-\beta)}-e \geq M, \\
\Rightarrow\ &w_2 \leq k_2-\tfrac{2(1-\gamma-p_B)(1-\gamma-\beta)-\beta p_B+(1-\gamma)k_2}{\beta}-\sqrt{\tfrac{4(1-\gamma)(1-\gamma-\beta)(M-M')}{\alpha\beta}+\left(\tfrac{2(1-\gamma-p_B)(1-\gamma-\beta)-\beta p_B+(1-\gamma)k_2}{\beta}\right)^2}, \text{(invalid)} \\
\Rightarrow\ &\text{or, } w_2 \geq k_2-\tfrac{2(1-\gamma-p_B)(1-\gamma-\beta)-\beta p_B+(1-\gamma)k_2}{\beta}+\sqrt{\tfrac{4(1-\gamma)(1-\gamma-\beta)(M-M')}{\alpha\beta}+\left(\tfrac{2(1-\gamma-p_B)(1-\gamma-\beta)-\beta p_B+(1-\gamma)k_2}{\beta}\right)^2}.
\end{aligned}$$

(ii) If $\widetilde{d}_1 = 0$, then we compare the overseas supplier's profits between Strategy O with counterfeiting and Strategy N, i.e., $\pi_2^{OC}(w_2)$ and π_2^N. If the overseas supplier decides to accept, then it should satisfy

$$\begin{aligned}
&\pi_2^{OC}(w_2) \geq \pi_2^N, \\
\Rightarrow\ &(w_2-k_2)\left(1-\tfrac{p_B}{1-\gamma}\right) \\
&+\alpha(w_2-k_2)\left(\tfrac{2(1-\gamma-\beta)(1-\gamma-p_B)-\beta p_B+(1-\gamma)k_2+\beta(w_2-k_2)}{2(1-\gamma)(1-\gamma-\beta)}\right)+\alpha\left(\tfrac{\beta p_B-(1-\gamma)k_2+\beta(w_2-k_2)}{2(1-\gamma)}\right)\tfrac{\beta p_B-(1-\gamma)k_2-\beta(w_2-k_2)}{2\beta(1-\gamma-\beta)}-e \geq K, \\
\Rightarrow\ &w_2 \leq k_2-\tfrac{2(1+\frac{1}{\alpha})(1-\gamma-p_B)(1-\gamma-\beta)-\beta p_B+(1-\gamma)k_2}{\beta}-\sqrt{\tfrac{4(1-\gamma)(1-\gamma-\beta)(K-M')}{\alpha\beta}+\left(\tfrac{2(1+\frac{1}{\alpha})(1-\gamma-p_B)(1-\gamma-\beta)-\beta p_B+(1-\gamma)k_2}{\beta}\right)^2}, \text{(invalid)} \\
\Rightarrow\ &w_2 \geq k_2-\tfrac{2(1+\frac{1}{\alpha})(1-\gamma-p_B)(1-\gamma-\beta)-\beta p_B+(1-\gamma)k_2}{\beta}+\sqrt{\tfrac{4(1-\gamma)(1-\gamma-\beta)(K-M')}{\alpha\beta}+\left(\tfrac{2(1+\frac{1}{\alpha})(1-\gamma-p_B)(1-\gamma-\beta)-\beta p_B+(1-\gamma)k_2}{\beta}\right)^2}.
\end{aligned}$$

We define the following notations:

$$\begin{aligned}
&w_2^{(0)} = \max\{w_2^{(0)\prime}, \underline{w}_2\}; \\
&w_2^{D(1)} = k_2-\tfrac{2(1-\gamma-p_B)(1-\gamma-\beta)-\beta p_B+(1-\gamma)k_2}{\beta}+\sqrt{\tfrac{4(1-\gamma)(1-\gamma-\beta)(M-M')}{\alpha\beta}+\left(\tfrac{2(1-\gamma-p_B)(1-\gamma-\beta)-\beta p_B+(1-\gamma)k_2}{\beta}\right)^2} < k_2, \\
&w_2^{O(1)} = k_2-\tfrac{2(1+\frac{1}{\alpha})(1-\gamma-p_B)(1-\gamma-\beta)-\beta p_B+(1-\gamma)k_2}{\beta}+\sqrt{\tfrac{4(1-\gamma)(1-\gamma-\beta)(K-M')}{\alpha\beta}+\left(\tfrac{2(1+\frac{1}{\alpha})(1-\gamma-p_B)(1-\gamma-\beta)-\beta p_B+(1-\gamma)k_2}{\beta}\right)^2} > k_2; \\
&w_2^{D(2)} = k_2+\tfrac{M}{\alpha(1-\frac{p_B}{1-\gamma})}, \\
&w_2^{O(2)} = k_2+\tfrac{K}{(1+\alpha)(1-\frac{p_B}{1-\gamma})};
\end{aligned}$$

where $w_2^{(0)\prime} = k_2+\frac{\beta p_B-(1-\gamma)k_2}{\beta}-\sqrt{\frac{4(1-\gamma)(1-\gamma-\beta)e}{\alpha\beta}} > k_2$, $\underline{w}_2 = k_2-\frac{2(1-\gamma-p_B)(1-\gamma-\beta)-\beta p_B+(1-\gamma)k_2}{\beta}$.

Thus, under $\underline{w}_2 < w_2 < w_2^{(0)}$, where $w_2^{(0)} = \max\{w_2^{(0)\prime}, \underline{w}_2\}$, we know that $w_2^{(0)} > k_2$, and

$$d_2(\widetilde{d}_1) = \begin{cases} d_2\left(\widetilde{d}_1 = 1\right) = 1, & \text{if } \max\{w_2^{D(1)}, \underline{w}_2, k_2\} \leq w_2 < w_2^{(0)}, \\ d_2\left(\widetilde{d}_1 = 1\right) = 0, & \text{if } \underline{w}_2 < w_2 < \max\{w_2^{D(1)}, \underline{w}_2, k_2\}, \\ d_2\left(\widetilde{d}_1 = 0\right) = 1, & \text{if } \max\{w_2^{O(1)}, \underline{w}_2, k_2\} \leq w_2 < w_2^{(0)}, \\ d_2\left(\widetilde{d}_1 = 0\right) = 0, & \text{if } \underline{w}_2 < w_2 < \max\{w_2^{O(1)}, \underline{w}_2, k_2\}. \end{cases}$$

Note that $w_2^{D(1)} < k_2 < w_2^{O(1)}$, and $w_2^{(0)} = \max\{w_2^{(0)\prime}, \underline{w}_2\}$, where $w_2^{(0)\prime} > k_2$, then we have:

$$d_2(\widetilde{d}_1) = \begin{cases} d_2\left(\widetilde{d}_1 = 1\right) = 1, & \text{if } \max\{k_2, \underline{w}_2\} \leq w_2 < w_2^{(0)}, \\ d_2\left(\widetilde{d}_1 = 1\right) = 0, & \text{if } \underline{w}_2 < w_2 < \max\{k_2, \underline{w}_2\}, \\ d_2\left(\widetilde{d}_1 = 0\right) = 1, & \text{if } \max\{w_2^{O(1)}, \underline{w}_2\} \leq w_2 < w_2^{(0)}, \\ d_2\left(\widetilde{d}_1 = 0\right) = 0, & \text{if } \underline{w}_2 < w_2 < \max\{w_2^{O(1)}, \underline{w}_2\}. \end{cases}$$

(2) Under $w_2 \geq \max\{w_2^{(0)\prime}, \underline{w_2}\}$, we discuss the decision d_2 for given $\widetilde{d_1}=1$ and $\widetilde{d_1}=0$, respectively.

(i) If $\widetilde{d_1}=1$, then we compare the overseas supplier's profits between Strategy D without counterfeiting and Strategy H, i.e., $\pi_2^{D\dagger}(w_2)$ and π_2^H. If the overseas supplier decides to accept the wholesale contract, then it should satisfy

$$\begin{aligned}
&\pi_2^{D\dagger}(w_2) \geq \pi_2^H,\\
\Rightarrow\ &\alpha(w_2-k_2)\left(1-\tfrac{p_B}{1-\gamma}\right) \geq M,\\
\Rightarrow\ &w_2 \geq w_2^{D(2)}\text{, where } w_2^{D(2)} = k_2 + \tfrac{M}{\alpha\left(1-\frac{p_B}{1-\gamma}\right)}.
\end{aligned}$$

(ii) If $\widetilde{d_1}=0$, then we compare the overseas supplier's profits between Strategy O without counterfeiting and Strategy N, i.e., $\pi_2^{O\dagger}(w_2)$ and π_2^N. If the overseas supplier decides to accept the wholesale contract, then it should satisfy

$$\begin{aligned}
&\pi_2^{O\dagger}(w_2) \geq \pi_2^N,\\
\Rightarrow\ &(w_2-k_2)\left(1-\tfrac{p_B}{1-\gamma}\right)+\alpha(w_2-k_2)\left(1-\tfrac{p_B}{1-\gamma}\right) \geq K,\\
\Rightarrow\ &w_2 \geq w_2^{O(2)}\text{, where } w_2^{O(2)} = k_2 + \tfrac{K}{(1+\alpha)\left(1-\frac{p_B}{1-\gamma}\right)}.
\end{aligned}$$

Thus, under $w_2 \geq w_2^{(0)}$, where $w_2^{(0)} = \max\{w_2^{(0)\prime}, \underline{w_2}\} > k_2$, we obtain

$$d_2(\widetilde{d_1}) = \begin{cases}
d_2\left(\widetilde{d_1}=1\right)=1, & \text{if } w_2 \geq \max\{w_2^{D(2)}, w_2^{(0)}\},\\
d_2\left(\widetilde{d_1}=1\right)=0, & \text{if } w_2^{(0)} < w_2 < \max\{w_2^{D(2)}, w_2^{(0)}\},\\
d_2\left(\widetilde{d_1}=0\right)=1, & \text{if } w_2 \geq \max\{w_2^{O(2)}, w_2^{(0)}\},\\
d_2\left(\widetilde{d_1}=0\right)=0, & \text{if } w_2^{(0)} < w_2 < \max\{w_2^{O(2)}, w_2^{(0)}\}.
\end{cases}$$

Step 2: We derive the best response function of the home supplier $d_1(\widetilde{d_2})$ to the overseas supplier's action $\widetilde{d_2} \in \{0,1\}$ as follows:

$$d_1(\widetilde{d_2}) = \begin{cases}
d_1\left(\widetilde{d_2}=1\right)=1, & \text{if } w_1 \geq k_1,\\
d_1\left(\widetilde{d_2}=0\right)=1, & \text{if } w_1 \geq k_1,\\
d_1\left(\widetilde{d_2}=1\right)=0, & \text{if } w_1 < k_1,\\
d_1\left(\widetilde{d_2}=0\right)=0, & \text{if } w_1 < k_1.
\end{cases}$$

Step 3: Given best response functions $d_1(\widetilde{d_2})$ and $d_2(\widetilde{d_1})$, we obtain the following fixed point (d_1^*, d_2^*) that satisfies $(d_1(\widetilde{d_2}), \widetilde{d_2}) = (\widetilde{d_1}, d_2(\widetilde{d_1}))$. Thus, the optimal decisions of the two suppliers are

$$(d_1^*, d_2^*) = \begin{cases}
(1,1), & \text{if } w_1 \geq k_1, \max\{k_2, \underline{w_2}\} \leq w_2 < w_2^{(0)} \text{ or } w_2 \geq \max\{w_2^{D(2)}, w_2^{(0)}\},\\
(1,0), & \text{if } w_1 \geq k_1, \underline{w_2} < w_2 < \max\{k_2, \underline{w_2}\} \text{ or } w_2^{(0)} < w_2 < \max\{w_2^{D(2)}, w_2^{(0)}\},\\
(0,1), & \text{if } w_1 < k_1, \max\{w_2^{O(1)}, \underline{w_2}\} \leq w_2 < w_2^{(0)} \text{ or } w_2 \geq \max\{w_2^{O(2)}, w_2^{(0)}\},\\
(0,0), & \text{if } w_1 < k_1, \underline{w_2} < w_2 < \max\{w_2^{O(1)}, \underline{w_2}\} \text{ or } w_2^{(0)} < w_2 < \max\{w_2^{O(2)}, w_2^{(0)}\}.
\end{cases}$$

Part 2. We discuss the brand-name firm's optimal wholesale prices, (w_1, w_2).

Substituting (d_1^*, d_2^*) into the profit functions of the brand-name firm, we analyze the optimal wholesale price under each possible sourcing strategy.

$$\pi_B^H(w_1) = (p_B - w_1)(1-p_B) + \alpha(p_B - w_1 - t)\left(1 - \tfrac{(2-\beta)p_B - k_2}{2(1-\beta)}\right);$$

$$\pi_B^D = \begin{cases} \pi_B^{DC}(w_1,w_2) = (p_B - w_1)(1-p_B) \\ +\alpha(p_B - w_2)\left(\frac{2(1-\gamma-\beta)(1-\gamma-p_B)-\beta p_B+(1-\gamma)k_2+\beta(w_2-k_2)}{2(1-\gamma)(1-\gamma-\beta)}\right), & \text{if } w_1 \geq k_1, \max\{k_2, \underline{w}_2\} \leq w_2 < w_2^{(0)}, \\ \pi_B^{D\dagger}(w_1,w_2) = (p_B - w_1)(1-p_B) + \alpha(p_B - w_2)(1-\frac{p_B}{1-\gamma}), & \text{if } w_1 \geq k_1, w_2 \geq \max\{w_2^{D(2)}, w_2^{(0)}\}; \end{cases}$$

$$\pi_B^O = \begin{cases} \pi_B^{OC}(w_2) = (p_B - w_2 - t)\left(1-\frac{p_B}{1-\gamma}\right) \\ +\alpha(p_B - w_2)\left(\frac{2(1-\gamma-\beta)(1-\gamma-p_B)-\beta p_B+(1-\gamma)k_2+\beta(w_2-k_2)}{2(1-\gamma)(1-\gamma-\beta)}\right), & \text{if } \max\{w_2^{O(1)}, \underline{w}_2\} \leq w_2 < w_2^{(0)}, \\ \pi_B^{O\dagger}(w_2) = (p_B - w_2 - t)(1-\frac{p_B}{1-\gamma}) + \alpha(p_B - w_2)(1-\frac{p_B}{1-\gamma}), & \text{if } w_2 \geq \max\{w_2^{O(2)}, w_2^{(0)}\}; \end{cases}$$

$$\pi_B^N(w_1,w_2) = 0.$$

Next, we derive the optimal wholesale prices under each sourcing strategy. As $\pi_B(w_1,w_2)$ decreases in w_1, then the optimal wholesale price of the home supplier that the brand-name firm is willing to offer is equal to the production cost, that is, $w_1^H = k_1$ under Strategy H, and $w_1^D = k_1$ under Strategy D.

With Strategy D, we have the following observations.

(1) Under Strategy D without counterfeiting, as $\pi_B^D(w_1,w_2)$ decreases in w_2, then the optimal wholesale price of the overseas supplier that the brand-name firm is willing to offer is the lower bound of the feasible regions, i.e., $w_2^{D\dagger *} = \max\{w_2^{D(2)}, w_2^{(0)}\}$.

(2) Under Strategy D with counterfeiting, by taking the first-order derivative of the profit function $\pi_B^{DC}(w_1,w_2)$ with respect to w_2, we obtain

$$\frac{\partial(\pi_B^{DC}(w_2))}{\partial(w_2)} = -\alpha\left(\frac{2(1-\gamma-\beta)(1-\gamma-p_B)-\beta p_B+(1-\gamma)k_2+\beta(w_2-k_2)}{2(1-\gamma)(1-\gamma-\beta)}\right) + \alpha(p_B - w_2)\left(\frac{\beta}{2(1-\gamma)(1-\gamma-\beta)}\right).$$

Then, from $\frac{\partial(\pi_B^{DC}(w_2))}{\partial(w_2)} = 0$, we obtain the critical point,

$$\hat{w}_2^{DC} = k_2 - \frac{2x(1-\gamma-\beta-p_B)+\beta k_2+(1-\gamma)k_2}{2\beta} = k_2 - \frac{2(1-\gamma-p_B)(1-\gamma-\beta)-\beta p_B+(1-\gamma)k_2}{2\beta} + \frac{\beta(p_B-k_2)}{2\beta}.$$

If $\hat{w}_2^{DC} < w_2^{(0)}$, then, the optimal wholesale price is $w_2^{DC*} = \max\{k_2, \underline{w}_2, \hat{w}_2^{DC}\}$. As $\hat{w}_2^{DC} > \underline{w}_2$, then, $w_2^{DC*} = \max\{k_2, \hat{w}_2^{DC}\}$. We need to compare the profits of Strategy D with and without counterfeiting.

If $\hat{w}_2^{DC} \geq w_2^{(0)}$, then the optimal wholesale price is $w_2^{DC*} = w_2^{(0)}$. But this profit is dominated by the Strategy D without counterfeiting.

With Strategy O, we have the following observations.

(1) Under Strategy O without counterfeiting, as $\pi_B^O(w_2)$ decreases in w_2, then the optimal wholesale price of the overseas supplier that the brand-name firm is willing to offer is the lower bound of the feasible regions, i.e., $w_2^{O\dagger *} = \max\{w_2^{O(2)}, w_2^{(0)}\}$.

(2) Under Strategy O with counterfeiting, by taking the first order derivative of the profit function $\pi_B^{OC}(w_2)$ with respect to w_2, we obtain

$$\frac{\partial(\pi_B^{OC}(w_2))}{\partial(w_2)} = -\left(1-\frac{p_B}{1-\gamma}\right) - \alpha\left(\frac{2(1-\gamma-\beta)(1-\gamma-p_B)-\beta p_B+(1-\gamma)k_2+\beta(w_2-k_2)}{2(1-\gamma)(1-\gamma-\beta)}\right) + \alpha(p_B - w_2)\left(\frac{\beta}{2(1-\gamma)(1-\gamma-\beta)}\right).$$

Then, from $\frac{\partial(\pi_B^{OC}(w_2))}{\partial(w_2)}=0$, we obtain the critical point,

$\hat{w}_2^{OC}=k_2-\frac{2(1-\gamma)(1-\gamma-\beta-p_B)+\beta k_2+(1-\gamma)k_2}{2\beta}-\frac{(1-\gamma-p_B)(1-\gamma-\beta)}{\alpha\beta}=k_2-\frac{2(1+\frac{1}{\alpha})(1-\gamma-p_B)(1-\gamma-\beta)-\beta p_B+(1-\gamma)k_2}{2\beta}+\frac{\beta(p_B-k_2)}{2\beta}.$

If $\hat{w}_2^{OC}<w_2^{(0)}$, then the optimal wholesale price is $w_2^{OC*}=\max\{w_2^{O(1)},\underline{w}_2,\hat{w}_2^{OC}\}$. We need to compare the profits under Strategy O with and without counterfeiting.

If $\hat{w}_2^{OC}\geq w_2^{(0)}$, then the optimal wholesale price is $w_2^{OC*}=w_2^{(0)}$. But this profit is dominated by the Strategy O without counterfeiting.

Recall that

$w_2^{D(2)}=k_2+\frac{M}{\alpha(1-\frac{p_B}{1-\gamma})}$;

$w_2^{O(2)}=k_2+\frac{K}{(1+\alpha)(1-\frac{p_B}{1-\gamma})}$;

$w_2^{O(1)}=k_2-\frac{2(1+\frac{1}{\alpha})(1-\gamma-p_B)(x-\beta)-\beta p_B+(1-\gamma)k_2}{\beta}+\sqrt{\frac{4(1-\gamma)(1-\gamma-\beta)(K-M')}{\alpha\beta}+\left(\frac{2(1+\frac{1}{\alpha})(1-\gamma-p_B)(1-\gamma-\beta)-\beta p_B+(1-\gamma)k_2}{\beta}\right)^2}$;

$w_2^{(0)}=\max\{w_2^{(0)\prime},\underline{w}_2\}$;

where $\underline{w}_2=k_2-\frac{2(x-p_B)(x-\beta)-\beta p_B+xk_2}{\beta}$, $w_2^{(0)\prime}=k_2+\frac{\beta p_B-(1-\gamma)k_2}{\beta}-\sqrt{\frac{4(1-\gamma)(1-\gamma-\beta)e}{\alpha\beta}}$; and

$$\hat{w}_2^{DC}=k_2-\frac{2(1-\gamma-p_B)(1-\gamma-\beta)-\beta p_B+(1-\gamma)k_2}{2\beta}+\frac{\beta(p_B-k_2)}{2\beta}>\underline{w}_2;$$
$$\hat{w}_2^{OC}=k_2-\frac{2(1+\frac{1}{\alpha})(1-\gamma-p_B)(1-\gamma-\beta)-\beta p_B+(1-\gamma)k_2}{2\beta}+\frac{\beta(p_B-k_2)}{2\beta}.$$

We observe that $w_2^{O(1)}$, $\underline{w}_2$, $\hat{w}_2^{DC}$ and $\hat{w}_2^{OC}$ are independent of e; $w_2^{D(2)}$, $w_2^{O(2)}$ and $w_2^{(0)\prime}$ are dependent of e. Furthermore, we know that w_2^{DC*} and w_2^{OC*} are independent of e. Thus, we obtain the optimal profit functions for each sourcing strategy:

$$\pi_B^H=(p_B-k_1)(1-p_B)+\alpha(p_B-w_1-t)\left(1-\frac{(2-\beta)p_B-k_2}{2(1-\beta)}\right);$$

$$\pi_B^D=\begin{cases}\pi_B^{DC}(w_2^{DC*})=(p_B-k_1)(1-p_B)\\ +\alpha(p_B-w_2^{DC*})\left(\frac{2(1-\gamma-\beta)(1-\gamma-p_B)-\beta p_B+(1-\gamma)k_2+\beta\left(w_2^{DC*}-k_2\right)}{2(1-\gamma)(1-\gamma-\beta)}\right), & \text{if } \max\{k_2,\hat{w}_2^{DC}\}\leq w_2^{(0)},\\ \pi_B^{D\dagger}\left(w_2^{D\dagger*}\right)=(p_B-k_1)(1-p_B)+\alpha\left(p_B-w_2^{D\dagger*}\right)(1-\frac{p_B}{1-\gamma});\end{cases}$$

$$\pi_B^O=\begin{cases}\pi_B^{OC}\left(w_2^{OC*}\right)=(p_B-w_2^{OC*}-t)\left(1-\frac{p_B}{1-\gamma}\right)\\ +\alpha(p_B-w_2^{OC*})\left(\frac{2(1-\gamma-\beta)(1-\gamma-p_B)-\beta p_B+(1-\gamma)k_2+\beta\left(w_2^{OC*}-k_2\right)}{2(1-\gamma)(1-\gamma-\beta)}\right), & \text{if } \max\{w_2^{O(1)},\underline{w}_2,\hat{w}_2^{OC}\}\leq w_2^{(0)},\\ \pi_B^{O\dagger}\left(w_2^{O\dagger*}\right)=\left(p_B-w_2^{O\dagger*}-t\right)(1-\frac{p_B}{1-\gamma})+\alpha\left(p_B-w_2^{O\dagger*}\right)(1-\frac{p_B}{1-\gamma});\end{cases}$$

where $w_2^{DC*}=\max\{k_2,\hat{w}_2^{DC}\}$, $w_2^{D\dagger*}=\max\{w_2^{D(2)},w_2^{(0)\prime},\underline{w}_2\}$, $w_2^{OC*}=\max\{w_2^{O(1)},\underline{w}_2,\hat{w}_2^{OC}\}$, and $w_2^{O\dagger*}=\max\{w_2^{O(2)},w_2^{(0)\prime},\underline{w}_2\}$.

We next compare strategies D and O, respectively. We define $\Pi_{B2}^D(w_2^{DC*})$, $\Pi_{B2}^O(w_2^{OC*})$ are the brand-name firm's profit from the overseas market under Strategy D, Strategy O, respectively; that is, $\Pi_{B2}^D(w_2^{DC*})=\alpha(p_B-w_2^{DC*})\left(\frac{2(1-\gamma-\beta)(1-\gamma-p_B)-\beta p_B+(1-\gamma)k_2+\beta\left(w_2^{DC*}-k_2\right)}{2(1-\gamma)(1-\gamma-\beta)}\right)$; $\Pi_{B2}^O(w_2^{OC*})=\alpha(p_B-w_2^{OC*})\left(\frac{2(1-\gamma-\beta)(1-\gamma-p_B)-\beta p_B+(1-\gamma)k_2+\beta\left(w_2^{OC*}-k_2\right)}{2(1-\gamma)(1-\gamma-\beta)}\right)$.

Under Strategy D:

$$\pi_B^D = \begin{cases} \pi_B^{DC}\left(w_2^{DC*}\right) = \left(p_B - k_1\right)\left(1 - p_B\right) \\ +\alpha\left(p_B - w_2^{DC*}\right)\left(\frac{2(1-\gamma-\beta)(1-\gamma-p_B)-\beta p_B+(1-\gamma)k_2+\beta\left(w_2^{DC*}-k_2\right)}{2(1-\gamma)(1-\gamma-\beta)}\right), & \text{if } \max\{k_2, \hat{w}_2^{DC}\} \le w_2^{(0)\prime}, \\ \pi_B^{D\dagger}\left(w_2^{D\dagger*}\right) = \left(p_B - k_1\right)\left(1 - p_B\right) + \alpha\left(p_B - w_2^{D\dagger*}\right)\left(1 - \frac{p_B}{1-\gamma}\right). \end{cases}$$

Then,

$$\begin{aligned} &\pi_B^{D\dagger}\left(w_2^{D\dagger*}\right) \ge \pi_B^{DC}\left(w_2^{DC*}\right), \\ \Rightarrow\ &\alpha\left(p_B - w_2^{D\dagger*}\right)\left(1 - \tfrac{p_B}{1-\gamma}\right) \ge \Pi_{B2}^D\left(w_2^{DC*}\right), \\ \Rightarrow\ &w_2^{D\dagger*}(e) \le p_B - \frac{\Pi_{B2}^D\left(w_2^{DC*}\right)}{\alpha\left(1-\frac{p_B}{1-\gamma}\right)}. \end{aligned}$$

Under Strategy O:

$$\pi_B^O = \begin{cases} \pi_B^{OC}\left(w_2^{OC*}\right) = \left(p_B - w_2^{OC*} - t\right)\left(1 - \frac{p_B}{1-\gamma}\right) \\ +\alpha\left(p_B - w_2^{OC*}\right)\left(\frac{2(1-\gamma-\beta)(1-\gamma-p_B)-\beta p_B+(1-\gamma)k_2+\beta\left(w_2^{OC*}-k_2\right)}{2(1-\gamma)(1-\gamma-\beta)}\right), & \text{if } \max\{w_2^{O(1)}, \underline{w}_2, \hat{w}_2^{OC}\} \le w_2^{(0)\prime}, \\ \pi_B^{O\dagger}\left(w_2^{O\dagger*}\right) = \left(p_B - w_2^{O\dagger*} - t\right)\left(1 - \frac{p_B}{1-\gamma}\right) + \alpha\left(p_B - w_2^{O\dagger*}\right)\left(1 - \frac{p_B}{1-\gamma}\right). \end{cases}$$

$$\begin{aligned} &\pi_B^{O\dagger}\left(w_2^{O\dagger*}\right) \ge \pi_B^{OC}\left(w_2^{OC*}\right), \\ \Rightarrow\ &(1+\alpha)\left(p_B - w_2^{O\dagger*}\right)\left(1 - \tfrac{p_B}{1-\gamma}\right) - t\left(1 - \tfrac{p_B}{1-\gamma}\right) \ge \left(p_B - w_2^{OC*} - t\right)\left(1 - \tfrac{p_B}{1-\gamma}\right) + \Pi_{B2}^O\left(w_2^{OC*}\right), \\ \Rightarrow\ &(1+\alpha)\left(p_B - w_2^{O\dagger*}\right)\left(1 - \tfrac{p_B}{1-\gamma}\right) \ge \left(p_B - w_2^{OC*}\right)\left(1 - \tfrac{p_B}{1-\gamma}\right) + \Pi_{B2}^O\left(w_2^{OC*}\right), \\ \Rightarrow\ &w_2^{O\dagger*}(e) \le p_B - \frac{\left(p_B - w_2^{OC*}\right)\left(1-\frac{p_B}{1-\gamma}\right) + \Pi_{B2}^O\left(w_2^{OC*}\right)}{(1+\alpha)\left(1-\frac{p_B}{1-\gamma}\right)}. \end{aligned}$$

Thus, we summarize our notations for comparison as below:

$$\begin{aligned} &w_2^{D(2)} = k_2 + \frac{M}{\alpha\left(1-\frac{p_B}{1-\gamma}\right)}; \\ &w_2^{O(2)} = k_2 + \frac{K}{(1+\alpha)\left(1-\frac{p_B}{1-\gamma}\right)}; \\ &w_2^{O(1)} = k_2 - \frac{2(1+\frac{1}{\alpha})(1-\gamma-p_B)(1-\gamma-\beta)-\beta p_B+(1-\gamma)k_2}{\beta} + \sqrt{\frac{4(1-\gamma)(1-\gamma-\beta)(K-M')}{\alpha\beta} + \left(\frac{2(1+\frac{1}{\alpha})(1-\gamma-p_B)(1-\gamma-\beta)-\beta p_B+(1-\gamma)k_2}{\beta}\right)^2}; \\ &\hat{w}_2^{DC} = k_2 - \frac{2(1-\gamma-p_B)(1-\gamma-\beta)-\beta p_B+(1-\gamma)k_2}{2\beta} + \frac{\beta(p_B-k_2)}{2\beta}, \\ &\hat{w}_2^{OC} = k_2 - \frac{2(1+\frac{1}{\alpha})(1-\gamma-p_B)(1-\gamma-\beta)-\beta p_B+(1-\gamma)k_2}{2\beta} + \frac{\beta(p_B-k_2)}{2\beta}; \\ &\Pi_{B2}^D(w_2^{DC*}) = \alpha\left(p_B - w_2^{DC*}\right)\left(\frac{2(1-\gamma-\beta)(1-\gamma-p_B)-\beta p_B+(1-\gamma)k_2+\beta\left(w_2^{DC*}-k_2\right)}{2(1-\gamma)(1-\gamma-\beta)}\right), \\ &\Pi_{B2}^O(w_2^{OC*}) = \alpha\left(p_B - w_2^{OC*}\right)\left(\frac{2(1-\gamma-\beta)(1-\gamma-p_B)-\beta p_B+(1-\gamma)k_2+\beta\left(w_2^{OC*}-k_2\right)}{2(1-\gamma)(1-\gamma-\beta)}\right). \end{aligned} \tag{16}$$

Then, we have the following optimal wholesale price w_2 for Strategy D and Strategy O, respectively.

(a) Under Strategy D, $w_2^D = w_2^{DC*}$ and $s^* = 1$, if $\max\{k_2, \hat{w}_2^{DC}\} \le w_2^{(0)\prime}$ and $w_2^{D*} \le p_B - \frac{\Pi_{B2}^D\left(w_2^{DC*}\right)}{\alpha\left(1-\frac{p_B}{1-\gamma}\right)}$; otherwise, $w_2^D = w_2^{D\dagger*}$ and $s^* = 0$;

(b) Under Strategy O, $w_2^O = w_2^{OC*}$ and $s^* = 1$, if $\max\{w_2^{O(1)}, \underline{w}_2, \hat{w}_2^{OC}\} \le w_2^{(0)\prime}$ and $w_2^{O\dagger*} \le p_B - \frac{\left(p_B - w_2^{OC*}\right)\left(1-\frac{p_B}{1-\gamma}\right) + \Pi_{B2}^O\left(w_2^{OC*}\right)}{(1+\alpha)\left(1-\frac{p_B}{1-\gamma}\right)}$; otherwise, $w_2^O = w_2^{O\dagger*}$ and $s^* = 0$;

where $w_2^{DC*} = \max\{k_2, \hat{w}_2^{DC}\}$, $w_2^{D\dagger*} = \max\{w_2^{D(2)}, w_2^{(0)\prime}, \underline{w}_2\}$; $w_2^{OC*} = \max\{w_2^{O(1)}, \underline{w}_2, \hat{w}_2^{OC}\}$, $w_2^{O\dagger*} = \max\{w_2^{O(2)}, w_2^{(0)\prime}, \underline{w}_2\}$. ■

B.10 Proofs For Extension 3: Endogenous Brand-Name Product and Counterfeit Prices

B.10.1 Proof of Lemma A1.

Note that when the demand of the brand-name product is $m_{B2} = 0$, it is not dual sourcing or single sourcing from the overseas supplier, because there is no market share for the brand-name firm in the overseas market. Thus, in order to focus on the cases of strategies D or O with $m_{B2} > 0$ and to examine the conditions to effectively prevent counterfeiting, in this extension, we assume that the brand-name firm has a positive market share in the overseas market, and it is possible for the overseas supplier to sell counterfeits under optimal retail prices.

It is convenient for us to define below notations:

$$\begin{aligned} \hat{p}_B^D &= \tfrac{2(1-\gamma)(1-\gamma-\beta)(1+w_1)+\alpha(2(1-\gamma-\beta)(1-\gamma)+(1-\gamma-\beta)k_2)+2\alpha(1-\gamma)w_2}{4(1-\gamma)(1-\gamma-\beta)+\alpha(4(1-\gamma-\beta)+2\beta)}, \\ \hat{p}_B^O &= \tfrac{2(1-\gamma-\beta)(1-\gamma+t)+\alpha(2(1-\gamma-\beta)(1-\gamma)+(1-\gamma-\beta)k_2)+(2(1-\gamma-\beta)+2\alpha(1-\gamma))w_2}{4(1-\gamma-\beta)+\alpha(4(1-\gamma-\beta)+2\beta)}. \end{aligned} \tag{17}$$

We assume the penalty from law enforcement e is not very high such that $w_2 < h_0(e)$, where $h_0(e) = \frac{(2(1-\gamma)+k_2)(1-\gamma-\beta)\beta-(\sqrt{\frac{4\beta(1-\gamma)(1-\gamma-\beta)e}{\alpha}}+(1-\gamma-\beta)k_2)(2(1-\gamma)-\beta)}{2\beta(1-\gamma-\beta)}$. This condition guarantees that it is possible for the overseas supplier to sell counterfeits.

Below, the proof includes two parts for strategies D and O, respectively.

Part 1: With Strategy D, we derive the counterfeiting prevention condition, and compare the counterfeiting prevention condition between this extension and the base model.

Under Strategy D, we assume $w_2 > (\underline{h}_D(w_1))^+$, where $\underline{h}_D(w_1) = \frac{(2(1-\gamma)(1+w_1)(2(1-\gamma)-\beta)-(4(1-\gamma)(1-\gamma-\beta)+\alpha(2(1-\gamma)-\beta))(2(1-\gamma)+k_2))(1-\gamma-\beta)}{\beta(4(1-\gamma)(1-\gamma-\beta)+2\alpha\gamma(2(1-\gamma)-\beta))}$. Under this condition, $\hat{p}_B^D < \frac{2(1-\gamma)(1-\gamma-\beta)+(1-\gamma)k_2+\beta(w_2-k_2)}{2(1-\gamma)-\beta}$ holds, where $\hat{p}_B^D$ is defined in Equation (17). It implies that the brand-name firm has a positive market share in the overseas market with Strategy D.

Step 1: Given p_B, we derive the overseas supplier's profit $s = 0$ and $s = 1$, respectively.

(1) If the overseas supplier does not sell the counterfeit in the market, i.e., $s = 0$, we know:

$$\begin{aligned} \pi_B^D(p_B, s=0) &= (p_B - w_1)(1 - p_B) + \alpha(p_B - w_2)(1 - \tfrac{p_B}{1-\gamma}), \\ \pi_2^D(p_B, s=0) &= \alpha(w_2 - k_2)(1 - \tfrac{p_B}{1-\gamma}). \end{aligned}$$

(2) If the overseas supplier sell the counterfeit in the market, i.e., $s = 1$, then, the brand-name firm firstly decides on the retail price p_B for the brand-name product, then the overseas supplier decides on the retail price p_2 for the counterfeit. Their profits are as follows.

$$\begin{aligned} \pi_B^D(p_B, p_2, s=1) &= (p_B - w_1)(1 - p_B) + \alpha(p_B - w_2)\left(1 - \tfrac{p_B - p_2}{1-\gamma-\beta}\right), \\ \pi_2^D(p_B, p_2, s=1) &= \alpha(w_2 - k_2)\left(1 - \tfrac{p_B - p_2}{1-\gamma-\beta}\right) + \alpha(p_2 - k_2)\left(\tfrac{p_B - p_2}{1-\gamma-\beta} - \tfrac{p_2}{\beta}\right)^+ - e. \end{aligned}$$

If both the brand-name firm and the overseas supplier get positive overseas market share, i.e., $m_{B2} = \alpha\left(1 - \frac{p_B - p_2}{\gamma-\beta}\right) > 0$, and $m_2 = \alpha\left(\frac{p_B - p_2}{1-\gamma-\beta} - \frac{p_2}{\beta}\right) > 0$, then, $p_B - (1-\gamma-\beta) < p_2 < \frac{\beta p_B}{1-\gamma}$. The profit of the overseas supplier is

$$\pi_2^D(p_B, p_2) = \alpha(w_2 - k_2)\left(1 - \tfrac{p_B - p_2}{1-\gamma-\beta}\right) + \alpha(p_2 - k_2)\left(\tfrac{p_B - p_2}{1-\gamma-\beta} - \tfrac{p_2}{\beta}\right) - e.$$

By taking the first order derivative of $\pi_2^D(p_B, p_2)$ with respect to p_2, we have,

$$\frac{\partial(\pi_2^D(p_B,p_2))}{\partial(p_2)} = \alpha\left(\frac{p_B+k_2-2p_2+(w_2-k_2)}{1-\gamma-\beta} - \frac{2p_2-k_2}{\beta}\right) = \alpha\left(\frac{p_B-2p_2+w_2}{1-\gamma-\beta} - \frac{2p_2-k_2}{\beta}\right).$$

From $\frac{\partial\left(\pi_2^D(p_B,p_2)\right)}{\partial(p_2)} = 0$, we obtain the critical point $\hat{p}_2^D = \frac{\beta p_B+(1-\gamma)k_2+\beta(w_2-k_2)}{2(1-\gamma)}$. Next, we need to check whether $\hat{p}_2^D$ is in the feasible region $p_B - (1-\gamma-\beta) < p_2 < \frac{\beta p_B}{1-\gamma}$. From $p_B - (1-\gamma-\beta) < \hat{p}_2^D < \frac{\beta p_B}{1-\gamma}$, we obtain, $\frac{(1-\gamma)k_2+\beta(w_2-k_2)}{\beta} < p_B < \frac{2(1-\gamma)(1-\gamma-\beta)+(1-\gamma)k_2+\beta(w_2-k_2)}{2(1-\gamma)-\beta}$. Recall that we assume the brand-name firm has a positive market share in the overseas market, i.e., $m_{B2} > 0$. Thus, with Strategy D, if the overseas supplier sells the counterfeit, i.e., $s = 1$, the optimal retail price p_2 for the counterfeit is

$$p_2^{D*} = \begin{cases} \frac{\beta p_B}{1-\gamma}, & \text{if } p_B \leq \frac{(1-\gamma)k_2+\beta(w_2-k_2)}{\beta}, \text{[note that } m_2 = 0] \\ \hat{p}_2^D, & \text{if } \frac{(1-\gamma)k_2+\beta(w_2-k_2)}{\beta} < p_B < \frac{2(1-\gamma)(1-\gamma-\beta)+(1-\gamma)k_2+\beta(w_2-k_2)}{2(1-\gamma)-\beta}, \text{[note that } m_2 > 0] \end{cases}$$

and the overseas supplier's profit is

$$\pi_2^D(p_B, s=1) = \begin{cases} \pi_2^{DC1} = \alpha(w_2-k_2)\left(1 - \frac{p_B-p_2^{D*}}{1-\gamma-\beta}\right) - e, & \text{if } p_B \leq \frac{(1-\gamma)k_2+\beta(w_2-k_2)}{\beta}, \\ \hat{\pi}_2^{DC} = \alpha(w_2-k_2)\left(1 - \frac{p_B-p_2^{D*}}{1-\gamma-\beta}\right) & \\ +\alpha(p_2^*-k_2)\left(\frac{p_B-p_2^{D*}}{1-\gamma-\beta} - \frac{p_2}{\beta}\right) - e, & \text{if } \frac{(1-\gamma)k_2+\beta(w_2-k_2)}{\beta} < p_B < \frac{2(1-\gamma)(1-\gamma-\beta)+(1-\gamma)k_2+\beta(w_2-k_2)}{2(1-\gamma)-\beta}, \end{cases}$$

and the brand-name firm's profit is

$$\pi_B^D(p_B, s=1)$$
$$= \begin{cases} \pi_B^{DC1} = (p_B-w_1)(1-p_B) + \alpha(p_B-w_2)\left(1 - \frac{p_B-p_2^*}{1-\gamma-\beta}\right), & \text{if } p_B < \frac{(1-\gamma)k_2+\beta(w_2-k_2)}{\beta}, \\ \hat{\pi}_B^{DC} = (p_B-w_1)(1-p_B) & \\ +\alpha(p_B-w_2)\left(\frac{2(1-\gamma-\beta)(\gamma-p_B)-\beta p_B+(1-\gamma)k_2+\beta(w_2-k_2)}{2\gamma(1-\gamma-\beta)}\right), & \text{if } \frac{(1-\gamma)k_2+\beta(w_2-k_2)}{\beta} < p_B < \frac{2(1-\gamma)(1-\gamma-\beta)+(1-\gamma)k_2+\beta(w_2-k_2)}{2(1-\gamma)-\beta}. \end{cases}$$

Step 2: The overseas supplier decides on whether to sell the counterfeit, $s^*(p_B)$.

For the overseas supplier, if $\pi_2^D(p_B, s=1) > \pi_2^D(p_B, s=0)$, she decides to sell the counterfeit. Otherwise, she does not sell the counterfeit. Recall that when $s = 0$, the overseas supplier's profit is

$$\pi_2^D(p_B, s=0) = \alpha(w_2-k_2)\left(1 - \frac{p_B}{1-\gamma}\right).$$

Note that given p_B, for the overseas supplier, there are below two scenarios.

(1) If $p_B < \frac{(1-\gamma)k_2+\beta(w_2-k_2)}{\beta}$, then, the overseas supplier's profit of counterfeiting is $\pi_2^D(p_B, s=1) = \pi_B^{DC1}$, which implies $p_2^{D*} = \frac{\beta p_B}{1-\gamma}$. Then, we know: the optimal decision is $s^* = 0$, because $\pi_2^D(p_B, s=0) > \pi_2^{DC1}$ always holds.

(2) If $\frac{(1-\gamma)k_2+\beta(w_2-k_2)}{\beta} < p_B < \frac{2(1-\gamma)(1-\gamma-\beta)+(1-\gamma)k_2+\beta(w_2-k_2)}{2(1-\gamma)-\beta}$, then, the overseas supplier's profit of counterfeiting is $\pi_2^D(p_B, s=1) = \hat{\pi}_2^{DC}$, which implies $p_2^{D*} = \hat{p}_2^D(p_B) = \frac{\beta p_B+(1-\gamma)k_2+\beta(w_2-k_2)}{2(1-\gamma)}$. Then, the optimal decision is $s = 0$ if $\pi_2^D(p_B, s=0) > \hat{\pi}_2^{DC}$, which means

$$\alpha(w_2-k_2)\left(1 - \frac{p_B}{1-\gamma}\right) > \alpha(w_2-k_2)\left(1 - \frac{p_B-\hat{p}_2^D}{1-\gamma-\beta}\right) + \left(\alpha\left(\hat{p}_2^D-k_2\right)\left(\frac{p_B-\hat{p}_2^D}{1-\gamma-\beta} - \frac{\hat{p}_2^D}{\beta}\right) - e\right),$$
$$\Rightarrow \alpha(w_2-k_2)\left(1 - \frac{p_B}{1-\gamma}\right) > \alpha(w_2-k_2)\left(\frac{2(1-\gamma-\beta)(1-\gamma) - (2(1-\gamma)-\beta)p_B + (1-\gamma)k_2 + \beta(w_2-k_2)}{2(1-\gamma)(1-\gamma-\beta)}\right)$$
$$+ \left(\alpha\left(\frac{\beta p_B - (1-\gamma)k_2 + \beta(w_2-k_2)}{2(1-\gamma)}\right)\frac{\beta p_B - (1-\gamma)k_2 - \beta(w_2-k_2)}{2\beta(1-\gamma-\beta)} - e\right),$$
$$\Rightarrow x_{low} < p_B < x_{high},$$

where $x_{low} = \frac{-\sqrt{\frac{4\beta(1-\gamma)(1-\gamma-\beta)e}{\alpha}}+((1-\gamma)k_2+\beta(w_2-k_2))}{\beta}$, $x_{high} = \frac{\sqrt{\frac{4\beta(1-\gamma)(1-\gamma-\beta)e}{\alpha}}+((1-\gamma)k_2+\beta(w_2-k_2))}{\beta}$.

Note that $x_{low} < \frac{\gamma k_2+\beta(w_2-k_2)}{\beta} < x_{high}$. Recall that we assume $w_2 < h_0(e)$, where $h_0(e) = \frac{(2(1-\gamma)+k_2)(1-\gamma-\beta)\beta-(\sqrt{\frac{4\beta(1-\gamma)(1-\gamma-\beta)e}{\alpha}}+(1-\gamma-\beta)k_2)(2(1-\gamma)-\beta)}{2\beta(1-\gamma-\beta)}$, it implies that $x_{high} < \frac{2(1-\gamma)(1-\gamma-\beta)+(1-\gamma)k_2+\beta(w_2-k_2)}{2(1-\gamma)-\beta}$. Then, if $\frac{(1-\gamma)k_2+\beta(w_2-k_2)}{\beta} < p_B < x_{high}$, the optimal decision is $s^*=0$; if $x_{high} < p_B < \frac{2\gamma(1-\gamma-\beta)+(1-\gamma)k_2+\beta(w_2-k_2)}{2(1-\gamma)-\beta}$, the optimal decision is $s^*=1$, and $\pi_B^D(p_B, s=1) = \hat{\pi}_B^{DC}$.

Thus, combining these two scenarios, the overseas supplier's optimal decision of counterfeiting is

$$s^*(p_B) = \begin{cases} 0, & \text{if } p_B \le x_{high}, \text{ [note that } m_2 = 0] \\ 1, & \text{if } x_{high} < p_B < \frac{2(1-\gamma)(1-\gamma-\beta)+(1-\gamma)k_2+\beta(w_2-k_2)}{2(1-\gamma)-\beta}. \text{ [note that } m_2 > 0] \end{cases}$$

Subsequently, the brand-name firm's profit is

$$\begin{aligned} &\pi_B^D(p_B) \\ &= \begin{cases} \pi_B^D(p_B, s=0) = (p_B - w_1)(1-p_B) + \alpha(p_B - w_2)(1 - \frac{p_B}{1-\gamma}), & \text{if } p_B \le x_{high}, \\ \pi_B^D(p_B, s=1) = (p_B - w_1)(1-p_B) & \\ +\alpha(p_B - w_2)\left(\frac{2(1-\gamma-\beta)(1-\gamma-p_B)-\beta p_B+(1-\gamma)k_2+\beta(w_2-k_2)}{2\gamma(1-\gamma-\beta)}\right), & \text{if } x_{high} < p_B < \frac{2(1-\gamma)(1-\gamma-\beta)+(1-\gamma)k_2+\beta(w_2-k_2)}{2(1-\gamma)-\beta}. \end{cases} \end{aligned}$$

Step 3: The brand-name firm decides on the optimal retail price p_B^{D*}. We discuss possible cases as follow.

(1) If $p_B \le x_{high}$, which means $s=0$, the brand-name firm's profit is

$$\pi_B^D(p_B, s=0) = (p_B - w_1)(1-p_B) + \alpha(p_B - w_2)(1 - \frac{p_B}{1-\gamma}).$$

In this case, only the brand-name firm decides on the optimal price p_B.

$$\begin{aligned} \frac{\partial(\pi_B^D(p_B))}{\partial(p_B)} &= ((1-p_B)-(p_B-w_1)) + \alpha((1-\frac{p_B}{1-\gamma}) - \frac{p_B-w_2}{1-\gamma}) \\ &= (1-2p_B+w_1) + \alpha(\frac{1-\gamma-2p_B+w_2}{1-\gamma}) \\ &= \frac{(1+w_1)(1-\gamma)-2p_B(1-\gamma)+\alpha(1-\gamma+w_2)+\alpha(-2p_B)}{1-\gamma}. \end{aligned}$$

From the first order condition, i.e., $\frac{\partial(\pi_B^D(p_B))}{\partial(p_B)} = 0$, the critical point of the optimal retail price is

$$p_B^{D0} = \frac{(1+w_1)(1-\gamma)+\alpha(1-\gamma+w_2)}{2(\alpha+1-\gamma)}.$$

We check whether this critical point is in the feasible region. From $p_B^{D0} \le x_{high}$, we have

$$\begin{aligned} &\frac{(1+w_1)(1-\gamma)+\alpha(1-\gamma+w_2)}{2(\alpha+1-\gamma)} \le \frac{\sqrt{\frac{4\beta(1-\gamma)(1-\gamma-\beta)e}{\alpha}}+((1-\gamma)k_2+\beta(w_2-k_2))}{\beta}, \\ &\Rightarrow w_2 \ge h_{D1}(w_1,e), \text{ where } h_{D1}(w_1,e) = \frac{((1+w_1)(1-\gamma)+\alpha(1-\gamma))\beta-2(\alpha+1-\gamma)(\sqrt{\frac{4\beta(1-\gamma)(1-\gamma-\beta)e}{\alpha}}+(1-\gamma-\beta)k_2)}{\beta(2(1-\gamma)+\alpha)}. \end{aligned}$$

Thus, with $s=0$, the brand-name firm's optimal retail price is

$$p_B^{D*}(s=0) = \begin{cases} p_B^{D0}, & \text{if } w_2 \ge h_{D1}(w_1,e), \\ x_{high}, & \text{if } w_2 < h_{D1}(w_1,e). \end{cases}$$

(2) If $x_{high} < p_B < \frac{2(1-\gamma)(1-\gamma-\beta)+(1-\gamma)k_2+\beta(w_2-k_2)}{2(1-\gamma)-\beta}$, which means $s=1$, the brand-name firm's profit is

$$\pi_B^D(p_B, s=1) = \hat{\pi}_B^{DC} = (p_B - w_1)(1-p_B) + \alpha(p_B - w_2)\left(\frac{2(1-\gamma-\beta)(1-\gamma-p_B)-\beta p_B+(1-\gamma)k_2+\beta(w_2-k_2)}{2\gamma(1-\gamma-\beta)}\right).$$

By taking the derivative of the first order condition, the critical point of the optimal retail price is

$$\hat{p}_B^D = \frac{2(1-\gamma)(1-\gamma-\beta)(1+w_1)+\alpha(2(1-\gamma-\beta)(1-\gamma)+(1-\gamma-\beta)k_2)+2\alpha(1-\gamma)w_2}{4(1-\gamma)(1-\gamma-\beta)+\alpha(4(1-\gamma-\beta)+2\beta)}.$$

We check whether this critical point $\hat{p}_B^D$ is in the feasible region of $[x_{high}, \frac{2(1-\gamma)(1-\gamma-\beta)+(1-\gamma)k_2+\beta(w_2-k_2)}{2(1-\gamma)-\beta}]$. Recall that $w_2 > (\underline{h}_D(w_1))^+$, which implies $\hat{p}_B^D < \frac{2(1-\gamma)(1-\gamma-\beta)+(1-\gamma)k_2+\beta(w_2-k_2)}{2(1-\gamma)-\beta}$ holds.

From $x_{high} < \hat{p}_B^D$, we have:

$$\frac{\sqrt{\frac{4\beta(1-\gamma)(1-\gamma-\beta)e}{\alpha}}+((1-\gamma)k_2+\beta(w_2-k_2))}{\beta} < \frac{2(1-\gamma)(1-\gamma-\beta)(1+w_1)+\alpha(2(1-\gamma-\beta)(1-\gamma)+(1-\gamma-\beta)k_2)+2\alpha(1-\gamma)w_2}{4(1-\gamma)(1-\gamma-\beta)+\alpha(4(1-\gamma-\beta)+2\beta)},$$

$\Rightarrow w_2 < h_{D2}(w_1,e)$,

where $h_{D2}(w_1,e) = \frac{(2(1-\gamma)(1-\gamma-\beta)(1+w_1)+\alpha(2(1-\gamma)+k_2)(1-\gamma-\beta))\beta-\left(\sqrt{\frac{4\beta(1-\gamma)(1-\gamma-\beta)e}{\alpha}}+(1-\gamma-\beta)k_2\right)(4(1-\gamma)(1-\gamma-\beta)+\alpha(4(1-\gamma-\beta)+2\beta))}{2\beta(2(1-\gamma)+\alpha)(1-\gamma-\beta)}$.

Thus, with $s=1$, the brand-name firm's optimal retail price is

$$p_B^{D*}(s=1) = \begin{cases} x_{high}, & \text{if } w_2 \geq h_{D2}(w_1,e), \\ \hat{p}_B^D, & \text{if } \underline{h}_D(w_1) < w_2 < h_{D2}(w_1,e). \end{cases}$$

Based on the above discussions, the brand-name firm chooses p_B^* to maximize her profit by making a comparison between $\pi_B^D(s=0)$ and $\hat{\pi}_B^{DC}(s=1)$ in overleaping region.

Note that $h_{D2}(w_1,e) < h_{D1}(w_1,e)$. Then, the optimal retail price of the brand-name firm is

$$p_B^{D*} = \begin{cases} p_B^{D0}, & \text{if } w_2 \geq h_{D1}(w_1,e), \text{[note that } m_2 = 0] \\ x_{high}, & \text{if } h_{D2}(w_1,e) \leq w_2 < h_{D1}(w_1,e), \text{[note that } m_2 = 0] \\ \hat{p}_B^D, & \text{if } (\underline{h}_D(w_1))^+ < w_2 < h_{D2}(w_1,e), \text{[note that } m_2 > 0] \end{cases}$$

where $x_{high} = \frac{\sqrt{\frac{4\beta(1-\gamma)(1-\gamma-\beta)e}{\alpha}}+((1-\gamma)k_2+\beta(w_2-k_2))}{\beta}$, $p_B^{D0} = \frac{(1+w_1)(1-\gamma)+\alpha(1-\gamma+w_2)}{2(\alpha+1-\gamma)}$, $\hat{p}_B^D = \frac{2(1-\gamma)(1-\gamma-\beta)(1+w_1)+\alpha(2(1-\gamma-\beta)\gamma+(1-\gamma-\beta)k_2)+2\alpha(1-\gamma)w_2}{4(1-\gamma)(1-\gamma-\beta)+\alpha(4(1-\gamma-\beta)+2\beta)}$.

Thus, the condition to prevent counterfeiting is $w_2 \geq w_2^{D,endog}$, where $w_2^{D,endog} = h_{D2}(w_1,e)$. That is to say, under Strategy D, $s^* = 0$ if $\hat{p}_B^D \leq \frac{\sqrt{\frac{4\beta(1-\gamma)(1-\gamma-\beta)e}{\alpha}}+((1-\gamma)k_2+\beta(w_2-k_2))}{\beta}$, where $\hat{p}_B^D$ is defined in Equation (17).

Part 2: With Strategy O, we derive the counterfeiting prevention condition, and compare the counterfeiting prevention condition between this extension and the base model.

Under Strategy O, we assume $w_2 > (\underline{h}_O)^+$, where $\underline{h}_O = \frac{(2(1-\gamma+\alpha)(2(1-\gamma)-\beta)-(4(1-\gamma-\beta)+\alpha(2(1-\gamma)-\beta))(2(1-\gamma)+k_2))(1-\gamma-\beta)}{\beta(2(1-\gamma-\beta)(2\gamma+\beta)+2\alpha\gamma(2(1-\gamma)-\beta))}$. Under this condition, $\hat{p}_B^O < \frac{2(1-\gamma)(1-\gamma-\beta)+(1-\gamma)k_2+\beta(w_2-k_2)}{2(1-\gamma)-\beta}$ holds, where $\hat{p}_B^O$ is defined in Equation (17). It implies that the brand-name firm has a positive market share in the overseas market with Strategy O.

Step 1: Given p_B, we derive the overseas supplier's profit $s=0$ and $s=1$, respectively.

(1) If the overseas supplier does not sell the counterfeit in the market, i.e., $s=0$, we know:

$$\pi_B^O(p_B, s=0) = (p_B - w_2 - t)\,(1 - \frac{p_B}{1-\gamma}) + \alpha\,(p_B - w_2)\,(1 - \frac{p_B}{1-\gamma}),$$
$$\pi_2^O(p_B, s=0) = (w_2 - k_2)\,(1 - \frac{p_B}{1-\gamma}) + \alpha\,(w_2 - k_2)\,(1 - \frac{p_B}{1-\gamma}).$$

(2) If the overseas supplier sells the counterfeit in the market, i.e., $s=1$, then, the brand-name firm firstly decides on the retail price p_B for the brand-name product, then the overseas supplier decides on the retail price p_2 for the counterfeit. Their profits are as follows.

$$\pi_B^O(p_B,p_2,s=1)=(p_B-w_2-t)\left(1-\tfrac{p_B}{1-\gamma}\right)+\alpha(p_B-w_2)\left(1-\tfrac{p_B-p_2}{1-\gamma-\beta}\right),$$
$$\pi_2^O(p_B,p_2,s=1)=(w_2-k_2)\left(1-\tfrac{p_B}{1-\gamma}\right)+\alpha(w_2-k_2)\left(1-\tfrac{p_B-p_2}{1-\gamma-\beta}\right)+\alpha(p_2-k_2)\left(\tfrac{p_B-p_2}{1-\gamma-\beta}-\tfrac{p_2}{\beta}\right)^+-e.$$

Similar with the discussion in Strategy D, we derive the optimal retail price p_2 for the overseas supplier under Strategy O by backward deduction. Thus, with Strategy O, if the overseas supplier sells the counterfeit, i.e., $s=1$, we have $\hat{p}_2^O=\frac{\beta p_B+(1-\gamma)k_2+\beta(w_2-k_2)}{2(1-\gamma)}$, and the optimal retail price is

$$p_2^{O*}=\begin{cases}\frac{\beta p_B}{1-\gamma}, & \text{if } p_B<\frac{(1-\gamma)k_2+\beta(w_2-k_2)}{\beta}, \text{[note that } m_2=0]\\ \hat{p}_2^O, & \text{if } \frac{(1-\gamma)k_2+\beta(w_2-k_2)}{\beta}<p_B<\frac{2(1-\gamma)(1-\gamma-\beta)+(1-\gamma)k_2+\beta(w_2-k_2)}{2(1-\gamma)-\beta}, \text{[note that } m_2>0]\end{cases}$$

and the overseas supplier's profit is

$$\begin{aligned}&\pi_2^O(p_B,s=1)\\&=\begin{cases}\pi_2^{OC1}=(w_2-k_2)\left(1-\frac{p_B}{1-\gamma}\right)+\alpha(w_2-k_2)\left(1-\frac{p_B-p_2^{O*}}{1-\gamma-\beta}\right)-e, & \text{if } p_B<\frac{(1-\gamma)k_2+\beta(w_2-k_2)}{\beta},\\ \hat{\pi}_2^{OC}=(w_2-k_2)\left(1-\frac{p_B}{1-\gamma}\right) & \\ +\alpha(w_2-k_2)\left(1-\frac{p_B-p_2^{O*}}{1-\gamma-\beta}\right)+\left(\alpha(p_2^*-k_2)\left(\frac{p_B-p_2^{O*}}{1-\gamma-\beta}-\frac{p_2^{O*}}{\beta}\right)-e\right), & \text{if } \frac{(1-\gamma)k_2+\beta(w_2-k_2)}{\beta}<p_B<\frac{2(1-\gamma)(1-\gamma-\beta)+(1-\gamma)k_2+\beta(w_2-k_2)}{2(1-\gamma)-\beta},\end{cases}\end{aligned}$$

and the brand-name firm's profit is

$$\begin{aligned}&\pi_B^O(p_B,s=1)\\&=\begin{cases}\pi_B^{OC1}=(p_B-w_2-t)(1-p_B)+\alpha(p_B-w_2)\left(1-\frac{p_B-p_2^{O*}}{1-\gamma-\beta}\right), & \text{if } p_B<\frac{(1-\gamma)k_2+\beta(w_2-k_2)}{\beta},\\ \hat{\pi}_B^{OC}=(p_B-w_2-t)(1-p_B) & \\ +\alpha(p_B-w_2)\left(\frac{2(1-\gamma-\beta)(1-\gamma-p_B)-\beta p_B+(1-\gamma)k_2+\beta(w_2-k_2)}{2(1-\gamma)(1-\gamma-\beta)}\right), & \text{if } \frac{(1-\gamma)k_2+\beta(w_2-k_2)}{\beta}<p_B<\frac{2(1-\gamma)(1-\gamma-\beta)+(1-\gamma)k_2+\beta(w_2-k_2)}{2(1-\gamma)-\beta}.\end{cases}\end{aligned}$$

Step 2: The overseas supplier decides on whether to sell the counterfeit, $s^*(p_B)$.

For the overseas supplier, if $\pi_2^O(p_B,s=1)>\pi_2^O(p_B,s=0)$, she decides to sell the counterfeit. Otherwise, she does not sell the counterfeit. Recall that when $s=0$, the overseas supplier's profit is

$$\pi_2^O(p_B,s=0)=(w_2-k_2)(1-\frac{p_B}{1-\gamma})+\alpha(w_2-k_2)(1-\frac{p_B}{1-\gamma}).$$

Similarly, we obtain,

$$s^*(p_B)=\begin{cases}0, & \text{if } p_B\le x_{high}, \text{[note that } m_2=0]\\ 1, & \text{if } x_{high}<p_B<\frac{2(1-\gamma)(1-\gamma-\beta)+(1-\gamma)k_2+\beta(w_2-k_2)}{2(1-\gamma)-\beta}. \text{[note that } m_2>0]\end{cases}$$

Step 3: The brand-name firm decides on the optimal retail price p_B^{O*} to maximize her profit. We discuss possible cases as follows.

(1) If $p_B\le x_{high}$, which means $s=0$, the brand-name firm's profit is

$$\pi_B^O(p_B,s=0)=(p_B-w_2-t)(1-\frac{p_B}{1-\gamma})+\alpha(p_B-w_2)(1-\frac{p_B}{1-\gamma}),$$

In this case, only the brand-name firm decides on the optimal price p_B.

$$\begin{aligned}\frac{\partial(\pi_B^O(p_B))}{\partial(p_B)} &= ((1-\frac{p_B}{1-\gamma})-\frac{p_B-w_2-t}{1-\gamma})+\alpha((1-\frac{p_B}{1-\gamma})-\frac{p_B-w_2}{1-\gamma})\\ &= \frac{(\gamma-2p_B+w_2+t)}{1-\gamma}+\alpha(\frac{1-\gamma-2p_B+w_2}{1-\gamma})\\ &= \frac{(1+\alpha)(1-\gamma+w_2)+t+(1+\alpha)(-2p_B)}{1-\gamma}.\end{aligned}$$

From the first order condition, i.e., $\frac{\partial(\pi_B^O(p_B))}{\partial(p_B)}=0$, we have,

$$p_B^{O0}=\frac{(1+\alpha)(1-\gamma+w_2)+t}{2(1+\alpha)}.$$

We check whether this critical point p_B^{O0} is in the feasible region. From $p_B^{O0}\le x_{high}$, we have

$$\begin{aligned}&\frac{(1+\alpha)(1-\gamma+w_2)+t}{2(1+\alpha)}\le\frac{\sqrt{\frac{4\beta(1-\gamma)(1-\gamma-\beta)e}{\alpha}}+((1-\gamma)k_2+\beta(w_2-k_2))}{\beta},\\ \Rightarrow\ & w_2\ge h_{O1}(e),\ \text{where } h_{O1}(e)=\frac{((1+\alpha)(1-\gamma)+t)\beta-2(1+\alpha)(\sqrt{\frac{4\beta(1-\gamma)(1-\gamma-\beta)e}{\alpha}}+(1-\gamma-\beta)k_2)}{\beta(1+\alpha)}.\end{aligned}$$

Thus, with $s=0$, the brand-name firm's optimal retail price is as follows:

$$p_B^{O*}(s=0)=\begin{cases}p_B^{O0}, & \text{if } w_2\ge h_{O1}(e),\\ x_{high}, & \text{if } w_2<h_{O1}(e).\end{cases}$$

(2) If $x_{high}<p_B<\frac{2(1-\gamma)(1-\gamma-\beta)+(1-\gamma)k_2+\beta(w_2-k_2)}{2(1-\gamma)-\beta}$, which means $s=1$, the brand-name firm's profit is

$$\pi_B^O(p_B,s=1)=\hat{\pi}_B^{OC}=(p_B-w_2-t)\left(1-\frac{p_B}{1-\gamma}\right)+\alpha(p_B-w_2)\left(\frac{2(1-\gamma-\beta)(1-\gamma-p_B)-\beta p_B+(1-\gamma)k_2+\beta(w_2-k_2)}{2(1-\gamma)(1-\gamma-\beta)}\right).$$

By taking the first order derivative of $\pi_B^O(p_B)$ with respect to p_B, we have,

$$\begin{aligned}\frac{\partial\left(\pi_B^O(p_B)\right)}{\partial(p_B)} &= \left(1-\frac{2p_B-w_2-t}{1-\gamma}\right)+\alpha\left(\left(\frac{2(1-\gamma-\beta)(1-\gamma-p_B)-\beta p_B+(1-\gamma)k_2+\beta(w_2-k_2)}{2(1-\gamma)(1-\gamma-\beta)}\right)+(p_B-w_2)\frac{-2(1-\gamma-\beta)-\beta}{2(1-\gamma)(1-\gamma-\beta)}\right)\\ &= \frac{1-\gamma-2p_B+w_2+t}{1-\gamma}+\alpha\left(\frac{2(1-\gamma-\beta)(1-\gamma-2p_B+w_2)-2\beta p_B+\beta w_2+(1-\gamma)k_2+\beta(w_2-k_2)}{2(1-\gamma)(1-\gamma-\beta)}\right).\end{aligned}$$

From the first order condition, i.e., $\frac{\partial\left(\pi_B^O(p_B)\right)}{\partial(p_B)}=0$, we have,

$$\hat{p}_B^O=\frac{2(1-\gamma-\beta)(1-\gamma+t)+\alpha(2(1-\gamma-\beta)(1-\gamma)+(1-\gamma-\beta)k_2)+(2(1-\gamma-\beta)+2\alpha(1-\gamma))w_2}{4(1-\gamma-\beta)+\alpha(4(1-\gamma-\beta)+2\beta)}.$$

We check whether this critical point $\hat{p}_B^O$ is in the feasible region of $[x_{high},\frac{2(1-\gamma)(1-\gamma-\beta)+(1-\gamma)k_2+\beta(w_2-k_2)}{2(1-\gamma)-\beta}]$. Recall that with Strategy O, $w_2>(\underline{h}_O)^+$, which implies that $\hat{p}_B^O<\frac{2(1-\gamma)(1-\gamma-\beta)+(1-\gamma)k_2+\beta(w_2-k_2)}{2(1-\gamma)-\beta}$.

From $x_{high}<\hat{p}_B^O$,

$$\begin{aligned}&\frac{\sqrt{\frac{4\beta(1-\gamma)(1-\gamma-\beta)e}{\alpha}}+((1-\gamma)k_2+\beta(w_2-k_2))}{\beta}<\frac{2(1-\gamma-\beta)(1-\gamma+t)+\alpha(2(1-\gamma-\beta)(1-\gamma)+(1-\gamma-\beta)k_2)+(2(1-\gamma-\beta)+2\alpha(1-\gamma))w_2}{4(1-\gamma-\beta)+\alpha(4(1-\gamma-\beta)+2\beta)},\\ \Rightarrow\ & w_2<h_{O2}(e),\\ &\text{where } h_{O2}(e)=\frac{(2(1-\gamma-\beta)(1-\gamma+t)+\alpha(2(1-\gamma-\beta)(1-\gamma)+(1-\gamma-\beta)k_2))\beta-\left(\sqrt{\frac{4\beta(1-\gamma)(1-\gamma-\beta)e}{\alpha}}+(1-\gamma-\beta)k_2\right)(4(1-\gamma-\beta)+\alpha(4(1-\gamma-\beta)+2\beta))}{2\beta(1+\alpha)(1-\gamma-\beta)}.\end{aligned}$$

Thus, with $s=1$, the brand-name firm's optimal retail price is

$$p_B^{O*}(s=1)=\begin{cases}x_{high}, & \text{if } w_2\ge h_{O2}(e),\\ \hat{p}_B^O, & \text{if } \underline{h}_O<w_2<h_{O2}(e).\end{cases}$$

Based on the above discussions, the brand-name firm chooses p_B^* to maximize her profit by making a comparison between $\pi_B^O(s=0)$ and $\pi_B^O(s=1)$ in overleaping region.

Note that $h_{O2}(e) < h_{O1}(e)$. Then, the optimal retail price of the brand-name firm is

$$p_B^{O*} = \begin{cases} p_B^{O0}, & \text{if } w_2 \geq h_{O1}(e), \text{[note that } m_2 = 0] \\ x_{high}, & \text{if } h_{O2}(e) \leq w_2 < h_{O1}(e), \text{[note that } m_2 = 0] \\ \hat{p}_B^O, & \text{if } (\underline{h}_O)^+ < w_2 < h_{O2}(e), \text{[note that } m_2 > 0] \end{cases}$$

where $x_{high} = \frac{\sqrt{\frac{4\beta(1-\gamma)(1-\gamma-\beta)e}{\alpha}}+((1-\gamma)k_2+\beta(w_2-k_2))}{\beta}$, $p_B^{O0} = \frac{(1+\alpha)(1-\gamma+w_2)+t}{2(1+\alpha)}$, $\hat{p}_B^O = \frac{2(1-\gamma-\beta)(1-\gamma)+\alpha(2(1-\gamma-\beta)(1-\gamma+t)+(1-\gamma-\beta)k_2)+(2(1-\gamma-\beta)+2\alpha(1-\gamma))w_2}{4(1-\gamma-\beta)+\alpha(4(1-\gamma-\beta)+2\beta)}$.

Thus, the condition to prevent counterfeiting is $w_2 \geq w_2^{O,endog}$, where $w_2^{O,endog} = h_{O2}(e)$. That is to say, under Strategy O, $s^* = 0$ if $\hat{p}_B^O \leq \frac{\sqrt{\frac{4\beta(1-\gamma)(1-\gamma-\beta)e}{\alpha}}+((1-\gamma)k_2+\beta(w_2-k_2))}{\beta}$, where $\hat{p}_B^O$ is defined in Equation (17).

Thus, we have the results. ■

B.10.2 Proof of Proposition EC.1.

Part 1: With Strategy D, the overseas supplier is prevented from counterfeiting if $w_2^{D,endog} = h_{D2}(w_1, e)$. Then, we compare the threshold with the counterfeiting prevention condition under our base case. Recall that under our base case, the counterfeiting is prevented if $w_2 \geq w_2^{(0)}$, where $w_2^{(0)} = k_2 + \frac{\alpha(p_2-k_2)(\frac{p_B-p_2}{1-\gamma-\beta}-\frac{p_2}{\beta})-e}{\alpha\left(\frac{p_B-p_2}{1-\gamma-\beta}-\frac{p_B}{1-\gamma}\right)}$.

By making a comparison between the thresholds, that is, $w_2^{D,endog}$ and $w_2^{(0)}$, we obtain,

$$w_2^{D,endog} < w_2^{(0)},$$

$$\Rightarrow \frac{(2(1-\gamma)(1-\gamma-\beta)(1+w_1)+\alpha(2(1-\gamma)+k_2)(1-\gamma-\beta))\beta-\left(\sqrt{\frac{4\beta(1-\gamma)(1-\gamma-\beta)e}{\alpha}}+(1-\gamma-\beta)k_2\right)(4(1-\gamma)(1-\gamma-\beta)+\alpha(4(1-\gamma-\beta)+2\beta))}{2\beta(2(1-\gamma)+\alpha)(1-\gamma-\beta)} < k_2 + \frac{\alpha(p_2-k_2)(\frac{p_B-p_2}{1-\gamma-\beta}-\frac{p_2}{\beta})-e}{\alpha\left(\frac{p_B-p_2}{1-\gamma-\beta}-\frac{p_B}{1-\gamma}\right)},$$

$$\Rightarrow \frac{e}{\alpha\left(\frac{p_B-p_2}{1-\gamma-\beta}-\frac{p_B}{1-\gamma}\right)} - \frac{\left(\sqrt{\frac{4\beta(1-\gamma)(1-\gamma-\beta)e}{\alpha}}\right)(4(1-\gamma)(1-\gamma-\beta)+\alpha(4(1-\gamma-\beta)+2\beta))}{2\beta(2(1-\gamma)+\alpha)(1-\gamma-\beta)} + \frac{(2(1-\gamma)(1+w_1)+\alpha(2(1-\gamma)+k_2))\beta-k_2(4(1-\gamma)(1-\gamma-\beta)+\alpha(4(1-\gamma-\beta)+2\beta))}{2\beta(2(1-\gamma)+\alpha)(1-\gamma-\beta)}$$
$$< k_2 + \frac{\alpha(p_2-k_2)(\frac{p_B-p_2}{1-\gamma-\beta}-\frac{p_2}{\beta})}{\alpha\left(\frac{p_B-p_2}{1-\gamma-\beta}-\frac{p_B}{1-\gamma}\right)}.$$

We define $e_1^{D,endog}$ and $e_2^{D,endog}$ as two solutions of e satisfying $w_2^{D,endog} = w_2^{(0)}$, where $e_1^{D,endog} \leq e_2^{D,endog}$. Note that if $\frac{(2(1-\gamma)(1+w_1)+\alpha(2(1-\gamma)+k_2))\beta-k_2(4(1-\gamma)(1-\gamma-\beta)+\alpha(4(1-\gamma-\beta)+2\beta))}{2\beta(2(1-\gamma)+\alpha)(1-\gamma-\beta)} - k_2 - \frac{\alpha(p_2-k_2)(\frac{p_B-p_2}{1-\gamma-\beta}-\frac{p_2}{\beta})}{\alpha\left(\frac{p_B-p_2}{1-\gamma-\beta}-\frac{p_B}{1-\gamma}\right)} < 0$, then, the two solutions for $w_2^{D,endog} = w_2^{(0)}$ must exist and satisfy $e_1^{D,endog} < 0$ and $e_2^{D,endog} > 0$.

Thus, from $w_2^{D,endog} < w_2^{(0)}$, we have, $(e_1^{D,endog})^+ < e < (e_2^{D,endog})^+$.

Part 2: With Strategy O, the overseas supplier is prevented from counterfeiting if $w_2^{O,endog} = h_{O2}(e)$. Then, we compare the threshold with the counterfeiting prevention condition under our base case. Recall that under our base case, the counterfeiting is prevented if $w_2 \geq w_2^{(0)}$, where $w_2^{(0)} = k_2 + \frac{\alpha(p_2-k_2)(\frac{p_B-p_2}{1-\gamma-\beta}-\frac{p_2}{\beta})-e}{\alpha\left(\frac{p_B-p_2}{1-\gamma-\beta}-\frac{p_B}{1-\gamma}\right)}$).

By making a comparison between the thresholds, that is, $w_2^{O,endog}$ and $w_2^{(0)}$, we obtain,

$$w_2^{O,endog} < w_2^{(0)},$$

$$\Rightarrow \frac{(2(1-\gamma-\beta)(1-\gamma+t)+\alpha(2(1-\gamma-\beta)(1-\gamma)+(1-\gamma-\beta)k_2))\beta-\left(\sqrt{\frac{4\beta(1-\gamma)(1-\gamma-\beta)e}{\alpha}}+(1-\gamma-\beta)k_2\right)(4(1-\gamma-\beta)+\alpha(4(1-\gamma-\beta)+2\beta))}{2\beta(1+\alpha)(1-\gamma-\beta)} < k_2 + \frac{\alpha(p_2-k_2)(\frac{p_B-p_2}{1-\gamma-\beta}-\frac{p_2}{\beta})-e}{\alpha\left(\frac{p_B-p_2}{1-\gamma-\beta}-\frac{p_B}{1-\gamma}\right)},$$

$$\Rightarrow \frac{e}{\alpha\left(\frac{p_B-p_2}{1-\gamma-\beta}-\frac{p_B}{1-\gamma}\right)} - \frac{\left(\sqrt{\frac{4\beta(1-\gamma)(1-\gamma-\beta)e}{\alpha}}\right)(4(1-\gamma-\beta)+\alpha(4(1-\gamma-\beta)+2\beta))}{2\beta(1+\alpha)(1-\gamma-\beta)} + \frac{(2(1-\gamma+t)+\alpha(2(1-\gamma)+k_2))\beta-k_2(4(1-\gamma-\beta)+\alpha(4(1-\gamma-\beta)+2\beta))}{2\beta(1+\alpha)}$$
$$< k_2 + \frac{\alpha(p_2-k_2)(\frac{p_B-p_2}{1-\gamma-\beta}-\frac{p_2}{\beta})}{\alpha\left(\frac{p_B-p_2}{1-\gamma-\beta}-\frac{p_B}{1-\gamma}\right)}.$$

We define $e_1^{O,endog}$ and $e_2^{O,endog}$ as two real-value solutions of e satisfying $w_2^{O,endog} = w_2^{(0)}$, where $e_1^{O,endog} \leq e_2^{O,endog}$. Note that if $\frac{(2(1-\gamma+t)+\alpha(2(1-\gamma)+k_2))\beta-k_2(4(1-\gamma-\beta)+\alpha(4(1-\gamma-\beta)+2\beta))}{2\beta(1+\alpha)} - k_2 - \frac{\alpha(p_2-k_2)(\frac{p_B-p_2}{1-\gamma-\beta}-\frac{p_2}{\beta})}{\alpha\left(\frac{p_B-p_2}{1-\gamma-\beta}-\frac{p_B}{1-\gamma}\right)} < 0$, then, the two solutions for $w_2^{O,endog} = w_2^{(0)}$ must exist and satisfy $e_1^{O,endog} < 0$ and $e_2^{O,endog} > 0$.

Thus, from $w_2^{O,endog} < w_2^{(0)}$, we have, $(e_1^{O,endog})^+ < e < (e_2^{O,endog})^+$.

To summarize, based on the discussions under strategies D and O, we have the following sufficient conditions:

(i) Under Strategy D, if $(e_1^{D,endog})^+ < e < (e_2^{D,endog})^+$, then, $w_2^{D,endog} < w_2^{(0)}$;

(ii) under Strategy O, if $(e_1^{O,endog})^+ < e < (e_2^{O,endog})^+$, then, $w_2^{O,endog} < w_2^{(0)}$;

where

$$\begin{aligned}
&w_2^{D,endog} = \frac{(2(1-\gamma)(1-\gamma-\beta)(1+w_1)+\alpha(2(1-\gamma)+k_2)(1-\gamma-\beta))\beta-\left(\sqrt{\frac{4\beta(1-\gamma)(1-\gamma-\beta)e}{\alpha}}+(1-\gamma-\beta)k_2\right)(4(1-\gamma)(1-\gamma-\beta)+\alpha(4(1-\gamma-\beta)+2\beta))}{2\beta(2(1-\gamma)+\alpha)(1-\gamma-\beta)},\\
&e_1^{D,endog} \text{ and } e_2^{D,endog} \text{ are the solutions of } e \text{ satisfying } w_2^{D,endog} = w_2^{(0)}, \text{ and } e_1^{D,endog} \leq e_2^{D,endog};\\
&w_2^{O,endog} = \frac{(2(1-\gamma-\beta)(1-\gamma+t)+\alpha(2(1-\gamma-\beta)(1-\gamma)+(1-\gamma-\beta)k_2))\beta-\left(\sqrt{\frac{4\beta(1-\gamma)(1-\gamma-\beta)e}{\alpha}}+(1-\gamma-\beta)k_2\right)(4(1-\gamma-\beta)+\alpha(4(1-\gamma-\beta)+2\beta))}{2\beta(1+\alpha)(1-\gamma-\beta)},\\
&e_1^{O,endog} \text{ and } e_2^{O,endog} \text{ are the solutions of } e \text{ satisfying } w_2^{O,endog} = w_2^{(0)}, \text{ and } e_1^{O,endog} \leq e_2^{O,endog}.
\end{aligned} \tag{18}$$

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