Converting Counterfeiters in Emerging Markets to Authorized Suppliers: A New Anti-Counterfeiting Measure

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Abstract: In recent years, "super fakes," i.e., high-quality counterfeits, have gained popularity. The ability of super fake manufacturers to produce high-quality products has inspired a novel anti-counterfeiting measure: *converting counterfeiters to authorized suppliers.* We develop a game-theoretic model to examine the interactions between a brand-name firm with a licit home supplier and a counterfeiter who can be potentially converted to an authorized overseas supplier. Our analysis leads to three main results. First, the brand-name firm can convert the counterfeiter to a licit supplier through either dual sourcing or single sourcing when the cost differential between two suppliers is moderate to high, or the quality perception differential between brand-name products produced by two suppliers is low. However, this conversion does not necessarily prevent counterfeiting unless the penalty from law enforcement is stringent. Our paper recommends that brand-name firms strategically use their wholesale pricing and sourcing decisions to establish socially responsible supply chain operations and prevent counterfeiting. Second, while dual sourcing is known for its risk diversification, our study identifies another benefit previously not reported in the literature: mitigating counterfeit risks. Dual sourcing can be more effective than single sourcing as the quality perception of super fakes approaches that of brand-name products, and its effectiveness becomes more pronounced when the brand-name firm offers wholesale price contracts in a sequential order. Conversely, single sourcing is preferable under conditions of high cost differential and low quality perception differential. Lastly, converting counterfeiters to authorized suppliers can reduce consumer surplus and does not improve social surplus unless authorities enforce high penalties on counterfeiters or the cost differential between two suppliers is substantial.

Key words: Anti-counterfeiting; super fakes; global operations management; game theory

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1 Introduction

Counterfeits are illegal products that imitate and infringe on the brands of genuine items. Globalization has significantly expanded the range of products being counterfeited, extending from luxury goods (e.g., fashion apparel and watches) to other consumer products (e.g., electronics and stationery). According to the

Organisation for Economic Cooperation and Development (OECD), in 2019 alone, imports of fake products into the European Union (EU) increased to 6.8% of all imports, and globally, the value of counterfeit and pirated merchandise reached USD 509 billion, representing about 3.3% of world trade (UNECE 2023).

Although industries and governments have been actively working to combat counterfeiting, companies continue to suffer significant trademark infringement from counterfeiters. For years, the luxury goods industry has invested heavily in fighting against counterfeiters by encouraging governments to strengthen regulations and law enforcement to seize counterfeit products, and running public awareness campaigns on the risks of purchasing counterfeits (Fontana et al. 2019). In the United States, the government has been advocating for stronger global law enforcement on trademarks and intellectual property (IP) rights. These measures aim to drive counterfeiters out of the market. However, in some developing economies, despite the passage of anti-counterfeiting laws, enforcement remains weak. For instance, the Turkish parliament passed regulations against counterfeits in 2016, which include prison terms and steep fines. Nonetheless, these laws have not effectively curbed counterfeiting, and their overall impact remains uncertain (Smith 2018). In China, local governments have established laws to crack down on fake products, but in some cases, the incentive to enforce these laws is insufficient. For example, the “fake shoe market” has become an invisible pillar of the local economy in Putian, China (Chen 2017).

With advancements in manufacturing technology, the quality of counterfeits is steadily improving (Yao 2014). As quoted by Alibaba’s Jack Ma, “fake products today are of better quality and better price than the real names” (Dou 2016). In recent years, many counterfeiters have demonstrated the capability to produce high-quality products. Turkish counterfeit Louis Vuitton (LV) bags, for instance, are notorious for being high-end ”genuine fakes” because they are made from leather sourced from the same suppliers as genuine bags and crafted by experienced artisans. This makes these high-quality fake bags difficult to distinguish from the originals (Letsch 2011, Iredale 2024). Similarly, Chinese imitations, which were once known for their low quality, are now being produced as “super fakes”: products of such high quality that even experts have trouble differentiating them from genuine items (Mau 2018). The Economist (2022) points out that the quality of counterfeit consumer goods has reached unprecedented levels.

Due to the improved manufacturing capabilities of counterfeiters, we observe a practice that some brand-name firms outsource their production to counterfeiters, *converting them to authorized suppliers*. In the luxury goods industry, for example, Balenciaga’s Triple S sneakers were initially made in Italy but are now produced in factories in Putian, China. A Balenciaga official explained that the key reason for outsourcing to China was that Chinese factories “have the savoir-faire and capacity to produce a lighter shoe” (Silbert 2018). In the consumer goods industry, Japanese stationery maker Kokuyo partnered with the Chinese “shanzhai” stationery brand Gambol, which had previously imitated Kokuyo’s famous Campus brand and sold the knockoff at much lower prices in over 5,000 retail stores in China. By collaborating with its counterfeiter, Kokuyo has successfully increased its market share in China (Sugawara 2015). Similarly, Honda established a joint venture with Hainan Sundiro Motorcycle CO., a company that used to produce Honda knockoffs. The motivation behind this collaboration is that the Chinese company can produce parts at one-fourth the cost of Honda's production (Zaun and Leggett 2001).

Converting counterfeiters into authorized overseas suppliers has various implications. For brand-name firms, outsourcing to overseas suppliers who were previously counterfeiters may have two benefits. First, if counterfeiters can produce high-quality products at low costs, it will save production costs for brand-name firms. Second, converting counterfeiters to authorized suppliers may help brand-name firms mitigate the risks of counterfeiting in overseas markets and possibly capture larger market shares. This is because former counterfeiters, now authorized suppliers, may become less likely to produce counterfeit goods. However, not all brand-name firms favor such partnerships due to potential damage to their brand reputation. For instance, some consumers in China expressed dissatisfaction when Balenciaga moved the production of their Triple S sneakers to Putian, a city notoriously known for its counterfeit producers (Pan 2018). Similarly, despite the FDA's assurance about the quality of drugs regardless of manufacturing locations, there are still consumers concerned about generic drugs made in India (Villa et al. 2023). For counterfeiters, becoming authorized suppliers for brand-name firms can help them secure profits and avoid lawsuits and penalties from law enforcement if they stop counterfeiting. However, counterfeiters may not always be willing to become authorized suppliers. For example, according to our conversations with leather counterfeit producers in the Grand Bazaar of Istanbul in Turkey, some counterfeiters of high-quality fake bags are not willing to become authorized suppliers for genuine brands due to a significant cut in their profit margin in the contracts offered by brand-name firms. Thus, brand-name firms need prescriptions regarding when and how they can convert counterfeiters to licit suppliers and reduce the risk of counterfeiting.

Motivated by the above observations, in this work, we consider the strategy of converting counterfeiters who are capable of producing high-quality products to authorized overseas suppliers. We build a game-theoretical model and study the following three research questions:

[1] Considering the option of converting a counterfeiter to an authorized overseas supplier, what is the equilibrium sourcing strategy for a brand-name firm? Under what conditions is the counterfeiter willing to be authorized as an overseas supplier?

[2] Under what conditions would a counterfeiter cease selling counterfeits upon becoming an authorized overseas supplier? Specifically, which sourcing strategy effectively mitigates the risk of counterfeiting?

[3] Does converting the counterfeiter to an authorized overseas supplier benefit consumers or society?

To examine these questions, we develop a model that captures interactions between a brand-name firm with a home supplier in the home market and a counterfeiter who produces "super fakes" in the overseas market. The brand-name firm may outsource production to the home supplier and/or the counterfeiter via wholesale-price contracts, selling the brand-name product in both home and overseas markets. If the counterfeiter sells counterfeits in the overseas market (regardless of whether the counterfeiter accepts or rejects the contract from the brand-name firm), the counterfeiter risks facing the penalty from law enforcement. In our paper, we assume that the counterfeiter sells non-deceptive counterfeits, meaning consumers know they are purchasing counterfeits at the time of sale (e.g., Grossman and Shapiro 1988, Zhang et al. 2012, Cho et al. 2015, Gao et al. 2016, Yi et al. 2022). As observed from our motivating examples, counterfeiters capable of producing high-quality products are generally non-deceptive. Thus, brand-name firms consider them as potential suppliers rather than deceptive counterfeiters who lack such capability.

The brand-name firm's possible sourcing strategies can be classified into four types: single sourcing from the home supplier, dual sourcing, single sourcing from the overseas supplier, and no sourcing. We develop the conditions that lead to each of these four sourcing strategies as game equilibrium. We also explore key modeling features that affect the equilibrium sourcing strategies and the effectiveness of preventing counterfeit sales. These factors include the cost differential between the home supplier and the overseas supplier converted from the counterfeiter, the consumers' quality perception differential between brand-name products produced by two suppliers (to capture the loss of brand value when the brand-name firm sources from the overseas supplier that was previously a counterfeiter), the consumers' quality perception of a counterfeit product, and the penalty from law enforcement. Our main findings are summarized as follows:

(1) Equilibrium sourcing strategies: When the cost differential between two suppliers or the penalty from law enforcement is intermediate to high, the brand-name firm can convert the counterfeiter through dual sourcing or single sourcing. Otherwise, the brand-name firm may lack the incentive to convert the counterfeiter, owing to minimal cost savings and the necessity for a significantly high wholesale price to incentivize the counterfeiter. Furthermore, as the quality perception differential decreases, which reflects a smaller loss in brand value from converting the counterfeiter, the counterfeiter will be more likely to be converted through dual sourcing or single sourcing.

(2) Preventing counterfeit sales: Our results demonstrate that although an intermediate to high cost differential or a low quality perception differential makes the conversion more likely, it does not necessarily prevent the overseas supplier from engaging in counterfeiting. The penalty from law enforcement becomes the primary factor in further preventing the authorized overseas supplier from counterfeiting. Additionally, we identify another benefit of dual sourcing — mitigating counterfeit risks — in addition to its role of risk diversification studied in the literature. It can be more effective than single sourcing as the quality perception of super fakes approaches that of brand-name products, and its effectiveness becomes even more pronounced when the brand-name firm offers wholesale contracts in a sequential manner. Conversely, single sourcing is preferable under conditions of high cost differential and low quality perception differential. As the quality perception of counterfeit products approaches that of brand-name products, as seen with super fakes, the threat of counterfeiting increases. Our study shows that the brand-name firm can strategically set wholesale prices to prevent counterfeiting, which is an effective anti-counterfeiting strategy for super fakes.

(3) Impacts on consumer and social surplus: We find that converting the counterfeiter to an overseas supplier may hurt consumer surplus and does not always improve social surplus. This is because consumers tend to prefer products from the home supplier over those from the overseas supplier. More importantly, converting the counterfeiter reduces competition in the overseas market, leading to a surplus loss. When the penalty from law enforcement or the cost differential between two suppliers is high, converting the counterfeiter benefits society. Otherwise, caution should be exercised with this conversion strategy.

The rest of this paper is organized as follows. Section 2 reviews the related literature. In Sections 3 and 4, we present the model, the equilibrium, and main results and insights of the paper. Section 5 discusses the impacts on the profits of firms, consumer surplus, and social surplus. Section 6 explores two extensions of our base model. Section 7 concludes the paper. We present a total of four extensions to the base model in E-Companion A, and all proofs are relegated to E-Companion B.

2 Literature Review

The literature on product counterfeiting is extensive. Grossman and Shapiro (1988) examine the status and quality attributes of brand-name products, analyzing both the positive and normative effects of counterfeiting. Qian (2008), using panel data from Chinese shoe companies, finds that original producers tend to offer higher quality products at higher prices in response to counterfeit entry. Qian (2014) discusses the dual roles of counterfeits as both advertising and substitutes for brand-name products of varying quality levels. Qian et al. (2015) explore strategies for authentic firms to combat counterfeiters by enhancing experiential quality (e.g., functionality) and searchable quality (e.g., appearance). Gao et al. (2016) analyze the entry decisions of copycats, considering both physical resemblance and product quality features. Pun and DeYong (2017) study the competition between authentic manufacturers and copycat firms in the context of impatient consumers with strategic behavior. Jin et al. (2023) discuss the impacts of copycatting and network externality on a retailer's strategic inventory. Chen et al. (2022) find that the benefits of consumer wealth status signaling to the firm are neutralized by the presence of counterfeits. Yuan et al. (2022) discuss how manufacturers determine information disclosure strategies about product fit, considering the potential risk of supplier copycatting. Gao et al. (2023) consider the direct and indirect online channels of an authentic luxury brand, showing that sharing the same online channel with counterfeiters can improve consumer surplus. Peinkofer and Jin (2023) empirically examine the impact of counterfeit products on consumer perception of product quality. Ding et al. (2024) find that the branded firm may benefit from the counterfeit competition by considering a price signaling game. In our paper, we focus on the emerging "supper fakes" produced by non-deceptive counterfeiters, who have the capability to produce brand-name products of high quality. Apart from the competition between the brand-name firm and the counterfeiter analyzed in the literature, we investigate the "collaboration" between the brand-name firm and the counterfeiter, i.e., the brand-name firm converts the counterfeiter to an authorized overseas supplier by adopting different sourcing strategies. In addition, we differentiate the brand-name products produced by the licit home supplier and the overseas supplier converted from the counterfeiter by using consumers' quality perception differential, which captures the loss of brand value when the brand-name firm sources from the overseas supplier that was previously a counterfeiter. This new element of quality perception differential is not considered in the literature. Furthermore, we allow the authorized overseas supplier to decide, endogenously, whether to sell counterfeits.

There are several strategies to combat counterfeiting discussed in the literature. Grossman and Shapiro (1988) examine the effect of enforcement policy to combat foreign counterfeits with low quality for international trade. Zhang et al. (2012) investigate strategies for brand-name companies to fight non-deceptive counterfeiting by raising consumers' awareness of intellectual properties and the potential harm of counterfeits or pushing the government for enforcement. Cho et al. (2015) study the effectiveness of different approaches for a brand-name firm competing with deceptive and non-deceptive counterfeiters, including law enforcement efforts and consumer education. Gao et al. (2016) show that higher quality and enhancement of status image through advertising can prevent the copycat from entering the market. Yi et al. (2022) discuss the supply chain members' anti-counterfeiting efforts such as enforcing the closure of factories supplying counterfeits and educating consumers. Li et al. (2023) examine the customer-to-customer platform's inspection service to detect counterfeits. Gao and Wu (2023) consider the retailers who sell both authentic products and counterfeits, and the manufacturer's strategic response to regulation, such as a penalty for counterfeit sales imposed by regulators. There are also papers considering anti-counterfeiting technologies that help consumers distinguish genuine products from fakes. Gao (2018) examine how pharmaceutical firms adopt overt anti-counterfeiting technologies to increase the fixed entry cost to combat deceptive counterfeiters. Pun et al. (2021) and Shen et al. (2022) discuss the value of blockchain technology adoption to combat deceptive counterfeits. Yao et al. (2023) analyze the effect of an authentic company's anti-counterfeit technology and regulatory authorities' law enforcement on combating deceptive counterfeit products. Unlike the existing literature, we study the innovative anti-counterfeiting measure that the brand-name firm can use to combat the counterfeiter by converting her to an authorized overseas supplier. Even after the counterfeiter is converted by accepting the contract, she may still decide to sell counterfeits in the overseas market. We further examine the effectiveness of this new measure in preventing counterfeiting and improving welfare.

Related to counterfeiting, gray markets (or parallel importing) are unauthorized channels in which retailers sell brand-name products (e.g., Ahmadi and Yang (2000), Hu et al. (2013), Ahmadi et al. (2015, 2017), Autrey et al. (2015), Shao et al. (2016)). Unlike counterfeits produced or sold by unauthorized imitators, products in gray markets are genuine and sourced from authorized sellers. Recent work by Wang et al. (2020) provides an overview of this topic.

Our paper is also related to the literature on firms' global sourcing decisions. Feng and Lu (2012) consider production cost and contract negotiations between manufacturers and suppliers. Wu and Zhang (2014) consider supply lead time to capture the trade-off between cost and responsiveness. Sun et al. (2010) study the firm's technology outsourcing strategy to a foreign firm with imitation risk. Berry and Kaul (2015) empirically examine how foreign knowledge-seeking impacts the firm's global sourcing choices between offshore integration and offshore outsourcing. Guo et al. (2016) consider socially conscious consumers and analyze a buyer's sourcing decision between a responsible supplier and a supplier with violation risk. Orsdemir et al. (2019) study how firms decide between vertical integration and horizontal sourcing under corporate social and environmental responsibility violation risk of suppliers and demand externalities. Hu et al. (2020) study when an innovator may source from a competitor-supplier under technical and non-technical innovation spillover risks. Pun and Hou (2022) consider a manufacturer's outsourcing decision of production tasks to a supplier with imitation risk. Skowronski and Benton (2017) empirically evaluate how brand-name firms protect IP from poaching when outsourcing to suppliers in countries with weak IP rights. A stream of literature considers sourcing decisions with suppliers who potentially sell products through a direct channel to consumers (e.g., Arya et al. 2007, Li et al. 2014, and Ha et al. 2016). Different from the above literature, we focus on sourcing decisions in the setting where a counterfeiter produces and sells super fakes, and may be converted to an authorized overseas supplier. In our paper, the counterfeiter decides on whether to accept the brand-name firm's contract for being converted to an authorized supplier, and whether to sell the counterfeit in the overseas market even after the conversion. When selling the counterfeit, the counterfeiter faces a law enforcement penalty from the local government in the overseas market.

Our paper is related to the literature about the coopetition relationship between upstream and downstream firms. Qi et al. (2024), and the references therein, present a comprehensive review of the coopetition literature. Ha et al. (2016) investigate a scenario that the upstream manufacturer sells directly to consumers, thereby competing with its retail partners. Mantovani and Ruiz-Aliseda (2016) examine a setting when firms collaborate with producers formed by their components. Pei et al. (2023) consider manufacturers adopting the "coopetitive" strategy by opening a platform for operations involving after-sales service. Different from these studies, our research focuses on the brand-name firm who can offer a wholesale price to convert the counterfeiter to operate in a "coopetitive" manner. This coopetitive strategy helps the brand-name firm to not only enjoy the cost advantage benefit, but also to mitigate the competition in the overseas market. Our study contributes to existing research on anti-counterfeiting by depicting a new coopetition relationship between two firms and by showing a comprehensive analysis of the impact from the strategy adoption.

In summary, our model incorporates the recent trend of high-quality counterfeits to examine interactions between the brand-name firm and two types of potential suppliers, one of which is converted from the counterfeiter. To the best of our knowledge, our paper is the first to study combating counterfeiting through conversion. Based on our model, we derive novel insights into anti-counterfeiting and global sourcing.

3 Model

In this section, we describe the model setting and the sequence of events before formulating the consumer utility and expected profit for each firm. Table 1 presents the notation used in the model.

Table 1 Model Notation

Decision Variables	
w_i	Wholesale price for supplier $i \in \{1,2\}$
d_i	Whether supplier i accepts the contract, $d_i \in \{0,1\}$, $i \in \{1,2\}$
s	Whether the overseas supplier sells counterfeits, $s \in \{0,1\}$
Parameters	
θ	Taste of consumers, $\theta \sim U[0,1]$
α	Overseas market size, $\alpha > 0$
q_j	Perceived quality of product $j \in \{B,2\}$
β	Quality perception of the counterfeit, $\beta \in (0,1)$
γ	Quality perception differential between brand-name products produced by the home and overseas suppliers, $\gamma \in [0, 1-\beta)$
p_j	Retail price of product $j \in \{B,2\}$
k_i	Unit cost of supplier $i \in \{1,2\}$, $k_1 \geq k_2 > 0$
Δ	Cost differential between two suppliers, $\Delta = k_1 - k_2$
e	Expected penalty from law enforcement for counterfeiting, $e \geq 0$
t	Unit transportation cost between markets, $t > 0$
Profits and Demands	
π_B	Expected profit of the brand-name firm
π_i	Expected profit of supplier $i \in \{1,2\}$
m_{Bi}	Demand of the brand-name product in market $i \in \{1,2\}$
m_2	Demand of the counterfeit in the overseas market

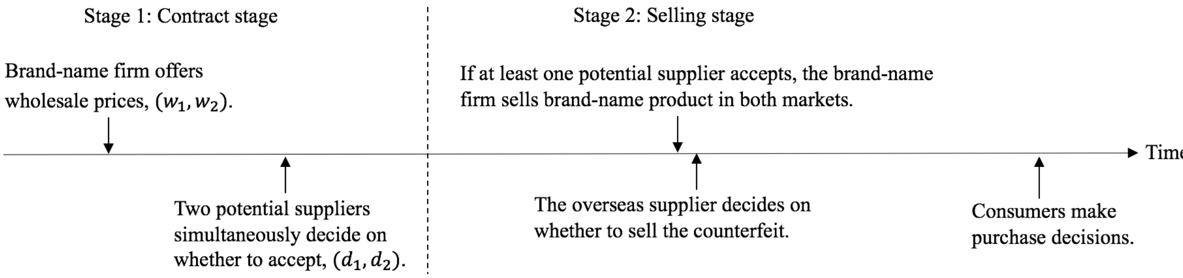
3.1 Model Setting and Sequence of Events

We consider a setting where a brand-name firm ('he') potentially outsources production to a home supplier and/or an overseas supplier converted from a counterfeiter, selling the brand-name product to consumers in both home and overseas markets. The size of the home market is normalized to 1, and the overseas market size is $\alpha (> 0)$. Each market has a potential supplier ('she'). Subscript $i = 1$ indicates the home supplier and $i = 2$ indicates the counterfeiter who can be converted to an authorized overseas supplier. The marginal cost of supplier i is k_i, with $k_1 \geq k_2 > 0$, reflecting the lower marginal cost of the counterfeiter. We define $\Delta = k_1 - k_2$ as the cost differential between two suppliers.

The brand-name firm engages suppliers through wholesale-price contracts. As shown in Figure 1, the model features two stages of decisions: the contract stage (stage 1) and the selling stage (stage 2). At the beginning of the first stage, the brand-name firm offers a wholesale price w_i to supplier i. Both suppliers then simultaneously decide whether to accept the contracts. We extend the analysis by featuring sequential contract offering in Section 6.1. Let $d_i \in \{0,1\}$ denote supplier i's contract acceptance decision, where $d_i = 0$ denotes rejection, and $d_i = 1$ represents acceptance. We assume that if a supplier is indifferent between accepting and rejecting the contract, she will accept it.

In the second stage, if at least one supplier accepts the contract, the brand-name firm sells the brand-name product at a retail price p_B in the two markets. To comply with anti-dumping laws, the retail price of the brand-name product is set to be equal in both markets (Macrory 2005, Park et al. 2016 and Park et al. 2017). Note that $p_B \geq k_i$ for $i \in \{1,2\}$ ensures that the brand-name firm secures non-negative marginal profit when any potential supplier accepts the contract.

Figure 1 Sequence of Decisions and Events

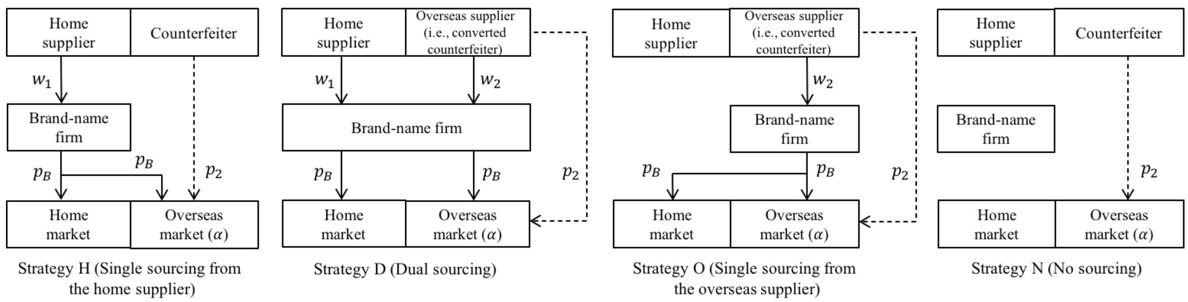


In our paper, the overseas supplier may decide to sell counterfeits regardless of her contract acceptance decision. Specifically, in the second stage, the overseas supplier decides on whether to sell counterfeits in the overseas market at the retail price, p_2. We use $s \in \{0,1\}$ to denote this decision; $s = 1$ means she sells counterfeits, and $s = 0$ means she does not. If the overseas supplier sells counterfeits, she risks being caught and paying a penalty due to law enforcement. Based on practical observations, we assume the penalty from law enforcement in the home market is high enough to prevent the home supplier from producing or selling counterfeits. Conversely, the penalty from law enforcement in the overseas market may not be sufficient to fully prevent counterfeiting, and we denote the expected penalty as e (≥ 0), accounting for the probability of being caught. To avoid the uninteresting case where the overseas supplier never sell counterfeits if she rejects the contract, we assume the penalty is not too high, i.e., $e < \frac{\alpha(\beta p_B - p_2)(p_2 - k_2)}{(1-\beta)\beta}$. In Online Supplement A.4, we consider a scenario where the law enforcement penalty depends on the revenue from selling counterfeits and show that our main findings still hold. We assume the same penalty regardless of whether the counterfeiter is converted to an authorized supplier or not. When an "additional" monitoring system is employed by the brand-name firm towards the authorized overseas supplier, the firm can impose an extra penalty and incur a monitoring cost to discourage the overseas supplier from selling counterfeits. Depending on this cost-benefit trade-off, the additional monitoring system may make the conversion strategy more or less favorable.

We use $j = B$ to denote the brand-name product and $j = 2$ for the counterfeit. At the end of stage 2, consumers in each market make their purchase decisions after observing the prices of the brand-name and counterfeit products. For tractability, we assume that retail prices of the brand-name product and the counterfeit, p_B and p_2, are exogenous. Such an assumption has been adopted in the counterfeiting literature (e.g., Gao et al. 2016, Gao et al. 2023). Additionally, our analysis of retail price data for 30 newly launched luxury brand products over three months supports this assumption, as brand-name firms do not adjust prices solely to compete with overseas counterfeits. To check robustness, in Section 6.2 and Online Supplement A.3, we extend our base model to consider endogenous retail price(s).

Figure 2 depicts the resulting four supply chain structures based on the brand-name firm's sourcing strategies: single sourcing from the home supplier (H), dual sourcing (D), single sourcing from the overseas supplier (O), and no sourcing (N).

Figure 2 Supply Chain Structures



Note. Solid lines represent the product flow of the authentic brand-name product and dash lines represent the product flow of the counterfeit.

Strategy H: When the home supplier accepts the contract and the overseas supplier rejects the contract, the brand-name firm satisfies demands in both home and overseas markets with the product sourced from the home supplier. He potentially competes with the counterfeiter if the overseas supplier sells the counterfeit in the overseas market.

Strategy D: When both the home supplier and the overseas supplier accept their contracts, the brand-name firm satisfies the home (overseas) market demand with the product sourced from the home (overseas) supplier. The overseas supplier may or may not sell the counterfeit. Since the brand-name product from the overseas supplier has a lower perceived quality than the one made by the home supplier, consumers would opt for the one made by the home supplier if these two brand-name products are available in the same market. Thus, there is no cross-market shipment in our model under this strategy.

Strategy O: When the home supplier rejects the contract and the overseas supplier accepts the contract, the brand-name firm satisfies demands in both home and overseas markets with the product produced by the authorized overseas supplier. The overseas supplier may or may not sell the counterfeit.

Strategy N: When both suppliers reject their contracts, the brand-name firm does not enter markets. The overseas supplier may sell the counterfeit in the overseas market. This strategy is unlikely in practice, and later we show that it is not an equilibrium strategy.

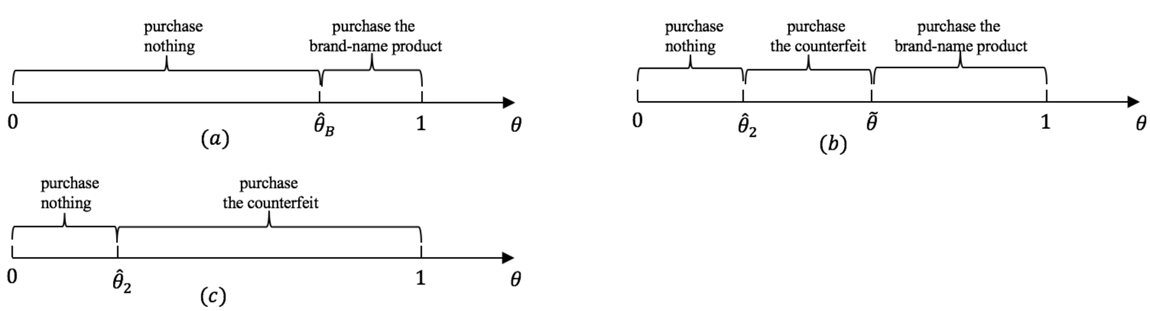
3.2 Consumer Utility

In our model, each consumer demands at most one unit of the product and does not purchase across markets. Consumers make their purchase decisions to maximize their utility. A consumer's utility from purchasing product $j \in \{B, 2\}$ is given by $u_j = \theta q_j - p_j$, where θ denotes the consumer's taste, uniformly distributed between 0 and 1, i.e., $\theta \sim U[0,1]$; q_j denotes the "perceived quality" of product j, and p_j is the retail price of product j. Note that perceived quality can differ from actual quality. For example, socially conscious consumers may be reluctant to purchase from a brand with an infamous reputation, which lowers the perceived value despite the actual quality. Because our study examines non-deceptive counterfeiting and the manufacturing location information is usually available to consumers through product packages, they know whether the product is purchased from a counterfeiter or a brand-name firm and where it is manufactured. For example, Balenciaga's shoe labels distinguish whether they were made in Italy or China, allowing consumers to identify the source (Pan 2018). Thus, the perceived quality can be directly and knowingly derived.

For simplicity, we normalize the actual quality of the brand-name product to 1. The perceived quality of the brand-name product can be different from its actual quality, especially when produced by an overseas supplier converted from a counterfeiter. To capture the possible loss in brand value due to sourcing from a former counterfeiter, we assume no perception discount for the brand-name product produced by the home supplier, and we use γ to represent consumers' quality perception differential between the brand-name products produced by the home and overseas suppliers. Thus, the perceived qualities of the brand-name product produced by the home and overseas suppliers are $q_B = 1$ and $q_B = 1-\gamma$, respectively. For the counterfeit product, even if the actual product quality is not inferior to the authentic one (Wang 2023), consumers perceive it as of lower value. We define a perceived quality discount factor $\beta \in (0,1)$ such that $q_2 = \beta \times 1$. In other words, although high-quality counterfeits imitate the design of brand-name products and are made with premium materials, consumers who knowingly purchase counterfeits receive only a proportion of the brand value from the brand-name products. We further assume $\gamma \leq 1-\beta$ so that $q_B \geq q_2$. To rule out uninteresting cases with zero demand for the brand-name firm or the counterfeiter, we assume $p_B < 1-\gamma$ and $\frac{p_2}{p_B} < \beta < 1-\gamma-(p_B-p_2)$; the former implies that $m_{B1}(d_1,1) > 0$, and the latter implies that $m_{B2}(d_1,1,1) > 0$ and $m_2(d_1,1,1) > 0$.

Strategy H: In the home market, only the brand-name product is available at price p_B. Consumers decide whether to purchase the brand-name product or not. A consumer with taste $\hat{\theta}_B = p_B$ is indifferent between buying and not buying, satisfying $\hat{\theta}_B q_B - p_B = 0$, where $q_B = 1$. Therefore, consumers with $\theta \in \left[\hat{\theta}_B, 1\right]$ purchase the brand-name product, and those with $\theta \in \left[0, \hat{\theta}_B\right)$ purchase nothing (Figure 3(a)).

Figure 3 Consumer Utility Thresholds



Note. (a) The home or overseas market with only the brand-name product; (b) the overseas market with both the brand-name product and the counterfeit; (c) the overseas market with only the counterfeit.

In the overseas market, if the counterfeiter sells the counterfeit, both the brand-name product at price p_B and the counterfeit at price p_2 are available. Consumers decide whether to purchase the brand-name product, the counterfeit, or nothing. A consumer indifferent between the counterfeit and not purchasing has taste $\hat{\theta}_2 = \frac{p_2}{\beta}$, satisfying $\hat{\theta}_2\beta - p_2 = 0$. A consumer indifferent between the brand-name product and the counterfeit has taste $\tilde{\theta} = \frac{p_B - p_2}{1-\beta}$, satisfying $\tilde{\theta} - p_B = \tilde{\theta}\beta - p_2$. Thus, consumers with $\theta \in [\tilde{\theta}, 1]$ purchase the brand-name product, those with $\theta \in [\hat{\theta}_2, \tilde{\theta})$ purchase the counterfeit, and those with $\theta \in [0, \hat{\theta}_2)$ purchase nothing (Figure 3(b)). If the overseas supplier does not sell counterfeits, the scenario mirrors the home market. Consumers with $\theta \in [\hat{\theta}_B, 1]$ purchase the brand-name product, and those with $\theta \in [0, \hat{\theta}_B)$ purchase nothing, where $\hat{\theta}_B = p_B$ (Figure 3(a)).

Strategy D and Strategy O: The brand-name firm sells the brand-name product at p_B in two markets. In the home market, consumers with $\theta \in [\hat{\theta}_B, 1]$ purchase the brand-name product, and those with $\theta \in [0, \hat{\theta}_B)$ purchase nothing, where $\hat{\theta}_B = \frac{p_B}{q_B}$ (Figure 3(a)). Under Strategy D, $q_B = 1$; and under Strategy O, $q_B = 1 - \gamma$. In the overseas market, if the overseas supplier does not sell the counterfeit, consumers with $\theta \in [\hat{\theta}_B, 1]$purchase the brand-name product, and those with $\theta \in [0, \hat{\theta}_B)$ purchase nothing, where $\hat{\theta}_B = \frac{p_B}{1-\gamma}$ (Figure 3(a)). If the overseas supplier sells counterfeits, as illustrated in Figure 3(b), consumers with $\theta \in [\tilde{\theta}, 1]$ purchase the brand-name product, where $\tilde{\theta} = \frac{p_B - p_2}{1-\gamma-\beta}$. Consumers with $\theta \in [\hat{\theta}_2, \tilde{\theta})$ purchase the counterfeit, where $\hat{\theta}_2 = \frac{p_2}{\beta}$. Consumers with $\theta \in [0, \hat{\theta}_2)$ purchase nothing.

Strategy N: In the home market, no products are available for purchase. In the overseas market, only the counterfeit with price p_2 is available. If the counterfeiter sells the counterfeit, consumers with $\theta \in [\hat{\theta}_2, 1]$ purchase the counterfeit, and those with $\theta \in [0, \hat{\theta}_2)$ purchase nothing, where $\hat{\theta}_2 = \frac{p_2}{\beta}$(Figure 3(c)). If the counterfeiter does not sell the counterfeit, no products are available for purchase.

To summarize, given the home supplier's contract acceptance decision d_1, and the counterfeiter's contract acceptance and counterfeit selling decisions d_2 and s, we derive demands for the brand-name product and the counterfeit. Let $(x)^+$ denote $\max(x, 0)$. The demand for the brand-name product in the home market, m_{B1}, is given below:

$$m_{B1}(d_1, d_2) = \begin{cases} 1 - p_B, & \text{if } d_1 = 1, \\ 1 - \frac{p_B}{1-\gamma}, & \text{if } d_1 = 0 \text{ and } d_2 = 1, \\ 0, & \text{otherwise.} \end{cases} \tag{1}$$

The demand for the brand-name product in the overseas market is

$$m_{B2}(d_1, d_2, s) = \begin{cases} \alpha\left(1 - \frac{p_B - p_2}{1-\beta}\right), & \text{if } d_1 = 1,\, d_2 = 0 \text{ and } s = 1, \\ \alpha\left(1 - p_B\right), & \text{if } d_1 = 1,\, d_2 = 0 \text{ and } s = 0, \\ \alpha\left(1 - \frac{p_B - p_2}{1-\gamma-\beta}\right), & \text{if } d_2 = 1 \text{ and } s = 1, \\ \alpha\left(1 - \frac{p_B}{1-\gamma}\right), & \text{if } d_2 = 1 \text{ and } s = 0, \\ 0, & \text{otherwise.} \end{cases} \tag{2}$$

The demand for the counterfeit in the overseas market is

$$m_2(d_1, d_2, s) = \begin{cases} \alpha\left(\frac{p_B - p_2}{1-\beta} - \frac{p_2}{\beta}\right), & \text{if } d_1 = 1,\, d_2 = 0 \text{ and } s = 1, \\ \alpha\left(1 - \frac{p_2}{\beta}\right), & \text{if } d_1 = 0,\, d_2 = 0 \text{ and } s = 1, \\ \alpha\left(\frac{p_B - p_2}{1-\gamma-\beta} - \frac{p_2}{\beta}\right), & \text{if } d_2 = 1 \text{ and } s = 1, \\ 0, & \text{otherwise.} \end{cases} \tag{3}$$

3.3 Expected Profits of Firms

In this subsection, we derive the expected profits of the brand-name firm, the home supplier, and the overseas supplier. We use $t(>0)$ to denote the unit transportation cost borne by the brand-name firm when products are shipped across different markets. This occurs if the brand-name firm chooses Strategy H (using the home supplier for production and selling the product to the overseas market) or Strategy O (using the overseas supplier for production and selling the product to the home market).

Given the wholesale prices and players' decisions (w_1, w_2, d_1, d_2, s), the brand-name firm's expected profit π_B is given by:

$$\begin{aligned}\pi_B(w_1, w_2, d_1, d_2, s) = &\left(p_B - d_1 w_1 - (1-d_1)\, d_2\,(w_2+t)\right) m_{B1}(d_1, d_2)\\ &+ \left(p_B - d_1(1-d_2)(w_1+t) - d_2 w_2\right) m_{B2}(d_1, d_2, s),\end{aligned}$$

where $m_{B1}(d_1, d_2)$ and $m_{B2}(d_1, d_2, s)$ are given in equations (1) and (2), respectively. The two terms represent the expected profits of the brand-name firm in the home and overseas market, respectively.

The home supplier's expected profit π_1 is given as

$$\pi_1(w_1, d_1, d_2, s) = d_1(w_1 - k_1)\, m_{B1}(d_1, d_2) + (1-d_2)\, d_1(w_1 - k_1)\, m_{B2}(d_1, d_2, s).$$

The two terms represent the expected profits of the home supplier from producing the product for the brand-name firm to sell in the home and overseas markets, respectively, if she accepts the contract. If she rejects the contract, we normalize her profit to zero. This assumption can be easily relaxed by considering an outside option with a positive profit for the supplier.

The overseas supplier's expected profit π_2 is given as

$$\begin{aligned}\pi_2(w_2, d_1, d_2, s) = &d_2\left((1-d_1)(w_2-k_2)\, m_{B1}(d_1, d_2) + (w_2-k_2)\, m_{B2}(d_1, d_2, s)\right)\\ &+ s\left((p_2 - k_2)\, m_2(d_1, d_2, s) - e\right),\end{aligned}$$

where $m_2(d_1, d_2, s)$ is given in Equation (3). The first term represents the expected profit of the overseas supplier from producing brand-name products for the home and overseas markets if she accepts the contract. The second term represents the expected profit of the overseas supplier from counterfeiting if she decides to sell counterfeits in the overseas market.

4 Equilibrium Analysis

In this section, we use backward induction to analyze the sequential game between the brand-name firm and the two potential suppliers, as depicted in Figure 1. In Section 4.1, we analyze the contract acceptance decisions of the home and overseas suppliers, as well as the overseas supplier's decision on whether to sell counterfeits. In Section 4.2, we derive the profit of the brand-name firm in the second stage under each possible sourcing strategy and discuss the optimal wholesale prices the brand-name firm should offer to maximize his profit. In Section 4.3, we identify the conditions under which a particular sourcing strategy arises in equilibrium in the first stage and analyze the impact of different factors on the equilibrium. In Section 4.4, we investigate the conditions under which counterfeiting can be prevented in the equilibrium and compare the effectiveness of different sourcing strategies on preventing counterfeiting.

4.1 Suppliers' Best Responses

In the second stage, the overseas supplier determines whether to sell counterfeits. She does not engage in counterfeiting if she generates a higher profit without counterfeiting, i.e., $\pi_2(s=0) \geq \pi_2(s=1)$. The overseas supplier's counterfeiting decision is formalized in Lemma 1.

LEMMA 1. *For given* (w_1, w_2, d_1, d_2),

$$s^*(w_1, w_2, d_1, d_2) = \begin{cases} 0, & \text{if } d_2 = 1 \text{ and } w_2 \geq w_2^{(0)}, \\ 1, & \text{if } d_2 = 1 \text{ and } k_2 \leq w_2 < w_2^{(0)}, \text{ or if } d_2 = 0, \end{cases}$$

where $w_2^{(0)} = k_2 + \frac{(p_2-k_2)(\frac{p_B-p_2}{1-\gamma-\beta} - \frac{p_2}{\beta}) - e}{\frac{p_B-p_2}{1-\gamma-\beta} - \frac{p_B}{1-\gamma}}$.

Lemma 1 implies that after accepting the contract and becoming an authorized supplier ($d_2 = 1$), the overseas supplier is prevented from selling counterfeits ($s^* = 0$) under strategies D and O only when the wholesale price w_2 is high enough, i.e., $w_2 \geq w_2^{(0)}$. If w_2 is low or if the overseas supplier rejects the contract offered ($d_2 = 0$), the overseas supplier sells counterfeits in the overseas market ($s^* = 1$).

Next, given w_1 and w_2, we derive the home and overseas suppliers' optimal contract acceptance decisions. By evaluating the difference in each potential supplier's expected profit between accepting and rejecting the contract, we obtain their optimal decisions as follows:

$$(d_1^*(w_1,w_2), d_2^*(w_1,w_2)) = \begin{cases} (1,1), & \text{if } w_1 \geq k_1,\ w_2 \geq k_2, \\ (1,0), & \text{if } w_1 \geq k_1,\ w_2 < k_2, \\ (0,1), & \text{if } w_1 < k_1,\ \min\{w_2^{(0)}, w_2^{O(1)}\} \leq w_2 < w_2^{(0)} \text{ or } w_2 \geq \max\{w_2^{(0)}, w_2^{O(2)}\}, \\ (0,0), & \text{if } w_1 < k_1,\ w_2 < \min\{w_2^{(0)}, w_2^{O(1)}\} \text{ or } w_2^{(0)} \leq w_2 < \max\{w_2^{(0)}, w_2^{O(2)}\}, \end{cases} \tag{4}$$

where $w_2^{O(1)} = k_2 + \frac{\alpha(p_2-k_2)\left(1-\frac{p_B-p_2}{1-\gamma-\beta}\right)}{(1-\frac{p_B}{1-\gamma})+\alpha(1-\frac{p_B-p_2}{1-\gamma-\beta})}$, $w_2^{O(2)} = k_2 + \frac{\alpha(p_2-k_2)\left(1-\frac{p_2}{\beta}\right)-e}{(1+\alpha)(1-\frac{p_B}{1-\gamma})}$.

Equation (4) indicates that if the wholesale price w_1 for the home supplier is greater than or equal to her production cost k_1, the home supplier accepts the contract. In this case (i.e., when $w_1 \geq k_1$), the overseas supplier accepts the contract if the wholesale price w_2 is greater than or equal to the production cost k_2. When the home supplier rejects the contract, the overseas supplier may reject the contract even when the wholesale price w_2 is higher than k_2, particularly when e is high that leads to the condition $w_2^{O(2)} > w_2^{(0)}$.

4.2 Brand-Name Firm's Optimal Wholesale Prices

In this subsection, we determine the optimal wholesale prices. The brand-name firm chooses wholesale prices w_1 and w_2 to maximize his expected profit by solving the following program:

$$\max_{w_1,w_2} \pi_B(w_1, w_2);\ \text{s.t. } (4)\,.$$

Lemma 2 presents the optimal wholesale prices of the brand-name firm under each possible sourcing strategy. For the analysis below, we develop thresholds for the penalty cost from law enforcement (e). Specifically, e_{D1} (respectively e_{O1}) represents the penalty cost threshold that makes the profits under Strategy D (respectively O) with counterfeiting equal to the same strategy without counterfeiting. As shown in Equation (8) of E-Companion B, e_{D1} and e_{O1} are derived independently of the cost advantage of the overseas supplier (Δ).

LEMMA 2. *The optimal wholesale price(s) of the brand-name firm, which will be accepted by the home and overseas suppliers, satisfies the following:*
(a) under Strategy H, $w_1^H = k_1$*;*
(b) under Strategy D, $w_1^D = k_1$*, and (i)* $w_2^D = k_2$ *and* $s^* = 1$*, if* $e < e_{D1}$*; (ii)* $w_2^D = w_2^{(0)} (> k_2)$ *and* $s^* = 0$*, if* $e \geq e_{D1}$*;*
(c) under Strategy O, (i) $w_2^O = w_2^{O(1)} (> k_2)$ *and* $s^* = 1$*, if* $e < e_{O1}$*; (ii)* $w_2^O = \max\{w_2^{(0)}, w_2^{O(2)}\} (> w_2^{O(1)})$ *and* $s^* = 0$*, if* $e \geq e_{O1}$*.*

In leader-follower games with wholesale-price contracts, the leader can generally extract all the benefits and set a wholesale price equal to the marginal production cost (Cachon 2003). In our model, under strategies H and D, the brand-name firm sources from the home supplier by providing the wholesale price equal to the supplier's marginal cost, i.e., $w_1^H = w_1^D = k_1$. For the overseas supplier, however, there exists the option to sell counterfeit products and compete with the brand-name firm in the overseas market. Therefore, to successfully convert her to an authorized supplier, the brand-name firm may need to offer a wholesale price premium that exceeds the marginal cost k_2. Specifically, the brand-name firm may set $w_2^D > k_2$ and $w_2^O > k_2$, as detailed in the scenarios of Lemma 2 b(ii) and c.

Under strategies D and O, the brand-name firm has two options regarding the wholesale price w_2. He can offer a relatively low wholesale price sufficient to convert the counterfeiter into an authorized supplier while still competing with her as she sells counterfeits in the overseas market, as described in Lemma 2 b(i) and c(i). Alternatively, the brand-name firm can offer a sufficiently high wholesale price to prevent the overseas supplier from engaging in counterfeiting post-conversion, as detailed in Lemma 2 b(ii) and c(ii).

The decision between offering low and high wholesale prices, which leads to the aforementioned two scenarios, depends on the penalty e imposed by law enforcement. Specifically, when e is low ($e < e_{D1}$ in Lemma 2b(i), or $e < e_{O1}$ in Lemma 2c(i)), implying that the profit from selling counterfeits is high, the brand-name firm would need to offer a significantly high wholesale price to prevent the overseas supplier from counterfeiting. This significantly high wholesale price could reduce the brand-name firm's profits. Consequently, the brand-name firm might prefer to compete with the counterfeiter by offering a lower wholesale price as long as it is sufficient to convert her. Conversely, when e is high ($e \geq e_{D1}$ in Lemma 2b(ii), or $e \geq e_{O1}$ in Lemma 2c(ii)), full prevention of counterfeiting becomes feasible and potentially more profitable for the brand-name firm. In these cases, the brand-name firm opts to offer a high wholesale price for both conversion and prevention of counterfeiting.

4.3 Equilibrium Sourcing Strategies

When the brand-name firm decides on his sourcing strategy while facing the counterfeiter who can potentially be converted to an overseas supplier, he considers the following tradeoff: reducing the production cost by sourcing from a low-cost and authorized overseas supplier versus losing brand value by sourcing from a former counterfeiter. If the overall benefit from cost reduction exceeds the overall loss in brand value, the brand-name firm prefers converting the counterfeiter, leading to the preference of strategies D and O; otherwise, the brand-name firm is better off sourcing only from the home supplier, i.e., Strategy H. Converting the counterfeiter to an authorized supplier does not guarantee that the overseas supplier will not produce and sell counterfeits. Under strategies D and O, the brand-name firm's wholesale price decision becomes critical. He now has to offer a premium wholesale price to attract the counterfeiter to accept the contract. While offering a sufficiently high wholesale price mitigates competition by preventing the overseas supplier from counterfeiting after conversion, a relatively smaller wholesale price premium that is sufficient to utilize the counterfeiter as an authentic supplier would still allow her to sell counterfeits in the overseas market.

We next present a proposition that compares the optimal profits of the brand-name firm under the four possible sourcing strategies. The derivations rely on the threshold functions developed for the penalty cost (e) that change with the overseas supplier's cost advantage (Δ). Specifically, f_{DH} represents the threshold of the penalty cost that equates the profits for strategies D and H; f_{DO1} (respectively f_{DO2}) represents the threshold of the penalty cost that equates the profits from Strategy D with counterfeiting (respectively without counterfeiting) and Strategy O without counterfeiting (respectively with counterfeiting). f_{DO3} and f_{DO4} equate the profits of strategies D and O without counterfeiting; these functions differ because of the two different optimal wholesale prices w_2^* featured in Proposition 1(c). Moreover, threshold Δ_{DH} represents the value of overseas supplier's cost advantage, independent of the penalty cost e, that makes the profits of strategies D and H equal. Similarly, threshold Δ_{DO} (respectively Δ_{HO}) represents the value of overseas supplier's cost advantage that makes the profits of strategies D and O (respectively H and O) equal. All these thresholds are derived in equations (7) and (9) of E-Companion B. We denote the optimal wholesale prices for the home and overseas suppliers as w_1^* and w_2^*, respectively.

PROPOSITION 1. *The equilibrium sourcing strategy of the brand-name firm is as follows:*
(a) Strategy H with $w_1^* = k_1$ *if* $e < f_{DH}$ *and* $\Delta < \min\{\Delta_{DH}, \Delta_{HO}\}$*;*
(b) Strategy D with $w_1^* = k_1$*, and*

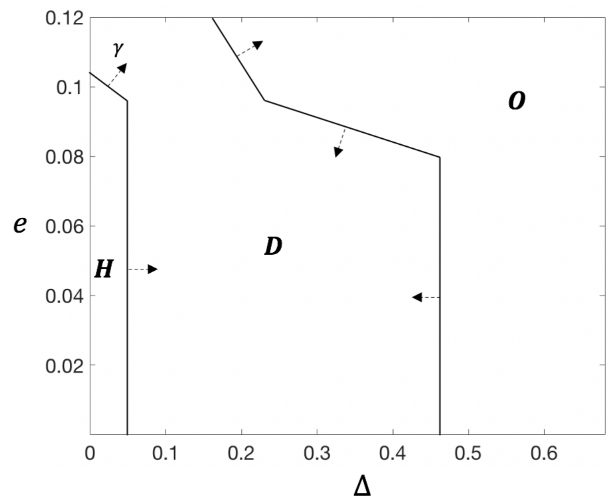
$$w_2^* = \begin{cases} k_2, & \text{if } e \leq \min\{e_{D1}, f_{DO1}\} \text{ and } \min\{\Delta_{DH}, \Delta_{DO}\} \leq \Delta < \Delta_{DO}, \\ w_2^{(0)}, & \text{if } \max\{e_{D1}, f_{DH}, f_{DO2}\} \leq e \leq \min\{e_3, f_{DO3}\}, \text{ or if } e \geq \max\{e_3, f_{DO4}\}; \end{cases}$$

(c) Strategy O with

$$w_2^* = \begin{cases} w_2^{O(1)}, & \text{if } e \leq \min\{e_{O1}, f_{DO2}\} \text{ and } \Delta > \max\{\Delta_{HO}, \Delta_{DO}\}, \\ \max\{w_2^{(0)}, w_2^{O(2)}\}, & \text{if } \max\{e_{O1}, f_{DO1}, f_{DO3}\} < e \leq e_3, \text{ or if } e_3 < e < f_{DO4}. \end{cases}$$

Proposition 1 presents three possible equilibrium outcomes: Strategy H, Strategy D, and Strategy O. Strategy N is not an equilibrium strategy because it is dominated by Strategy H, under which the brand-name firm can earn a non-negative profit from the home market. Figure 4 illustrates how the equilibrium sourcing strategy varies with respect to the cost differential between two suppliers (Δ) and the penalty from law enforcement in the overseas market (e). Additionally, the figure illustrates the shifts in regional boundaries as the quality perception differential (γ) increases.

Figure 4 Equilibrium Sourcing Strategy Relative to the Cost Differential Between Two Suppliers (Δ) and Penalty from Law Enforcement in the Overseas Market (e). (Dashed arrows show changes in threshold lines with an increasing γ. In this example, $p_B = 0.7$, $p_2 = 0.15$, $k_2 = 0.02$, $\alpha = 5$, $\beta = 0.3$, $\gamma = 0.01$, $t = 0.01$.)



When the cost differential (Δ) is high, indicating a significant cost advantage for the counterfeiter, the benefit from sourcing through the overseas supplier who is converted from the counterfeiter becomes substantial. Consequently, the brand-name firm is willing to offer a high wholesale price w_2^* to incentivize the counterfeiter to accept the contract. Such a wholesale contract with a price premium can successfully convert the counterfeiter into an authorized overseas supplier. If the cost advantage is substantial, the brand-name firm chooses to source exclusively from the overseas supplier, leading to Strategy O. This approach is reflected in industry practices where brand-name firms turn solely to overseas suppliers due to lower production costs. For example, the luxury brand Balenciaga transferred production of its Triple S product line from Italy to factories in Putian, China, due to lower labor costs. If the cost advantage is not too high but moderate, the brand-name firm adopts dual sourcing from both home and overseas suppliers, leading to Strategy D. This strategy enables the brand-name firm to potentially mitigate competition in the overseas market without over-relying on the overseas supplier. For instance, Kokuyo collaborates with the Chinese company Gambol to produce and sell their notebooks in China while maintaining their home supplier (Sugawara 2015). When the cost advantage for the counterfeiter is small, the brand-name firm prefers to source from the home supplier, leading to Strategy H.

When the penalty from law enforcement (e) is high, the counterfeiter earns a lower expected profit from selling counterfeits. This situation makes the counterfeiter more inclined to be converted to an authorized supplier. Consequently, the wholesale price w_2^* does not need to be very high to attract the counterfeiter. We show that the brand-name firm can convert the counterfeiter to an authorized supplier through strategies D and O, and a high e enables successful conversion at a lower level of cost differential Δ. Otherwise, the brand-name firm prefers sourcing exclusively from the home supplier, leading to Strategy H, as illustrated in Figure 4. This scenario elucidates why some counterfeiters of leather luxury products in the Grand Bazaar of Turkey are reluctant to become authorized suppliers for brand-name firms, as observed during our field study interviews. First, production costs for high-quality counterfeits are close to those of genuine products, resulting in small values of Δ. Second, law enforcement penalties for counterfeiting in Turkey are modest, and enforcement typically occurs only upon reporting, leading to small values of e. Some brand-name firms may even tolerate counterfeiters, viewing them as a strategy to enhance brand recognition (Letsch 2011).

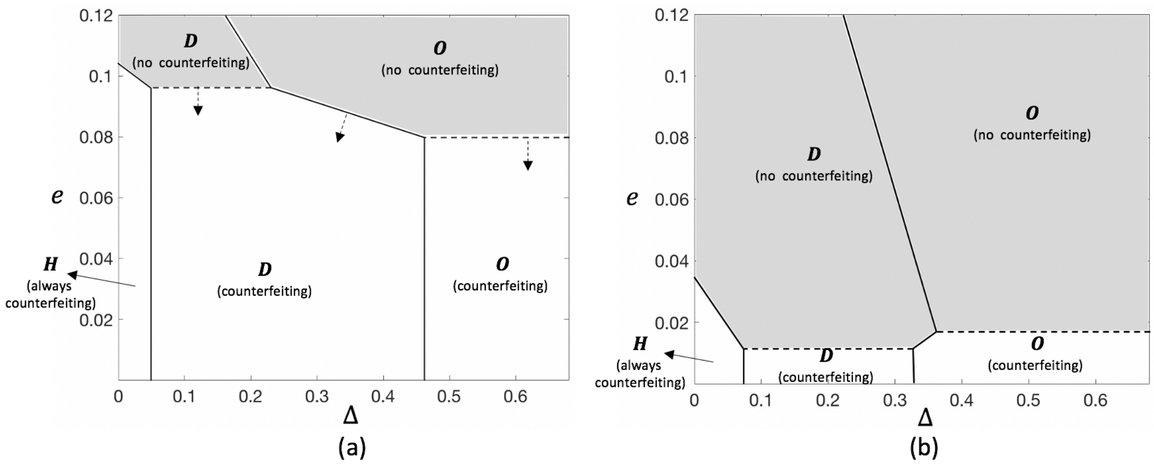
Dashed arrows in Figure 4 illustrate the impact of increasing values of the quality perception differential between two suppliers (γ). As γ decreases, implying that consumers do not perceive much difference in the quality of products produced by home and overseas suppliers, the region where Strategy H is preferred becomes smaller, and the regions where strategies D and O are preferred expand, depending on the value of e. There are two implications from a smaller value of γ. First, the brand-name firm becomes more willing to source from the overseas supplier due to potentially higher demand for his product as γ decreases, which can be seen from the demand function in equations (1) and (2). Second, the counterfeiter has a higher incentive to accept the contract as it can secure larger demand under strategies D and O, and the brand-name firm may be more willing to pay a higher wholesale price to convert, as evidenced by the fact that $w_2^* = w_2^{O(1)}$ increases as γ decreases under Strategy O. Therefore, when γ decreases, the brand-name firm becomes less likely to source only from the home supplier with Strategy H and is more likely to adopt Strategy D or Strategy O. This dynamic underpins why brand-name firms, when selecting suppliers in developing countries, engage in meticulous selection processes to ensure the quality standards of overseas production and implement transparency through information sharing and educational campaigns. These measures aim to enhance consumers' perceived quality of products produced in emerging markets.

4.4 Measures to Prevent Counterfeiting

When the brand-name firm successfully converts the counterfeiter to an authorized overseas supplier, the overseas supplier may still decide to sell counterfeits in the overseas market. This section investigates the conditions under which counterfeiting is prevented in the equilibrium, i.e., $s^* = 0$, under strategies D and O. We focus on the impact of the key model parameters on the counterfeiting decision.

The shaded areas in Figure 5 represent the regions in which the overseas supplier chooses not to sell counterfeits in the equilibrium under strategies D and O. Figure 5 shows that four factors have a non-monotone and interdependent influence on the counterfeiting decision. These four critical factors are (i) the cost differential between two suppliers (Δ), (ii) the penalty from law enforcement in the overseas market (e), (iii) the quality perception differential between brand-name products produced by the home and overseas suppliers (γ), and (iv) consumers' quality perception of the counterfeit (β).

Figure 5 Equilibrium Sourcing Strategy Relative to the Cost Differential Between Two Suppliers (Δ) and Penalty from Law Enforcement in the Overseas Market (e). (Shadow areas indicate that counterfeiting is prevented. Dashed arrows show changes in threshold lines with an increasing γ. In this example, $p_B = 0.7$, $p_2 = 0.15$, $k_2 = 0.02$, $\alpha = 5$, $\gamma = 0.01$, $t = 0.01$. In panel (a), $\beta = 0.3$; in panel (b), $\beta = 0.35$.)



Cost differential and penalty from law enforcement to counterfeiting: It is intuitive that counterfeiting can be prevented if the cost differential between two suppliers (Δ) and the penalty to counterfeiting (e) are high. In Figure 5, the horizontal dashed lines designate the minimum penalty that needs to be imposed by local authorities so that the overseas supplier does not engage in counterfeiting. In panel (a), the minimum penalty to prevent counterfeiting under Strategy D ($e = 0.096$ in this case) is higher than that under Strategy O ($e = 0.081$ in this case). This makes single sourcing more effective in mitigating counterfeiting than dual sourcing in this numerical illustration. More generally, Proposition 2 below compares the effectiveness of dual sourcing and single sourcing in preventing counterfeit sales.

Quality perception differential between home and overseas suppliers: In panel (a) of Figure 5, the dashed arrows illustrate the impact of an increasing quality perception differential (γ). The main effect can be described as follows: As γ increases, indicating a declining perception of the overseas product's quality relative to the home supplier, the brand-name firm has a stronger incentive to prevent the overseas supplier from counterfeiting under strategies D and O. Correspondingly, the horizontal dashed lines indicating the minimum penalty required to stop counterfeiting decrease, implying a reduced reliance on authorities to enforce penalties on counterfeiters. Building on the findings in Section 4.3, a smaller γ leads to an interesting dynamic where the brand-name firm is more likely to convert the counterfeiter to the overseas supplier using strategies D or O, but it is less effective in preventing counterfeiting. This seemingly counter-intuitive result arises because a lower γ enhances the brand-name firm's incentive to engage the overseas supplier due to the potential profit increase when consumers value the product produced by the overseas supplier higher. However, once the conversion is successful, the brand-name firm's profit gains from having the overseas supplier authorized — even if she continues to sell counterfeits — diminish the urgency to fully prevent counterfeiting. This is because completely preventing counterfeiting would require setting a much higher wholesale price, which could compress the brand-name firm's profit margin. Therefore, the economic benefits of conversion outweigh the immediate need to stop counterfeiting under smaller values of γ. In this case, a higher penalty becomes necessary to prevent the overseas supplier from counterfeiting. The impact of quality perception differential on the effectiveness of strategies D and O in preventing counterfeiting is formalized in the next proposition, which is established through a threshold expression $\widehat{\gamma}$ derived in Equation (10) of E-Companion B. Here, we describe a strategy as more effective in preventing counterfeiting if it requires a lower minimum penalty e.

PROPOSITION 2. *If $\gamma < \widehat{\gamma}$, then Strategy O is more effective in preventing the overseas supplier from counterfeiting than Strategy D. Otherwise, Strategy D is more effective than Strategy O.*

Proposition 2 indicates that when the quality perception differential (γ) is small, the likelihood of the overseas supplier engaging in counterfeiting post-conversion is smaller under Strategy O than under Strategy D. The rationale is as follows: Strategy O involves the brand-name firm relying exclusively on the overseas supplier for production, thereby granting the overseas supplier a larger production volume than Strategy D. This reduces the overseas supplier's incentive to sell counterfeits when the value of being an authorized supplier is enhanced as consumers place greater value on products produced by the overseas supplier.

Quality perception of the counterfeit: The comparison between panel (a) (small β) and panel (b) (large β) in Figure 5 illustrates the effect of a quality perception improvement in the counterfeit. As the quality perception of super fakes approaches that of brand-name products, notable shifts are observed in panel (b) relative to panel (a). First, the brand-name firm not only converts the counterfeiter to an authorized overseas supplier (with expanded regions for strategies D and O in panel (b)) but also prevents her from counterfeiting (with expanded shaded areas in panel (b)). Second, the brand-name firm does not have to rely on high penalties imposed by authorities to prevent counterfeiting, as shown by the lowered horizontal dashed lines representing the minimum penalty required. Third, Strategy H becomes less desirable for the brand-name firm (with a smaller region for Strategy H in panel (b)). The reason for this outcome is that as the quality perception of the counterfeit approaches that of the brand-name product, the threat from counterfeiting becomes significant, which enhances the benefits for the brand-name firm in mitigating the competition from the counterfeiter in the overseas market. Without the option to convert the counterfeiter, the brand-name firm might have to rely on conventional anti-counterfeiting measures like imposing higher penalties. However, with the conversion option, this can be effectively managed through a strategic wholesale contract that both converts the counterfeiter and prevents the sale of counterfeit goods. This innovative approach facilitates

Table 2 Equilibrium Sourcing Strategies with the Option to Convert the Counterfeiter

Sourcing \Impact	**Cost differential between two suppliers** (Δ)	**Penalty** (e)	**Quality perception differential** (γ)	**Quality perception of the counterfeit** (β)
D (no counterfeiting)	Low & Intermediate	High	High	High
D (counterfeiting)	Intermediate	Low & Intermediate	Low	Low
O (no counterfeiting)	Intermediate & High	Intermediate & High	High	High
O (counterfeiting)	High	Low	Low	Low
H	Low	Low & Intermediate	High	Low

counterfeiting prevention even in the absence of stringent law enforcement, highlighting the strategic anti-counterfeiting value of the conversion strategy in dealing with super fakes.

To summarize, the brand-name firm has three possible equilibrium sourcing strategies, as outlined in Table 2. Our analysis yields important implications for global brand-name firms facing the risk of counterfeits, along with recommendations for policymakers. First, while an intermediate to high cost differential (Δ) or a low quality perception differential (γ) between the home and the overseas suppliers facilitates the offer of a wholesale contract that converts the counterfeiter to an authorized overseas supplier, this alone is insufficient to stop counterfeiting. A significant penalty (e) is essential to dissuade the authorized overseas supplier from engaging in counterfeit activities. Second, we identify another benefit of dual sourcing not identified in prior publications: mitigating counterfeit risks. This is in addition to its role of risk diversification well reported in the literature. Dual sourcing can be more effective than single sourcing as the quality perception of super fakes approaches that of brand-name products. Conversely, single sourcing is preferable under conditions of high Δ and low γ. Third, as super fakes are approaching in quality perception to brand-name products (larger values of β), it potentially becomes more likely for the brand-name firm to both convert the counterfeiter into an authorized supplier and prevent counterfeiting. The finding has critical recommendations for policymakers. With super fakes, the reliance on stringent penalties can be reduced if brand-name firms strategically set wholesale prices to convert counterfeiters and prevent counterfeiting activities. Overall, our findings encourage global firms to reassess their sourcing contracts to convert counterfeiters into authorized suppliers and prevent their involvement in counterfeit operations.

5 Impact of Conversion

In this section, we investigate the impact of converting the counterfeiter to an authorized supplier on firms, consumers, and society. In Section 5.1, we introduce the benchmark case. Sections 5.2 and 5.3 further compare the base model with the benchmark case in terms of profits, consumer surplus, and social surplus.

5.1 Benchmark

We first present a benchmark case where the brand-name firm does not have the option to convert the counterfeiter to an authorized overseas supplier and, therefore, sources only from the home supplier. We use "$-$" to denote the benchmark case.

In the benchmark case, the optimal decision of the counterfeiter about whether to sell the counterfeit is

$$\bar{s}^*(\bar{w}_1) = \begin{cases} 0, & \text{if } \bar{\pi}_2(\bar{s}=0) \geq \bar{\pi}_2(\bar{s}=1), \\ 1, & \text{otherwise.} \end{cases}$$

From $\bar{\pi}_B(\bar{w}_1)$ and $\bar{\pi}_1(\bar{w}_1)$, we obtain that the brand-name firm's optimal wholesale price for the home supplier is $\bar{w}_1^* = k_1$, and the home supplier would accept the contract, i.e., $\bar{d}_1^* = 1$. That is to say, in the benchmark case, the brand-name firm's sourcing strategy is equivalent to Strategy H, in which the overseas supplier rejects the contract and sells counterfeits in the overseas market to compete with the brand-name firm. We denote the equilibrium profits of the brand-name firm, the home supplier, and the counterfeiter as $\bar{\pi}_B^*$, $\bar{\pi}_1^*$, and $\bar{\pi}_2^*$, respectively, in this benchmark case. The equilibrium profits of the brand-name firm, the home supplier, and the overseas supplier in the base model are denoted by π_B^j, π_1^j, and π_2^j, respectively, with $j \in \{H, D, O\}$ representing the equilibrium sourcing strategy.

5.2 Comparison of Profits

By comparing equilibrium profits of firms in the base model with that in the benchmark case, we obtain Corollary 1.

COROLLARY 1. *(a) For the brand-name firm,* $\pi_B^H = \bar{\pi}_B^*$, $\pi_B^D \geq \bar{\pi}_B^*, \pi_B^O \geq \bar{\pi}_B^*$.
(b) For the overseas supplier, $\pi_2^H = \pi_2^D = \bar{\pi}_2^*$, $\pi_2^O \geq \bar{\pi}_2^*$.
(c) For the home supplier, $\pi_1^H = \pi_1^D = \pi_1^O = \bar{\pi}_1^*$.

Corollary 1 shows that converting the counterfeiter brings a win-win outcome for the brand-name firm and the counterfeiter. It is straightforward that when Strategy H is the equilibrium strategy, the profit of each firm in the base model is the same as that in the benchmark case. In the following, we focus on the cases in which strategies D or O is the equilibrium strategy in the base model. Corollary 1(a) shows that, compared with the benchmark case, strategies D and O benefit the brand-name firm. By converting the counterfeiter to an authorized overseas supplier, the brand-name firm not only takes advantage of the low production cost of the overseas supplier but also might expand his market share in the overseas market by mitigating the competition with the counterfeiter. This is consistent with practical examples of Japanese stationery makers Kokuyo (Sugawara 2015) and Honda (Zaun and Leggett 2001). Corollary 1(b) shows that, compared with the benchmark case, converting the counterfeiter through Strategy D does not affect the counterfeiter's profit. This is because, with the home supplier being available, the brand-name firm sets the wholesale price w_2^* so that the overseas supplier is indifferent between accepting and rejecting the contract, under which the overseas supplier obtains the same profit as the counterfeiter in the benchmark case. However, converting the counterfeiter through Strategy O can bring extra profit benefit over the benchmark setting. This is because the profit of the overseas supplier obtained under Strategy O equals the profit that the counterfeiter would have obtained if she sells the counterfeits in the overseas market as a monopoly, which is larger than her profit in the benchmark. Corollary 1(c) is intuitive because, in equilibrium, the brand-name firm can always source from the home supplier at the marginal cost, k_1, in both the benchmark case and the base model with either Strategy H or Strategy D.

5.3 Comparison of Consumer and Social Surplus

In the benchmark case without the option to convert the counterfeiter, in equilibrium, the consumer surplus in the home market, $\overline{CS}_1$, and that in the overseas market, $\overline{CS}_2$, are given as follows:

$$\overline{CS}_1 = \int_{p_B}^{1} (\theta - p_B) d\theta, \quad \overline{CS}_2 = \alpha \int_{\frac{p_2}{\beta}}^{\frac{p_B - p_2}{1-\beta}} (\theta\beta - p_2) d\theta + \alpha \int_{\frac{p_B - p_2}{1-\beta}}^{1} (\theta - p_B) d\theta.$$

In particular, the first term in $\overline{CS}_2$ represents the surplus of consumers who purchase the counterfeit, and the second term is the surplus of consumers who purchase the brand-name product. Let $\overline{CS}$ denote the total consumer surplus in the benchmark case, i.e., $\overline{CS} = \overline{CS}_1 + \overline{CS}_2$. Following the literature such as Grossman and Shapiro (1988) and Yi et al. (2022), we define social welfare as the sum of the profits of firms, including the brand-name firm as well as the home and overseas suppliers, and the total consumer welfare from two markets; that is, $\overline{SS} = \bar{\pi}_B^* + \bar{\pi}_1^* + \bar{\pi}_2^* + \overline{CS}$.

In the base model with the option to convert the counterfeiter, in the equilibrium, under Strategy H, the consumer surplus in each market, CS_1^H and CS_2^H, the total consumer surplus in the two markets, CS^H, and the social surplus, SS^H, are the same as those in the benchmark case. Let CS_1^D and CS_2^D denote the consumer surplus in each market under Strategy D in the equilibrium:

$$CS_1^D = \int_{p_B}^{1} (\theta - p_B) d\theta,$$

$$CS_2^D = \begin{cases} \alpha \int_{\frac{p_B}{1-\gamma}}^{1} (\theta(1-\gamma) - p_B) d\theta, & \text{if } s^* = 0, \\ \alpha \int_{\frac{p_2}{\beta}}^{\frac{p_B - p_2}{1-\gamma-\beta}} (\theta\beta - p_2) d\theta + \alpha \int_{\frac{p_B - p_2}{1-\gamma-\beta}}^{1} (\theta(1-\gamma) - p_B) d\theta, & \text{if } s^* = 1. \end{cases}$$

Let CS_1^O and CS_2^O denote the consumer surplus in each market under Strategy O in the equilibrium:

$$CS_1^O = \int_{\frac{p_B}{1-\gamma}}^{1} (\theta(1-\gamma) - p_B) d\theta,$$

$$CS_2^O = \begin{cases} \alpha \int_{\frac{p_B}{1-\gamma}}^{1} (\theta(1-\gamma) - p_B) d\theta, & \text{if } s^* = 0, \\ \alpha \int_{\frac{p_2}{\beta}}^{\frac{p_B - p_2}{1-\gamma-\beta}} (\theta\beta - p_2) d\theta + \alpha \int_{\frac{p_B - p_2}{1-\gamma-\beta}}^{1} (\theta(1-\gamma) - p_B) d\theta, & \text{if } s^* = 1. \end{cases}$$

Let $CS^D(CS^O)$ denotes the total consumer surplus under Strategy D(O) in the equilibrium, that is, $CS^D = CS_1^D + CS_2^D$, and $CS^O = CS_1^O + CS_2^O$. Similarly, $SS^D(SS^O)$ denotes the social surplus under Strategy D(O) in the equilibrium: $SS^D = \pi_B^D + \pi_1^D + \pi_2^D + CS^D$ and $SS^O = \pi_B^O + \pi_1^O + \pi_2^O + CS^O$.

PROPOSITION 3. *(a) In the home market,* $CS_1^H = CS_1^D = \overline{CS}_1$*,* $CS_1^O \leq \overline{CS}_1$*.*
(b) In the overseas market, $CS_2^H = \overline{CS}_2, CS_2^D = CS_2^O \leq \overline{CS}_2$*.*
(c) In the two markets, $CS^H = \overline{CS}, CS^D \leq \overline{CS}$*,* $CS^O \leq \overline{CS}$*.*

Proposition 3(a) shows that, compared with the benchmark case, Strategy D has no impact on consumer surplus in the home market, whereas Strategy O reduces consumer surplus in the home market. This reduction is due to the quality perception differential γ; i.e., consumer surplus is reduced in the home market when consumers purchase products produced by the overseas supplier under Strategy O. Proposition 3(b) shows that both strategies D and O decrease consumer surplus in the overseas market. In the benchmark case, the brand-name firm competes with the counterfeiter in the overseas market. This competition benefits consumers by providing them with the option to purchase the counterfeit at a relatively low price. However, under strategies D and O, overseas consumers purchasing brand-name products produced by the overseas supplier suffer a unit loss in surplus due to the quality perception differential. In addition, if the overseas supplier does not sell the counterfeit, the lack of competition in the overseas market further impairs consumer surplus. Proposition 3(c) shows that the total consumer surplus in the two markets is lower in the base model than in the benchmark case.

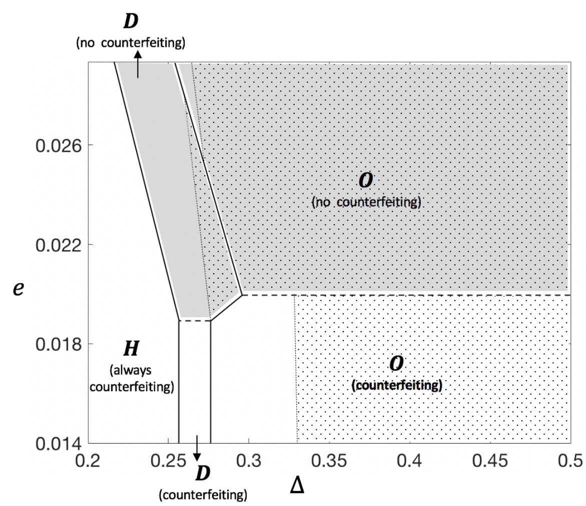
Next, we analyze the impact of converting the counterfeiter on social surplus. According to Corollary 1 and Proposition 3, compared with the benchmark case, converting the counterfeiter benefits the profits of firms but leads to a reduction in consumer surplus. Therefore, the comparison of social surplus between our base model and the benchmark case depends on whether the gain in profits or the loss in consumer surplus dominates. This is formalized in Proposition 4, which employs thresholds $\bar{\Delta}_D$, $\bar{\Delta}_O$, e_1' and e_2' defined in Equation (11) in E-Companion B.

PROPOSITION 4. *(a)* $SS^H = \overline{SS}$*.*
(b) When counterfeiting is prevented in the equilibrium, $SS^D > \overline{SS}$ *if* $e > (e_1')^+$*, and* $SS^O > \overline{SS}$ *if* $e > (e_2')^+$*. When counterfeiting is not prevented,* $SS^D > \overline{SS}$ *if* $\Delta > \bar{\Delta}_D$*, and* $SS^O > \overline{SS}$ *if* $\Delta > \bar{\Delta}_O$*.*

Proposition 4 shows that neither Strategy D nor Strategy O necessarily improves social surplus compared with the benchmark case. Specifically, when the authorized overseas supplier does not sell counterfeits in the equilibrium, both strategies D and O improve social surplus when e is high. The reason is that, compared with the benchmark case, under strategies D and O, as e increases, the loss in consumer surplus remains the same, whereas the gain in profits increases as the counterfeiter's profit in the benchmark case becomes lower with higher e. When e is sufficiently high, the gain in profits dominates the loss in consumer surplus. Thus, strategies D and O benefit society with a high e, and this benefit increases in e. When the authorized overseas supplier sells counterfeits in the equilibrium, strategies D and O improve social welfare when Δ is high. This is because as Δ increases, the profit benefit of converting the counterfeiter increases due to the lower production cost (even though the counterfeit still exists). When Δ is sufficiently high, the benefit outweighs the loss experienced by consumers. It is noteworthy that our social surplus definition does not consider the penalty imposed on the counterfeiter by law enforcement to be redistributed into society. If the penalty is redistributed, the social surplus may become $SS + e$, and this alternative definition does not qualitatively alter our findings.

We describe the scenario in which converting the counterfeiter leads to higher profits at the brand-name firm and the overseas supplier, and an increase in social surplus as the "triple-win" outcome. This triple-win outcome is shown in Corollary 1 and Proposition 4, and is depicted with the dotted areas in Figure 6.

Figure 6 Triple-Win Outcome. (Shadow areas indicate that counterfeiting is prevented. Dotted areas indicate the triple-win outcome. In this example, $p_B = 0.71$, $p_2 = 0.043$, $k_2 = 0.02$, $\alpha = 5$, $\beta = 0.09$, $\gamma = 0.11$, $t = 0.01$.)



The above analysis of consumer surplus and social surplus provides insights for governments on whether they should encourage converting counterfeiters to authorized overseas suppliers. When the penalty from law enforcement or the cost differential between two suppliers is high, converting counterfeiters should be encouraged because it benefits brand-name firms, overseas suppliers, and society. In this case, inter-governmental cooperation is helpful in reinforcing law enforcement to implement the conversion strategy towards counterfeiters in emerging markets. However, when the penalty from law enforcement or the cost differential between two suppliers is relatively low, caution should be taken about converting counterfeiters.

6 Extensions

In this section, we consider two extensions of our base model: (1) the brand-name firm *sequentially* offers wholesale contracts to two potential suppliers; (2) counterfeit price is determined *endogenously*.

6.1 Extension 1: Sequential Contract Offering

This extension examines the case when the brand-name firm offers wholesale price contracts sequentially (rather than simultaneously) as depicted in Figure A1. The overseas supplier first decides whether to accept the contract, followed by the home supplier. This sequential setting allows us to explore scenarios where, if the overseas supplier rejects the contract, the brand-name firm may then turn to the home supplier. We solve this problem by backward induction. Optimal wholesale prices of the brand-name firm under each possible sourcing strategy are formalized in Lemma 3, which employs penalty cost thresholds $e_D^{1\prime}$, $\hat{e}_2, \hat{e}_3$ derived in equations (12) and (13) of E-Companion B.

LEMMA 3. *The optimal wholesale price(s) of the brand-name firm, which will be accepted by the home and overseas suppliers, satisfies the following: let* $\widehat{w}_2 = p_B - \frac{(p_B-k_2-\Delta)(1-p_B)}{1-\frac{p_B}{1-\gamma}} - t$,
(a) under Strategy H, $w_1^H = k_1$;
(b) under Strategy D, $w_1^D = k_1$, *and (i)* $w_2^D = \max\{w_2^{O(1)}, \widehat{w}_2\}$ *and* $s^* = 1$, *if* $e < e'_{D1}$; *(ii)* $w_2^D = \max\{w_2^{(0)}, w_2^{O(2)}, \widehat{w}_2\}$ *and* $s^* = 0$, *if* $e \geq e'_{D1}$;
(c) under Strategy O, (i) $w_2^O = w_2^{O(1)}$ *and* $s^* = 1$, *if* $\Delta > \Delta_0$ *and* $e < \max\{e_{O1}, \hat{e}_2\}$; *(ii)* $w_2^O = \max\{w_2^{(0)}, w_2^{O(2)}\}$ *and* $s^* = 0$, *if* $e \geq \max\{e_{O1}, \hat{e}_2, \hat{e}_3\}$.

Compared with Lemma 2 in our base model, the possible optimal wholesale prices w_2 exhibit similar dependence on the penalty e. Lemma 3 shows that, under sequential contract offering, the overseas supplier can leverage the first-mover advantage to secure a higher wholesale price under dual sourcing, i.e., $w_2^D = \max\{w_2^{O(1)}, \widehat{w}_2\}$ where $w_2^{O(1)} > k_2$ and $\widehat{w}_2 > k_2$, particularly when e is small. This implies that converting the counterfeiter becomes more costly under the sequential setting.

By comparing potential profits under each sourcing strategy, we get the equilibrium presented in Proposition 5. Δ'_{DH}, Δ'_0 and Δ_0 are the thresholds of Δ, which are developed independently of the penalty cost. f'_{DH} (respectively f'_{DO2}) are the threshold of the penalty cost that equates the profits of strategies D with no counterfeiting and H (respectively strategies D with no counterfeiting and O without counterfeiting). These thresholds are derived in Equation (14) of E-Companion B.

PROPOSITION 5. *The equilibrium sourcing strategy of the brand-name firm is as follows:*
(a) Strategy H with $w_1^* = k_1$ *if* $e < f'_{DH}$ *and* $\Delta < \min\{\Delta'_{DH}, \Delta_0\}$;
(b) Strategy D with $w_1^* = k_1$, *and*

$$w_2^* = \begin{cases} w_2^{O(1)}, & \text{if } e < e'_{D1} \text{ and } \Delta'_{DH} \leq \Delta \leq \Delta_0, \\ \max\{w_2^{(0)}, w_2^{O(2)}\}, & \text{if } e \geq \max\{e'_{D1}, f'_{DH}\} \text{ and } \Delta \leq \Delta_0, \text{ or if } f'_{DO2} \leq e \leq \max\{\hat{e}_2, \hat{e}_3\} \text{ and } \Delta > \Delta_0; \end{cases}$$

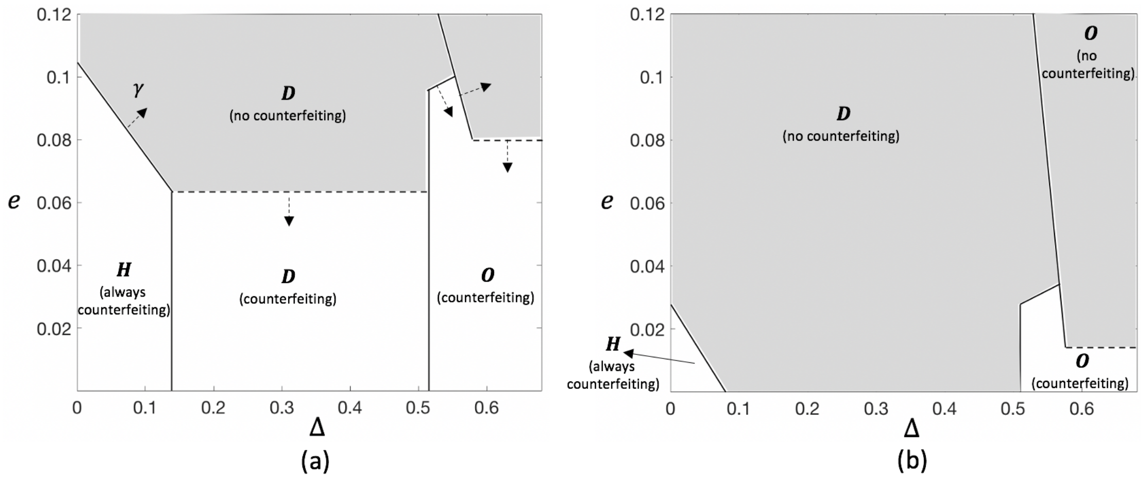
(c) Strategy O with

$$w_2^* = \begin{cases} w_2^{O(1)}, & \text{if } e < \min\{\hat{e}_2, f'_{DO2}\} \text{ and } \Delta_0 < \Delta \leq \Delta'_0, \text{ or if } e < e_{O1} \text{ and } \Delta > \Delta'_0, \\ \max\{w_2^{(0)}, w_2^{O(2)}\}, & \text{if } e \geq \max\{e_{O1}, \hat{e}_2, \hat{e}_3\}. \end{cases}$$

Figure 7(a) displays the regions corresponding to each optimal sourcing strategy. Consistent with earlier findings, the same three optimal sourcing strategies emerge in our equilibrium analysis. The brand-name firm prefers to convert the counterfeiter into an authorized overseas supplier under strategies D and O.

Strategy D is preferred at intermediate values of the cost differential Δ, and Strategy O is preferred when Δ is high. The dashed arrows indicate that as the quality perception differential γ decreases, the regions for strategies D and O expand at the expense of Strategy H. When both the cost differential Δ and the penalty cost e are small, however, the brand-name firm's optimal sourcing strategy changes to Strategy H.

Figure 7 Equilibrium Sourcing Strategy Relative to the Cost Differential Between Two Suppliers (Δ) and Penalty from Law Enforcement in the Overseas Market (e). (Shadow areas indicate that counterfeiting is prevented. Dashed arrows show changes in threshold lines with an increasing γ. In this example, $p_B = 0.7$, $p_2 = 0.15$, $k_2 = 0.02$, $\alpha = 5$, $\gamma = 0.01$, $t = 0.01$. In panel (a), $\beta = 0.3$; in panel (b), $\beta = 0.35$.)



We make three observations regarding the prevention of counterfeiting. First, the horizontal dashed line ($e = 0.061$) separating the region of counterfeiting and no counterfeiting under Strategy D in Figure 7(a) is lower than that in Figure 5(a) of our base model. This implies that once the counterfeiter is converted under the sequential setting, she is less likely to sell counterfeits under Strategy D. Second, while Strategy O has been identified as more effective under the simultaneous setting, Figure 5 shows that Strategy D is more effective under the sequential setting. Third, the comparison of shaded areas between panels (a) and (b) reveals that a higher β (higher consumers' perceived quality of super fakes) encourages the brand-name firm to convert the counterfeiter to an authorized supplier. This is especially true when the cost differential between two suppliers Δ exceeds a certain threshold, which is consistent with our earlier findings.

6.2 Extension 2: Endogenous Counterfeit Price

This section examines endogenous price setting for counterfeits. If the overseas supplier determines to sell counterfeit products (i.e., $s = 1$), regardless of which strategy is employed (H, D, O or N), she sets p_2 endogenously to maximize her profits. We use backward induction to solve this revised problem; details are presented in Online Supplement A.2.

LEMMA 4. *For given (w_1, w_2), under either Strategy D or Strategy O,*

$$s^*(w_2) = \begin{cases} 0, & \text{if } w_2 \geq \max\{w_2^{(0)\prime}, \underline{w}_2\}, \\ 1, & \text{if } \underline{w}_2 < w_2 < \max\{w_2^{(0)\prime}, \underline{w}_2\}, \end{cases}$$

where $w_2^{(0)\prime} = k_2 + \frac{\beta p_B-(1-\gamma)k_2}{\beta} - \sqrt{\frac{4(1-\gamma)(1-\gamma-\beta)e}{\alpha\beta}}$, $\underline{w}_2 = k_2 - \frac{2(1-\gamma-p_B)(1-\gamma-\beta)-\beta p_B+(1-\gamma)k_2}{\beta}$. *In particular, when* $s^*(w_2) = 1$, *the optimal retail price of the counterfeit product is* $p_2^*(w_2) = \frac{\beta p_B+(1-\gamma)k_2+\beta(w_2-k_2)}{2(1-\gamma)}$.

Lemma 4 shows that the overseas supplier can be prevented from selling counterfeits, i.e., $s^*(w_2) = 0$, only when the brand-name firm offers a high enough wholesale price w_2. When the overseas supplier's optimal decision is to sell counterfeits in the overseas market ($s^*(w_2) = 1$), the retail price of the counterfeit $p_2^*(w_2)$ increases in w_2. A higher w_2 triggers a higher order quantity from the brand-name firm to the overseas supplier. Moreover, as p_2 increases, the quantity of counterfeits sold decreases. Thus, as w_2 increases, the overseas supplier's profit from producing the brand-name product increases, whereas the profit from selling the counterfeit may increase or decrease.

Lemma 5 presents optimal wholesale prices provided under each possible sourcing strategy, which employs notations $\hat{w}_2^{DC}$, $\hat{w}_2^{OC}$, $w_2^{D(2)}$, $w_2^{O(1)}$ and $w_2^{O(2)}$, $\Pi_{B2}^{O}\left(w_2^{OC*}\right)$ and $\Pi_{B2}^{D}\left(w_2^{DC*}\right)$ described in Equation (16) of E-Companion B.

LEMMA 5. *The optimal wholesale prices of the brand-name firm, which will be accepted by the home and overseas suppliers, satisfy the following:*
(a) under Strategy H, $w_1^H = k_1$;
(b) under Strategy D, $w_1^D = k_1$, *and (i)* $w_2^D = w_2^{DC*}$ *and* $s^* = 1$, *if* $w_2^{DC*} \leq w_2^{(0)\prime}$ *and* $w_2^{D\dagger*} \leq p_B - \frac{\Pi_{B2}^{D}\left(w_2^{DC*}\right)}{\alpha(1-\frac{p_B}{1-\gamma})}$; *(ii) otherwise,* $w_2^D = w_2^{D\dagger*}$ *and* $s^* = 0$;
(c) under Strategy O, (i) $w_2^O = w_2^{OC*}$ *and* $s^* = 1$, *if* $w_2^{OC*} \leq w_2^{(0)\prime}$ *and* $w_2^{O\dagger*} \leq p_B - \frac{\left(p_B-w_2^{OC*}\right)(1-\frac{p_B}{1-\gamma})+\Pi_{B2}^{O}\left(w_2^{OC*}\right)}{(1+\alpha)(1-\frac{p_B}{1-\gamma})}$;
(ii) otherwise, $w_2^O = w_2^{O\dagger*}$ *and* $s^* = 0$;
where $w_2^{DC*} = \max\{k_2, \hat{w}_2^{DC}\}$, $w_2^{D\dagger*} = \max\{w_2^{D(2)}, w_2^{(0)\prime}, \underline{w}_2\}$, $w_2^{OC*} = \max\{w_2^{O(1)}, \underline{w}_2, \hat{w}_2^{OC}\}$ *and* $w_2^{O\dagger*} = \max\{w_2^{O(2)}, w_2^{(0)\prime}, \underline{w}_2\}$.

The Online Supplement A.2 presents a numerical illustration of when each strategy is optimal under the endogenous counterfeit price setting (Figure A2). The results are similar to those presented in the setting with exogenous price in Figure 5. Our analysis with endogenous counterfeit pricing yields two insights. First, the ability to set the price of the counterfeit enables the overseas supplier to maximize the profit from selling counterfeits, making counterfeiting more attractive. Second, the brand-name firm has a smaller range of acceptable wholesale prices to offer the overseas supplier in the pursuit of eliminating counterfeiting.

In Online Supplement A.3, we present an additional extension where both the brand-name firm's retail price p_B and the overseas supplier's counterfeit price p_2 are endogenous. This analysis shows that counterfeiting can be eliminated when the retail price p_B is less than a threshold price under strategies D and O. This additional extension also proves that, when the penalty from law enforcement e is not high, it becomes easier for the brand-name firm (in comparison to the setting with exogenous p_B and p_2) to prevent the overseas supplier from counterfeiting by choosing the retail price optimally.

7 Conclusions

High-quality counterfeits, known as "super fakes," are increasingly prevalent in emerging markets. We have developed a game-theoretic model based on a two-tier supply chain that captures the interactions between a brand-name firm with a legitimate home supplier and a counterfeiter capable of producing these super fakes, who could potentially be converted to an authorized overseas supplier. Our study outlines three primary sourcing strategies for the brand-name firm: dual sourcing (Strategy D), single sourcing from the overseas supplier (Strategy O), and single sourcing from the home supplier (Strategy H). The effectiveness of each strategy varies based on specific problem parameters, with strategies D and O potentially facilitating the conversion of the counterfeiter into an authorized supplier. Our analysis explores how various factors, such as the penalty from law enforcement, the cost differential between two suppliers, the quality perception differential between the brand-name products produced by two suppliers, and the quality perception of counterfeits affect the brand-name firm's choice of sourcing strategy and the prevalence of counterfeit sales. This research contributes to the understanding of socially responsible supply chain operations by examining the impacts of converting counterfeiters on firms, consumers, and society at large, offering valuable insights for managing supply chains in emerging markets susceptible to high-quality counterfeiting.

Our paper offers insights and recommendations for global brand-name firms facing counterfeiting risks, as well as guidance for policymakers. First, the brand-name firm is positioned to convert the counterfeiter into an authorized supplier when the cost differential between home and overseas suppliers exceeds an intermediate level. If the cost differential is intermediate, dual sourcing is adopted; if the overseas supplier has a significant cost advantage, single sourcing is preferred. Under both strategies, the brand-name firm offers a high wholesale price to the counterfeiter, which does not need to be excessively high if the penalty for counterfeiting is moderate to high. Additionally, a smaller quality perception differential, indicating a lesser loss in brand value upon converting the counterfeiter, increases the likelihood of conversion. Our findings suggest that brand-name firms should strategically use wholesale contracts to convert counterfeiters.

Second, converting the counterfeiter to an authorized overseas supplier does not automatically prevent counterfeit sales, as the authorized overseas supplier might still engage in such activities. Our study delineates the conditions under which an overseas supplier chooses to sell or refrain from selling counterfeits. While a moderate to high cost differential and/or a low quality perception differential facilitate conversion, they do not ensure cessation of counterfeiting. The enforcement of penalties becomes crucial in preventing counterfeiting. As the quality perception of super fakes approaches that of genuine brand-name products, the brand-name firm gains a stronger incentive to completely prevent counterfeiting by strategically setting wholesale prices. This strategy lessens the reliance on traditional anti-counterfeiting measures, rather it encourages global firms to offer effective contracts to prevent counterfeit sales.

Third, we identify another benefit of dual sourcing that has not been reported earlier: mitigating counterfeit risks, in addition to its widely-reported role of risk diversification. It can be more effective than single sourcing as the quality perception of super fakes approaches that of brand-name products; its effectiveness becomes more pronounced when the brand-name firm offers wholesale price contracts in a sequential order. Conversely, single sourcing is preferable under conditions of high cost differential and low quality perception differential, as it requires lower penalties from local authorities.

Fourth, converting counterfeit producers in overseas markets to authorized suppliers can negatively affect consumer surplus and does not necessarily enhance social surplus. This outcome arises because consumers may prefer the product from the home supplier, and conversion also removes competition in the overseas market, thereby reducing consumer choice and surplus. Society benefits when either penalties from law enforcement or the cost differential between suppliers is high, suggesting a complex interplay between market dynamics and regulatory actions.

Our study introduces two extensions. The first examines the potential of offering wholesale contracts sequentially. While our primary conclusions remain valid, we find that dual sourcing outperforms single sourcing in terms of mitigating counterfeiting risk as it requires a smaller penalty to be effective. In the second extension, we explore the brand-name firm's ability to set retail prices, demonstrating that this pricing flexibility can be leveraged to prevent counterfeiting.

Future research could extend these findings in several ways. Our analysis adopts a risk-neutral perspective when considering the potential for counterfeiting. Incorporating a risk-averse perspective of the counterfeiting overseas supplier may further improve our findings and identify the necessary and sufficient conditions in a granular way. Additionally, our study assumes that the home supplier earns no profit if she rejects the brand-name firm's contract. Investigating scenarios where the home supplier has alternatives, such as producing for other companies upon rejecting the contract, could provide further insights.

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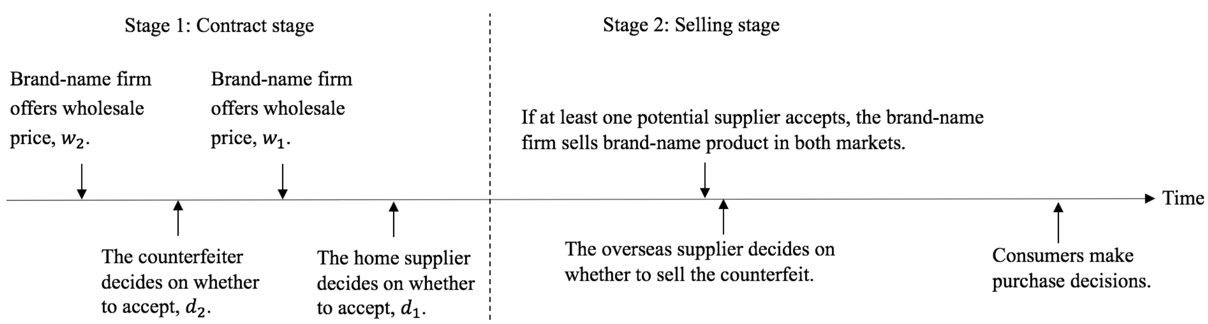
E-Companion
Converting Counterfeiters in Emerging Markets to Authorized Suppliers: A New Anti-Counterfeiting Measure

E-Companion A Extensions

A.1 Extension 1: Sequential Contract Offering

In this extension, based on the decision sequence in Figure A1, we conduct backward deduction to solve our problem. The procedures are as follows: firstly, we discuss the overseas supplier's counterfeiting decision $s(w_1, w_2, d_1, d_2)$ given $d_2 = 1$; secondly, we discuss the home supplier's acceptance decision $d_1(w_1, w_2, d_2)$; thirdly, we discuss the optimal wholesale price decision $w_1(w_2, d_2)$; fourthly, we discuss the overseas supplier's acceptance decision $d_2(w_2)$; lastly, we discuss the optimal wholesale price decision w_2.

Figure A1 Sequence of Decisions and Events (Extension 1)



A.2 Extension 2: Endogenous Counterfeit Price

In this extension, we examine the price-setting capability of the counterfeiter. We conduct the analysis by backward induction. First, for a given sourcing strategy, we derive the profit expressions and discuss the optimal counterfeiting decision of the overseas supplier, s^*.

Under each possible sourcing strategy, we obtain the profit expressions for each firm, and discuss the optimal retail price p_2^* of the counterfeit with $s = 1$. In particular, if the counterfeiter sells the counterfeits, we focus on the case when the brand-name firm has a positive market share in the overseas market, i.e., $m_{B2} > 0$. The overseas supplier decides whether to sell the counterfeit, $s^*(w_2)$ by comparing $\pi_2(w_2, s = 1)$ and $\pi_2(w_2, s = 0)$. If $\pi_2(w_2, s = 1) > \pi_2(w_2, s = 0)$, she decides to sell the counterfeit; otherwise, she does not sell the counterfeit. Recall that $e < \frac{\alpha(\beta p_B - k_2)^2}{4\beta(1-\beta)}$. Thus, under strategies H and N, the counterfeiter always sells the counterfeit products. Under strategies D and O, the overseas supplier decisions on selling the counterfeit only when w_2 is not high, which is summarized in Lemma 4.

Second, we derive the best response functions of the overseas and home suppliers, $(d_1^*(w_1, w_2), d_2^*(w_1, w_2))$. For the analysis below, it is convenient to define the following notations: $M = \frac{\alpha(\beta p_B - k_2)^2}{4\beta(1-\beta)} - e$, $M' = \frac{\alpha(\beta p_B - (1-\gamma)k_2)^2}{4(1-\gamma)\beta(1-\gamma-\beta)} - e$, and $K = \frac{\alpha(\beta - k_2)^2}{4\beta} - e$.

Given w_1 and w_2, we derive the home and overseas suppliers' optimal contract acceptance decisions. By evaluating the difference in each potential supplier's expected profit between accepting and rejecting the contract, we obtain the optimal decisions of the two suppliers:

$$(d_1^*(w_1,w_2),d_2^*(w_1,w_2)) = \begin{cases} (1,1), & \text{if } w_1 \geq k_1,\ \max\{k_2,\underline{w}_2\} \leq w_2 < w_2^{(0)} \text{ or } w_2 \geq \max\{w_2^{D(2)},w_2^{(0)}\}, \\ (1,0), & \text{if } w_1 \geq k_1,\ \underline{w}_2 < w_2 < \max\{k_2,\underline{w}_2\} \text{ or } w_2^{(0)} < w_2 < \max\{w_2^{D(2)},w_2^{(0)}\}, \\ (0,1), & \text{if } w_1 < k_1,\ \max\{w_2^{O(1)},\underline{w}_2\} \leq w_2 < w_2^{(0)} \text{ or } w_2 \geq \max\{w_2^{O(2)},w_2^{(0)}\}, \\ (0,0), & \text{if } w_1 < k_1,\ \underline{w}_2 < w_2 < \max\{w_2^{O(1)},\underline{w}_2\} \text{ or } w_2^{(0)} < w_2 < \max\{w_2^{O(2)},w_2^{(0)}\}, \end{cases}$$

where $w_2^{D(2)} = k_2 + \frac{M}{\alpha(1-\frac{p_B}{1-\gamma})}$, $w_2^{O(1)} = k_2 - \frac{2(1+\frac{1}{\alpha})(1-\gamma-p_B)(1-\gamma-\beta)-\beta p_B+(1-\gamma)k_2}{\beta} + \sqrt{\frac{4(1-\gamma)(1-\gamma-\beta)(K-M')}{\alpha\beta} + \left(\frac{2(1+\frac{1}{\alpha})(1-\gamma-p_B)(1-\gamma-\beta)-\beta p_B+(1-\gamma)k_2}{\beta}\right)^2}$, $w_2^{O(2)} = k_2 + \frac{K}{(1+\alpha)(1-\frac{p_B}{1-\gamma})}$, and $w_2^{(0)} = \max\{w_2^{(0)\prime},\underline{w}_2\}$.

Third, we discuss the optimal wholesale prices (w_1,w_2) that the brand-name firm would offer under each sourcing strategy. Substituting $(d_1^*(w_1,w_2),d_2^*(w_1,w_2))$ into the profit functions of the brand-name firm, we analyze the optimal wholesale price under each possible sourcing strategy.

$$\pi_B^H(w_1) = (p_B-w_1)(1-p_B) + \alpha(p_B-w_1-t)\left(1-\frac{(2-\beta)p_B-k_2}{2(1-\beta)}\right);$$

$$\pi_B^D = \begin{cases} \pi_B^{DC}(w_1,w_2) = (p_B-w_1)(1-p_B) & \\ +\alpha(p_B-w_2)\left(\frac{2(1-\gamma-\beta)(1-\gamma-p_B)-\beta p_B+(1-\gamma)k_2+\beta(w_2-k_2)}{2(1-\gamma)(1-\gamma-\beta)}\right), & \text{if } w_1 \geq k_1,\ \max\{k_2,\underline{w}_2\} \leq w_2 < w_2^{(0)}, \\ \pi_B^{D\dagger}(w_1,w_2) = (p_B-w_1)(1-p_B) + \alpha(p_B-w_2)(1-\frac{p_B}{1-\gamma}), & \text{if } w_1 \geq k_1,\ w_2 \geq \max\{w_2^{D(2)},w_2^{(0)}\}; \end{cases}$$

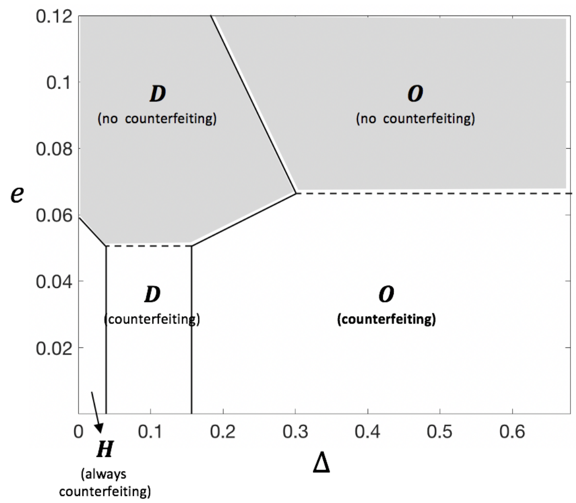
$$\pi_B^O = \begin{cases} \pi_B^{OC}(w_2) = (p_B-w_2-t)\left(1-\frac{p_B}{1-\gamma}\right) & \\ +\alpha(p_B-w_2)\left(\frac{2(1-\gamma-\beta)(1-\gamma-p_B)-\beta p_B+(1-\gamma)k_2+\beta(w_2-k_2)}{2(1-\gamma)(1-\gamma-\beta)}\right), & \text{if } \max\{w_2^{O(1)},\underline{w}_2\} \leq w_2 < w_2^{(0)}, \\ \pi_B^{O\dagger}(w_2) = (p_B-w_2-t)(1-\frac{p_B}{1-\gamma}) + \alpha(p_B-w_2)(1-\frac{p_B}{1-\gamma}), & \text{if } w_2 \geq \max\{w_2^{O(2)},w_2^{(0)}\}; \end{cases}$$

$$\pi_B^N(w_1,w_2) = 0.$$

By analyzing the brand-name firm's profit under each sourcing strategy, we obtain the optimal wholesale prices in Lemma 5.

Finally, we obtain the equilibrium by comparing the brand-name firm's optimal profits among different sourcing strategies. We provide the numerical analysis about the equilibrium under the setting with the price-setting flexibility. Figure A2 illustrates how the equilibrium sourcing strategy varies with respect to the cost differential between two suppliers (Δ) and the penalty from law enforcement in the overseas market (e). We observe that in this extension, the equilibrium is similar to that developed under the base model which has been depicted in Figure 5.

Figure A2 Equilibrium Sourcing Strategy Relative to the Cost Differential Between Two Suppliers (Δ) and Penalty from Law Enforcement in the Overseas Market (e). (Shadow areas indicate that counterfeiting is prevented. In this example, $p_B = 0.7$, $k_2 = 0.02$, $\alpha = 5$, $\beta = 0.3$, $\gamma = 0.01$, $t = 0.01$.)



A.3 Extension 3: Endogenous Brand-Name Product and Counterfeit Prices

Our base model assumes retail prices p_B and p_2 are exogenously determined. This extension explores the implications of endogenizing retail prices. Solving the game with endogenous retail prices alongside endogenous sourcing decisions introduces analytical challenges. For tractability, we focus on optimizing retail pricing decisions for given wholesale prices w_1 and w_2 under strategies D and O, respectively. Specifically, we examine scenarios where the wholesale price contracts have already been structured to convert the counterfeiter through either dual sourcing or single sourcing from the overseas supplier, and it is possible for the authorized overseas supplier to sell counterfeits. The subsequent analysis investigates the conditions that the overseas supplier is prevented from selling counterfeits, considering the dynamics of endogenized retail pricing decisions.

Under Strategy D or Strategy O, the sequence of events unfolds as follows: First, the brand-name firm sets the retail price p_B of the brand-name product. Subsequently, the overseas supplier decides whether to sell counterfeits, s. If she opts to sell counterfeits in the overseas market, i.e., $s = 1$, she then determines the retail price of the counterfeit p_2. We employ backward induction to solve the game, with details provided in E-Companion B.

Endogenously setting their retail prices under competition in the overseas market introduces more interactions among players. Specifically, the endogenous retail price p_B provides the brand-name firm an additional lever to prevent counterfeiting through price competition. At the same time, it allows the overseas supplier the opportunity to adjust her retail price p_2. When retail price p_B is low enough, counterfeiting can be prevented as competition leads to zero market share for the counterfeit product. In the following lemma, we outline the conditions under which the overseas supplier does not sell counterfeits. We define $\hat{p}_B^D$ and $\hat{p}_B^O$ in Equation (17) in E-Companion B.

LEMMA A1. *Given* (w_1, w_2)*, (i) under Strategy D,* $s^* = 0$ *if* $\hat{p}_B^D \leq \frac{\sqrt{\frac{4\beta(1-\gamma)(1-\gamma-\beta)e}{\alpha}} + ((1-\gamma)k_2 + \beta(w_2 - k_2))}{\beta}$*; (ii) under Strategy O,* $s^* = 0$ *if* $\hat{p}_B^O \leq \frac{\sqrt{\frac{4\beta(1-\gamma)(1-\gamma-\beta)e}{\alpha}} + ((1-\gamma)k_2 + \beta(w_2 - k_2))}{\beta}$.

Recall that in our base model, the overseas supplier's profit from selling counterfeits does not depend on the wholesale price w_2. However, Lemma A1 implies that wholesale price w_2 may affect the optimal retail price when $\hat{p}_B^D$ or $\hat{p}_B^O$ is adopted to prevent counterfeit sales, which in turn affects the overseas supplier's profit from counterfeiting.

In the following, we compare the conditions with respect to (w_1, w_2) under which the overseas supplier is prevented from counterfeiting under endogenous retail prices with those from our base model under exogenous retail prices. Recall from Lemma 1 that, when retail prices are exogenous, the brand-name firm is able to prevent counterfeiting by setting a sufficiently high wholesale price $w_2 \geq w_2^{(0)}$ under strategies D and O. When retail prices are endogenously determined, counterfeiting is prevented if $w_2 \geq w_2^{D,endog}$ under Strategy D or if $w_2 \geq w_2^{O,endog}$ under Strategy O. The following proposition provides the sufficient conditions about the comparison between $w_2^{D,endog}$ and$w_2^{O,endog}$ with $w_2^{(0)}$, respectively. We define the thresholds $e_1^{D,endog}$, $e_1^{O,endog}$, $e_2^{D,endog}$ and $e_2^{O,endog}$ in Equation (18) of E-Companion B.

PROPOSITION EC.1. *For given* (w_1, w_2)*,*

(a) under Strategy D, $w_2^{D,endog} < w_2^{(0)}$ *if* $(e_1^{D,endog})^+ < e < (e_2^{D,endog})^+$*;*

(b) under Strategy O, $w_2^{O,endog} < w_2^{(0)}$ *if* $(e_1^{O,endog})^+ < e < (e_2^{O,endog})^+$.

Proposition EC.1 indicates that if the penalty from law enforcement e is not high, it becomes easier for the brand-name firm to prevent the overseas supplier from counterfeiting if he can choose the retail price optimally. Specifically, in this case, a wholesale price w_2, which satisfies $w_2^{D,endog} \leq w_2 < w_2^{(0)}$ under Strategy D or $w_2^{O,endog} \leq w_2 < w_2^{(0)}$ under Strategy O, can prevent counterfeit sales under the optimal retail prices, whereas it cannot prevent counterfeiting under fixed retail prices. This occurs because the optimal retail price of the brand-name firm increases with w_2. When the wholesale price w_2 is lower, the brand-name firm chooses a lower retail price. Consequently, the potentially intense price competition discourages the overseas supplier from selling counterfeits. This result confirms that the flexibility to adjust retail prices is a valuable leverage for the brand-name firm to prevent counterfeit sales.

A.4 Extension 4: Revenue-Dependent Penalty for Counterfeiting

In this section, our model is extended to consider a different law enforcement penalty, which depends on the revenue from selling counterfeits.

Denote the probability of a counterfeiter getting caught as ϕ, where $\phi \in (0, 1)$, we examine the effect of the revenue related penalty for counterfeiting: after getting caught, the counterfeiter pays the penalty from law enforcement e and gets her investment of counterfeiting confiscated, which means she cannot sell and produce the counterfeit in the market. Thus, the overseas supplier's expected profit π_2 is given as

$$\begin{aligned}\pi_2\left(w_2,d_1,d_2,s\right) =& d_2\left(\left(1-d_1\right)\left(w_2-k_2\right)m_{B1}(d_1,d_2)+\left(w_2-k_2\right)m_{B2}(d_1,d_2,s)\right)\\ &+s\left(\left(1-\phi\right)\left(p_2-k_2\right)m_2(d_1,d_2,s)-\phi e\right),\end{aligned} \tag{5}$$

where $m_{B1}(d_1,d_2)$, $m_{B2}(d_1,d_2,s)$ and $m_2(d_1,d_2,s)$ are given in equations (1)-(3), respectively. For the second line of Equation (5), the first term represents the expected profit of selling the counterfeit, and the second term represents the expected penalty from law enforcement. In this extension, to avoid the uninteresting case where the counterfeiter never sell counterfeits if she rejects the contract, we assume the penalty is not too high, that is, $e<\frac{\alpha(\beta p_B-p_2)(p_2-k_2)(1-\phi)}{(1-\beta)\beta\phi}$. For the analysis below, it is convenient to define the following notations:

$$\begin{aligned}
&M_p=\alpha(1-\phi)\left(p_2-k_2\right)\left(\tfrac{p_B-p_2}{1-\beta}-\tfrac{p_2}{\beta}\right)-\phi e,\ w_2^{D(2)}=k_2+\tfrac{M_p}{\alpha(1-\frac{p_B}{1-\gamma})},\\
&M_p'=\alpha(1-\phi)\left(p_2-k_2\right)\left(\tfrac{p_B-p_2}{1-\gamma-\beta}-\tfrac{p_2}{\beta}\right)-\phi e,\ w_2^{(0)}=k_2+\tfrac{M_p'}{\alpha\left(\frac{p_B-p_2}{1-\gamma-\beta}-\frac{p_B}{1-\gamma}\right)},\\
&K_p=\alpha(1-\phi)\left(p_2-k_2\right)\left(1-\tfrac{p_2}{\beta}\right)-\phi e,\qquad w_2^{O(2)}=k_2+\tfrac{K_p}{(1+\alpha)(1-\frac{p_B}{1-\gamma})},\\
&K_p-M_p'=\alpha(1-\phi)\left(p_2-k_2\right)\left(1-\tfrac{p_B-p_2}{1-\gamma-\beta}\right),\ \ w_2^{O(1)}=k_2+\tfrac{K_p-M_p'}{(1-\frac{p_B}{1-\gamma})+\alpha(1-\frac{p_B-p_2}{1-\gamma-\beta})}.
\end{aligned}$$

Similar to the analysis in Section 4, in this extension, if $w_2<w_2^{(0)}$, after being converted, she will choose to sell counterfeits in the overseas market. Further, by evaluating the difference in each potential supplier's expected profit between accepting and rejecting the contract, we obtain the best response function of two potential suppliers. As a result, the optimal decisions of two suppliers are

$$(d_1^*,d_2^*)=\begin{cases}(1,1), & \text{if } w_1\geq k_1,\ k_2\leq w_2<w_2^{(0)} \text{ or } w_2\geq w_2^{(0)},\\ (1,0), & \text{if } w_1\geq k_1,\ w_2<k_2,\\ (0,1), & \text{if } w_1<k_1,\ \min\{w_2^{(0)},w_2^{O(1)}\}\leq w_2<w_2^{(0)} \text{ or } w_2\geq \max\{w_2^{(0)},w_2^{O(2)}\},\\ (0,0), & \text{if } w_1<k_1,\ w_2<\min\{w_2^{(0)},w_2^{O(1)}\} \text{ or } w_2^{(0)}\leq w_2<\max\{w_2^{(0)},w_2^{O(2)}\}.\end{cases}$$

Thus, for each possible sourcing strategy, the optimal wholesale price(s) of the brand-name firm, which will be accepted by the home or overseas suppliers, satisfies the following:

(a) under Strategy H, $w_1^H=k_1$;

(b) under Strategy D, $w_1^D=k_1$ and

(i) $w_2^D=k_2$ and $s^*=1$, if $e<e_{D1}$;

(ii) $w_2^D=\max\{w_2^{(0)},w_2^{D(2)}\}$ and $s^*=0$, if $e\geq e_{D1}$;

(c) under Strategy O,

(i) $w_2^O=w_2^{O(1)}$ and $s^*=1$, if $e<e_{O1}$;

(ii) $w_2^O=\max\{w_2^{(0)},w_2^{O(2)}\}$ and $s^*=0$, if $e\geq e_{O1}$;

where e_{D1} and e_{O1} are defined as

$$\begin{aligned}
e_{D1}&=\left((1-\phi)(p_2-k_2)-\left(\tfrac{(p_B-k_2)(\frac{p_B-p_2}{1-\gamma-\beta}-\frac{p_B}{1-\gamma})}{1-\frac{p_B}{1-\gamma}}\right)\tfrac{\beta}{1-\gamma}\right)\tfrac{\alpha(\beta p_B-(1-\gamma)p_2)}{(1-\gamma-\beta)\beta\phi},\\
e_{O1}&=\left((1-\phi)(p_2-k_2)-\left(\tfrac{\alpha(p_B-w_2^{OC*})(\frac{p_B-p_2}{1-\gamma-\beta}-\frac{p_B}{1-\gamma})}{(1+\alpha)(1-\frac{p_B}{1-\gamma})}+w_2^{OC*}-k_2\right)\tfrac{\beta}{1-\gamma}\right)\tfrac{\alpha(\beta p_B-(1-\gamma)p_2)}{(1-\gamma-\beta)\beta\phi},
\end{aligned}$$

and $w_2^{OC*} = k_2 + \frac{\alpha(1-\phi)(p_2-k_2)(1-\frac{p_B-p_2}{1-\gamma-\beta})}{(1-\frac{p_B}{1-\gamma})+\alpha(1-\frac{p_B-p_2}{1-\gamma-\beta})}$.

By further making comparisons among different scenarios and using the approach in Lemma B2, we have the following equilibrium results. We define thresholds in the below Equation (6): $R = \frac{\alpha(\beta p_B-(1-\gamma)p_2)}{(1-\gamma-\beta)\beta}$, and

$$
\begin{aligned}
&\Delta_{DH} = p_B - k_2 - \frac{(p_B-k_2)(1-\frac{p_B-p_2}{1-\gamma-\beta})}{(1-\frac{p_B-p_2}{1-\beta})} - t;\\
&\Delta_{DO} = p_B - k_2 - \frac{\left(p_B-w_2^{OC*}-t\right)(1-\frac{p_B}{1-\gamma})-\alpha\left(w_2^{OC*}-k_2\right)(1-\frac{p_B-p_2}{1-\gamma-\beta})}{(1-p_B)};\\
&\Delta_{HO} = p_B - k_2 - \frac{\left(p_B-w_2^{OC*}-t\right)(1-\frac{p_B}{1-\gamma})+\alpha\left(p_B-w_2^{OC*}\right)(1-\frac{p_B-p_2}{1-\gamma-\beta})+\alpha t(1-\frac{p_B-p_2}{1-\beta})}{(1-p_B)+\alpha(1-\frac{p_B-p_2}{1-\beta})};\\
&f_{DH} = ((1-\phi)(p_2-k_2) - (x_{DH}(\Delta)-k_2)\tfrac{\beta}{1-\gamma})\tfrac{R}{\phi}, \text{ where } x_{DH}(\Delta) = p_B - \frac{(p_B-k_2-\Delta-t)(1-\frac{p_B-p_2}{1-\beta})}{(1-\frac{p_B}{1-\gamma})};\\
&f_{DO1} = ((1-\phi)(p_2-k_2) - (x_{DO1}(\Delta)-k_2)\tfrac{\beta}{1-\gamma})\tfrac{R}{\phi}, \text{ where } x_{DO1}(\Delta) = p_B - \frac{(p_B-k_2-\Delta)(1-p_B)+\alpha(p_B-k_2)(1-\frac{p_B-p_2}{1-\gamma-\beta})+t(1-\frac{p_B}{1-\gamma})}{(1+\alpha)(1-\frac{p_B}{1-\gamma})};\\
&f_{DO2} = ((1-\phi)(p_2-k_2) - (x_{DO2}(\Delta)-k_2)\tfrac{\beta}{1-\gamma})\tfrac{R}{\phi}, \text{ where } x_{DO2}(\Delta) = p_B - \frac{\left(p_B-w_2^{OC*}-t\right)((1-\frac{p_B}{1-\gamma})+\alpha(1-\frac{p_B-p_2}{1-\gamma-\beta}))-(p_B-k_2-\Delta)(1-p_B)}{\alpha(1-\frac{p_B}{1-\gamma})};\\
&f_{DO3} = ((1-\phi)(p_2-k_2) - (x_{DO3}(\Delta)-k_2)\tfrac{\beta}{1-\gamma})\tfrac{R}{\phi}, \text{ where } x_{DO3}(\Delta) = p_B - \frac{(p_B-k_2-\Delta)(1-p_B)}{(1-\frac{p_B}{1-\gamma})} - t;\\
&f_{DO4} = \frac{(p_B-\frac{(p_B-k_2-\Delta)(1-p_B)}{1-\frac{p_B}{1-\gamma}}-k_2-t(1-\frac{p_B}{1-\gamma}))\beta(1-\frac{p_B}{1-\gamma})-(1-\phi)\alpha(p_2-k_2)(1-\frac{p_2}{\beta})\beta+(1-\phi)\alpha(1-\gamma)(1-\frac{p_B}{1-\gamma})(p_2-k_2)}{\left(\frac{(1-\gamma)(1-\gamma-\beta)(1-\frac{p_B}{1-\gamma})}{(\beta p_B-(1-\gamma)p_2)}-1\right)\beta\phi};
\end{aligned}
\tag{6}
$$

where $w_2^{OC*} = k_2 + \frac{\alpha(1-\phi)(p_2-k_2)(1-\frac{p_B-p_2}{1-\gamma-\beta})}{(1-\frac{p_B}{1-\gamma})+\alpha(1-\frac{p_B-p_2}{1-\gamma-\beta})}$.

The equilibrium sourcing strategy of the brand-name firm is as follows:

(a) Strategy H with $w_1^* = k_1$ if $e < f_{DH}$ and $\Delta < \min\{\Delta_{DH}, \Delta_{HO}\}$;

(b) Strategy D with $w_1^* = k_1$, and

$$
w_2^* = \begin{cases} k_2, & \text{if } e \le \min\{e_{D1}, f_{DO1}\} \text{ and } \min\{\Delta_{DH}, \Delta_{DO}\} \le \Delta < \Delta_{DO};\\ w_2^{(0)}, & \text{if } \max\{e_{D1}, f_{DH}, f_{DO2}\} \le e \le \min\{\frac{(1-\phi)e_3}{\phi}, f_{DO3}\}, \text{ or if } e > \max\{\frac{(1-\phi)e_3}{\phi}, f_{DO4}\}; \end{cases}
$$

(c) Strategy O with

$$
w_2^* = \begin{cases} w_2^{O(1)}, & \text{if } e < \min\{e_{O1}, f_{DO2}\} \text{ and } \Delta > \max\{\Delta_{HO}, \Delta_{DO}\};\\ \max\{w_2^{(0)}, w_2^{O(2)}\}, & \text{if } \max\{e_{O1}, f_{DO1}, f_{DO3}\} \le e \le \frac{(1-\phi)e_3}{\phi}, \text{ or if } \frac{(1-\phi)e_3}{\phi} < e < f_{DO4}, \end{cases}
$$

where e_3 is defined in Equation (7).

In this extension, the equilibrium is similar to that in the base model. We find that the consumer surplus under each optimal strategy is the same as that in the base model, while the social surplus can be lower or higher than that in the base model.

E-Companion B Proofs of Analytical Results

B.1 Proof of Lemma 1.

This proof has two steps: (1) we derive the profit expressions under each possible strategy; (2) we focus on the discussion about the counterfeiter or the authorised overseas supplier about whether to sell the counterfeit.

Step 1: Under each possible sourcing strategy, we obtain the profit expression of each firm as below.

Strategy H: Given wholesale prices w_1 and w_2, the home supplier accepts the contract and the counterfeiter rejects the contract, i.e., $d_1 = 1$ and $d_2 = 0$. Thus, the brand-name firm only sources from the home supplier.

(1) If the counterfeiter sells the counterfeit in the overseas market, i.e., $s = 1$, the expected profits of the brand-name firm, the home and overseas suppliers are given below:

$$\begin{aligned}
&\pi_B^H(w_1) = (p_B - w_1)(1 - p_B) + \alpha(p_B - w_1 - t)\left(1 - \tfrac{p_B - p_2}{1-\beta}\right), \\
&\pi_1^H(w_1) = (w_1 - k_1)\left((1 - p_B) + \alpha\left(1 - \tfrac{p_B - p_2}{1-\beta}\right)\right), \\
&\pi_2^H = \alpha(p_2 - k_2)\left(\tfrac{p_B - p_2}{1-\beta} - \tfrac{p_2}{\beta}\right) - e.
\end{aligned}$$

(2) If the counterfeiter does not sell the counterfeit, i.e., $s = 0$, the brand-name firm is the monopoly in the overseas market. Thus, their profits expressions are:

$$\pi_B^H(w_1) = (p_B - w_1)(1 - p_B) + \alpha(p_B - w_1 - t)(1 - p_B), \quad \pi_1^H(w_1) = (1+\alpha)(w_1 - k_1)(1 - p_B), \quad \pi_2^H = 0.$$

Strategy D: Given wholesale prices w_1 and w_2, the home supplier and the counterfeiter accept their contracts, respectively, i.e., $d_1 = 1$ and $d_2 = 1$. Then, the counterfeiter is converted to an authorized overseas supplier. Thus, their profit expressions are as follows.

(1) If the overseas supplier sells the counterfeit in the overseas market, i.e., $s = 1$:

$$\begin{aligned}
&\pi_B^D(w_1, w_2) = (p_B - w_1)(1 - p_B) + \alpha(p_B - w_2)\left(1 - \tfrac{p_B - p_2}{1-\gamma-\beta}\right), \\
&\pi_1^D(w_1) = (w_1 - k_1)(1 - p_B), \\
&\pi_2^D(w_2) = \alpha(w_2 - k_2)\left(1 - \tfrac{p_B - p_2}{1-\gamma-\beta}\right) + \alpha(p_2 - k_2)\left(\tfrac{p_B - p_2}{1-\gamma-\beta} - \tfrac{p_2}{\beta}\right) - e.
\end{aligned}$$

(2) If the overseas supplier does not sell the counterfeit, i.e., $s = 0$:

$$\begin{aligned}
&\pi_B^D(w_1, w_2) = (p_B - w_1)(1 - p_B) + \alpha(p_B - w_2)\left(1 - \tfrac{p_B}{1-\gamma}\right), \\
&\pi_1^D(w_1) = (w_1 - k_1)(1 - p_B), \\
&\pi_2^D(w_2) = \alpha(w_2 - k_2)\left(1 - \tfrac{p_B}{1-\gamma}\right).
\end{aligned}$$

Strategy O: Given wholesale prices w_1 and w_2, the home supplier rejects the contract and the counterfeiter accepts the contract, i.e., $d_1 = 0$ and $d_2 = 1$. Then, the counterfeiter is converted to an authorized overseas supplier. Thus, their profit expressions are as follows.

(1) If the overseas supplier sells the counterfeit in the overseas market, i.e., $s = 1$:

$$\begin{aligned}
&\pi_B^O(w_2) = (p_B - w_2 - t)\left(1 - \tfrac{p_B}{1-\gamma}\right) + \alpha(p_B - w_2)\left(1 - \tfrac{p_B - p_2}{1-\gamma-\beta}\right), \\
&\pi_1^O = 0, \quad \pi_2^O(w_2) = (w_2 - k_2)\left(1 - \tfrac{p_B}{1-\gamma}\right) + \alpha(w_2 - k_2)\left(1 - \tfrac{p_B - p_2}{1-\gamma-\beta}\right) + \alpha(p_2 - k_2)\left(\tfrac{p_B - p_2}{1-\gamma-\beta} - \tfrac{p_2}{\beta}\right) - e.
\end{aligned}$$

(2) If the overseas supplier does not sell the counterfeit, i.e., $s = 0$:

$$\begin{aligned}
&\pi_B^O(w_2) = (p_B - w_2 - t)\left(1 - \tfrac{p_B}{1-\gamma}\right) + \alpha(p_B - w_2)\left(1 - \tfrac{p_B}{1-\gamma}\right), \\
&\pi_1^O = 0, \quad \pi_2^O(w_2) = (1+\alpha)(w_2 - k_2)\left(1 - \tfrac{p_B}{1-\gamma}\right).
\end{aligned}$$

Strategy N: Given wholesale prices w_1 and w_2, the home supplier and the counterfeiter reject their contracts, respectively, i.e., $d_1 = 0$ and $d_2 = 0$. Under this strategy, the brand-name firm does not have suppliers, and later we will show that it is not an equilibrium strategy.

(1) If the counterfeiter sells the counterfeit in the overseas market, i.e., $s = 1$:

$$\pi_B^N = 0, \quad \pi_1^N = 0, \quad \pi_2^N = \alpha\,(p_2 - k_2)\left(1 - \frac{p_2}{\beta}\right) - e.$$

(2) If the counterfeiter does not sell the counterfeit, i.e., $s = 0$:

$$\pi_B^N = 0, \quad \pi_1^N = 0, \quad \pi_2^N = 0.$$

For the analysis below, it is convenient to define the following notations:

$$\begin{aligned}
&M = \alpha\,(p_2 - k_2)\,(\tfrac{p_B - p_2}{1-\beta} - \tfrac{p_2}{\beta}) - e, && w_2^{D(2)} = k_2 + \tfrac{M}{\alpha(1-\frac{p_B}{1-\gamma})},\\
&M' = \alpha\,(p_2 - k_2)\,(\tfrac{p_B - p_2}{1-\gamma-\beta} - \tfrac{p_2}{\beta}) - e, && w_2^{(0)} = k_2 + \tfrac{M'}{\alpha\left(\frac{p_B - p_2}{1-\gamma-\beta} - \frac{p_B}{1-\gamma}\right)},\\
&K = \alpha\,(p_2 - k_2)\,(1 - \tfrac{p_2}{\beta}) - e, && w_2^{O(2)} = k_2 + \tfrac{K}{(1+\alpha)(1-\frac{p_B}{1-\gamma})},\\
&K - M' = \alpha\,(p_2 - k_2)\,(1 - \tfrac{p_B - p_2}{1-\gamma-\beta}), && w_2^{O(1)} = k_2 + \tfrac{K - M'}{(1-\frac{p_B}{1-\gamma}) + \alpha(1-\frac{p_B - p_2}{1-\gamma-\beta})}.
\end{aligned}$$

With the assumption $0 \le e < \alpha(p_2 - k_2)(\frac{p_B - p_2}{1-\beta} - \frac{p_2}{\beta})$, we know, $M > 0$, $M' > 0$ and $K > 0$. Note that $M < M'$.

Step 2: We discuss whether the counterfeiting exists.

In the following, we make a comparison between $\pi_2^H\,(s = 1)$ and $\pi_2^H\,(s = 0)$. There are two scenarios depending on d_2.

1. When the counterfeiter does not accept the contract, i.e., $d_2 = 0$, which means she is not converted to an authorized overseas supplier, we have the below discussion.

(1) Under Strategy H, if the counterfeiter sells the counterfeit in the overseas market, her profit is $\pi_2^H\,(w_2, s = 1) = \alpha\,(p_2 - k_2)\,(\frac{p_B - p_2}{1-\beta} - \frac{p_2}{\beta}) - e$.

(2) Under Strategy N, if the counterfeiter sells the counterfeit in the overseas market, her profit is $\pi_2^N\,(w_2, s = 1) = \alpha\,(p_2 - k_2)\,(1 - \frac{p_2}{\beta}) - e$.

Note that we assume $0 \le e < \alpha(p_2 - k_2)(\frac{p_B - p_2}{1-\beta} - \frac{p_2}{\beta})$. Thus, when the counterfeiter does not accept the contract, she will sell the counterfeit in the overseas market.

2. When the counterfeiter accepts the contract, i.e., $d_2 = 1$, which means she becomes an authorized overseas supplier, we have the below discussion.

(1) Under Strategy D, if the overseas supplier does not sell the counterfeit in the overseas market, her profit is $\pi_2^D\,(w_2, s = 0) = \alpha\,(w_2 - k_2)\,(1 - \frac{p_B}{1-\gamma})$. If the overseas supplier sells the counterfeit in the overseas market, her profit is $\pi_2^D\,(w_2, s = 1) = \alpha\,(w_2 - k_2)\,(1 - \frac{p_B - p_2}{1-\gamma-\beta}) + (\alpha\,(p_2 - k_2)\,(\frac{p_B - p_2}{1-\gamma-\beta} - \frac{p_2}{\beta}) - e)$.

Then, from $\pi_2^D\,(w_2, s = 0) \ge \pi_2^D\,(w_2, s = 1)$, we obtain, $w_2 \ge w_2^{(0)}$, where $w_2^{(0)} = k_2 + \frac{M'}{\alpha(\frac{p_B - p_2}{1-\gamma-\beta} - \frac{p_B}{1-\gamma})}$.

(2) Under Strategy O, if the overseas supplier does not sell the counterfeit in the overseas market, her profit is $\pi_2^O(w_2, s=0) = (w_2 - k_2)(1 - \frac{p_B}{1-\gamma}) + \alpha(w_2 - k_2)(1 - \frac{p_B}{1-\gamma})$. If the overseas supplier sells the counterfeit in the overseas market, her profit is $\pi_2^O(w_2, s=1) = (w_2 - k_2)(1 - \frac{p_B}{1-\gamma}) + \alpha(w_2 - k_2)(1 - \frac{p_B - p_2}{1-\gamma-\beta}) + (\alpha(p_2 - k_2)(\frac{p_B - p_2}{1-\gamma-\beta} - \frac{p_2}{\beta}) - e)$.

Then, from $\pi_2^O(w_2, s=0) \geq \pi_2^O(w_2, s=1)$, we obtain, $w_2 \geq w_2^{(0)}$.

Thus, if the wholesale price of w_2 satisfies $w_2 < w_2^{(0)}$, then even the counterfeiter is converted to an authorized overseas supplier, she would still sell counterfeits in the overseas market, i.e., $s(w_1, w_2, d_1) = 1$ with $d_2 = 1$. ■

B.2 Proof of Lemma 2.

This proof has two steps: (1) we derive the best response of two suppliers; (2) we discuss the possible optimal wholesale prices offered by the brand-name firm under each sourcing strategy. In order to differentiate the cases that the overseas supplier sells counterfeits, we use the superscripts "D†", "O†" to denote the Strategy D without counterfeiting, Strategy O without counterfeiting, respectively; and use the superscripts "DC", "OC" to denote the Strategy D with counterfeiting, Strategy O with counterfeiting, respectively.

Step 1: We derive the best responses of the overseas and home suppliers.

With each sourcing strategy, the overseas supplier's profit function is as follows:

$$\pi_2^H(w_2) = \alpha(p_2 - k_2)\left(\frac{p_B - p_2}{1-\beta} - \frac{p_2}{\beta}\right) - e,$$

$$\pi_2^D = \begin{cases} \pi_2^{DC}(w_2) = \alpha(w_2 - k_2)\left(1 - \frac{p_B - p_2}{1-\gamma-\beta}\right) + \left(\alpha(p_2 - k_2)\left(\frac{p_B - p_2}{1-\gamma-\beta} - \frac{p_2}{\beta}\right) - e\right), & \text{if } w_2 < w_2^{(0)}, \\ \pi_2^{D\dagger}(w_2) = \alpha(w_2 - k_2)(1 - \frac{p_B}{1-\gamma}), & \text{if } w_2 \geq w_2^{(0)}, \end{cases}$$

$$\pi_2^O = \begin{cases} \pi_2^{OC}(w_2) = (w_2 - k_2)\left(1 - \frac{p_B}{1-\gamma}\right) & \\ \qquad + \alpha(w_2 - k_2)\left(1 - \frac{p_B - p_2}{1-\gamma-\beta}\right) + \left(\alpha(p_2 - k_2)\left(\frac{p_B - p_2}{1-\gamma-\beta} - \frac{p_2}{\beta}\right) - e\right), & \text{if } w_2 < w_2^{(0)}, \\ \pi_2^{O\dagger}(w_2) = (w_2 - k_2)\left(1 - \frac{p_B}{1-\gamma}\right) + \alpha(w_2 - k_2)\left(1 - \frac{p_B}{1-\gamma}\right), & \text{if } w_2 \geq w_2^{(0)}, \end{cases}$$

$$\pi_2^N = \alpha(p_2 - k_2)(1 - \frac{p_2}{\beta}) - e.$$

1.1 Below, we discuss the conditions for overseas supplier's accepting.

(1) Under $w_2 < w_2^{(0)}$, where $w_2^{(0)} = k_2 + \frac{M'}{\alpha(\frac{p_B - p_2}{1-\gamma-\beta} - \frac{p_B}{1-\gamma})}$, we discuss for a given belief on the home supplier's contact decision $\widetilde{d_1} = 1$ and $\widetilde{d_1} = 0$, respectively.

(i) If $\widetilde{d_1} = 1$, then, we compare the overseas supplier's profits between Strategy D with counterfeiting and Strategy H, i.e., $\pi_2^{DC}(w_2)$ and π_2^H. If the overseas supplier decides to accept, then it should satisfy

$$\begin{aligned} &\pi_2^{DC}(w_2) \geq \pi_2^H, \\ \Rightarrow\ &\alpha(w_2 - k_2)\left(1 - \frac{p_B - p_2}{1-\gamma-\beta}\right) + M' \geq M, \\ \Rightarrow\ &w_2 \geq k_2 + \frac{M - M'}{\alpha\left(1 - \frac{p_B - p_2}{1-\gamma-\beta}\right)}. \end{aligned}$$

Note that $w_2 \geq k_2$. As $M < M'$, then, we have, $w_2 \geq k_2$.

(ii) If $\widetilde{d_1} = 0$, then, we compare the overseas supplier's profits between Strategy O with counterfeiting and Strategy N, i.e., $\pi_2^{OC}(w_2)$ and π_2^N. If the overseas supplier decides to accept, then it should satisfy

$$\begin{aligned}
&\pi_2^{OC}(w_2) \geq \pi_2^N, \\
\Rightarrow\ & (w_2 - k_2)\left(1 - \tfrac{p_B}{1-\gamma}\right) + \alpha(w_2 - k_2)\left(1 - \tfrac{p_B - p_2}{1-\gamma-\beta}\right) + M' \geq K, \\
\Rightarrow\ & w_2 \geq w_2^{O(1)}, \text{ where } w_2^{O(1)} = k_2 + \frac{K - M'}{\left(1 - \frac{p_B}{1-\gamma}\right) + \alpha\left(1 - \frac{p_B - p_2}{1-\gamma-\beta}\right)} = k_2 + \frac{\alpha(p_2 - k_2)\left(1 - \frac{p_B - p_2}{1-\gamma-\beta}\right)}{\left(1 - \frac{p_B}{1-\gamma}\right) + \alpha\left(1 - \frac{p_B - p_2}{1-\gamma-\beta}\right)}.
\end{aligned}$$

Thus, in the case of $w_2 < w_2^{(0)}$, we obtain

$$d_2(\widetilde{d_1}) = \begin{cases}
d_2\left(\widetilde{d_1} = 1\right) = 1, & \text{if } k_2 \leq w_2 < w_2^{(0)}, \\
d_2\left(\widetilde{d_1} = 1\right) = 0, & \text{if } w_2 < k_2, \\
d_2\left(\widetilde{d_1} = 0\right) = 1, & \text{if } \min\{w_2^{O(1)}, w_2^{(0)}\} \leq w_2 < w_2^{(0)}, \\
d_2\left(\widetilde{d_1} = 0\right) = 0, & \text{if } w_2 < \min\{w_2^{O(1)}, w_2^{(0)}\}.
\end{cases}$$

(2) Under $w_2 \geq w_2^{(0)}$, we discuss for given $\widetilde{d_1} = 1$ and $\widetilde{d_1} = 0$, respectively.

(i) If $\widetilde{d_1} = 1$, then, we compare the overseas supplier's profits between Strategy D without counterfeiting and Strategy H, i.e., $\pi_2^{D\dagger}(w_2)$ and π_2^H. If the overseas supplier decides to accept, then it should satisfy

$$\begin{aligned}
&\pi_2^{D\dagger}(w_2) \geq \pi_2^H, \\
\Rightarrow\ & \alpha(w_2 - k_2)\left(1 - \tfrac{p_B}{1-\gamma}\right) \geq M, \\
\Rightarrow\ & w_2 \geq w_2^{D(2)}, \text{ where } w_2^{D(2)} = k_2 + \frac{M}{\alpha\left(1 - \frac{p_B}{1-\gamma}\right)}.
\end{aligned}$$

(ii) If $\widetilde{d_1} = 0$, then, we compare the overseas supplier's profits between Strategy O without counterfeiting and Strategy N, i.e., $\pi_2^{O\dagger}(w_2)$ and π_2^N. If the overseas supplier decides to accept, then it should satisfy

$$\begin{aligned}
&\pi_2^{O\dagger}(w_2) \geq \pi_2^N, \\
\Rightarrow\ & (w_2 - k_2)\left(1 - \tfrac{p_B}{1-\gamma}\right) + \alpha(w_2 - k_2)\left(1 - \tfrac{p_B}{1-\gamma}\right) \geq K, \\
\Rightarrow\ & w_2 \geq w_2^{O(2)}, \text{ where } w_2^{O(2)} = k_2 + \frac{K}{(1+\alpha)\left(1 - \frac{p_B}{1-\gamma}\right)}.
\end{aligned}$$

Thus, in the case of $w_2 \geq w_2^{(0)}$, we obtain

$$d_2(\widetilde{d_1}) = \begin{cases}
d_2\left(\widetilde{d_1} = 1\right) = 1, & \text{if } w_2 \geq \max\{w_2^{D(2)}, w_2^{(0)}\}, \\
d_2\left(\widetilde{d_1} = 1\right) = 0, & \text{if } w_2^{(0)} < w_2 < \max\{w_2^{D(2)}, w_2^{(0)}\}, \\
d_2\left(\widetilde{d_1} = 0\right) = 1, & \text{if } w_2 \geq \max\{w_2^{O(2)}, w_2^{(0)}\}, \\
d_2\left(\widetilde{d_1} = 0\right) = 0, & \text{if } w_2^{(0)} < w_2 < \max\{w_2^{O(2)}, w_2^{(0)}\}.
\end{cases}$$

1.2 Similarly, we derive the best response function of the home supplier $d_1(\widetilde{d_2})$ to the overseas supplier's action $\widetilde{d_2} \in \{0, 1\}$ as follows:

$$d_1(\widetilde{d_2}) = \begin{cases}
d_1\left(\widetilde{d_2} = 1\right) = 1, & \text{if } w_1 \geq k_1, \\
d_1\left(\widetilde{d_2} = 0\right) = 1, & \text{if } w_1 \geq k_1, \\
d_1\left(\widetilde{d_2} = 1\right) = 0, & \text{if } w_1 < k_1, \\
d_1\left(\widetilde{d_2} = 0\right) = 0, & \text{if } w_1 < k_1.
\end{cases}$$

1.3 Given best response functions $d_1(\widetilde{d_2})$ and $d_2(\widetilde{d_1})$, we obtain the following fixed point (d_1^*, d_2^*) that satisfies $(d_1(\widetilde{d_2}), \widetilde{d_2}) = (\widetilde{d_1}, d_2(\widetilde{d_1}))$. Thus, the optimal decisions of two suppliers are

$$(d_1^*, d_2^*) = \begin{cases} (1,1), & \text{if } w_1 \geq k_1,\, k_2 \leq w_2 < w_2^{(0)} \text{ or } w_2 \geq \max\{w_2^{D(2)}, w_2^{(0)}\}, \\ (1,0), & \text{if } w_1 \geq k_1,\, w_2 < k_2 \text{ or } w_2^{(0)} \leq w_2 < \max\{w_2^{D(2)}, w_2^{(0)}\}, \\ (0,1), & \text{if } w_1 < k_1,\, \min\{w_2^{O(1)}, w_2^{(0)}\} \leq w_2 < w_2^{(0)} \text{ or } w_2 \geq \max\{w_2^{O(2)}, w_2^{(0)}\}, \\ (0,0), & \text{if } w_1 < k_1,\, w_2 < \min\{w_2^{O(1)}, w_2^{(0)}\} \text{ or } w_2^{(0)} \leq w_2 < \max\{w_2^{O(2)}, w_2^{(0)}\}, \end{cases}$$

where

$$\begin{aligned} &M = \alpha\,(p_2 - k_2)\,(\tfrac{p_B - p_2}{1-\beta} - \tfrac{p_2}{\beta}) - e, \;\; w_2^{D(2)} = k_2 + \tfrac{M}{\alpha(1-\frac{p_B}{1-\gamma})}; \\ &M' = \alpha\,(p_2 - k_2)\,(\tfrac{p_B - p_2}{1-\gamma-\beta} - \tfrac{p_2}{\beta}) - e, \; w_2^{(0)} = k_2 + \tfrac{M'}{\alpha\left(\frac{p_B-p_2}{1-\gamma-\beta} - \frac{p_B}{1-\gamma}\right)}; \\ &K = \alpha\,(p_2 - k_2)\,(1 - \tfrac{p_2}{\beta}) - e, \qquad w_2^{O(2)} = k_2 + \tfrac{K}{(1+\alpha)(1-\frac{p_B}{1-\gamma})}; \\ &K - M' = \alpha\,(p_2 - k_2)\,(1 - \tfrac{p_B - p_2}{1-\gamma-\beta}), \;\; w_2^{O(1)} = k_2 + \tfrac{K - M'}{(1-\frac{p_B}{1-\gamma}) + \alpha(1 - \frac{p_B - p_2}{1-\gamma-\beta})}. \end{aligned}$$

Note that $w_2^{O(1)}$ is independent on e; and $w_2^{(0)}$, $w_2^{O(2)}$ and $w_2^{D(2)}$ are dependent on e.

Step 2: We derive the optimal wholesale price(s) with each case.

Substituting (d_1^*, d_2^*) into the profit functions of the brand-name firm, we analyze the optimal wholesale price under each possible sourcing strategy.

$$\pi_B^H\,(w_1) = (p_B - w_1)\,(1 - p_B) + \alpha\,(p_B - w_1 - t)\,(1 - \tfrac{p_B - p_2}{1-\beta}), \text{ if } w_1 \geq k_1,$$

$$\pi_B^D = \begin{cases} \pi_B^{DC}\,(w_1, w_2) = (p_B - w_1)\,(1 - p_B) + \alpha\,(p_B - w_2)\,(1 - \frac{p_B - p_2}{1-\gamma-\beta}), \text{ if } w_1 \geq k_1,\, k_2 \leq w_2 < w_2^{(0)}, \\ \pi_B^{D\dagger}\,(w_1, w_2) = (p_B - w_1)\,(1 - p_B) + \alpha\,(p_B - w_2)\,(1 - \frac{p_B}{1-\gamma}), \quad \text{if } w_1 \geq k_1,\, w_2 \geq \max\{w_2^{D(2)}, w_2^{(0)}\}, \end{cases}$$

$$\pi_B^O = \begin{cases} \pi_B^{OC}\,(w_2) = (p_B - w_2 - t)\,(1 - \frac{p_B}{1-\gamma}) + \alpha\,(p_B - w_2)\,(1 - \frac{p_B - p_2}{1-\gamma-\beta}), \text{ if } \min\{w_2^{O(1)}, w_2^{(0)}\} \leq w_2 < w_2^{(0)}, \\ \pi_B^{O\dagger}\,(w_2) = (p_B - w_2 - t)\,(1 - \frac{p_B}{1-\gamma}) + \alpha\,(p_B - w_2)\,(1 - \frac{p_B}{1-\gamma}), \quad \text{if } w_2 \geq \max\{w_2^{O(2)}, w_2^{(0)}\}, \end{cases}$$

$$\pi_B^N = 0.$$

In the following, we have two steps: (1) firstly check the feasible region of π_B under each case; (2) then make a comparison between $\pi_B^{D\dagger}$ and π_B^{DC}, $\pi_B^{O\dagger}$ and π_B^{OC}, respectively.

2.1 We first check the feasibility of π_B under our assumption of $e < \bar{e}$, where $\bar{e} = \alpha(p_2 - k_2)(\frac{p_B - p_2}{1-\beta} - \frac{p_2}{\beta})$.

Recall that $w_2^{O(1)}$ is independent on e; and $w_2^{(0)}$, $w_2^{O(2)}$ and $w_2^{D(2)}$ are dependent on e. From the conditions of the brand-name firm's profit expression under each possible strategy, we know:

$$\begin{aligned} &w_2^{O(1)} < w_2^{(0)}(e) \quad\;\; \Rightarrow e < e_1, \text{ where } e_1 = \left(p_2 - k_2 - \frac{\alpha(p_2-k_2)(1-\frac{p_B-p_2}{1-\gamma-\beta})}{(1-\frac{p_B}{1-\gamma})+\alpha(1-\frac{p_B-p_2}{1-\gamma-\beta})}\frac{\beta}{1-\gamma}\right)\frac{\alpha(\beta p_B-(1-\gamma)p_2)}{(1-\gamma-\beta)\beta}; \\ &w_2^{D(2)}(e) < w_2^{(0)}(e) \Rightarrow e < e_2, \text{ where } e_2 = \frac{(1-\gamma)(p_2-k_2)\alpha(1-\frac{p_B}{1-\gamma})-\alpha(p_2-k_2)(\frac{p_B-p_2}{1-\beta}-\frac{p_2}{\beta})\beta}{\left(\frac{(1-\gamma)(1-\gamma-\beta)(1-\frac{p_B}{1-\gamma})}{(\beta p_B-(1-\gamma)p_2)}-1\right)\beta}; \\ &w_2^{O(2)}(e) < w_2^{(0)}(e) \Rightarrow e < e_3, \text{ where } e_3 = \frac{(1-\gamma)(p_2-k_2)(1+\alpha)(1-\frac{p_B}{1-\gamma})-\alpha(p_2-k_2)(1-\frac{p_2}{\beta})\beta}{\left(\frac{(1-\gamma)(1-\gamma-\beta)(1+\alpha)(1-\frac{p_B}{1-\gamma})}{\alpha(\beta p_B-(1-\gamma)p_2)}-1\right)\beta}. \end{aligned} \tag{7}$$

Note that $e_2 \geq \bar{e} > e_3 \geq e_1$. When $\gamma = 0$, we have, $e_2 = \bar{e} > e_3 = e_1$. Thus, with the assumption of $e < \bar{e}$ in our base model, if $w_2^{D(2)} > w_2^{(0)}$, then $e > e_2$ and $w_2 > w_2^{D(2)}$. As $e > e_2$ is out of the feasible region, it implies that under Strategy D with $e < \bar{e}$, the feasible condition is $w_2 > w_2^{(0)}$.

Note that for Strategy O with counterfeiting, i.e., $\pi_B^{OC}(w_2)$, the condition $\min\{w_2^{O(1)}, w_2^{(0)}\} \leq w_2 < w_2^{(0)}$ is non-empty if $e < e_1$.

As $\pi_B(w_1, w_2)$ decreases in w_1, then, the optimal wholesale price of the home supplier that the brand-name firm is willing to offer is equal to the production cost, that is, $w_1^H = k_1$ with Strategy H, and $w_1^D = k_1$ with Strategy D.

As $\pi_B(w_1, w_2)$ decreases in w_2, then, the optimal wholesale price of the overseas supplier that the brand-name firm is willing to offer is the lower bound of the feasible regions. We use $*$ to indicate the optimal wholesale decision of these cases. Then, the optimal wholesale prices w_2 for these cases are $w_2^{DC*} = k_2$, $w_2^{D\dagger*} = w_2^{(0)}$, $w_2^{OC*} = w_2^{O(1)}$, $w_2^{O\dagger*} = \max\{w_2^{O(2)}, w_2^{(0)}\}$, respectively.

Thus, we have following profit expression under each case:

$$\pi_B^H = (p_B - k_1)(1 - p_B) + \alpha (p_B - k_1 - t)(1 - \tfrac{p_B - p_2}{1-\beta}),$$

$$\pi_B^D = \begin{cases} \pi_B^{DC}\left(w_2^{DC*}\right) = (p_B - k_1)(1 - p_B) + \alpha \left(p_B - w_2^{DC*}\right)(1 - \tfrac{p_B - p_2}{\gamma - \beta}), \\ \pi_B^{D\dagger}\left(w_2^{D\dagger*}\right) = (p_B - k_1)(1 - p_B) + \alpha \left(p_B - w_2^{D\dagger*}\right)(1 - \tfrac{p_B}{\gamma}), \end{cases}$$

$$\pi_B^O = \begin{cases} \pi_B^{OC}\left(w_2^{OC*}\right) = \left(p_B - w_2^{OC*} - t\right)(1 - \tfrac{p_B}{\gamma}) + \alpha \left(p_B - w_2^{OC*}\right)(1 - \tfrac{p_B - p_2}{\gamma - \beta}), \text{ if } e < e_1, \\ \pi_B^{O\dagger}\left(w_2^{O\dagger*}\right) = \left(p_B - w_2^{O\dagger*} - t\right)(1 - \tfrac{p_B}{\gamma}) + \alpha \left(p_B - w_2^{O\dagger*}\right)(1 - \tfrac{p_B}{\gamma}), \end{cases}$$

$$\pi_B^N = 0,$$

where $w_2^{DC*} = k_2$, $w_2^{D\dagger*} = w_2^{(0)}$, $w_2^{OC*} = w_2^{O(1)}$, $w_2^{O\dagger*} = \max\{w_2^{O(2)}, w_2^{(0)}\}$.

2.2 Then, we make comparisons for strategies D and O, respectively.

Under Strategy D:

$$\pi_B^D = \begin{cases} \pi_B^{DC}\left(w_2^{DC*}\right) = (p_B - k_1)(1 - p_B) + \alpha \left(p_B - w_2^{DC*}\right)(1 - \tfrac{p_B - p_2}{1-\gamma-\beta}), \\ \pi_B^{D\dagger}\left(w_2^{D\dagger*}\right) = (p_B - k_1)(1 - p_B) + \alpha \left(p_B - w_2^{D\dagger*}\right)(1 - \tfrac{p_B}{1-\gamma}). \end{cases}$$

Then,

$$\begin{aligned} & \pi_B^{D\dagger}\left(w_2^{D\dagger*}\right) \geq \pi_B^{DC}\left(w_2^{DC*}\right), \\ \Rightarrow\ & \left(p_B - w_2^{D*}\right)(1 - \tfrac{p_B}{1-\gamma}) \geq \left(p_B - w_2^{DC*}\right)(1 - \tfrac{p_B - p_2}{1-\gamma-\beta}), \\ \Rightarrow\ & w_2^{D*} \leq \frac{p_B(\frac{p_B - p_2}{1-\gamma-\beta} - \frac{p_B}{1-\gamma}) + w_2^{DC*}(1 - \frac{p_B - p_2}{1-\gamma-\beta})}{1 - \frac{p_B}{1-\gamma}} = p_B - \frac{\left(p_B - w_2^{DC*}\right)(1 - \frac{p_B - p_2}{1-\gamma-\beta})}{1 - \frac{p_B}{1-\gamma}}, \\ \Rightarrow\ & e \geq e_{D1} \text{ , where } e_{D1} = \left(p_2 - k_2 - \left(\frac{(p_B - k_2)(\frac{p_B - p_2}{1-\gamma-\beta} - \frac{p_B}{1-\gamma})}{1 - \frac{p_B}{1-\gamma}}\right)\frac{\beta}{1-\gamma}\right)\frac{\alpha(\beta p_B - (1-\gamma)p_2)}{(1-\gamma-\beta)\beta}. \end{aligned}$$

Under Strategy O:

$$\pi_B^O = \begin{cases} \pi_B^{OC}\left(w_2^{OC*}\right) = \left(p_B - w_2^{OC*} - t\right)(1 - \tfrac{p_B}{1-\gamma}) + \alpha \left(p_B - w_2^{OC*}\right)(1 - \tfrac{p_B - p_2}{1-\gamma-\beta}), \text{ if } e < e_1, \\ \pi_B^{O\dagger}\left(w_2^{O\dagger*}\right) = \left(p_B - w_2^{O\dagger*} - t\right)(1 - \tfrac{p_B}{1-\gamma}) + \alpha \left(p_B - w_2^{O\dagger*}\right)(1 - \tfrac{p_B}{1-\gamma}). \end{cases}$$

Then,

$$\begin{aligned}
&\pi_B^{O\dagger}\left(w_2^{O\dagger*}\right) \geq \pi_B^{OC}\left(w_2^{OC*}\right),\\
\Rightarrow\ &\alpha(p_B - w_2^{OC*})(\tfrac{p_B-p_2}{1-\gamma-\beta} - \tfrac{p_B}{1-\gamma}) \geq (w_2^{O\dagger*} - w_2^{OC*})(1+\alpha)(1-\tfrac{p_B}{1-\gamma}),\\
\Rightarrow\ &w_2^{O\dagger*} \leq \frac{\alpha(p_B - w_2^{OC*})(\frac{p_B-p_2}{1-\gamma-\beta} - \frac{p_B}{1-\gamma})}{(1+\alpha)(1-\frac{p_B}{1-\gamma})} + w_2^{OC*},\\
\Rightarrow\ &e \geq e_{O1}\text{ , where } e_{O1} = \left(p_2 - k_2 - (\frac{\alpha(p_B - w_2^{OC*})(\frac{p_B-p_2}{1-\gamma-\beta} - \frac{p_B}{\gamma})}{(1+\alpha)(1-\frac{p_B}{1-\gamma})} + w_2^{OC*} - k_2)\frac{\beta}{1-\gamma}\right)\frac{\alpha(\beta p_B - (1-\gamma)p_2)}{(1-\gamma-\beta)\beta}.
\end{aligned}$$

Then, based on above discussion, we have the following optimal wholesale price w_2 for Strategy D and Strategy O, respectively. Note that $e_{O1} < e_1$.

(a) Under Strategy D, (i) $w_2^D = k_2$ and $s^* = 1$, if $e < e_{D1}$; (ii) $w_2^D = w_2^{(0)}$ and $s^* = 0$, if $e \geq e_{D1}$;

(b) under Strategy O, (i) $w_2^O = w_2^{O(1)}$ and $s^* = 1$, if $e < e_{O1}$; (ii) $w_2^O = \max\{w_2^{O(2)}, w_2^{(0)}\}$ and $s^* = 0$, if $e \geq e_{O1}$;

where

$$\begin{aligned}
e_{D1} &= \left(p_2 - k_2 - (\frac{(p_B-k_2)(\frac{p_B-p_2}{1-\gamma-\beta} - \frac{p_B}{1-\gamma})}{1-\frac{p_B}{1-\gamma}})\frac{\beta}{1-\gamma}\right)\frac{\alpha(\beta p_B-(1-\gamma)p_2)}{(1-\gamma-\beta)\beta},\\
e_{O1} &= \left(p_2 - k_2 - (\frac{\alpha(p_B - w_2^{OC*})(\frac{p_B-p_2}{1-\gamma-\beta} - \frac{p_B}{\gamma})}{(1+\alpha)(1-\frac{p_B}{1-\gamma})} + w_2^{OC*} - k_2)\frac{\beta}{1-\gamma}\right)\frac{\alpha(\beta p_B-(1-\gamma)p_2)}{(1-\gamma-\beta)\beta};
\end{aligned} \tag{8}$$

and $w_2^{OC*} = k_2 + \frac{\alpha(p_2-k_2)(1-\frac{p_B-p_2}{1-\gamma-\beta})}{(1-\frac{p_B}{1-\gamma})+\alpha(1-\frac{p_B-p_2}{1-\gamma-\beta})}$.

Thus, we have the results. ■

B.3 Proof of Proposition 1.

Recall that the brand-name firm's optimal profit under each case is as follows:

$$\pi_B^H = (p_B - k_1)(1-p_B) + \alpha(p_B - k_1 - t)(1 - \tfrac{p_B-p_2}{1-\beta}),$$

$$\pi_B^D = \begin{cases} \pi_B^{D\dagger}\left(w_2^{D\dagger*}\right) = (p_B-k_1)(1-p_B) + \alpha\left(p_B - w_2^{D\dagger*}\right)(1-\frac{p_B}{1-\gamma}), & \text{if } e \geq e_{D1},\\ \pi_B^{DC}\left(w_2^{DC*}\right) = (p_B-k_1)(1-p_B) + \alpha\left(p_B - w_2^{DC*}\right)(1-\frac{p_B-p_2}{1-\gamma-\beta}), & \text{if } e < e_{D1},\end{cases}$$

$$\pi_B^O = \begin{cases} \pi_B^{O\dagger}\left(w_2^{O\dagger*}\right) = \left(p_B - w_2^{O\dagger*} - t\right)(1-\frac{p_B}{1-\gamma}) + \alpha\left(p_B - w_2^{O\dagger*}\right)(1-\frac{p_B}{1-\gamma}), & \text{if } e \geq e_{O1},\\ \pi_B^{OC}\left(w_2^{OC*}\right) = \left(p_B - w_2^{OC*} - t\right)(1-\frac{p_B}{1-\gamma}) + \alpha\left(p_B - w_2^{OC*}\right)(1-\frac{p_B-p_2}{1-\gamma-\beta}), & \text{if } e < e_{O1},\end{cases}$$

$$\pi_B^N = 0,$$

where $w_2^{DC*} = k_2$, $w_2^{D\dagger*} = w_2^{(0)}$, $w_2^{OC*} = w_2^{O(1)}$, $w_2^{O\dagger*} = \max\{w_2^{O(2)}, w_2^{(0)}\}$.

Note that $w_2^{D\dagger*}$ and $w_2^{O\dagger*}$ are dependent on e, and w_2^{DC*} and w_2^{OC*} are independent on e.

In the following, before we analyze the comparison results in our equilibrium, we have below lemma for the general comparison results.

LEMMA B2. *The equilibrium sourcing strategy of the brand-name firm is as follows:*

(a) Strategy H with $w_1^* = k_1$ *if* $e < \min\{f_{DH}, f_{HO}\}$ *and* $\Delta < \min\{\Delta_{DH}, \Delta_{HO}\}$*;*

(b) Strategy D with $w_1^* = k_1$*, and*

$$w_2^* = \begin{cases} w_2^{DC*}, & \textit{if } e < \min\{e_{D1}, f_{DO1}\} \textit{ and } \min\{\Delta_{DH}, \Delta_{DO}\} \leq \Delta < \Delta_{DO},\\ w_2^{D\dagger*}, & \textit{if } \max\{e_{D1}, f_{DH}, f_{DO2}\} \leq e \leq \min\{e_3, f_{DO3}\}, \textit{ or if } e > \max\{e_3, f_{DO4}\};\end{cases}$$

(c) Strategy O with

$$w_2^* = \begin{cases} w_2^{OC*}, & \textit{if } e < \min\{e_{O1}, f_{DO2}\} \textit{ and } \Delta > \max\{\Delta_{HO}, \Delta_{DO}\}, \\ w_2^{O\dagger *}, & \textit{if } \max\{e_{O1}, f_{DO1}, f_{DO3}, f_{HO}\} \le e < e_3, \textit{ or if } \max\{e_3, f_{HO}\} < e < f_{DO4}. \end{cases}$$

where the thresholds are derived by

$$\begin{array}{lll}
\pi_B^H > \pi_B^{O\dagger} & \Rightarrow w_2^{O*} > p_B - \frac{(p_B - k_2 - \Delta)((1-p_B) + \alpha(1-\frac{p_B-p_2}{1-\beta})) - \alpha t(1-\frac{p_B-p_2}{1-\beta}) + t(1-\frac{p_B}{\gamma})}{(1+\alpha)(1-\frac{p_B}{\gamma})}, & \Rightarrow e < f_{HO}, \\
\pi_B^H > \pi_B^{OC} & \Rightarrow \Delta < p_B - k_2 - \frac{\left(p_B - w_2^{OC*} - t\right)(1-\frac{p_B}{\gamma}) + \alpha\left(p_B - w_2^{OC*}\right)(1-\frac{p_B-p_2}{\gamma-\beta}) + \alpha t(1-\frac{p_B-p_2}{1-\beta})}{(1-p_B) + \alpha(1-\frac{p_B-p_2}{1-\beta})}, & \Rightarrow \Delta < \Delta_{HO}, \\
\pi_B^H > \pi_B^{DC} & \Rightarrow \Delta < p_B - k_2 - \frac{\left(p_B - w_2^{DC*}\right)(1-\frac{p_B-p_2}{1-\gamma-\beta})}{1-\frac{p_B-p_2}{1-\beta}} - t, & \Rightarrow \Delta < \Delta_{DH}, \\
\pi_B^H > \pi_B^{D\dagger} & \Rightarrow w_2^{D\dagger *} > p_B - \frac{(p_B - k_2 - \Delta - t)(1-\frac{p_B-p_2}{1-\beta})}{1-\frac{p_B}{1-\gamma}}, & \Rightarrow e < f_{DH}, \\
\pi_B^{D\dagger} > \pi_B^{O\dagger} & \Rightarrow w_2^{O\dagger *} > p_B - \frac{\alpha(w_2^{O*} - w_2^{D*})(1-\frac{p_B}{1-\gamma}) + (p_B - k_2 - \Delta)(1-p_B)}{(1-\frac{p_B}{1-\gamma})} - t, & \Rightarrow e < f_{DO3}, or, e > f_{DO4}, \\
\pi_B^{D\dagger} > \pi_B^{OC} & \Rightarrow w_2^{D\dagger *} < p_B - \frac{\left(p_B - w_2^{OC*} - t\right)(1-\frac{p_B}{1-\gamma}) + \alpha\left(p_B - w_2^{OC*}\right)(1-\frac{p_B-p_2}{1-\gamma-\beta}) - (p_B - k_2 - \Delta)(1-p_B)}{\alpha(1-\frac{p_B}{1-\gamma})}, & \Rightarrow e > f_{DO2}, \\
\pi_B^{D\dagger} > \pi_B^{DC} & \Rightarrow w_2^{D\dagger *} < \frac{p_B(\frac{p_B-p_2}{1-\gamma-\beta} - \frac{p_B}{1-\gamma}) + w_2^{DC*}(1-\frac{p_B-p_2}{1-\gamma-\beta})}{1-\frac{p_B}{1-\gamma}} = p_B - \frac{\left(p_B - w_2^{DC*}\right)(1-\frac{p_B-p_2}{1-\gamma-\beta})}{1-\frac{p_B}{1-\gamma}}, & \Rightarrow e > e_{D1}, \\
\pi_B^{DC} > \pi_B^{O\dagger} & \Rightarrow w_2^{O\dagger *} > p_B - \frac{(p_B - k_2 - \Delta)(1-p_B) + \alpha\left(p_B - w_2^{DC*}\right)(1-\frac{p_B-p_2}{1-\gamma-\beta}) + t(1-\frac{p_B}{1-\gamma})}{(1+\alpha)(1-\frac{p_B}{1-\gamma})}, & \Rightarrow e < f_{DO1}, \\
\pi_B^{DC} > \pi_B^{OC} & \Rightarrow \Delta < p_B - k_2 - \frac{\left(p_B - w_2^{OC*} - t\right)(1-\frac{p_B}{1-\gamma}) - \alpha\left(w_2^{OC*} - w_2^{DC*}\right)(1-\frac{p_B-p_2}{1-\gamma-\beta})}{1-p_B}, & \Rightarrow \Delta < \Delta_{DO}, \\
\pi_B^{O\dagger} > \pi_B^{OC} & \Rightarrow w_2^{O\dagger *} < \frac{\alpha(p_B - w_2^{OC*})(\frac{p_B-p_2}{1-\gamma-\beta} - \frac{p_B}{1-\gamma})}{(1+\alpha)(1-\frac{p_B}{1-\gamma})} + w_2^{OC*}, & \Rightarrow e > e_{O1}.
\end{array}$$

Proof of Lemma B2: There are three steps to make comparisons about the brand-name firm's profits: (1) we compare Strategy *D* without counterfeiting, Strategy *D* with counterfeiting, and Strategy *O* without counterfeiting, Strategy *O* with counterfeiting; (2) we compare every strategy with Strategy H; (3) we summarize the whole conditions for each Strategy.

1. We make comparisons between strategies D and O. Then, we have four cases to compare:

(1.1) $\pi_B^{D\dagger}$ and $\pi_B^{O\dagger}$:

$$\begin{cases} \pi_B^{D\dagger} = (p_B - k_1)(1-p_B) + \alpha\left(p_B - w_2^{D\dagger *}\right)(1-\frac{p_B}{1-\gamma}), & \text{if } e \ge e_{D1}, \\ \pi_B^{O\dagger} = \left(p_B - w_2^{O\dagger *} - t\right)(1-\frac{p_B}{1-\gamma}) + \alpha\left(p_B - w_2^{O\dagger *}\right)(1-\frac{p_B}{1-\gamma}), & \text{if } e \ge e_{O1}. \end{cases}$$

Thus,

$$\begin{aligned} & \pi_B^{O\dagger} > \pi_B^{D\dagger}, \\ \Rightarrow & \left(p_B - w_2^{O\dagger *} - t\right)(1-\frac{p_B}{1-\gamma}) > \alpha(w_2^{O\dagger *} - w_2^{D\dagger *})(1-\frac{p_B}{1-\gamma}) + (p_B - k_2 - \Delta)(1-p_B), \\ \Rightarrow & w_2^{O\dagger *} < p_B - \frac{\alpha(w_2^{O\dagger *} - w_2^{D*})(1-\frac{p_B}{1-\gamma}) + (p_B - k_2 - \Delta)(1-p_B)}{(1-\frac{p_B}{1-\gamma})} - t. \end{aligned}$$

(1.2) $\pi_B^{D\dagger}$ and π_B^{OC}:

$$\begin{cases} \pi_B^{D\dagger} = (p_B - k_1)(1-p_B) + \alpha\left(p_B - w_2^{D\dagger *}\right)(1-\frac{p_B}{1-\gamma}), & \text{if } e \ge e_{D1}, \\ \pi_B^{OC} = (p_B - w_2^{OC*} - t)(1-\frac{p_B}{1-\gamma}) + \alpha(p_B - w_2^{OC*})(1-\frac{p_B-p_2}{1-\gamma-\beta}), & \text{if } e < e_{O1}. \end{cases}$$

Thus,

$$\begin{aligned} & \pi_B^{OC} > \pi_B^{D\dagger}, \\ \Rightarrow & (p_B - w_2^{OC*} - t)(1-\frac{p_B}{1-\gamma}) + \alpha(p_B - w_2^{OC*})(1-\frac{p_B-p_2}{1-\gamma-\beta}) > (p_B - k_2 - \Delta)(1-p_B) + \alpha\left(p_B - w_2^{D\dagger *}\right)(1-\frac{p_B}{\gamma}), \\ \Rightarrow & w_2^{D\dagger *} > p_B - \frac{\left(p_B - w_2^{OC*} - t\right)(1-\frac{p_B}{1-\gamma}) + \alpha\left(p_B - w_2^{OC*}\right)(1-\frac{p_B-p_2}{1-\gamma-\beta}) - (p_B - k_2 - \Delta)(1-p_B)}{\alpha(1-\frac{p_B}{1-\gamma})}. \end{aligned}$$

(1.3) π_B^{DC} and $\pi_B^{O\dagger}$:

$$\begin{cases} \pi_B^{DC} = (p_B - k_1)\,(1-p_B) + \alpha\,(p_B - w_2^{DC*})\,(1-\frac{p_B-p_2}{1-\gamma-\beta}), & \text{if } e < e_{D1}, \\ \pi_B^{O\dagger} = \left(p_B - w_2^{O\dagger *} - t\right)(1-\frac{p_B}{1-\gamma}) + \alpha\left(p_B - w_2^{O\dagger *}\right)(1-\frac{p_B}{1-\gamma}), & \text{if } e \geq e_{O1}. \end{cases}$$

Thus,

$$\begin{aligned} &\pi_B^{O\dagger} > \pi_B^{DC}, \\ \Rightarrow\ &(1+\alpha)\left(p_B - w_2^{O\dagger *}\right)(1-\tfrac{p_B}{1-\gamma}) - t(1-\tfrac{p_B}{1-\gamma}) > (p_B-k_1)\,(1-p_B) + \alpha\,(p_B - w_2^{DC*})\,(1-\tfrac{p_B-p_2}{1-\gamma-\beta}), \\ \Rightarrow\ &w_2^{O\dagger *} < p_B - \frac{(p_B-k_2-\Delta)(1-p_B)+\alpha\left(p_B-w_2^{DC*}\right)(1-\frac{p_B-p_2}{1-\gamma-\beta})+t(1-\frac{p_B}{1-\gamma})}{(1+\alpha)(1-\frac{p_B}{1-\gamma})}. \end{aligned}$$

(1.4) π_B^{DC} and π_B^{OC}:

$$\begin{cases} \pi_B^{DC} = (p_B - k_1)\,(1-p_B) + \alpha\,(p_B - w_2^{DC*})\,(1-\frac{p_B-p_2}{1-\gamma-\beta}), & \text{if } e < e_{D1}, \\ \pi_B^{OC} = (p_B - w_2^{OC*} - t)\,(1-\frac{p_B}{1-\gamma}) + \alpha\,(p_B - w_2^{OC*})\,(1-\frac{p_B-p_2}{1-\gamma-\beta}), & \text{if } e < e_{O1}. \end{cases}$$

Thus,

$$\begin{aligned} &\pi_B^{OC} > \pi_B^{DC}, \\ \Rightarrow\ &(p_B - w_2^{OC*} - t)\,(1-\tfrac{p_B}{1-\gamma}) > (p_B-k_2-\Delta)\,(1-p_B) + \alpha\,(w_2^{OC*} - w_2^{DC*})\,(1-\tfrac{p_B-p_2}{1-\gamma-\beta}), \\ \Rightarrow\ &\Delta > p_B - k_2 - \frac{\left(p_B-w_2^{OC*}-t\right)(1-\frac{p_B}{1-\gamma})-\alpha\left(w_2^{OC*}-w_2^{DC*}\right)(1-\frac{p_B-p_2}{1-\gamma-\beta})}{1-p_B}. \end{aligned}$$

2. We make comparisons for Strategy D and Strategy O, and Strategy H:

(2.1) $\pi_B^{D\dagger}$ and π_B^{H}:

$$\begin{cases} \pi_B^{H} = (p_B - k_1)\,(1-p_B) + \alpha\,(p_B - k_1 - t)\,(1-\frac{p_B-p_2}{1-\beta}), \\ \pi_B^{D\dagger} = (p_B - k_1)\,(1-p_B) + \alpha\left(p_B - w_2^{D\dagger *}\right)(1-\frac{p_B}{1-\gamma}), & \text{if } e \geq e_{D1}. \end{cases}$$

Thus,

$$\begin{aligned} &\pi_B^{D\dagger} > \pi_B^{H}, \\ \Rightarrow\ &\left(p_B - w_2^{D\dagger *}\right)(1-\tfrac{p_B}{1-\gamma}) > (p_B-k_2-\Delta-t)\,(1-\tfrac{p_B-p_2}{1-\beta}), \\ \Rightarrow\ &w_2^{D\dagger *} < p_B - \frac{(p_B-k_2-\Delta-t)(1-\frac{p_B-p_2}{1-\beta})}{1-\frac{p_B}{1-\gamma}}. \end{aligned}$$

(2.2) π_B^{DC} and π_B^{H}:

$$\begin{cases} \pi_B^{H} = (p_B - k_1)\,(1-p_B) + \alpha\,(p_B - k_1 - t)\,(1-\frac{p_B-p_2}{1-\beta}), \\ \pi_B^{DC} = (p_B - k_1)\,(1-p_B) + \alpha\,(p_B - w_2^{DC*})\,(1-\frac{p_B-p_2}{1-\gamma-\beta}), & \text{if } e < e_{D1}. \end{cases}$$

Thus,

$$\begin{aligned} &\pi_B^{DC} > \pi_B^{H}, \\ \Rightarrow\ &(p_B - w_2^{DC*})\,(1-\tfrac{p_B-p_2}{1-\gamma-\beta}) > (p_B-k_2-\Delta-t)\,(1-\tfrac{p_B-p_2}{1-\beta}), \\ \Rightarrow\ &\Delta > p_B - k_2 - \frac{\left(p_B-w_2^{DC*}\right)(1-\frac{p_B-p_2}{1-\gamma-\beta})}{1-\frac{p_B-p_2}{1-\beta}} - t. \end{aligned}$$

(2.3) $\pi_B^{O\dagger}$ and π_B^{H}:

$$\begin{cases} \pi_B^{H} = (p_B - k_1)\,(1-p_B) + \alpha\,(p_B - k_1 - t)\,(1-\frac{p_B-p_2}{1-\beta}), \\ \pi_B^{O\dagger} = \left(p_B - w_2^{O\dagger *} - t\right)(1-\frac{p_B}{1-\gamma}) + \alpha\left(p_B - w_2^{O\dagger *}\right)(1-\frac{p_B}{1-\gamma}), & \text{if } e \geq e_{O1}. \end{cases}$$

Thus,

$$
\begin{aligned}
&\pi_B^{O\dagger} > \pi_B^H, \\
\Rightarrow\ & (1+\alpha)\left(p_B - w_2^{O\dagger*}\right)(1-\tfrac{p_B}{1-\gamma}) - t(1-\tfrac{p_B}{1-\gamma}) > (p_B - k_2 - \Delta)\left((1-p_B) + \alpha(1-\tfrac{p_B-p_2}{1-\beta})\right) - \alpha t(1-\tfrac{p_B-p_2}{1-\beta}), \\
\Rightarrow\ & w_2^{O\dagger*} < p_B - \frac{(p_B-k_2-\Delta)((1-p_B)+\alpha(1-\frac{p_B-p_2}{1-\beta}))-\alpha t(1-\frac{p_B-p_2}{1-\beta})+t(1-\frac{p_B}{1-\gamma})}{(1+\alpha)(1-\frac{p_B}{1-\gamma})}.
\end{aligned}
$$

(2.4) π_B^{OC} and π_B^H:

$$
\begin{cases}
\pi_B^H = (p_B - k_1)(1-p_B) + \alpha(p_B - k_1 - t)(1-\frac{p_B-p_2}{1-\beta}), \\
\pi_B^{OC} = (p_B - w_2^{OC*} - t)(1-\frac{p_B}{1-\gamma}) + \alpha(p_B - w_2^{OC*})(1-\frac{p_B-p_2}{1-\gamma-\beta}), \text{ if } e < e_{O1}.
\end{cases}
$$

Thus,

$$
\begin{aligned}
&\pi_B^{OC} > \pi_B^H, \\
\Rightarrow\ & (p_B - w_2^{OC*} - t)(1-\tfrac{p_B}{1-\gamma}) + \alpha(p_B - w_2^{OC*})(1-\tfrac{p_B-p_2}{1-\gamma-\beta}) > (p_B - k_2 - \Delta)\left((1-p_B) + \alpha(1-\tfrac{p_B-p_2}{1-\beta})\right) - \alpha t(1-\tfrac{p_B-p_2}{1-\beta}), \\
\Rightarrow\ & \Delta > p_B - k_2 - \frac{\left(p_B-w_2^{OC*}-t\right)(1-\frac{p_B}{1-\gamma})+\alpha\left(p_B-w_2^{OC*}\right)(1-\frac{p_B-p_2}{1-\gamma-\beta})+\alpha t(1-\frac{p_B-p_2}{1-\beta})}{(1-p_B)+\alpha(1-\frac{p_B-p_2}{1-\beta})}.
\end{aligned}
$$

3. Therefore, we obtain the results as follows.

(3.1) The conditions for $\pi_B^* = \pi_B^{D\dagger}$ are $e \geq e_{D1}$, and

$$
\begin{aligned}
&\pi_B^{D\dagger} > \pi_B^{O\dagger} \Rightarrow w_2^{O\dagger*} > p_B - \frac{\alpha(w_2^{O\dagger*}-w_2^{D\dagger*})(1-\frac{p_B}{1-\gamma})+(p_B-k_2-\Delta)(1-p_B)}{(1-\frac{p_B}{1-\gamma})} - t, && \Rightarrow e < f_{DO3}, or, e > f_{DO4}, \\
&\pi_B^{D\dagger} > \pi_B^{OC} \Rightarrow w_2^{D\dagger*} < p_B - \frac{\left(p_B-w_2^{OC*}-t\right)(1-\frac{p_B}{1-\gamma})+\alpha\left(p_B-w_2^{OC*}\right)(1-\frac{p_B-p_2}{1-\gamma-\beta})-(p_B-k_2-\Delta)(1-p_B)}{\alpha(1-\frac{p_B}{1-\gamma})}, && \Rightarrow e > f_{DO2}, \\
&\pi_B^{D\dagger} > \pi_B^{H} \Rightarrow w_2^{D\dagger*} < p_B - \frac{(p_B-k_2-\Delta-t)(1-\frac{p_B-p_2}{1-\beta})}{1-\frac{p_B}{1-\gamma}}, && \Rightarrow e > f_{DH};
\end{aligned}
$$

(3.2) the conditions for $\pi_B^* = \pi_B^{DC}$ are $e < e_{D1}$, and

$$
\begin{aligned}
&\pi_B^{DC} > \pi_B^{O\dagger} \Rightarrow w_2^{O\dagger*} > p_B - \frac{(p_B-k_2-\Delta)(1-p_B)+\alpha\left(p_B-w_2^{DC*}\right)(1-\frac{p_B-p_2}{1-\gamma-\beta})+t(1-\frac{p_B}{1-\gamma})}{(1+\alpha)(1-\frac{p_B}{1-\gamma})}, && \Rightarrow e < f_{DO1}, \\
&\pi_B^{DC} > \pi_B^{OC} \Rightarrow \Delta < p_B - k_2 - \frac{\left(p_B-w_2^{OC*}-t\right)(1-\frac{p_B}{1-\gamma})-\alpha\left(w_2^{OC*}-w_2^{DC*}\right)(1-\frac{p_B-p_2}{1-\gamma-\beta})}{1-p_B}, && \Rightarrow \Delta < \Delta_{DO}, \\
&\pi_B^{DC} > \pi_B^{H} \Rightarrow \Delta > p_B - k_2 - \frac{\left(p_B-w_2^{DC*}\right)(1-\frac{p_B-p_2}{1-\gamma-\beta})}{1-\frac{p_B-p_2}{1-\beta}} - t, && \Rightarrow \Delta > \Delta_{DH};
\end{aligned}
$$

(3.3) the conditions for $\pi_B^* = \pi_B^{O}$ are $e \geq e_{O1}$, and

$$
\begin{aligned}
&\pi_B^{O\dagger} > \pi_B^{D\dagger} \Rightarrow w_2^{O\dagger*} < p_B - \frac{\alpha(w_2^{O\dagger*}-w_2^{D\dagger*})(1-\frac{p_B}{1-\gamma})+(p_B-k_2-\Delta)(1-p_B)}{(1-\frac{p_B}{1-\gamma})} - t, && \Rightarrow e > f_{DO3}, or, e < f_{DO4}, \\
&\pi_B^{O\dagger} > \pi_B^{DC} \Rightarrow w_2^{O\dagger*} < p_B - \frac{(p_B-k_2-\Delta)(1-p_B)+\alpha\left(p_B-w_2^{DC*}\right)(1-\frac{p_B-p_2}{1-\gamma-\beta})+t(1-\frac{p_B}{1-\gamma})}{(1+\alpha)(1-\frac{p_B}{1-\gamma})}, && \Rightarrow e > f_{DO1}, \\
&\pi_B^{O\dagger} > \pi_B^{H} \Rightarrow w_2^{O\dagger*} < p_B - \frac{(p_B-k_2-\Delta)((1-p_B)+\alpha(1-\frac{p_B-p_2}{1-\beta}))-\alpha t(1-\frac{p_B-p_2}{1-\beta})+t(1-\frac{p_B}{1-\gamma})}{(1+\alpha)(1-\frac{p_B}{1-\gamma})}, && \Rightarrow e > f_{HO};
\end{aligned}
$$

(3.4) the conditions for $\pi_B^* = \pi_B^{OC}$ are $e < e_{O1}$, and

$$
\begin{aligned}
&\pi_B^{OC} > \pi_B^{D\dagger} \Rightarrow w_2^{D*} > p_B - \frac{\left(p_B-w_2^{OC*}-t\right)(1-\frac{p_B}{1-\gamma})+\alpha\left(p_B-w_2^{OC*}\right)(1-\frac{p_B-p_2}{1-\gamma-\beta})-(p_B-k_2-\Delta)(1-p_B)}{\alpha(1-\frac{p_B}{1-\gamma})}, && \Rightarrow e < f_{DO2}, \\
&\pi_B^{OC} > \pi_B^{DC} \Rightarrow \Delta > p_B - k_2 - \frac{\left(p_B-w_2^{OC*}-t\right)(1-\frac{p_B}{1-\gamma})-\alpha\left(w_2^{OC*}-w_2^{DC*}\right)(1-\frac{p_B-p_2}{1-\gamma-\beta})}{1-p_B}, && \Rightarrow \Delta > \Delta_{DO}, \\
&\pi_B^{OC} > \pi_B^{H} \Rightarrow \Delta > p_B - k_2 - \frac{\left(p_B-w_2^{OC*}-t\right)(1-\frac{p_B}{1-\gamma})+\alpha\left(p_B-w_2^{OC*}\right)(1-\frac{p_B-p_2}{1-\gamma-\beta})+\alpha t(1-\frac{p_B-p_2}{1-\beta})}{(1-p_B)+\alpha(1-\frac{p_B-p_2}{1-\beta})}, && \Rightarrow \Delta > \Delta_{HO};
\end{aligned}
$$

(3.5) the conditions for $\pi_B^* = \pi_B^H$ are

$$\begin{array}{lll}
\pi_B^H > \pi_B^{O\dagger} \Rightarrow w_2^{O\dagger *} > p_B - \frac{(p_B-k_2-\Delta)((1-p_B)+\alpha(1-\frac{p_B-p_2}{1-\beta}))-\alpha t(1-\frac{p_B-p_2}{1-\beta})+t(1-\frac{p_B}{1-\gamma})}{(1+\alpha)(1-\frac{p_B}{\gamma})}, & & \Rightarrow e < f_{HO}, \\
\pi_B^H > \pi_B^{OC} \Rightarrow \Delta < p_B - k_2 - \frac{\left(p_B-w_2^{OC*}-t\right)(1-\frac{p_B}{1-\gamma})+\alpha\left(p_B-w_2^{OC*}\right)(1-\frac{p_B-p_2}{1-\gamma-\beta})+\alpha t(1-\frac{p_B-p_2}{1-\beta})}{(1-p_B)+\alpha(1-\frac{p_B-p_2}{1-\beta})}, & & \Rightarrow \Delta < \Delta_{HO}, \\
\pi_B^H > \pi_B^{DC} \Rightarrow \Delta < p_B - k_2 - \frac{\left(p_B-w_2^{DC*}\right)(1-\frac{p_B-p_2}{1-\gamma-\beta})}{1-\frac{p_B-p_2}{1-\beta}} - t, & & \Rightarrow \Delta < \Delta_{DH}, \\
\pi_B^H > \pi_B^{D\dagger} \Rightarrow w_2^{D\dagger *} > p_B - \frac{(p_B-k_2-\Delta-t)(1-\frac{p_B-p_2}{1-\beta})}{1-\frac{p_B}{1-\gamma}}, & & \Rightarrow e < f_{DH}.
\end{array}$$

By combining the conditions for each strategy, we have the results. ■

Following the general result in Lemma B2, we further derive the conditions of (Δ, e) for different wholesale price w_2^*. We define below thresholds: $w_2^{OC*} = k_2 + \frac{\alpha(p_2-k_2)(1-\frac{p_B-p_2}{1-\gamma-\beta})}{(1-\frac{p_B}{1-\gamma})+\alpha(1-\frac{p_B-p_2}{1-\gamma-\beta})}$, and $R = \frac{\alpha(\beta p_B-(1-\gamma)p_2)}{(1-\gamma-\beta)\beta}$,

$$\begin{aligned}
&\Delta_{DH} = p_B - k_2 - \frac{(p_B-k_2)(1-\frac{p_B-p_2}{1-\gamma-\beta})}{(1-\frac{p_B-p_2}{1-\beta})} - t; \\
&\Delta_{DO} = p_B - k_2 - \frac{\left(p_B-w_2^{OC*}-t\right)(1-\frac{p_B}{1-\gamma})-\alpha\left(w_2^{OC*}-k_2\right)(1-\frac{p_B-p_2}{1-\gamma-\beta})}{(1-p_B)}; \\
&\Delta_{HO} = p_B - k_2 - \frac{\left(p_B-w_2^{OC*}-t\right)(1-\frac{p_B}{1-\gamma})+\alpha\left(p_B-w_2^{OC*}\right)(1-\frac{p_B-p_2}{1-\gamma-\beta})+\alpha t(1-\frac{p_B-p_2}{1-\beta})}{(1-p_B)+\alpha(1-\frac{p_B-p_2}{1-\beta})}; \\
&f_{DH} = (p_2 - k_2 - (x_{DH}(\Delta) - k_2)\tfrac{\beta}{1-\gamma})R, \text{ where } x_{DH}(\Delta) = p_B - \frac{(p_B-k_2-\Delta-t)(1-\frac{p_B-p_2}{1-\beta})}{(1-\frac{p_B}{1-\gamma})}; \\
&f_{DO1} = (p_2 - k_2 - (x_{DO1}(\Delta) - k_2)\tfrac{\beta}{1-\gamma})R, \text{ where } x_{DO1}(\Delta) = p_B - \frac{(p_B-k_2-\Delta)(1-p_B)+\alpha(p_B-k_2)(1-\frac{p_B-p_2}{1-\gamma-\beta})+t(1-\frac{p_B}{1-\gamma})}{(1+\alpha)(1-\frac{p_B}{1-\gamma})}; \\
&f_{DO2} = (p_2 - k_2 - (x_{DO2}(\Delta) - k_2)\tfrac{\beta}{1-\gamma})R, \text{ where } x_{DO2}(\Delta) = p_B - \frac{\left(p_B-w_2^{OC*}-t\right)((1-\frac{p_B}{1-\gamma})+\alpha(1-\frac{p_B-p_2}{1-\gamma-\beta}))-(p_B-k_2-\Delta)(1-p_B)}{\alpha(1-\frac{p_B}{1-\gamma})}; \\
&f_{DO3} = (p_2 - k_2 - (x_{OD3}(\Delta) - k_2)\tfrac{\beta}{1-\gamma})R, \text{ where } x_{OD3}(\Delta) = p_B - \frac{(p_B-k_2-\Delta)(1-p_B)}{(1-\frac{p_B}{1-\gamma})} - t; \\
&f_{DO4} = \frac{(p_B - \frac{(p_B-k_2-\Delta)(1-p_B)}{1-\frac{p_B}{1-\gamma}} - k_2 - t(1-\frac{p_B}{1-\gamma}))\beta(1-\frac{p_B}{1-\gamma}) - \alpha(p_2-k_2)(1-\frac{p_2}{\beta})\beta + \alpha\gamma(1-\frac{p_B}{1-\gamma})(p_2-k_2)}{\left(\frac{(1-\gamma)(1-\gamma-\beta)(1-\frac{p_B}{1-\gamma})}{(\beta p_B-(1-\gamma)p_2)} - 1\right)\beta}; \\
&f_{HO1} = (p_2 - k_2 - (x_{HO1}(\Delta) - k_2)\tfrac{\beta}{1-\gamma})R, \text{ where } x_{HO1}(\Delta) = p_B - \frac{(p_B-k_2-\Delta)((1-p_B)+\alpha(1-\frac{p_B-p_2}{1-\beta}))-\alpha t(1-\frac{p_B-p_2}{1-\beta})+t(1-\frac{p_B}{1-\gamma})}{(1+\alpha)(1-\frac{p_B}{\gamma})}; \\
&f_{HO2} = \alpha\,(p_2 - k_2)\,(1 - \tfrac{p_2}{\beta}) - (x_{HO2}(\Delta) - k_2)(1+\alpha)(1 - \tfrac{p_B}{\gamma}), \text{ where } x_{HO2}(\Delta) = p_B - \frac{(p_B-k_2-\Delta)\left((1-p_B)+\alpha\left(1-\frac{p_B-p_2}{1-\beta}\right)\right)}{(1+\alpha)\left(1-\frac{p_B}{1-\gamma}\right)} \\
&-t(1 - \tfrac{p_B}{1-\gamma}) + t\alpha(1 - \tfrac{p_B-p_2}{1-\beta}).
\end{aligned} \tag{9}$$

Note that for the condition of $\pi_B^* = \pi_B^H$, $f_{DH} < f_{HO1}$, $f_{DH} < f_{HO2}$; for the condition of $\pi_B^* = \pi_B^{O\dagger}$, $f_{DO3} > f_{HO1}$, $f_{DO3} > f_{HO2}$. Thus, in our base case, the equilibrium sourcing strategy of the brand-name firm is as follows:

(a) Strategy H with $w_1^* = k_1$ if $e < f_{DH}$ and $\Delta < \min\{\Delta_{DH}, \Delta_{HO}\}$;

(b) Strategy D with $w_1^* = k_1$, and

$$w_2^* = \begin{cases} k_2, & \text{if } e \le \min\{e_{D1}, f_{DO1}\} \text{ and } \min\{\Delta_{DH}, \Delta_{DO}\} \le \Delta < \Delta_{DO}, \\ w_2^{(0)}, & \text{if } \max\{e_{D1}, f_{DH}, f_{DO2}\} \le e \le \min\{e_3, f_{DO3}\}, \text{ or if } e \ge \max\{e_3, f_{DO4}\}; \end{cases}$$

(c) Strategy O with

$$w_2^* = \begin{cases} w_2^{O(1)}, & \text{if } e \le \min\{e_{O1}, f_{DO2}\} \text{ and } \Delta > \max\{\Delta_{HO}, \Delta_{DO}\}, \\ \max\{w_2^{O(2)}, w_2^{(0)}\}, & \text{if } \max\{e_{O1}, f_{DO1}, f_{DO3}\} < e \le e_3, \text{ or if } e_3 < e < f_{DO4}; \end{cases}$$

where e_3 is defined in Equation (7). ■

B.4 Proof of Proposition 2.

Recall that $\pi_B^{D\dagger} \geq \pi_B^{DC}$ if $e \geq e_{D1}$, and $\pi_B^{O\dagger} \geq \pi_B^{OC}$ if $e \geq e_{O1}$, where

$$e_{D1} = \left(p_2 - k_2 - \left(\frac{(p_B-k_2)(\frac{p_B-p_2}{1-\gamma-\beta} - \frac{p_B}{1-\gamma})}{1-\frac{p_B}{1-\gamma}}\right)\frac{\beta}{1-\gamma}\right)\frac{\alpha(\beta p_B-(1-\gamma)p_2)}{(1-\gamma-\beta)\beta};$$
$$e_{O1} = \left(p_2 - k_2 - \left(\frac{\alpha(p_B-w_2^{OC*})(\frac{p_B-p_2}{1-\gamma-\beta} - \frac{p_B}{1-\gamma})}{(1+\alpha)(1-\frac{p_B}{1-\gamma})} + w_2^{OC*} - k_2\right)\frac{\beta}{1-\gamma}\right)\frac{\alpha(\beta p_B-(1-\gamma)p_2)}{(1-\gamma-\beta)\beta},$$

and $w_2^{OC*} = k_2 + \frac{\alpha(p_2-k_2)\left(1-\frac{p_B-p_2}{1-\gamma-\beta}\right)}{\left(1-\frac{p_B}{1-\gamma}\right)+\alpha\left(1-\frac{p_B-p_2}{1-\gamma-\beta}\right)}$.

Under Strategy D, if $e \geq e_{D1}$, the counterfeiter does not sell the counterfeit; under Strategy O, if $e \geq e_{O1}$, the counterfeiter does not sell the counterfeit. Thus, we compare thresholds e_{D1} and e_{O1}, to analyze which sourcing strategy helps prevent counterfeiting at a lower e. If $e_{D1} > e_{O1}$, it means that Strategy O is easier to prevent counterfeiting. Otherwise, Strategy D is easier to prevent counterfeiting.

Below we derive the condition of $e_{D1} > e_{O1}$.

$$\begin{aligned}
& e_{D1} > e_{O1}, \\
\Rightarrow\ & p_2 - k_2 - \left(\frac{(p_B-k_2)(\frac{p_B-p_2}{1-\gamma-\beta} - \frac{p_B}{1-\gamma})}{1-\frac{p_B}{1-\gamma}}\right)\frac{\beta}{1-\gamma} > (p_2-k_2) - \left(\frac{\alpha(p_B-w_2^{OC*})(\frac{p_B-p_2}{1-\gamma-\beta} - \frac{p_B}{1-\gamma})}{(1+\alpha)(1-\frac{p_B}{1-\gamma})} + w_2^{OC*} - k_2\right)\frac{\beta}{1-\gamma}, \\
\Rightarrow\ & \alpha > \frac{(p_B-k_2)\left(\frac{p_B-p_2}{1-\gamma-\beta} - \frac{p_B}{1-\gamma}\right)}{(p_2-k_2)\left(1-\frac{p_B-p_2}{1-\gamma-\beta}\right)}, \\
\Rightarrow\ & ((1-\gamma)(1-\gamma-\beta) - (1-\gamma)(p_B-p_2)) > \frac{(p_B-k_2)}{\alpha(p_2-k_2)}(\beta p_B - (1-\gamma)p_2), \\
\Rightarrow\ & ((1-\gamma)(1-\gamma-\beta) - (1-\gamma)(p_B-p_2)) > x(\beta p_B - (1-\gamma)p_2), \text{where } x = \frac{(p_B-k_2)}{\alpha(p_2-k_2)}, \\
\Rightarrow\ & (1-\gamma - \tfrac{\beta+p_B-(1+x)p_2}{2})^2 > x\beta p_B + (\tfrac{\beta+p_B-(1+x)p_2}{2})^2, \\
\Rightarrow\ & 1-\gamma > \tfrac{\beta+p_B-(1+x)p_2}{2} + \sqrt{x\beta p_B + (\tfrac{\beta+p_B-(1+x)p_2}{2})^2}, \\
& 1-\gamma < \tfrac{\beta+p_B-(1+x)p_2}{2} - \sqrt{x\beta p_B + (\tfrac{\beta+p_B-(1+x)p_2}{2})^2}, \text{invalid.}
\end{aligned}$$

Define

$$\widehat{\gamma} = 1 - \min\left\{\frac{\beta+p_B-\left(1+\frac{p_B-k_2}{\alpha(p_2-k_2)}\right)p_2}{2} + \sqrt{\frac{\beta p_B(p_B-k_2)}{\alpha(p_2-k_2)} + \frac{\left(\beta+p_B-\left(1+\frac{p_B-k_2}{\alpha(p_2-k_2)}\right)p_2\right)^2}{4}}, 1\right\}. \tag{10}$$

Thus, we obtain the result. ■

B.5 Proof of Corollary 1.

Note that in equilibrium of the base model, under Strategy H, the profit of each firm is the same as that under the benchmark, i.e., $\pi_1^H = \bar{\pi}_1^*$, $\pi_2^H = \bar{\pi}_2^*$, $\pi_B^H = \bar{\pi}_B^*$. In equilibrium, under Strategy D or Strategy O, the home supplier obtains zero profit, that is, $\pi_1^D = \pi_1^O = \bar{\pi}_1^* = 0$. Thus, in the following, we focus on comparing the profits of the brand-name firm, the overseas supplier, between the benchmark and Strategy D as well as Strategy O in equilibrium from Proposition 1, respectively. Note that we assume $0 \leq e < \alpha(p_2-k_2)(\frac{p_B-p_2}{1-\beta} - \frac{p_2}{\beta})$. Recall that

$$\begin{aligned}
M' &= \alpha\,(p_2-k_2)\,(\tfrac{p_B-p_2}{1-\gamma-\beta} - \tfrac{p_2}{\beta}) - e, & w_2^{(0)} &= k_2 + \frac{M'}{\alpha\left(\frac{p_B-p_2}{1-\gamma-\beta} - \frac{p_B}{1-\gamma}\right)}, \\
K &= \alpha\,(p_2-k_2)\,(1 - \tfrac{p_2}{\beta}) - e, & w_2^{O(2)} &= k_2 + \frac{K}{(1+\alpha)(1-\frac{p_B}{1-\gamma})}, \\
K - M' &= \alpha\,(p_2-k_2)\,(1 - \tfrac{p_B-p_2}{1-\gamma-\beta}), & w_2^{O(1)} &= k_2 + \frac{K-M'}{(1-\frac{p_B}{1-\gamma})+\alpha(1-\frac{p_B-p_2}{1-\gamma-\beta})}.
\end{aligned}$$

1.If there is no counterfeiting after conversion, i.e., $s^*=0$, then, we have the comparison of profits as follows.

For the brand-name firm:

$$\begin{aligned}
\pi_B^H &= (p_B-k_1)(1-p_B)+\alpha(p_B-k_1-t)\left(1-\tfrac{p_B-p_2}{1-\beta}\right),\\
\pi_B^D\left(w_2^*=w_2^{(0)}\right) &= (p_B-k_1)(1-p_B)+\alpha(p_B-w_2^*)\left(1-\tfrac{p_B}{1-\gamma}\right),\\
\pi_B^O\left(w_2^*=\max\{w_2^{O(2)},w_2^{(0)}\}\right) &= (p_B-w_2^*-t)\left(1-\tfrac{p_B}{1-\gamma}\right)+\alpha(p_B-w_2^*)\left(1-\tfrac{p_B}{1-\gamma}\right).
\end{aligned}$$

For the overseas supplier:

$$\begin{aligned}
\pi_2^H &= \alpha(p_2-k_2)\left(\tfrac{p_B-p_2}{1-\beta}-\tfrac{p_2}{\beta}\right)-e=M,\\
\pi_2^D\left(w_2^*=w_2^{(0)}\right) &= \alpha(w_2^*-k_2)\left(1-\tfrac{p_B}{1-\gamma}\right),\\
\pi_2^O\left(w_2^*=\max\{w_2^{O(1)},w_2^{(0)}\}\right) &= (w_2^*-k_2)\left(1-\tfrac{p_B}{1-\gamma}\right)+\alpha(w_2^*-k_2)\left(1-\tfrac{p_B}{1-\gamma}\right).
\end{aligned}$$

(1) When Strategy D is optimal,

for the brand-name firm, $\pi_B^D-\bar{\pi}_B^*\geq 0$;

for the overseas supplier, $\pi_2^D-\bar{\pi}_2^*=\alpha(w_2^*-k_2)\left(1-\dfrac{p_B}{1-\gamma}\right)-M\geq 0.$

(2) When Strategy O is optimal,

for the brand-name firm, $\pi_B^O-\bar{\pi}_B^*\geq 0$;

for the overseas supplier, $\pi_2^O-\bar{\pi}_2^*=(w_2^*-k_2)\left(1-\dfrac{p_B}{1-\gamma}\right)+\alpha(w_2^*-k_2)\left(1-\dfrac{p_B}{1-\gamma}\right)-M\geq 0.$

2. If there is counterfeiting after conversion, i.e., $s^*=0$, then, we have the comparison as follows.

For the brand-name firm:

$$\begin{aligned}
\pi_B^H &= (p_B-k_1)(1-p_B)+\alpha(p_B-k_1-t)\left(1-\tfrac{p_B-p_2}{1-\beta}\right),\\
\pi_B^D(w_2^*=k_2) &= (p_B-k_1)(1-p_B)+\alpha(p_B-w_2^*)\left(1-\tfrac{p_B-p_2}{1-\gamma-\beta}\right),\\
\pi_B^O\left(w_2^*=w_2^{O(1)}\right) &= (p_B-w_2^*-t)\left(1-\tfrac{p_B}{1-\gamma}\right)+\alpha(p_B-w_2^*)\left(1-\tfrac{p_B-p_2}{1-\gamma-\beta}\right).
\end{aligned}$$

For the overseas supplier:

$$\begin{aligned}
\pi_2^H &= \alpha(p_2-k_2)\left(\tfrac{p_B-p_2}{1-\beta}-\tfrac{p_2}{\beta}\right)-e=M,\\
\pi_2^D(w_2^*=k_2) &= \alpha(w_2^*-k_2)\left(1-\tfrac{p_B-p_2}{1-\gamma-\beta}\right)+\left(\alpha(p_2-k_2)\left(\tfrac{p_B-p_2}{1-\gamma-\beta}-\tfrac{p_2}{\beta}\right)-e\right),\\
\pi_2^O\left(w_2^*=w_2^{O(1)}\right) &= (w_2^*-k_2)\left(1-\tfrac{p_B}{1-\gamma}\right)+\alpha(w_2^*-k_2)\left(1-\tfrac{p_B-p_2}{1-\gamma-\beta}\right)+\left(\alpha(p_2-k_2)\left(\tfrac{p_B-p_2}{1-\gamma-\beta}-\tfrac{p_2}{\beta}\right)-e\right).
\end{aligned}$$

(1) When Strategy D is optimal,

for the brand-name firm, $\pi_B^D-\bar{\pi}_B^*\geq 0$;

for the overseas supplier, $\pi_2^D-\bar{\pi}_2^*=M'-M>0.$

(2) When Strategy O is optimal,

for the brand-name firm, $\pi_B^O - \bar{\pi}_B^* \geq 0$;
for the overseas supplier, $\pi_2^O - \bar{\pi}_2^* = (w_2^* - k_2)\left(1 - \dfrac{p_B}{1-\gamma}\right) + \alpha\,(w_2^* - k_2)\left(1 - \dfrac{p_B - p_2}{1-\gamma-\beta}\right) + M' - M > 0.$

Thus, based on the equilibrium in Proposition 1, under strategies D and O, for the brand-name firm, $\pi_B^D \geq \bar{\pi}_B^*$, $\pi_B^O \geq \bar{\pi}_B^*$, respectively; for the overseas supplier, $\pi_2^D \geq \bar{\pi}_2^*$, $\pi_2^O \geq \bar{\pi}_2^*$, respectively. ■

B.6 Proof of Proposition 3.

Recall that we have below thresholds of θ: $\widetilde{\theta} = \frac{p_B - p_2}{1-\beta}$, $\widetilde{\theta}' = \frac{p_B - p_2}{1-\gamma-\beta}$, $\hat{\theta}_B = \frac{p_B}{1-\gamma}$, and $\hat{\theta}_2 = \frac{p_2}{\beta}$.

Firstly, under the benchmark: in the equilibrium,
(1) consumer surplus in the home market is $\overline{CS}_1 = \frac{1-(p_B)^2}{2} - p_B\,(1-p_B) = \frac{(1-p_B)^2}{2}$;
(2) consumer surplus in the overseas market is

$$\overline{CS}_2 = \alpha\Big(\frac{\beta(\widetilde{\theta}^2 - (\hat{\theta}_2)^2)}{2} - p_2(\widetilde{\theta} - \hat{\theta}_2) + \frac{1-\widetilde{\theta}^2}{2} - p_B(1-\widetilde{\theta})\Big) = \alpha\Big(\frac{\beta\left(\widetilde{\theta} - \hat{\theta}_2\right)^2}{2} + \frac{1-\widetilde{\theta}^2}{2} - p_B\left(1-\widetilde{\theta}\right)\Big).$$

Secondly, under the base model: in the equilibrium,
(i) when Strategy H is optimal, consumer surplus in the home and overseas markets are as follows, respectively: $CS_1^H = \overline{CS}_1$, $CS_2^H = \overline{CS}_2$;
(ii) when Strategy D is optimal, consumer surplus in the home and overseas markets are as follows, respectively:

$$\begin{aligned}
&CS_1^D = \tfrac{1-(p_B)^2}{2} - p_B(1-p_B) = \tfrac{(1-p_B)^2}{2},\\
&CS_2^{D\dagger} = \alpha((1-\gamma)(\tfrac{1-(\hat{\theta}_B)^2}{2}) - p_B(1-\hat{\theta}_B)) = \alpha(1-\gamma)(\tfrac{(1-\hat{\theta}_B)^2}{2}),\\
&CS_2^{DC} = \alpha(\tfrac{\beta((\widetilde{\theta}')^2-(\hat{\theta}_2)^2)}{2} - p_2(\widetilde{\theta}' - \hat{\theta}_2) + \tfrac{1-(\widetilde{\theta}')^2}{2} - p_B(1-\widetilde{\theta}')) = \alpha(\tfrac{\beta(\widetilde{\theta}'-\hat{\theta}_2)^2}{2} + \tfrac{1-(\widetilde{\theta}')^2}{2} - p_B(1-\widetilde{\theta}'));
\end{aligned}$$

(iii) when Strategy O is optimal, consumer surplus in the home and overseas markets are as follows, respectively:

$$\begin{aligned}
&CS_1^O = (1-\gamma)(\tfrac{1-(\hat{\theta}_B)^2}{2}) - p_B(1-\hat{\theta}_B) = (1-\gamma)(\tfrac{(1-\hat{\theta}_B)^2}{2}),\\
&CS_2^{O\dagger} = \alpha((1-\gamma)(\tfrac{1-(\hat{\theta}_B)^2}{2}) - p_B(1-\hat{\theta}_B)) = \alpha(1-\gamma)(\tfrac{(1-\hat{\theta}_B)^2}{2}),\\
&CS_2^{OC} = \alpha(\tfrac{\beta((\widetilde{\theta}')^2-(\hat{\theta}_2)^2)}{2} - p_2(\widetilde{\theta}' - \hat{\theta}_2) + \tfrac{1-(\widetilde{\theta}')^2}{2} - p_B(1-\widetilde{\theta}')) = \alpha(\tfrac{\beta(\widetilde{\theta}'-\hat{\theta}_2)^2}{2} + \tfrac{1-(\widetilde{\theta}')^2}{2} - p_B(1-\widetilde{\theta}')).
\end{aligned}$$

Lastly, by comparing consumer surplus between the benchmark and Strategy D as well as Strategy O in equilibrium, respectively, we have the following results.

(1) In the home market, $CS_1^O \leq CS_1^D = \overline{CS}_1$. Because

$$CS_1^O - \overline{CS}_1 = (1-\gamma)(\tfrac{1-(\hat{\theta}_B)^2}{2}) - p_B(1-\hat{\theta}_B) - (\tfrac{1-(p_B)^2}{2} - p_B(1-p_B)) = \tfrac{(1-\gamma)(1-\hat{\theta}_B)^2}{2} - \tfrac{(1-p_B)^2}{2} \leq 0,$$

where the equality is achieved if $\gamma = 0$.

(2) In the overseas market, $CS_2^D = CS_2^O$. By comparing CS_2^D with $\overline{CS}_2$, we get the following results.

$$\begin{aligned}
CS_2^{D\dagger} - \overline{CS}_2 = {} & \alpha(\tfrac{(1-\gamma)(1-\hat{\theta}_B)^2}{2} - (\tfrac{\beta(\widetilde{\theta}-\hat{\theta}_2)^2}{2} + \tfrac{1-\widetilde{\theta}^2}{2} - p_B(1-\widetilde{\theta})) \\
= {} & -\tfrac{\alpha(\beta p_B - p_2)^2}{2\beta(1-\beta)} + \alpha(\tfrac{(1-\gamma)(1-\hat{\theta}_B)^2}{2} - \tfrac{(1-p_B)^2}{2}) < 0; \\
CS_2^{DC} - \overline{CS}_2 = {} & \alpha((\tfrac{\beta(\widetilde{\theta}'-\hat{\theta}_2)^2}{2} + \tfrac{1-(\widetilde{\theta}')^2}{2} - p_B(1-\widetilde{\theta}')) - (\tfrac{\beta(\widetilde{\theta}-\hat{\theta}_2)^2}{2} + \tfrac{1-\widetilde{\theta}^2}{2} - p_B(1-\widetilde{\theta})) \\
= {} & \tfrac{-\alpha(1-\beta)(\widetilde{\theta}' - \frac{p_B-p_2}{1-\beta})^2}{2} \leq 0.
\end{aligned}$$

The sign of $(CS_2^{DC} - \overline{CS}_2)$ is analyzed as follows. Note that the function $f(x) = \frac{\beta(x-\hat{\theta}_2)^2}{2} + \frac{1-(x)^2}{2} - p_B(1-x) = \frac{-(1-\beta)x^2+2(p_B-p_2)x+1-2p_B+\beta(\hat{\theta}_2)^2}{2} = \frac{-(1-\beta)(x-\frac{p_B-p_2}{1-\beta})^2+\frac{(p_B-p_2)^2}{1-\beta}+1-2p_B+\frac{p_2^2}{\beta}}{2}$ increases for $x < \frac{p_B-p_2}{1-\beta}$, and decreases for $x > \frac{p_B-p_2}{1-\beta}$. Recall $\widetilde{\theta}' = \frac{p_B-p_2}{1-\gamma-\beta} \geq \frac{p_B-p_2}{1-\beta}$. Thus, $CS_2^{DC} - \overline{CS}_2 \leq 0$.

Thus, in the overseas market, $CS_2^D = CS_2^O \leq \overline{CS}_2$.

For the total consumer surplus, $CS = CS_1 + CS_2$, thus, we know, $CS^O \leq CS^D \leq \overline{CS}$.

From above comparisons, we can obtain that in both the home and overseas markets, the consumer surplus loss increases in γ. When $\gamma = 0$, we have $CS_1^O = \overline{CS}_1$ and $CS_2^{DC} = \overline{CS}_2$. ■

B.7 Proof of Proposition 4.

Firstly, under the benchmark: in the equilibrium,
$\overline{SS} = \overline{CS}_1 + \overline{CS}_2 + \bar{\pi}_B^* + \bar{\pi}_1^* + \bar{\pi}_2^* = \frac{(1-p_B)^2}{2} + \alpha(\frac{\beta(\widetilde{\theta}-\hat{\theta}_2)^2}{2} + \frac{1-\widetilde{\theta}^2}{2} - p_B(1-\widetilde{\theta})) + (p_B - k_1)(1-p_B) + \alpha(p_B - k_1 - t)(1 - \frac{p_B-p_2}{1-\beta}) + (\alpha(p_2-k_2)(\frac{p_B-p_2}{1-\beta} - \frac{p_2}{\beta}) - e)$.

Secondly, under the base model: in the equilibrium,

(i) when Strategy D is optimal, the social surplus is
$SS^{D\dagger} = CS_1^D + CS_2^D + \pi_B^D + \pi_1^D + \pi_2^D = \frac{(1-p_B)^2}{2} + \alpha\gamma(\frac{(1-\hat{\theta}_B)^2}{2}) + (p_B-k_1)(1-p_B) + \alpha(p_B-k_2)\left(1 - \frac{p_B}{1-\gamma}\right)$,
$SS^{DC} = CS_1^{DC} + CS_2^{DC} + \pi_B^{DC} + \pi_1^{DC} + \pi_2^{DC} = \frac{(1-p_B)^2}{2} + \alpha(\frac{\beta(\widetilde{\theta}'-\hat{\theta}_2)^2}{2} + \frac{1-(\widetilde{\theta}')^2}{2} - p_B(1-\widetilde{\theta}')) + (p_B-k_1)(1-p_B) + \alpha(p_B-k_2)(1 - \frac{p_B-p_2}{1-\gamma-\beta}) + (\alpha(p_2-k_2)(\frac{p_B-p_2}{1-\gamma-\beta} - \frac{p_2}{\beta}) - e)$;

(ii) when Strategy O is optimal, the social surplus is
$SS^{O\dagger} = CS_1^O + CS_2^O + \pi_B^O + \pi_1^O + \pi_2^O = (1+\alpha)(1-\gamma)(\frac{(1-\hat{\theta}_B)^2}{2}) + (p_B-k_2-t)\left(1-\frac{p_B}{1-\gamma}\right) + \alpha(p_B-k_2)\left(1-\frac{p_B}{1-\gamma}\right)$,
$SS^{OC} = CS_1^{OC} + CS_2^{OC} + \pi_B^{OC} + \pi_1^{OC} + \pi_2^{OC} = (1-\gamma)(\frac{(1-\hat{\theta}_B)^2}{2}) + \alpha(\frac{\beta(\widetilde{\theta}'-\hat{\theta}_2)^2}{2} + \frac{1-(\widetilde{\theta}')^2}{2} - p_B(1-\widetilde{\theta}')) + (p_B-k_2-t)\left(1-\frac{p_B}{1-\gamma}\right) + \alpha(p_B-k_2)(1-\frac{p_B-p_2}{1-\gamma-\beta}) + (\alpha(p_2-k_2)(\frac{p_B-p_2}{1-\gamma-\beta} - \frac{p_2}{\beta}) - e)$.

Lastly, by comparing the social surplus between the benchmark and Strategy D as well as Strategy O in equilibrium, respectively, we have the following discussions. We define

$$\begin{aligned}
&\bar{\Delta}_D = \frac{\alpha(p_B-p_2)(\frac{p_B-p_2}{1-\gamma-\beta} - \frac{p_B-p_2}{1-\beta}) - g_1}{\alpha(1-\frac{p_B-p_2}{1-\beta})} - t, \\
&\bar{\Delta}_O = \frac{(p_B-k_2)(\frac{p_B}{1-\gamma} - p_B) + \alpha(p_B-p_2)(\frac{p_B-p_2}{1-\gamma-\beta} - \frac{p_B-p_2}{1-\beta}) - t(\alpha(1-\frac{p_B-p_2}{1-\beta}) - (1-\frac{p_B}{1-\gamma})) - g_2}{(1-p_B) + \alpha(1-\frac{p_B-p_2}{1-\beta})}; \\
&e_1' = \bar{e}_{D1} - g_1, \\
&e_2' = \bar{e}_{O1} - g_2,
\end{aligned} \tag{11}$$

where

$$\bar{e}_{D1} = \alpha(p_2-k_2)\left(\frac{p_B-p_2}{1-\beta}-\frac{p_2}{\beta}\right)-\alpha(\Delta+t)\left(1-\frac{p_B-p_2}{1-\beta}\right)-\alpha(p_B-k_2)\left(\frac{p_B-p_2}{1-\beta}-\frac{p_B}{1-\gamma}\right),$$

$$\bar{e}_{O1} = \alpha(p_2-k_2)\left(\frac{p_B-p_2}{1-\beta}-\frac{p_2}{\beta}\right)-\Delta(1-p_B)-(\Delta+t)\alpha\left(1-\frac{p_B-p_2}{1-\beta}\right)+(p_B-k_2)\left(\frac{p_B}{1-\gamma}-p_B\right)$$
$$-\alpha(p_B-k_2)\left(\frac{p_B-p_2}{1-\beta}-\frac{p_B}{1-\gamma}\right),$$

and

$$g_1 = \begin{cases} -\frac{\alpha(\beta p_B-p_2)^2}{2\beta(1-\beta)}+\alpha\left(\frac{(1-\gamma)(1-\hat{\theta}_B)^2}{2}-\frac{(1-p_B)^2}{2}\right), & \text{if } s^*=0,\\ \frac{-\alpha(1-\beta)(\widetilde{\theta}'-\frac{p_B-p_2}{1-\beta})^2}{2}, & \text{if } s^*=1,\end{cases}$$

$g_2 = g_1 + \left(\frac{(1-\gamma)(1-\frac{p_B}{1-\gamma})^2}{2}-\frac{(1-p_B)^2}{2}\right)$. Note that $g_1 \le 0$ and $g_2 \le 0$ represent the loss of consumer surplus under strategies D and O, respectively.

Then, we have the following comparisons about social welfare.

(1) If there is no counterfeiting after conversion, i.e., $s^*=0$, then: recall that $\widetilde{\theta} = \frac{p_B-p_2}{1-\beta}$, and $\hat{\theta}_B = \frac{p_B}{1-\gamma}$,

(i) when Strategy D is optimal,

$$\begin{aligned} SS^{D\dagger}-\overline{SS} &= (CS_1^D-\overline{CS}_1)+(CS_2^D-\overline{CS}_2)+(\pi_B^D+\pi_1^D+\pi_2^D)-(\bar{\pi}_B^*+\bar{\pi}_1^*+\bar{\pi}_2^*)\\ &= -\frac{\alpha(\beta p_B-p_2)^2}{2\beta(1-\beta)}+\alpha\left(\frac{(1-\gamma)(1-\hat{\theta}_B)^2}{2}-\frac{(1-p_B)^2}{2}\right)\\ &\quad+\alpha\left(1-\frac{p_B-p_2}{1-\beta}\right)(\Delta+t)+\alpha(p_B-k_2)\left(\frac{p_B-p_2}{1-\beta}-\frac{p_B}{1-\gamma}\right)-\left(\alpha(p_2-k_2)\left(\frac{p_B-p_2}{1-\beta}-\frac{p_2}{\beta}\right)-e\right);\end{aligned}$$

(ii) when Strategy O is optimal,

$$\begin{aligned} SS^{O\dagger}-\overline{SS} &= (CS_1^O-\overline{CS}_1)+(CS_2^O-\overline{CS}_2)+(\pi_B^O+\pi_1^O+\pi_2^O)-(\bar{\pi}_B^*+\bar{\pi}_1^*+\bar{\pi}_2^*)\\ &= -\frac{\alpha(\beta p_B-p_2)^2}{2\beta(1-\beta)}+(1+\alpha)\left(\frac{(1-\gamma)(1-\hat{\theta}_B)^2}{2}-\frac{(1-p_B)^2}{2}\right)\\ &\quad+\Delta(1-p_B)+(\Delta+t)\alpha\left(1-\frac{p_B-p_2}{1-\beta}\right)-(p_B-k_2)\left(\frac{p_B}{1-\gamma}-p_B\right)+\alpha(p_B-k_2)\left(\frac{p_B-p_2}{1-\beta}-\frac{p_B}{1-\gamma}\right)\\ &\quad-\left(\alpha(p_2-k_2)\left(\frac{p_B-p_2}{1-\beta}-\frac{p_2}{\beta}\right)-e\right).\end{aligned}$$

Since $e < \alpha(p_2-k_2)\left(\frac{p_B-p_2}{1-\beta}-\frac{p_2}{\beta}\right)$, then, we derive below conditions: $SS^{D\dagger} > \overline{SS}$ when $e > (e_1')^+$; $SS^{O\dagger} > \overline{SS}$ when $e > (e_2')^+$.

(2) If there is counterfeiting after conversion, i.e., $s^*=1$, then: recall that $\widetilde{\theta}' = \frac{p_B-p_2}{1-\gamma-\beta}$, and $\hat{\theta}_B = \frac{p_B}{1-\gamma}$,

(i) when Strategy *DC* is optimal,

$$\begin{aligned} SS^{DC}-\overline{SS} &= (CS_1^D-\overline{CS}_1)+(CS_2^D-\overline{CS}_2)+(\pi_B^D+\pi_1^D+\pi_2^D)-(\bar{\pi}_B^*+\bar{\pi}_1^*+\bar{\pi}_2^*)\\ &= \frac{-\alpha(1-\beta)(\widetilde{\theta}'-\frac{p_B-p_2}{1-\beta})^2}{2}\\ &\quad+\alpha\left(1-\frac{p_B-p_2}{1-\beta}\right)(\Delta+t)-\alpha(p_B-p_2)\left(\frac{p_B-p_2}{1-\gamma-\beta}-\frac{p_B-p_2}{1-\beta}\right);\end{aligned}$$

(ii) when Strategy *OC* is optimal,

$$\begin{aligned} SS^{OC}-\overline{SS} &= (CS_1^O-\overline{CS}_1)+(CS_2^O-\overline{CS}_2)+(\pi_B^O+\pi_1^O+\pi_2^O)-(\bar{\pi}_B^*+\bar{\pi}_1^*+\bar{\pi}_2^*)\\ &= \frac{-\alpha(1-\beta)(\widetilde{\theta}'-\frac{p_B-p_2}{1-\beta})^2}{2}+\left(\frac{(1-\gamma)(1-\hat{\theta}_B)^2}{2}-\frac{(1-p_B)^2}{2}\right)\\ &\quad+\Delta\left((1-p_B)+\alpha\left(1-\frac{p_B-p_2}{1-\beta}\right)\right)+t\alpha\left(1-\frac{p_B-p_2}{1-\beta}\right)-t\left(1-\frac{p_B}{1-\gamma}\right)-(p_B-k_2)\left(\frac{p_B}{1-\gamma}-p_B\right)\\ &\quad-\alpha(p_B-p_2)\left(\frac{p_B-p_2}{1-\gamma-\beta}-\frac{p_B-p_2}{1-\beta}\right).\end{aligned}$$

Thus, we obtain the conditions: $SS^{DC} > \overline{SS}$ when $\Delta > \bar{\Delta}_D$; $SS^{OC} > \overline{SS}$ when $\Delta > \bar{\Delta}_O$. ■

B.8 Proofs For Extension 1: Sequential Contract Offering

B.8.1 Proof of Lemma 3.

In order to differentiate the cases that the overseas supplier sells counterfeits, we call Strategy D without counterfeiting as Strategy D†, Strategy O without counterfeiting as Strategy O†; and call Strategy D with counterfeiting as Strategy D^{C}, Strategy O with counterfeiting as Strategy O^{C}.

Recall that

$$
\begin{array}{ll}
M' = \alpha\left(p_2-k_2\right)\left(\frac{p_B-p_2}{1-\gamma-\beta}-\frac{p_2}{\beta}\right)-e, & w_2^{(0)} = k_2+\frac{M'}{\alpha\left(\frac{p_B-p_2}{1-\gamma-\beta}-\frac{p_B}{1-\gamma}\right)}, \\
K = \alpha\left(p_2-k_2\right)\left(1-\frac{p_2}{\beta}\right)-e, & w_2^{O(2)} = k_2+\frac{K}{(1+\alpha)(1-\frac{p_B}{1-\gamma})}, \\
K-M' = \alpha\left(p_2-k_2\right)\left(1-\frac{p_B-p_2}{1-\gamma-\beta}\right), & w_2^{O(1)} = k_2+\frac{K-M'}{(1-\frac{p_B}{1-\gamma})+\alpha(1-\frac{p_B-p_2}{1-\gamma-\beta})}, \\
 & \widehat{w}_2 = p_B-\frac{(p_B-k_2-\Delta)(1-p_B)}{1-\frac{p_B}{1-\gamma}}-t.
\end{array}
$$

We observe that $w_2^{O(1)}$ is independent on e and Δ; $w_2^{(0)}$ and $w_2^{O(2)}$ are dependent on e; and $\widehat{w}_2$ is dependent on Δ.

Step 1: We derive the overseas supplier's counterfeiting decision $s(w_1,w_2,d_1,d_2)$. If the overseas supplier decides to sell counterfeits, then, it should satisfy: $\pi_2\left(s=1\right)\geq\pi_2\left(s=0\right)$ for $d_2=1$. That is,

$$
\begin{aligned}
&\max\{\pi_2\left(s=1;w_1,w_2,d_1=0,d_2=1\right),\pi_2\left(s=1;w_1,w_2,d_1=1,d_2=1\right)\} \\
\geq\ &\max\{\pi_2\left(s=0;w_1,w_2,d_1=0,d_2=1\right),\pi_2\left(s=0;w_1,w_2,d_1=1,d_2=1\right)\}.
\end{aligned}
$$

Note that with the assumption $0\leq e<\alpha(p_2-k_2)(\frac{p_B-p_2}{1-\beta}-\frac{p_2}{\beta})$, $\pi_2\left(s=1\right)\geq\pi_2\left(s=0\right)$ holds for $d_2=0$. Thus, we obtain,

$$
s^*\left(w_1,w_2,d_1,d_2\right)=\begin{cases}0, & \text{if } d_2=1 \text{ and } w_2\geq w_2^{(0)},\\ 1, & \text{if } d_2=1 \text{ and } k_2\leq w_2<w_2^{(0)}, \text{ or if } d_2=0.\end{cases}
$$

Step 2: We derive the home supplier's acceptance decision $d_1(w_1,w_2,d_2)$. If the home supplier decides to accept the contract, i.e., $d_1=1$, then, it should satisfy: $\pi_1\left(d_1=1\right)\geq\pi_1\left(d_1=0\right)$. That is,

$$
\begin{aligned}
&\max\{\pi_1\left(d_1=1;w_1,w_2,d_2=0\right),\pi_1\left(d_1=1;w_1,w_2,d_2=1,s=1\right),\pi_1\left(d_1=1;w_1,w_2,d_2=1,s=0\right)\} \\
\geq\ &\max\{\pi_1\left(d_1=0;w_1,w_2,d_2=0\right),\pi_1\left(d_1=0;w_1,w_2,d_2=1,s=1\right),\pi_1\left(d_1=0;w_1,w_2,d_2=1,s=0\right)\}.
\end{aligned}
$$

Thus, we obtain,

$$
d_1(w_1,w_2,d_2)=\begin{cases}1, & \text{if } w_1\geq k_1,\\ 0, & \text{otherwise.}\end{cases}
$$

Step 3: We derive the brand-name firm's optimal wholesale price $w_1(w_2,d_2)$.

$$
\pi_B\left(w_1;w_2,d_1=1,d_2\right)\geq\pi_B\left(w_1;w_2,d_1=0,d_2\right).
$$

Note that the brand-name firm's profit decreases in w_1.

Given $d_2=0$, we know, it should satisfy: $\pi_B\left(w_1;w_2,d_1=1,d_2=0\right)\geq\pi_B\left(w_1;w_2,d_1=0,d_2=0\right)$. That is, $k_1\leq w_1\leq p_B$. Thus, $w_1(w_2,d_2=0)=k_1$.

Given $d_2=1$, we know, it should satisfy:

$$\begin{aligned}&\max\{\pi_B\left(w_1;w_2,d_1=1,d_2=1,s=1\right),\pi_B\left(w_1;w_2,d_1=1,d_2=1,s=0\right)\}\\ \geq\ &\max\{\pi_B\left(w_1;w_2,d_1=0,d_2=1,s=1\right),\pi_B\left(w_1;w_2,d_1=0,d_2=1,s=0\right)\}.\end{aligned}$$

Then, from $\pi_B^D\geq\pi_B^O$, which means $(p_B-w_1)(1-p_B)\geq(p_B-w_2-t)(1-\frac{p_B}{1-\gamma})$, then, we obtain: $w_1\leq p_B-\frac{(p_B-w_2-t)(1-\frac{p_B}{1-\gamma})}{1-p_B}$. Note that $w_1\geq k_1$. From $p_B-\frac{(p_B-w_2-t)(1-\frac{p_B}{1-\gamma})}{1-p_B}\geq k_1$, we obtain, $w_2\geq\widehat{w}_2$, where $\widehat{w}_2=p_B-\frac{(p_B-k_2-\Delta)(1-p_B)}{1-\frac{p_B}{1-\gamma}}-t$, and $\widehat{w}_2<k_1$.

Thus, we have:

$$w_1(w_2,d_2)=\begin{cases}k_1, & \text{if } d_2=0,\\ & \text{or, if } d_2=1 \text{ and } w_2\geq\widehat{w}_2,\\ 0, & \text{otherwise.}\end{cases}$$

Step 4: We derive the overseas supplier's acceptance decision $d_2(w_2)$:

If the overseas supplier decides to accept the contract, i.e., $d_2=1$, then, it should satisfy:

$$\pi_2\left(d_2=1\right)\geq\pi_2\left(d_2=0\right).$$

That is,

$$\begin{aligned}&\max\{\pi_2\left(d_2=1;w_2,d_1=1,s=1\right),\pi_2\left(d_2=1;w_2,d_1=1,s=0\right)\\ &\qquad\pi_2\left(d_2=1;w_2,d_1=0,s=1\right),\pi_2\left(d_2=1;w_2,d_1=0,s=0\right)\}\\ \geq\ &\max\{\pi_2\left(d_2=0;w_2,d_1=1,s=1\right),\pi_2\left(d_2=0;w_2,d_1=1,s=0\right)\\ &\qquad\pi_2\left(d_2=0;w_2,d_1=0,s=1\right),\pi_2\left(d_2=0;w_2,d_1=0,s=0\right)\}.\end{aligned}$$

(1) For the case of $w_2<w_2^{(0)}$, we obtain

$$d_2(w_2;d_1)=\begin{cases}d_2\left(w_2;d_1=1\right)=1, & \text{if } \min\{w_2^{O(1)},w_2^{(0)}\}\leq w_2<w_{20}, \text{ and } w_2\geq\widehat{w}_2,\\ d_2\left(w_2;d_1=1\right)=0, & \text{if } w_2<\min\{w_2^{O(1)},w_2^{(0)}\},\\ d_2\left(w_2;d_1=0\right)=1, & \text{if } \min\{w_2^{O(1)},w_2^{(0)}\}\leq w_2<w_{20}, \text{ and } w_2<\widehat{w}_2,\\ d_2\left(w_2;d_1=0\right)=0, & \text{if } w_2<\min\{w_2^{O(1)},w_2^{(0)}\}.\end{cases}$$

(2) For the case of $w_2\geq w_2^{(0)}$, we obtain

$$d_2(w_2;d_1)=\begin{cases}d_2\left(w_2;d_1=1\right)=1, & \text{if } w_2\geq\max\{w_2^{O(2)},w_2^{(0)}\}, \text{ and } w_2\geq\widehat{w}_2,\\ d_2\left(w_2;d_1=1\right)=0, & \text{if } w_{20}<w_2<\max\{w_2^{O(2)},w_2^{(0)}\},\\ d_2\left(w_2;d_1=0\right)=1, & \text{if } w_2\geq\max\{w_2^{O(2)},w_2^{(0)}\}, \text{ and } w_2<\widehat{w}_2,\\ d_2\left(w_2;d_1=0\right)=0, & \text{if } w_{20}<w_2<\max\{w_2^{O(2)},w_2^{(0)}\}.\end{cases}$$

Thus, combined above discussions, the overseas supplier's optimal decision is

$$d_2(w_2;d_1)=\begin{cases}d_2\left(w_2;d_1=1\right)=1, & \text{if } w_2\geq\widehat{w}_2, \min\{w_2^{O(1)},w_2^{(0)}\}\leq w_2<w_{20} \text{ or } w_2\geq\max\{w_2^{O(2)},w_2^{(0)}\},\\ d_2\left(w_2;d_1=1\right)=0, & \text{if } w_2<\min\{w_2^{O(1)},w_2^{(0)}\} \text{ or } w_2^{(0)}<w_2<\max\{w_2^{O(2)},w_2^{(0)}\},\\ d_2\left(w_2;d_1=0\right)=1, & \text{if } w_2\geq\widehat{w}_2, \min\{w_2^{O(1)},w_2^{(0)}\}\leq w_2<w_2^{(0)} \text{ or } w_2\geq\max\{w_2^{O(2)},w_2^{(0)}\},\\ d_2\left(w_2;d_1=0\right)=0, & \text{if } w_2<\min\{w_2^{O(1)},w_2^{(0)}\} \text{ or } w_2^{(0)}<w_2<\max\{w_2^{O(2)},w_2^{(0)}\}.\end{cases}$$

Step 5: We derive the brand-name firm's optimal wholesale price w_2.

By substituting $d_2(w_2;d_1)$ into the brand-name firm's profit function, we obtain

$$\pi_B^H=\left(p_B-k_1\right)\left(1-p_B\right)+\alpha\left(p_B-k_1-t\right)\left(1-\frac{p_B-p_2}{1-\beta}\right),$$

$$\pi_B^D = \begin{cases} \pi_B^{D\dagger}(w_2) = (p_B - k_1)(1-p_B) + \alpha(p_B - w_2)(1 - \frac{p_B}{1-\gamma}), & \text{if } w_2 \geq \max\{w_2^{O(2)}, w_2^{(0)}\}, \text{ and } w_2 \geq \widehat{w}_2, \\ \pi_B^{DC}(w_2) = (p_B - k_1)(1 - \frac{p_B}{1-\gamma}) + \alpha(p_B - w_2)(1 - \frac{p_B - p_2}{1-\gamma-\beta}), & \text{if } \min\{w_2^{O(1)}, w_2^{(0)}\} \leq w_2 < w_2^{(0)}, \text{ and } w_2 \geq \widehat{w}_2, \end{cases}$$

$$\pi_B^O = \begin{cases} \pi_B^{O\dagger}(w_2) = (p_B - w_2 - t)(1 - \frac{p_B}{1-\gamma}) + \alpha(p_B - w_2)(1 - \frac{p_B}{1-\gamma}), & \text{if } w_2 \geq \max\{w_2^{O(2)}, w_2^{(0)}\}, \text{ and } w_2 < \widehat{w}_2, \\ \pi_B^{OC}(w_2) = (p_B - w_2 - t)(1 - \frac{p_B}{1-\gamma}) + \alpha(p_B - w_2)(1 - \frac{p_B - p_2}{1-\gamma-\beta}), & \text{if } \min\{w_2^{O(1)}, w_2^{(0)}\} \leq w_2 < w_2^{(0)}, \text{ and } w_2 < \widehat{w}_2, \end{cases}$$

Note that the brand-name firm's profit decreases in w_2. Then, the optimal wholesale price(s) of the brand-name firm, which will be accepted by the counterfeiter, satisfies the following:

(a) under Strategy D,

$$w_2^D = \begin{cases} w_2^{D\dagger *} = \max\{w_2^{O(2)}, w_2^{(0)}, \widehat{w}_2\}, & \text{if } s = 0, \\ w_2^{DC*} = \max\{w_2^{O(1)}, \widehat{w}_2\}, & \text{if } s = 1 \text{ and } \max\{w_2^{O(1)}, \widehat{w}_2\} < w_2^{(0)}, \end{cases}$$

(b) under Strategy O,

$$w_2^O = \begin{cases} w_2^{O\dagger *} = \max\{w_2^{O(2)}, w_2^{(0)}\}, & \text{if } s = 0 \text{ and } \max\{w_2^{O(2)}, w_2^{(0)}\} < \widehat{w}_2, \\ w_2^{OC*} = w_2^{O(1)}, & \text{if } s = 1 \text{ and } w_2^{O(1)} < \min\{w_2^{(0)}, \widehat{w}_2\}. \end{cases}$$

Recall that $w_2^{O(1)}$ is independent on e and Δ; $w_2^{(0)}$ and $w_2^{O(2)}$ are dependent on e; and $\widehat{w}_2$ is dependent on Δ. Then, we know the wholesale price $w_2^{D\dagger *}$ could be dependent on Δ and e, w_2^{DC*} could be dependent on Δ; $w_2^{O\dagger *}$ is dependent on e, w_2^{OC*} is independent on both e and Δ.

For Strategy D and Strategy O, the brand-name firm may offer different wholesale prices w_2 which helps prevent counterfeiting, below, we further check the feasible region of π_B under Strategy D and Strategy O. Then, there are four cases for the existence of possible strategies:

Case 1: $w_2^{O(1)} < \widehat{w}_2 < w_2^{(0)}$, in which both Strategy D^C and Strategy O^C are possible;

Case 2: $\widehat{w}_2 < w_2^{O(1)} < w_2^{(0)}$, in which only Strategy D^C is possible;

Case 3: $w_2^{O(1)} < w_2^{(0)} < \widehat{w}_2$, in which both Strategy O† and Strategy O^C are possible. In particular, only if $\max\{w_2^{O(2)}, w_2^{(0)}\} < \widehat{w}_2$, Strategy O† exists;

Case 4: $w_2^{O(1)} > w_2^{(0)}$, in which only Strategy O† is possible.

Note that

$$\begin{array}{ll} w_2^{O(1)} < w_2^{(0)}, & \Rightarrow e < e_1; \\ w_2^{O(1)} < \widehat{w}_2, & \Rightarrow \Delta > \Delta_0, \text{ where } \Delta_0 = (w_2^{OC*} - (p_B - \frac{(p_B-k_2)(1-p_B)}{1-\frac{p_B}{1-\gamma}}))\frac{1-\frac{p_B}{1-\gamma}}{1-p_B}; \\ \widehat{w}_2 < w_2^{(0)}, & \Rightarrow e < \hat{e}_2, \text{ where } \hat{e}_2 = (p_2 - k_2 - (\widehat{w}_2 - k_2)\frac{\beta}{1-\gamma})\frac{\alpha(\beta p_B - (1-\gamma)p_2)}{(1-\gamma-\beta)\beta}; \\ w_2^{O(2)} < \widehat{w}_2, & \Rightarrow e > \hat{e}_3, \text{ where } \hat{e}_3 = \alpha(p_2 - k_2)(1 - \frac{p_2}{\beta}) - (\widehat{w}_2 - k_2)(1+\alpha)(1 - \frac{p_B}{1-\gamma}). \end{array} \tag{12}$$

Thus, the feasible regions of each possible case are as follows:

Strategy D†: exists for all cases;

Strategy D^C: exists for case 1 and case 2, which means $e < \min\{e_1, \hat{e}_2\}$;

Strategy O†: exists for case 3 and case 4, which means $e > \max\{\hat{e}_2, \hat{e}_3\}$;

Strategy O^C: exists for case 1 and case 3, which means $e < e_1$ and $\Delta > \Delta_0$.

Note that it is easy to know that Strategy D^C and Strategy O† do not exist in the same feasible region.

Next, we make a comparison between Strategy D† and Strategy D^C, Strategy O† and Strategy O^C, respectively.

5.1 With Strategy D, we have

$$\pi_B^D = \begin{cases} \pi_B^{D\dagger}\left(w_2^{D\dagger*}\right) = (p_B - k_1)(1-p_B) + \alpha\left(p_B - w_2^{D\dagger*}\right)\left(1-\frac{p_B}{1-\gamma}\right), \\ \pi_B^{DC}\left(w_2^{DC*}\right) = (p_B - k_1)(1-p_B) + \alpha\left(p_B - w_2^{DC*}\right)\left(1-\frac{p_B-p_2}{1-\gamma-\beta}\right), \text{ if } e < \min\{e_1, \hat{e}_2\}, \end{cases}$$

and

$$\begin{aligned} &\pi_B^{D\dagger}\left(w_2^{D\dagger*}\right) \geq \pi_B^{DC}\left(w_2^{DC*}\right), \\ \Rightarrow\ &\left(p_B - w_2^{D\dagger*}\right)\left(1-\frac{p_B}{1-\gamma}\right) \geq \left(p_B - w_2^{DC*}\right)\left(1-\frac{p_B-p_2}{1-\gamma-\beta}\right), \\ \Rightarrow\ &w_2^{D\dagger*} \leq \frac{p_B\left(\frac{p_B-p_2}{1-\gamma-\beta}-\frac{p_B}{1-\gamma}\right)+w_2^{DC*}\left(1-\frac{p_B-p_2}{1-\gamma-\beta}\right)}{1-\frac{p_B}{1-\gamma}} = p_B - \frac{\left(p_B-w_2^{DC*}\right)\left(1-\frac{p_B-p_2}{1-\gamma-\beta}\right)}{1-\frac{p_B}{1-\gamma}}. \end{aligned}$$

Recall that $w_2^{DC*} = \max\{w_2^{O(1)}, \widehat{w}_2\}$, $w_2^{D\dagger*} = \max\{w_2^{O(2)}, w_2^{(0)}, \widehat{w}_2\}$. Note that when $\widehat{w}_2 > \max\{w_2^{O(2)}, w_2^{(0)}\}$ with Strategy D†, Strategy D^C does not exist. Thus, when Strategy D^C exists, that is, $e < \min\{e_1, \hat{e}_2\}$, the optimal wholesale price of Strategy D† is $w_2^{D\dagger*} = \max\{w_2^{O(2)}, w_2^{(0)}\}$, which is independent on Δ, and dependent on e.

If $w_2^{O(1)} > \widehat{w}_2$, then $w_2^{DC*} = w_2^{O(1)}$, which is independent on both e and Δ. Then,

$$\pi_B^{DC} > \pi_B^{D\dagger} \Rightarrow w_2^{D\dagger*} > \frac{p_B\left(\frac{p_B-p_2}{1-\gamma-\beta}-\frac{p_B}{1-\gamma}\right)+w_2^{O(1)}\left(1-\frac{p_B-p_2}{1-\gamma-\beta}\right)}{1-\frac{p_B}{1-\gamma}} = p_B - \frac{\left(p_B-w_2^{O(1)}\right)\left(1-\frac{p_B-p_2}{1-\gamma-\beta}\right)}{1-\frac{p_B}{1-\gamma}} \Rightarrow e < e_{D2}, \text{ [case 1, case 2]}$$

If $w_2^{O(1)} < \widehat{w}_2$, then $w_2^{DC*} = \widehat{w}_2$, which is dependent on Δ. Then,

$$\pi_B^{DC} > \pi_B^{D\dagger} \Rightarrow w_2^{D\dagger*} > \frac{p_B\left(\frac{p_B-p_2}{1-\gamma-\beta}-\frac{p_B}{1-\gamma}\right)+\widehat{w}_2\left(1-\frac{p_B-p_2}{1-\gamma-\beta}\right)}{1-\frac{p_B}{1-\gamma}} = p_B - \frac{(p_B-\widehat{w}_2)\left(1-\frac{p_B-p_2}{1-\gamma-\beta}\right)}{1-\frac{p_B}{1-\gamma}} \Rightarrow e < e_{D3}, \text{ [case 1, case 2]}$$

5.2 With Strategy O, similarly, for the comparison between Strategy O† and Strategy O^C, we know,

$$\pi_B^O = \begin{cases} \pi_B^{O\dagger}\left(w_2^{O\dagger*}\right) = \left(p_B - w_2^{O*} - t\right)\left(1-\frac{p_B}{1-\gamma}\right) + \alpha\left(p_B - w_2^{O\dagger*}\right)\left(1-\frac{p_B}{1-\gamma}\right), \\ \pi_B^{OC}\left(w_2^{OC*}\right) = \left(p_B - w_2^{OC*} - t\right)\left(1-\frac{p_B}{1-\gamma}\right) + \alpha\left(p_B - w_2^{OC*}\right)\left(1-\frac{p_B-p_2}{1-\gamma-\beta}\right), \end{cases}$$

and

$$\pi_B^{OC}\left(w_2^{OC*}\right) > \pi_B^{O\dagger}\left(w_2^{O\dagger*}\right) \Rightarrow w_2^{O\dagger*} > \frac{\alpha\left(p_B-w_2^{OC*}\right)\left(\frac{p_B-p_2}{1-\gamma-\beta}-\frac{p_B}{1-\gamma}\right)}{(1+\alpha)\left(1-\frac{p_B}{1-\gamma}\right)} + w_2^{OC*} \Rightarrow e < e_{O1}. \text{ [case 3]}$$

Then, based on above discussion, we have the following optimal wholesale price w_2 for Strategy D and Strategy O, respectively. Note that $e'_{D1} < \min\{e_1, \hat{e}_2\}$, $e_{O1} < e_1$.

(a) Under Strategy D, (i) $w_2^D = \max\{w_2^{O(1)}, \widehat{w}_2\}$ and $s^* = 1$, if $e < e'_{D1}$; (ii) $w_2^D = \max\{w_2^{(0)}, w_2^{O(2)}, \widehat{w}_2\}$ and $s^* = 0$, if $e \geq e'_{D1}$;

(b) under Strategy O, (i) $w_2^O = w_2^{O(1)}$ and $s^* = 1$, if $\Delta > \Delta_0$ and $e < \max\{e_{O1}, \hat{e}_2\}$; (ii) $w_2^O = \max\{w_2^{(0)}, w_2^{O(2)}\}$ and $s^* = 0$, if $e \geq \max\{e_{O1}, \hat{e}_2, \hat{e}_3\}$;

where

$$\begin{aligned} &e'_{D1} = \min\{e_{D2}, e_{D3}\}, \\ &e_{D2} \text{ is the threshold value of } e \text{ satisfying} \max\{w_2^{O(2)}, w_2^{(0)}\} = p_B - \frac{\left(p_B-w_2^{O(1)}\right)\left(1-\frac{p_B-p_2}{1-\gamma-\beta}\right)}{1-\frac{p_B}{1-\gamma}}, \\ &e_{D3} \text{ is the threshold value of } e \text{ satisfying} \max\{w_2^{O(2)}, w_2^{(0)}\} = p_B - \frac{(p_B-\widehat{w}_2)\left(1-\frac{p_B-p_2}{1-\gamma-\beta}\right)}{1-\frac{p_B}{1-\gamma}}, \\ &e_{O1} = \left(p_2 - k_2 - \left(\frac{\alpha\left(p_B-w_2^{OC*}\right)\left(\frac{p_B-p_2}{1-\gamma-\beta}-\frac{p_B}{\gamma}\right)}{(1+\alpha)\left(1-\frac{p_B}{1-\gamma}\right)} + w_2^{OC*} - k_2\right)\frac{\beta}{1-\gamma}\right)\frac{\alpha(\beta p_B-(1-\gamma)p_2)}{(1-\gamma-\beta)\beta}, \end{aligned} \tag{13}$$

and $w_2^{OC*} = k_2 + \frac{\alpha(p_2-k_2)\left(1-\frac{p_B-p_2}{1-\gamma-\beta}\right)}{\left(1-\frac{p_B}{1-\gamma}\right)+\alpha\left(1-\frac{p_B-p_2}{1-\gamma-\beta}\right)}$.

Thus, we have the results. ■

B.8.2 Proof of Proposition 5.

The brand-name firm makes comparisons among different strategies. Recall that the profits are as follows:

$$\pi_B^H = (p_B - k_1)(1-p_B) + \alpha(p_B - k_1 - t)(1 - \tfrac{p_B - p_2}{1-\beta}),$$

$$\pi_B^D = \begin{cases} \pi_B^{D\dagger}\left(w_2^{D\dagger *}\right) = (p_B - k_1)(1-p_B) + \alpha\left(p_B - w_2^{D\dagger *}\right)(1 - \frac{p_B}{1-\gamma}), & \text{if } e \geq e'_{D1}, \\ \pi_B^{DC}\left(w_2^{DC*}\right) = (p_B - k_1)(1-p_B) + \alpha\left(p_B - w_2^{DC*}\right)(1 - \frac{p_B - p_2}{1-\gamma-\beta}), & \text{if } e < e'_{D1}, \end{cases}$$

$$\pi_B^O = \begin{cases} \pi_B^{O\dagger}\left(w_2^{O\dagger *}\right) = \left(p_B - w_2^{O\dagger *} - t\right)(1 - \frac{p_B}{1-\gamma}) + \alpha\left(p_B - w_2^{O\dagger *}\right)(1 - \frac{p_B}{1-\gamma}), & \text{if } e \geq \max\{e_{O1}, \hat{e}_2, \hat{e}_3\}, \\ \pi_B^{OC}\left(w_2^{OC*}\right) = \left(p_B - w_2^{OC*} - t\right)(1 - \frac{p_B}{1-\gamma}) + \alpha\left(p_B - w_2^{OC*}\right)(1 - \frac{p_B - p_2}{1-\gamma-\beta}), & \text{if } \Delta > \Delta_0,\ e < \max\{e_{O1}, \hat{e}_2\}, \end{cases}$$

$$\pi_B^N = 0,$$

where $w_2^{DC*} = \max\{w_2^{O(1)}, \widehat{w}_2\}$, $w_2^{D\dagger *} = \max\{w_2^{O(2)}, w_2^{(0)}, \widehat{w}_2\}$; $w_2^{OC*} = w_2^{O(1)}$, $w_2^{O\dagger *} = \max\{w_2^{O(2)}, w_2^{(0)}\}$; and e_1, $\hat{e}_2$, $\hat{e}_3$, and Δ_0 are defined in Equation (12). Recall that Strategy D^C and Strategy $\mathrm{O}^\dagger$ do not exist at the same feasible region. Thus, there is no comparison between them.

Below, following the approach in Lemma B2, we derive the conditions for each possible strategy.

(1) The conditions for $\pi_B^* = \pi_B^{D\dagger}$ are $e \geq e'_{D1}$, and

$$\begin{aligned}
&\pi_B^{D\dagger} > \pi_B^{O\dagger} \Rightarrow w_2^{O\dagger *} > p_B - \frac{\alpha(w_2^{O\dagger *} - w_2^{D\dagger *})(1-\frac{p_B}{1-\gamma}) + (p_B - k_2 - \Delta)(1-p_B)}{(1-\frac{p_B}{1-\gamma})}, && \Rightarrow e < f'_{DO3}, \text{ [case 3, case 4]} \\
&\pi_B^{D\dagger} > \pi_B^{OC} \Rightarrow w_2^{D\dagger *} < p_B - \frac{\left(p_B - w_2^{OC*}\right)(1-\frac{p_B}{1-\gamma}) + \alpha\left(p_B - w_2^{OC*}\right)(1-\frac{p_B-p_2}{1-\gamma-\beta}) - (p_B - k_2 - \Delta)(1-p_B)}{\alpha(1-\frac{p_B}{1-\gamma})}, && \Rightarrow e > f'_{DO2}, \text{ [case 1, case 3]} \\
&\pi_B^{D\dagger} > \pi_B^{H} \Rightarrow w_2^{D\dagger *} < p_B - \frac{(p_B - k_2 - \Delta)(1-\frac{p_B-p_2}{1-\beta})}{1-\frac{p_B}{1-\gamma}}, && \Rightarrow e > f'_{DH},
\end{aligned}$$

(2) the conditions for $\pi_B^* = \pi_B^{DC}$ are $e < e'_{D1}$, and

$$\begin{aligned}
&\pi_B^{DC} > \pi_B^{OC} \Rightarrow \Delta < p_B - k_2 - \frac{\left(p_B - w_2^{OC*}\right)(1-\frac{p_B}{1-\gamma}) - \alpha\left(w_2^{OC*} - w_2^{DC*}\right)(1-\frac{p_B-p_2}{1-\gamma-\beta})}{1-p_B}, && \Rightarrow \Delta < \Delta'_{DO}, \text{ [case 1]} \\
&\pi_B^{DC} > \pi_B^{H} \Rightarrow \Delta > p_B - k_2 - \frac{\left(p_B - w_2^{DC*}\right)(1-\frac{p_B-p_2}{1-\gamma-\beta})}{1-\frac{p_B-p_2}{1-\beta}}, && \Rightarrow \Delta > \Delta'_{DH}, \text{ [case 1, case 2]}
\end{aligned}$$

(3) the conditions for $\pi_B^* = \pi_B^{O\dagger}$ are $e \geq \max\{e_{O1}, \hat{e}_2, \hat{e}_3\}$, and

$$\begin{aligned}
&\pi_B^{O\dagger} > \pi_B^{D\dagger} \Rightarrow w_2^{O\dagger *} < p_B - \frac{\alpha(w_2^{O\dagger *} - w_2^{D\dagger *})(1-\frac{p_B}{1-\gamma}) + (p_B - k_2 - \Delta)(1-p_B)}{(1-\frac{p_B}{1-\gamma})}, && \Rightarrow e > f'_{DO3}, \text{ [case 3, case 4]} \\
&\pi_B^{O\dagger} > \pi_B^{H} \Rightarrow w_2^{O\dagger *} < p_B - \frac{(p_B - k_2 - \Delta)((1-p_B) + \alpha(1-\frac{p_B-p_2}{1-\beta}))}{(1+\alpha)(1-\frac{p_B}{1-\gamma})}, && \Rightarrow e > f_{HO}, \text{ [case 3, case 4]}
\end{aligned}$$

(4) the conditions for $\pi_B^* = \pi_B^{OC}$ are $\Delta > \Delta_0$, $e < \max\{e_{O1}, \hat{e}_2\}$, and

$$\begin{aligned}
&\pi_B^{OC} > \pi_B^{D\dagger} \Rightarrow w_2^{D\dagger *} > p_B - \frac{\left(p_B - w_2^{OC*}\right)(1-\frac{p_B}{1-\gamma}) + \alpha\left(p_B - w_2^{OC*}\right)(1-\frac{p_B-p_2}{1-\gamma-\beta}) - (p_B - k_2 - \Delta)(1-p_B)}{\alpha(1-\frac{p_B}{1-\gamma})}, && \Rightarrow e < f'_{DO2}, \text{ [case 1, case 3]} \\
&\pi_B^{OC} > \pi_B^{DC} \Rightarrow \Delta > p_B - k_2 - \frac{\left(p_B - w_2^{OC*}\right)(1-\frac{p_B}{1-\gamma}) - \alpha\left(w_2^{OC*} - w_2^{DC*}\right)(1-\frac{p_B-p_2}{1-\gamma-\beta})}{1-p_B}, && \Rightarrow \Delta > \Delta'_{DO}, \text{ [case 1]} \\
&\pi_B^{OC} > \pi_B^{H} \Rightarrow \Delta > p_B - k_2 - \frac{\left(p_B - w_2^{OC*}\right)(1-\frac{p_B}{1-\gamma}) + \alpha\left(p_B - w_2^{OC*}\right)(1-\frac{p_B-p_2}{1-\gamma-\beta})}{(1-p_B) + \alpha(1-\frac{p_B-p_2}{1-\beta})}, && \Rightarrow \Delta > \Delta_{HO}, \text{ [case 1, case 3]}
\end{aligned}$$

(5) the conditions for $\pi_B^* = \pi_B^H$ are

$$\begin{aligned}
&\pi_B^H > \pi_B^{O\dagger} \Rightarrow w_2^{O\dagger *} > p_B - \frac{(p_B-k_2-\Delta)((1-p_B)+\alpha(1-\frac{p_B-p_2}{1-\beta}))}{(1+\alpha)(1-\frac{p_B}{1-\gamma})}, && \Rightarrow e < f_{HO}, \ \text{[case 3, case 4]}\\
&\pi_B^H > \pi_B^{OC} \Rightarrow \Delta < p_B - k_2 - \frac{\left(p_B-w_2^{OC*}\right)(1-\frac{p_B}{1-\gamma})+\alpha\left(p_B-w_2^{OC*}\right)(1-\frac{p_B-p_2}{1-\gamma-\beta})}{(1-p_B)+\alpha(1-\frac{p_B-p_2}{1-\beta})}, && \Rightarrow \Delta < \Delta_{HO}, \ \text{[case 1, case 3]}\\
&\pi_B^H > \pi_B^{DC} \Rightarrow \Delta < p_B - k_2 - \frac{\left(p_B-w_2^{DC*}\right)(1-\frac{p_B-p_2}{1-\gamma-\beta})}{1-\frac{p_B-p_2}{1-\beta}}, && \Rightarrow \Delta < \Delta'_{DH}, \ \text{[case 1, case 2]}\\
&\pi_B^H > \pi_B^{D\dagger} \Rightarrow w_2^{D\dagger *} > p_B - \frac{(p_B-k_2-\Delta)(1-\frac{p_B-p_2}{1-\beta})}{1-\frac{p_B}{1-\gamma}}, && \Rightarrow e < f'_{DH}.
\end{aligned}$$

Note that $f'_{DH} < f_{HO}$, $\max\{e_{O1}, \hat{e}_2, \hat{e}_3\} > f'_{DO3}$, $\max\{e_{O1}, \hat{e}_2, \hat{e}_3\} > f_{HO}$, and $\Delta_{HO} < \Delta_0$. Thus, we summarize the thresholds for comparisons, and are derived as follows:

$$\begin{aligned}
&e_{O1} < \hat{e}_2, && \Rightarrow \Delta < \Delta'_0,\\
&\Delta < p_B - k_2 - \frac{\left(p_B-w_2^{DC*}\right)(1-\frac{p_B-p_2}{1-\gamma-\beta})}{1-\frac{p_B-p_2}{1-\beta}}, && \Rightarrow \Delta < \Delta'_{DH},\\
&w_2^{D\dagger *} > p_B - \frac{(p_B-k_2-\Delta)(1-\frac{p_B-p_2}{1-\beta})}{1-\frac{p_B}{1-\gamma}}, && \Rightarrow e < f'_{DH},\\
&w_2^{D\dagger *} < p_B - \frac{\left(p_B-w_2^{OC*}\right)(1-\frac{p_B}{1-\gamma})+\alpha\left(p_B-w_2^{OC*}\right)(1-\frac{p_B-p_2}{1-\gamma-\beta})-(p_B-k_2-\Delta)(1-p_B)}{\alpha(1-\frac{p_B}{1-\gamma})}, && \Rightarrow e > f'_{DO2},\\
&\Delta < p_B - k_2 - \frac{\left(p_B-w_2^{OC*}\right)(1-\frac{p_B}{1-\gamma})-\alpha\left(w_2^{OC*}-w_2^{DC*}\right)(1-\frac{p_B-p_2}{1-\gamma-\beta})}{1-p_B}, && \Rightarrow \Delta < \Delta'_{DO},
\end{aligned} \tag{14}$$

where $w_2^{D\dagger *} = w_2^{O\dagger *} = \max\{w_2^{O(2)}, w_2^{(0)}\}$, $w_2^{DC*} = w_2^{OC*} = w_2^{O(1)}$.

The equilibrium sourcing strategy of the brand-name firm is as follows:

(a) Strategy H with $w_1^* = k_1$ if $e < f'_{DH}$ and $\Delta < \min\{\Delta'_{DH}, \Delta_0\}$;

(b) Strategy D with $w_1^* = k_1$, and

$$w_2^* = \begin{cases} w_2^{O(1)}, & \text{if } e < e'_{D1} \text{ and } \Delta'_{DH} \le \Delta \le \Delta_0;\\ \max\{w_2^{O(2)}, w_2^{(0)}\}, & \text{if } e \ge \max\{e'_{D1}, f'_{DH}\} \text{ and } \Delta \le \Delta_0,\\ & \text{or, if } f'_{DO2} \le e \le \max\{\hat{e}_2, \hat{e}_3\} \text{ and } \Delta > \Delta_0; \end{cases}$$

(c) Strategy O with

$$w_2^* = \begin{cases} w_2^{O(1)}, & \text{if } e < \min\{\hat{e}_2, f'_{DO2}\} \text{ and } \Delta_0 < \Delta \le \Delta'_0;\\ & \text{or, if } e < e_{O1} \text{ and } \Delta > \Delta'_0;\\ \max\{w_2^{O(2)}, w_2^{(0)}\}, & \text{if } e \ge \max\{e_{O1}, \hat{e}_2, \hat{e}_3\}. \end{cases}$$

Thus, by combining the conditions for each strategy, we obtain the results. ■

B.9 Proofs of Extension 2: Endogenous Counterfeit Price

B.9.1 Proof of Lemma 4.

Under each possible sourcing strategy, we obtain the profit expressions for each firm, and discuss the overseas supplier's counterfeiting decision, s^*, and the corresponding retail price of the counterfeit product if $s^* = 1$. Note that we focus on the case in which both the brand-name firm and the counterfeiter have positive market shares in the overseas market if the counterfeiter sells counterfeits.

Strategy H: Given wholesale prices w_1 and w_2, the home supplier accepts the contract and the counterfeiter rejects the contract, i.e., $d_1 = 1$ and $d_2 = 0$. Thus, the brand-name firm only sources from the home supplier.

(1) If the counterfeiter does not sell the counterfeit, i.e., $s=0$, the brand-name firm is the monopoly in the overseas market. Thus, their profit expressions are as follows:

$$\pi_B^H(w_1)=(p_B-w_1)(1-p_B)+\alpha(p_B-w_1-t)(1-p_B),\quad \pi_1^H(w_1)=(1+\alpha)(w_1-k_1)(1-p_B),\quad \pi_2^H=0.$$

(2) If the counterfeiter sells the counterfeit in the overseas market, i.e., $s=1$, the expected profits of the brand-name firm, the home and overseas suppliers are given below:

$$\begin{aligned}
&\pi_B^H(w_1)=(p_B-w_1)(1-p_B)+\alpha(p_B-w_1-t)\left(1-\tfrac{p_B-p_2}{1-\beta}\right),\\
&\pi_1^H(w_1)=(w_1-k_1)\left((1-p_B)+\alpha\left(1-\tfrac{p_B-p_2}{1-\beta}\right)\right),\\
&\pi_2^H(p_2)=\alpha(p_2-k_2)\left(\tfrac{p_B-p_2}{1-\beta}-\tfrac{p_2}{\beta}\right)-e.
\end{aligned}$$

If both the brand-name firm and the overseas supplier get positive overseas market share, i.e., $m_{B2}=\alpha\left(1-\frac{p_B-p_2}{1-\beta}\right)>0$, and $m_2=\alpha\left(\frac{p_B-p_2}{1-\beta}-\frac{p_2}{\beta}\right)>0$, then, $p_B-(1-\beta)<p_2<\beta p_B$.

In order to discuss the most interesting cases, we focus on $\frac{k_2}{p_B}<\beta<\frac{k_2+2(1-p_B)}{2-p_B}$, in which both the brand-name firm and the counterfeiter obtain positive market shares in the overseas market. The profit of the counterfeiter is

$$\pi_2^H(p_2)=\alpha(p_2-k_2)\left(\tfrac{p_B-p_2}{1-\beta}-\tfrac{p_2}{\beta}\right)-e.$$

By taking the first order derivative of $\pi_2^H(p_2)$ with respect to p_2, the optimal retail price of the counterfeit is $p_2^H=\frac{\beta p_B+k_2}{2}$. Substituting the expression of p_2^H into the profit functions, we obtain

$$\begin{aligned}
&\pi_B^H(w_1)=(p_B-w_1)(1-p_B)+\alpha(p_B-w_1-t)\left(1-\frac{(2-\beta)p_B-k_2}{2(1-\beta)}\right),\\
&\pi_1^H(w_1)=(w_1-k_1)\left((1-p_B)+\alpha\left(1-\frac{(2-\beta)p_B-k_2}{2(1-\beta)}\right)\right),\quad \pi_2^H=\frac{\alpha(\beta p_B-k_2)^2}{4\beta(1-\beta)}-e.
\end{aligned}$$

Recall that $e<\frac{\alpha(\beta p_B-k_2)^2}{4\beta(1-\beta)}$, resulting in $\pi_2^H(s=1)>\pi_2^H(s=0)$. It means that the counterfeiter always sells counterfeit products.

Strategy D: Given wholesale prices w_1 and w_2, both the home supplier and the counterfeiter accept their contracts, i.e., $d_1=1$ and $d_2=1$.

(1) If the overseas supplier does not sell the counterfeit in the market, i.e., $s=0$, the expected profits of the brand-name firm, the home and overseas suppliers are given below:

$$\begin{aligned}
&\pi_B^D(w_1,w_2)=(p_B-w_1)(1-p_B)+\alpha(p_B-w_2)\left(1-\tfrac{p_B}{1-\gamma}\right),\\
&\pi_1^D(w_1)=(w_1-k_1)(1-p_B),\\
&\pi_2^D(w_2)=\alpha(w_2-k_2)\left(1-\tfrac{p_B}{1-\gamma}\right).
\end{aligned}$$

(2) If the overseas supplier sells the counterfeit in the market, i.e., $s=1$, then, for given p_B for the brand-name product, the overseas supplier decides on the retail price p_2 for the counterfeit. Their profits are as follows:

$$\begin{aligned}
&\pi_B^D(w_1,w_2,p_2)=(p_B-w_1)(1-p_B)+\alpha(p_B-w_2)\left(1-\tfrac{p_B-p_2}{1-\gamma-\beta}\right),\\
&\pi_1^D(w_1)=(w_1-k_1)(1-p_B),\\
&\pi_2^D(w_2,p_2)=\alpha(w_2-k_2)\left(1-\tfrac{p_B-p_2}{1-\gamma-\beta}\right)+\alpha(p_2-k_2)\left(\tfrac{p_B-p_2}{1-\gamma-\beta}-\tfrac{p_2}{\beta}\right)-e.
\end{aligned}$$

If both the brand-name firm and the overseas supplier get positive overseas market share, i.e., $m_{B2} = \alpha\left(1-\frac{p_B-p_2}{1-\gamma-\beta}\right) > 0$, and $m_2 = \alpha\left(\frac{p_B-p_2}{1-\gamma-\beta}-\frac{p_2}{\beta}\right) > 0$, then, $p_B-(1-\gamma-\beta) < p_2 < \frac{\beta p_B}{1-\gamma}$. The profit of the overseas supplier is

$$\pi_2^D(w_2,p_2) = \alpha(w_2-k_2)\left(1-\frac{p_B-p_2}{1-\gamma-\beta}\right)+\alpha(p_2-k_2)\left(\frac{p_B-p_2}{1-\gamma-\beta}-\frac{p_2}{\beta}\right)-e.$$

By taking the first-order derivative of $\pi_2^D(w_2,p_2)$ with respect to p_2, we have,

$$\frac{\partial(\pi_2^D(w_2,p_2))}{\partial(p_2)} = \alpha\left(\frac{p_B+k_2-2p_2+(w_2-k_2)}{1-\gamma-\beta}-\frac{2p_2-k_2}{\beta}\right) = \alpha\left(\frac{p_B-2p_2+w_2}{1-\gamma-\beta}-\frac{2p_2-k_2}{\beta}\right).$$

From $\frac{\partial\left(\pi_2^D(w_2,p_2)\right)}{\partial(p_2)} = 0$, we obtain the critical point $\hat{p}_2 = \frac{\beta p_B+(1-\gamma)k_2+\beta(w_2-k_2)}{2(1-\gamma)}$. Next, we check whether $\hat{p}_2$ is in the feasible region $p_B-(1-\gamma-\beta) < p_2 < \frac{\beta p_B}{1-\gamma}$. From $p_B-(1-\gamma-\beta) < \hat{p}_2 < \frac{\beta p_B}{1-\gamma}$, we obtain,

$$\underline{w}_2 < w_2 < k_2+\frac{\beta p_B-(1-\gamma)k_2}{\beta},$$

where $\underline{w}_2 = k_2+\frac{2(1-\gamma)[p_B-(1-\gamma-\beta)]-(\beta p_B+(1-\gamma)k_2)}{\beta}$. Note that if $w_2 \le \underline{w}_2$, there is no market share for the counterfeiter in the overseas market.

We focus on the case when the brand-name firm has a positive market share in the overseas market, i.e., $m_{B2} > 0$. Thus, with Strategy D, if the overseas supplier sells the counterfeit, i.e., $s = 1$, the optimal retail price p_2 for the counterfeit is

$$p_2^D = \begin{cases} \frac{\beta p_B}{1-\gamma}, & \text{if } w_2 \ge k_2+\frac{\beta p_B-(1-\gamma)k_2}{\beta}, \text{ [note that } m_2=0] \\ \hat{p}_2, & \text{if } \underline{w}_2 < w_2 < k_2+\frac{\beta p_B-(1-\gamma)k_2}{\beta}, \text{ [note that } m_2>0] \end{cases} \tag{15}$$

and the overseas supplier's profit is

$$\pi_2^D(w_2,s=1) = \begin{cases} \pi_2^{DC1} = \alpha(w_2-k_2)\left(1-\frac{p_B-p_2^D}{1-\gamma-\beta}\right)-e, & \text{if } w_2 \ge k_2+\frac{\beta p_B-(1-\gamma)k_2}{\beta}, \\ \hat{\pi}_2^{DC} = \alpha(w_2-k_2)\left(1-\frac{p_B-p_2^D}{1-\gamma-\beta}\right) & \\ +\alpha(p_2^D-k_2)\left(\frac{p_B-p_2^D}{1-\gamma-\beta}-\frac{p_2^D}{\beta}\right)-e, & \text{if } \underline{w}_2 < w_2 < k_2+\frac{\beta p_B-(1-\gamma)k_2}{\beta}, \end{cases}$$

and the brand-name firm's profit is

$$\pi_B^D(w_1,w_2,s=1) = \begin{cases} \pi_B^{DC1} = (p_B-w_1)(1-p_B)+\alpha(p_B-w_2)\left(1-\frac{p_B-p_2^D}{1-\gamma-\beta}\right), & \text{if } w_2 \ge k_2+\frac{\beta p_B-(1-\gamma)k_2}{\beta}, \\ \hat{\pi}_B^{DC} = (p_B-w_1)(1-p_B) & \\ +\alpha(p_B-w_2)\left(\frac{2(1-\gamma-\beta)(1-\gamma-p_B)-\beta p_B+(1-\gamma)k_2+\beta(w_2-k_2)}{2(1-\gamma)(1-\gamma-\beta)}\right), & \text{if } \underline{w}_2 < w_2 < k_2+\frac{\beta p_B-(1-\gamma)k_2}{\beta}. \end{cases}$$

Next, the overseas supplier determines whether to sell the counterfeit, $s^*(w_2)$. For the overseas supplier, if $\pi_2^D(w_2,s=1) > \pi_2^D(w_2,s=0)$, she decides to sell the counterfeit; otherwise, she does not sell the counterfeit. Recall that when $s=0$, the overseas supplier's profit is

$$\pi_2^D(w_2,s=0) = \alpha(w_2-k_2)\left(1-\frac{p_B}{1-\gamma}\right).$$

Note that, given p_B, for the overseas supplier has the following two scenarios:

(i) If $w_2 \geq k_2 + \frac{\beta p_B-(1-\gamma)k_2}{\beta}$, then the overseas supplier's profit of counterfeiting is $\pi_2^D(w_2, s=1) = \pi_2^{DC1}$, which implies $p_2^D = \frac{\beta p_B}{1-\gamma}$. Then, we know that the optimal decision is $s^* = 0$ because $\pi_2^D(w_2, s=0) > \pi_2^{DC1}$ always holds.

(ii) If $\underline{w}_2 < w_2 < k_2 + \frac{\beta p_B-(1-\gamma)k_2}{\beta}$, then the overseas supplier's profit from counterfeiting is $\pi_2^D(w_2, s = 1) = \hat{\pi}_2^{DC}$, which implies $p_2^D = \hat{p}_2 = \frac{\beta p_B+(1-\gamma)k_2+\beta(w_2-k_2)}{2(1-\gamma)}$. Then, we know that the optimal decision is: $s = 0$ if $\pi_2^D(w_2, s=0) > \hat{\pi}_2^{DC}$, which means:

$$\begin{aligned}
&\alpha(w_2-k_2)(1-\frac{p_B}{1-\gamma}) > \alpha(w_2-k_2)\left(1-\frac{p_B-\hat{p}_2}{1-\gamma-\beta}\right) + \left(\alpha(\hat{p}_2-k_2)(\frac{p_B-\hat{p}_2}{1-\gamma-\beta}-\frac{\hat{p}_2}{\beta}) - e\right),\\
\Rightarrow &\alpha(w_2-k_2)(1-\frac{p_B}{1-\gamma}) > \alpha(w_2-k_2)\left(\frac{2(1-\gamma-\beta)(1-\gamma)-(2(1-\gamma)-\beta)p_B+(1-\gamma)k_2+\beta(w_2-k_2)}{2(1-\gamma)(1-\gamma-\beta)}\right)\\
&+\left(\alpha\left(\frac{\beta p_B-(1-\gamma)k_2+\beta(w_2-k_2)}{2(1-\gamma)}\right)\frac{\beta p_B-(1-\gamma)k_2-\beta(w_2-k_2)}{2\beta(1-\gamma-\beta)} - e\right),\\
\Rightarrow &w_2^{(0)\prime} < w_2 < w_2^{(0)\prime\prime},
\end{aligned}$$

where $w_2^{(0)\prime} = k_2 + \frac{\beta p_B-(1-\gamma)k_2}{\beta} - \sqrt{\frac{4(1-\gamma)(1-\gamma-\beta)e}{\alpha\beta}}$, $w_2^{(0)\prime\prime} = k_2 + \frac{\beta p_B-(1-\gamma)k_2}{\beta} + \sqrt{\frac{4(1-\gamma)(1-\gamma-\beta)e}{\alpha\beta}}$.

Note that $w_2^{(0)\prime} < k_2 + \frac{\beta p_B-(1-\gamma)k_2}{\beta} < w_2^{(0)\prime\prime}$. Thus, combining these two scenarios, the overseas supplier's optimal decision of counterfeiting is

$$s^*(w_2) = \begin{cases} 0, & \text{if } w_2 \geq \max\{w_2^{(0)\prime}, \underline{w}_2\}, \text{[note that } m_2 = 0]\\ 1, & \text{if } \underline{w}_2 < w_2 < \max\{w_2^{(0)\prime}, \underline{w}_2\}. \text{[note that } m_2 > 0] \end{cases}$$

Subsequently, the brand-name firm's profit is

$$\pi_B^D(w_2) = \begin{cases} \pi_B^D(w_2, s=0) = (p_B-w_1)(1-p_B) + \alpha(p_B-w_2)(1-\frac{p_B}{1-\gamma}), & \text{if } w_2 \geq \max\{w_2^{(0)\prime}, \underline{w}_2\},\\ \pi_B^D(w_2, s=1) = (p_B-w_1)(1-p_B) & \\ +\alpha(p_B-w_2)\left(\frac{2(1-\gamma-\beta)(1-\gamma-p_B)-\beta p_B+(1-\gamma)k_2+\beta(w_2-k_2)}{2(1-\gamma)(1-\gamma-\beta)}\right), & \text{if } \underline{w}_2 < w_2 < w_2^{(0)\prime}, \underline{w}_2\}. \end{cases}$$

Strategy O: Given wholesale prices w_1 and w_2, the home supplier rejects and the counterfeiter accepts their respective contracts, i.e., $d_1 = 0$ and $d_2 = 1$.

(1) If the overseas supplier does not sell the counterfeit in the market, i.e., $s = 0$, we know:

$$\begin{aligned}
&\pi_B^O(w_2, s=0) = (p_B-w_2-t)(1-\frac{p_B}{1-\gamma}) + \alpha(p_B-w_2)(1-\frac{p_B}{1-\gamma}),\\
&\pi_1^O = 0, \quad \pi_2^O(w_2, s=0) = (w_2-k_2)(1-\frac{p_B}{1-\gamma}) + \alpha(w_2-k_2)(1-\frac{p_B}{1-\gamma}).
\end{aligned}$$

(2) If the overseas supplier sells the counterfeit in the market, i.e., $s = 1$, then the overseas supplier determines the selling price p_2 for the counterfeit. Their profits are as follows.

$$\begin{aligned}
&\pi_B^O(w_2, p_2, s=1) = (p_B-w_2-t)\left(1-\frac{p_B}{1-\gamma}\right) + \alpha(p_B-w_2)\left(1-\frac{p_B-p_2}{1-\gamma-\beta}\right),\\
&\pi_1^O = 0, \quad \pi_2^O(w_2, p_2, s=1) = (w_2-k_2)\left(1-\frac{p_B}{1-\gamma}\right) + \alpha(w_2-k_2)\left(1-\frac{p_B-p_2}{1-\gamma-\beta}\right) + \alpha(p_2-k_2)\left(\frac{p_B-p_2}{1-\gamma-\beta}-\frac{p_2}{\beta}\right) - e.
\end{aligned}$$

Similar with the discussion in Strategy D, we derive the optimal retail price p_2 for the overseas supplier under Strategy O by backward deduction. Thus, with Strategy O, if the overseas supplier sells the counterfeit, i.e., $s = 1$, we have $\hat{p}_2 = \frac{\beta p_B+(1-\gamma)k_2+\beta(w_2-k_2)}{2(1-\gamma)}$, and the optimal retail price is

$$p_2^O = \begin{cases} \frac{\beta p_B}{1-\gamma}, & \text{if } w_2 \geq k_2 + \frac{\beta p_B-(1-\gamma)k_2}{\beta}, \text{[note that } m_2 = 0]\\ \hat{p}_2, & \text{if } \underline{w}_2 < w_2 < k_2 + \frac{\beta p_B-(1-\gamma)k_2}{\beta}, \text{[note that } m_2 > 0] \end{cases}$$

and the overseas supplier's profit is

$$\pi_2^O(w_2)=\begin{cases}\pi_2^{OC1}=(w_2-k_2)\left(1-\frac{p_B}{1-\gamma}\right)+\alpha\,(w_2-k_2)\left(1-\frac{p_B-p_2^O}{1-\gamma-\beta}\right)-e, & \text{if } w_2\geq k_2+\frac{\beta p_B-(1-\gamma)k_2}{\beta},\\ \hat{\pi}_2^{OC}=(w_2-k_2)\left(1-\frac{p_B}{1-\gamma}\right) & \\ +\alpha\,(w_2-k_2)\left(1-\frac{p_B-p_2^O}{1-\gamma-\beta}\right)+\left(\alpha\left(p_2^O-k_2\right)\left(\frac{p_B-p_2^O}{1-\gamma-\beta}-\frac{p_2^O}{\beta}\right)-e\right), & \text{if } \underline{w}_2<w_2<k_2+\frac{\beta p_B-(1-\gamma)k_2}{\beta},\end{cases}$$

and the brand-name firm's profit is

$$\pi_B^O(w_2)=\begin{cases}\pi_B^{OC1}=(p_B-w_2-t)\,(1-p_B)+\alpha\,(p_B-w_2)\left(1-\frac{p_B-p_2^O}{1-\gamma-\beta}\right), & \text{if } w_2\geq k_2+\frac{\beta p_B-(1-\gamma)k_2}{\beta},\\ \hat{\pi}_B^{OC}=(p_B-w_2-t)\,(1-p_B) & \\ +\alpha\,(p_B-w_2)\left(\frac{2(1-\gamma-\beta)(1-\gamma-p_B)-\beta p_B+(1-\gamma)k_2+\beta(w_2-k_2)}{2(1-\gamma)(1-\gamma-\beta)}\right), & \text{if } \underline{w}_2<w_2<k_2+\frac{\beta p_B-(1-\gamma)k_2}{\beta}.\end{cases}$$

Next, the overseas supplier determines whether to sell the counterfeit, $s^*(w_2)$. For the overseas supplier, if $\pi_2^O(w_2,s=1)>\pi_2^O(w_2,s=0)$, she decides to sell the counterfeit; otherwise, she does not sell the counterfeit. Recall that when $s=0$, the overseas supplier's profit is

$$\pi_2^O\,(w_2,s=0)=(w_2-k_2)\,(1-\frac{p_B}{1-\gamma})+\alpha\,(w_2-k_2)\,(1-\frac{p_B}{1-\gamma}).$$

Similarly, we obtain:

$$s^*(w_2)=\begin{cases}0, & \text{if } w_2\geq\max\{w_2^{(0)\prime},\underline{w}_2\}, \text{ [note that } m_2=0]\\ 1, & \text{if } \underline{w}_2<w_2<\max\{w_2^{(0)\prime},\underline{w}_2\}. \text{ [note that } m_2>0]\end{cases}$$

Subsequently, the brand-name firm's profit is

$$\pi_B^O(w_2)=\begin{cases}\pi_B^O\,(w_2,s=0)=(p_B-w_2-t)\,(1-\frac{p_B}{1-\gamma})+\alpha\,(p_B-w_2)\,(1-\frac{p_B}{1-\gamma}), & \text{if } w_2\geq\max\{w_2^{(0)\prime},\underline{w}_2\},\\ \pi_B^{OC}\,(w_2,s=1)=\hat{\pi}_B^{OC}=(p_B-w_2-t)\left(1-\frac{p_B}{1-\gamma}\right) & \\ +\alpha\,(p_B-w_2)\left(\frac{2(1-\gamma-\beta)(1-\gamma-p_B)-\beta p_B+(1-\gamma)k_2+\beta(w_2-k_2)}{2(1-\gamma)(1-\gamma-\beta)}\right), & \text{if } \underline{w}_2<w_2<\max\{w_2^{(0)\prime},\underline{w}_2\}.\end{cases}$$

Strategy N: Given wholesale prices w_1 and w_2, both the home supplier rejects and the counterfeiter reject their contracts, i.e., $d_1=0$ and $d_2=0$.

(1) If the counterfeiter does not enter the overseas market to sell the counterfeit, *i.e.*, $s=0$, then their profits are:

$$\pi_B^N\,(w_1,w_2)=0,\quad \pi_1^N(w_1)=0,\quad \pi_2^N=0.$$

(2) If the counterfeiter enters the overseas market to sell the counterfeit, *i.e.*, $s=1$, she is the monopoly in the overseas market and determines retail price p_2^N of the counterfeit and obtains the below profit:

$$\pi_2^N\,(p_2)=\alpha\,(p_2-k_2)\left(1-\frac{p_2}{\beta}\right)-e.$$

By taking the first-order derivative of $\pi_2^N(p_2)$ with respect to p_2, the optimal retail price of the counterfeit is $p_2^N=\frac{\beta+k_2}{2}$. Substituting the expression of p_2^N into Equation (3), we obtain $m_2=\alpha\left(1-\frac{\beta+k_2}{2\beta}\right)$. Thus, their profits are:

$$\pi_B^N\,(w_1,w_2)=0,\quad \pi_1^N=0,\quad \pi_2^N=\frac{\alpha(\beta-k_2)^2}{4\beta}-e.$$

Recall that $e < \frac{\alpha(\beta p_B - k_2)^2}{4\beta(1-\beta)}$, resulting in $\pi_2^N(s=1) > \pi_2^N(s=0)$. It means that the counterfeiter always sells the counterfeit products.

Based on above discussions, for given (w_1, w_2), under either Strategy D or Strategy O,

$$s^*(w_2) = \begin{cases} 0, & \text{if } w_2 \geq \max\{w_2^{(0)\prime}, \underline{w}_2\}, \\ 1, & \text{if } \underline{w}_2 < w_2 < \max\{w_2^{(0)\prime}, \underline{w}_2\}; \end{cases}$$

where $w_2^{(0)\prime} = k_2 + \frac{\beta p_B - (1-\gamma)k_2}{\beta} - \sqrt{\frac{4(1-\gamma)(1-\gamma-\beta)e}{\alpha\beta}}$, $\underline{w}_2 = k_2 - \frac{2(1-\gamma-p_B)(1-\gamma-\beta) - \beta p_B + (1-\gamma)k_2}{\beta}$. In particular, when $s^*(w_2) = 1$, the optimal retail price of the counterfeit product is $p_2^*(w_2) = \frac{\beta p_B + (1-\gamma)k_2 + \beta(w_2 - k_2)}{2(1-\gamma)}$. ■

B.9.2 Proof of Lemma 5.

There are two parts in this proof. In part 1, we analyze the suppliers' optimal participation decision by discussing the best response functions $(d_1^*(w_1, w_2), d_2^*(w_1, w_2))$. In part 2, we determine the optimal wholesale prices that the brand-name firm offers.

Part 1. We discuss the suppliers' best response functions $(d_1^*(w_1, w_2), d_2^*(w_1, w_2))$.

With each sourcing strategy, the overseas supplier's profit function is as follows:

$$\pi_2^H = \frac{\alpha(\beta p_B - k_2)^2}{4\beta(1-\beta)} - e = M.$$

$$\pi_2^D = \begin{cases} \pi_2^{DC}(w_2) = \alpha(w_2 - k_2)\left(\frac{2(1-\gamma-\beta)(1-\gamma-p_B) - \beta p_B + (1-\gamma)k_2 + \beta(w_2 - k_2)}{2(1-\gamma)(1-\gamma-\beta)}\right) & \\ +\alpha\left(\frac{\beta p_B - (1-\gamma)k_2 + \beta(w_2 - k_2)}{2(1-\gamma)}\right)\frac{\beta p_B - (1-\gamma)k_2 - \beta(w_2 - k_2)}{2\beta(1-\gamma-\beta)} - e, & \text{if } \underline{w}_2 < w_2 < \max\{w_2^{(0)\prime}, \underline{w}_2\}, \\ \pi_2^{D\dagger}(w_2) = \alpha(w_2 - k_2)(1 - \frac{p_B}{1-\gamma}), & \text{if } w_2 \geq \max\{w_2^{(0)\prime}, \underline{w}_2\}; \end{cases}$$

$$\pi_2^O = \begin{cases} \pi_2^{OC}(w_2) = (w_2 - k_2)\left(1 - \frac{p_B}{1-\gamma}\right) & \\ +\alpha(w_2 - k_2)\left(\frac{2(1-\gamma-\beta)(1-\gamma-p_B) - \beta p_B + (1-\gamma)k_2 + \beta(w_2 - k_2)}{2(1-\gamma)(1-\gamma-\beta)}\right) & \\ +\alpha\left(\frac{\beta p_B - (1-\gamma)k_2 + \beta(w_2 - k_2)}{2(1-\gamma)}\right)\frac{\beta p_B - (1-\gamma)k_2 - \beta(w_2 - k_2)}{2\beta(1-\gamma-\beta)} - e, & \text{if } \underline{w}_2 < w_2 < \max\{w_2^{(0)\prime}, \underline{w}_2\}, \\ \pi_2^{O\dagger}(w_2) = (p_B - w_2 - t)(1 - \frac{p_B}{1-\gamma}) + \alpha(p_B - w_2)(1 - \frac{p_B}{1-\gamma}), & \text{if } w_2 \geq \max\{w_2^{(0)\prime}, \underline{w}_2\}; \end{cases}$$

$$\pi_2^N = \frac{\alpha(\beta - k_2)^2}{4\beta} - e = K.$$

Recall that $M = \frac{\alpha(\beta p_B - k_2)^2}{4\beta(1-\beta)} - e$, $M' = \frac{\alpha(\beta p_B - (1-\gamma)k_2)^2}{4(1-\gamma)\beta(1-\gamma-\beta)} - e$ and $K = \frac{\alpha(\beta - k_2)^2}{4\beta} - e$. With the assumption $0 \leq e < \frac{\alpha(\beta p_B - k_2)^2}{4\beta(1-\beta)}$, we know that $0 < M < M' < K$.

Step 1: We first discuss the conditions for the overseas supplier's decision to accept the wholesale contract.

(1) Under $\underline{w}_2 < w_2 < \max\{w_2^{(0)\prime}, \underline{w}_2\}$, where $\underline{w}_2 = k_2 - \frac{2(1-\gamma-p_B)(1-\gamma-\beta) - \beta p_B + (1-\gamma)k_2}{\beta}$, $w_2^{(0)\prime} = k_2 + \frac{\beta p_B - (1-\gamma)k_2}{\beta} - \sqrt{\frac{4(1-\gamma)(1-\gamma-\beta)e}{\alpha\beta}}$, we discuss the decision d_2 for a given belief on the home supplier's contact decision $\widetilde{d_1} = 1$ and $\widetilde{d_1} = 0$, respectively.

(i) If $\widetilde{d}_1 = 1$, then we compare the overseas supplier's profits between Strategy D with counterfeiting and Strategy H, i.e., $\pi_2^{DC}(w_2)$ and π_2^H. If the overseas supplier decides to accept, then it should satisfy

$$
\begin{aligned}
&\pi_2^{DC}(w_2) \geq \pi_2^H,\\
\Rightarrow\ &\alpha(w_2-k_2)\left(\tfrac{2(1-\gamma-\beta)(1-\gamma-p_B)-\beta p_B+(1-\gamma)k_2+\beta(w_2-k_2)}{2(1-\gamma)(1-\gamma-\beta)}\right)+\alpha\left(\tfrac{\beta p_B-(1-\gamma)k_2+\beta(w_2-k_2)}{2(1-\gamma)}\right)\tfrac{\beta p_B-(1-\gamma)k_2-\beta(w_2-k_2)}{2\beta(1-\gamma-\beta)}-e \geq M,\\
\Rightarrow\ &w_2 \leq k_2-\tfrac{2(1-\gamma-p_B)(1-\gamma-\beta)-\beta p_B+(1-\gamma)k_2}{\beta}-\sqrt{\tfrac{4(1-\gamma)(1-\gamma-\beta)(M-M')}{\alpha\beta}+\left(\tfrac{2(1-\gamma-p_B)(1-\gamma-\beta)-\beta p_B+(1-\gamma)k_2}{\beta}\right)^2},\ \text{(invalid)}\\
\Rightarrow\ &\text{or, } w_2 \geq k_2-\tfrac{2(1-\gamma-p_B)(1-\gamma-\beta)-\beta p_B+(1-\gamma)k_2}{\beta}+\sqrt{\tfrac{4(1-\gamma)(1-\gamma-\beta)(M-M')}{\alpha\beta}+\left(\tfrac{2(1-\gamma-p_B)(1-\gamma-\beta)-\beta p_B+(1-\gamma)k_2}{\beta}\right)^2}.
\end{aligned}
$$

(ii) If $\widetilde{d}_1 = 0$, then we compare the overseas supplier's profits between Strategy O with counterfeiting and Strategy N, i.e., $\pi_2^{OC}(w_2)$ and π_2^N. If the overseas supplier decides to accept, then it should satisfy

$$
\begin{aligned}
&\pi_2^{OC}(w_2) \geq \pi_2^N,\\
\Rightarrow\ &(w_2-k_2)\left(1-\tfrac{p_B}{1-\gamma}\right)\\
&+\alpha(w_2-k_2)\left(\tfrac{2(1-\gamma-\beta)(1-\gamma-p_B)-\beta p_B+(1-\gamma)k_2+\beta(w_2-k_2)}{2(1-\gamma)(1-\gamma-\beta)}\right)+\alpha\left(\tfrac{\beta p_B-(1-\gamma)k_2+\beta(w_2-k_2)}{2(1-\gamma)}\right)\tfrac{\beta p_B-(1-\gamma)k_2-\beta(w_2-k_2)}{2\beta(1-\gamma-\beta)}-e \geq K,\\
\Rightarrow\ &w_2 \leq k_2-\tfrac{2(1+\frac{1}{\alpha})(1-\gamma-p_B)(1-\gamma-\beta)-\beta p_B+(1-\gamma)k_2}{\beta}-\sqrt{\tfrac{4(1-\gamma)(1-\gamma-\beta)(K-M')}{\alpha\beta}+\left(\tfrac{2(1+\frac{1}{\alpha})(1-\gamma-p_B)(1-\gamma-\beta)-\beta p_B+(1-\gamma)k_2}{\beta}\right)^2},\ \text{(invalid)}\\
\Rightarrow\ &w_2 \geq k_2-\tfrac{2(1+\frac{1}{\alpha})(1-\gamma-p_B)(1-\gamma-\beta)-\beta p_B+(1-\gamma)k_2}{\beta}+\sqrt{\tfrac{4(1-\gamma)(1-\gamma-\beta)(K-M')}{\alpha\beta}+\left(\tfrac{2(1+\frac{1}{\alpha})(1-\gamma-p_B)(1-\gamma-\beta)-\beta p_B+(1-\gamma)k_2}{\beta}\right)^2}.
\end{aligned}
$$

We define the following notations:

$$
\begin{aligned}
&w_2^{(0)} = \max\{w_2^{(0)\prime}, \underline{w}_2\};\\
&w_2^{D(1)} = k_2-\tfrac{2(1-\gamma-p_B)(1-\gamma-\beta)-\beta p_B+(1-\gamma)k_2}{\beta}+\sqrt{\tfrac{4(1-\gamma)(1-\gamma-\beta)(M-M')}{\alpha\beta}+\left(\tfrac{2(1-\gamma-p_B)(1-\gamma-\beta)-\beta p_B+(1-\gamma)k_2}{\beta}\right)^2} < k_2,\\
&w_2^{O(1)} = k_2-\tfrac{2(1+\frac{1}{\alpha})(1-\gamma-p_B)(1-\gamma-\beta)-\beta p_B+(1-\gamma)k_2}{\beta}+\sqrt{\tfrac{4(1-\gamma)(1-\gamma-\beta)(K-M')}{\alpha\beta}+\left(\tfrac{2(1+\frac{1}{\alpha})(1-\gamma-p_B)(1-\gamma-\beta)-\beta p_B+(1-\gamma)k_2}{\beta}\right)^2} > k_2;\\
&w_2^{D(2)} = k_2+\tfrac{M}{\alpha(1-\frac{p_B}{1-\gamma})},\\
&w_2^{O(2)} = k_2+\tfrac{K}{(1+\alpha)(1-\frac{p_B}{1-\gamma})};
\end{aligned}
$$

where $w_2^{(0)\prime} = k_2+\frac{\beta p_B-(1-\gamma)k_2}{\beta}-\sqrt{\frac{4(1-\gamma)(1-\gamma-\beta)e}{\alpha\beta}} > k_2$, $\underline{w}_2 = k_2-\frac{2(1-\gamma-p_B)(1-\gamma-\beta)-\beta p_B+(1-\gamma)k_2}{\beta}$.

Thus, under $\underline{w}_2 < w_2 < w_2^{(0)}$, where $w_2^{(0)} = \max\{w_2^{(0)\prime}, \underline{w}_2\}$, we know that $w_2^{(0)} > k_2$, and

$$
d_2(\widetilde{d}_1) = \begin{cases}
d_2\left(\widetilde{d}_1 = 1\right) = 1, & \text{if } \max\{w_2^{D(1)}, \underline{w}_2, k_2\} \leq w_2 < w_2^{(0)},\\
d_2\left(\widetilde{d}_1 = 1\right) = 0, & \text{if } \underline{w}_2 < w_2 < \max\{w_2^{D(1)}, \underline{w}_2, k_2\},\\
d_2\left(\widetilde{d}_1 = 0\right) = 1, & \text{if } \max\{w_2^{O(1)}, \underline{w}_2, k_2\} \leq w_2 < w_2^{(0)},\\
d_2\left(\widetilde{d}_1 = 0\right) = 0, & \text{if } \underline{w}_2 < w_2 < \max\{w_2^{O(1)}, \underline{w}_2, k_2\}.
\end{cases}
$$

Note that $w_2^{D(1)} < k_2 < w_2^{O(1)}$, and $w_2^{(0)} = \max\{w_2^{(0)\prime}, \underline{w}_2\}$, where $w_2^{(0)\prime} > k_2$, then we have:

$$
d_2(\widetilde{d}_1) = \begin{cases}
d_2\left(\widetilde{d}_1 = 1\right) = 1, & \text{if } \max\{k_2, \underline{w}_2\} \leq w_2 < w_2^{(0)},\\
d_2\left(\widetilde{d}_1 = 1\right) = 0, & \text{if } \underline{w}_2 < w_2 < \max\{k_2, \underline{w}_2\},\\
d_2\left(\widetilde{d}_1 = 0\right) = 1, & \text{if } \max\{w_2^{O(1)}, \underline{w}_2\} \leq w_2 < w_2^{(0)},\\
d_2\left(\widetilde{d}_1 = 0\right) = 0, & \text{if } \underline{w}_2 < w_2 < \max\{w_2^{O(1)}, \underline{w}_2\}.
\end{cases}
$$

(2) Under $w_2 \geq \max\{w_2^{(0)\prime}, \underline{w}_2\}$, we discuss the decision d_2 for given $\widetilde{d}_1 = 1$ and $\widetilde{d}_1 = 0$, respectively.

(i) If $\widetilde{d}_1 = 1$, then we compare the overseas supplier's profits between Strategy D without counterfeiting and Strategy H, i.e., $\pi_2^{D\dagger}(w_2)$ and π_2^H. If the overseas supplier decides to accept the wholesale contract, then it should satisfy

$$\begin{aligned}
&\pi_2^{D\dagger}(w_2) \geq \pi_2^H, \\
\Rightarrow\ &\alpha(w_2 - k_2)\left(1 - \tfrac{p_B}{1-\gamma}\right) \geq M, \\
\Rightarrow\ &w_2 \geq w_2^{D(2)}\text{ , where } w_2^{D(2)} = k_2 + \tfrac{M}{\alpha\left(1-\frac{p_B}{1-\gamma}\right)}.
\end{aligned}$$

(ii) If $\widetilde{d}_1 = 0$, then we compare the overseas supplier's profits between Strategy O without counterfeiting and Strategy N, i.e., $\pi_2^{O\dagger}(w_2)$ and π_2^N. If the overseas supplier decides to accept the wholesale contract, then it should satisfy

$$\begin{aligned}
&\pi_2^{O\dagger}(w_2) \geq \pi_2^N, \\
\Rightarrow\ &(w_2 - k_2)\left(1 - \tfrac{p_B}{1-\gamma}\right) + \alpha(w_2 - k_2)\left(1 - \tfrac{p_B}{1-\gamma}\right) \geq K, \\
\Rightarrow\ &w_2 \geq w_2^{O(2)}\text{ , where } w_2^{O(2)} = k_2 + \tfrac{K}{(1+\alpha)\left(1-\frac{p_B}{1-\gamma}\right)}.
\end{aligned}$$

Thus, under $w_2 \geq w_2^{(0)}$, where $w_2^{(0)} = \max\{w_2^{(0)\prime}, \underline{w}_2\} > k_2$, we obtain

$$d_2(\widetilde{d}_1) = \begin{cases}
d_2\left(\widetilde{d}_1 = 1\right) = 1, & \text{if } w_2 \geq \max\{w_2^{D(2)}, w_2^{(0)}\}, \\
d_2\left(\widetilde{d}_1 = 1\right) = 0, & \text{if } w_2^{(0)} < w_2 < \max\{w_2^{D(2)}, w_2^{(0)}\}, \\
d_2\left(\widetilde{d}_1 = 0\right) = 1, & \text{if } w_2 \geq \max\{w_2^{O(2)}, w_2^{(0)}\}, \\
d_2\left(\widetilde{d}_1 = 0\right) = 0, & \text{if } w_2^{(0)} < w_2 < \max\{w_2^{O(2)}, w_2^{(0)}\}.
\end{cases}$$

Step 2: We derive the best response function of the home supplier $d_1(\widetilde{d}_2)$ to the overseas supplier's action $\widetilde{d}_2 \in \{0, 1\}$ as follows:

$$d_1(\widetilde{d}_2) = \begin{cases}
d_1\left(\widetilde{d}_2 = 1\right) = 1, & \text{if } w_1 \geq k_1, \\
d_1\left(\widetilde{d}_2 = 0\right) = 1, & \text{if } w_1 \geq k_1, \\
d_1\left(\widetilde{d}_2 = 1\right) = 0, & \text{if } w_1 < k_1, \\
d_1\left(\widetilde{d}_2 = 0\right) = 0, & \text{if } w_1 < k_1.
\end{cases}$$

Step 3: Given best response functions $d_1(\widetilde{d}_2)$ and $d_2(\widetilde{d}_1)$, we obtain the following fixed point (d_1^*, d_2^*) that satisfies $(d_1(\widetilde{d}_2), \widetilde{d}_2) = (\widetilde{d}_1, d_2(\widetilde{d}_1))$. Thus, the optimal decisions of the two suppliers are

$$(d_1^*, d_2^*) = \begin{cases}
(1,1), & \text{if } w_1 \geq k_1,\ \max\{k_2, \underline{w}_2\} \leq w_2 < w_2^{(0)} \text{ or } w_2 \geq \max\{w_2^{D(2)}, w_2^{(0)}\}, \\
(1,0), & \text{if } w_1 \geq k_1,\ \underline{w}_2 < w_2 < \max\{k_2, \underline{w}_2\} \text{ or } w_2^{(0)} < w_2 < \max\{w_2^{D(2)}, w_2^{(0)}\}, \\
(0,1), & \text{if } w_1 < k_1,\ \max\{w_2^{O(1)}, \underline{w}_2\} \leq w_2 < w_2^{(0)} \text{ or } w_2 \geq \max\{w_2^{O(2)}, w_2^{(0)}\}, \\
(0,0), & \text{if } w_1 < k_1,\ \underline{w}_2 < w_2 < \max\{w_2^{O(1)}, \underline{w}_2\} \text{ or } w_2^{(0)} < w_2 < \max\{w_2^{O(2)}, w_2^{(0)}\}.
\end{cases}$$

Part 2. We discuss the brand-name firm's optimal wholesale prices, (w_1, w_2).

Substituting (d_1^*, d_2^*) into the profit functions of the brand-name firm, we analyze the optimal wholesale price under each possible sourcing strategy.

$$\pi_B^H(w_1) = (p_B - w_1)(1 - p_B) + \alpha(p_B - w_1 - t)\left(1 - \tfrac{(2-\beta)p_B - k_2}{2(1-\beta)}\right);$$

$$\pi_B^D=\begin{cases}\pi_B^{DC}(w_1,w_2)=(p_B-w_1)(1-p_B) & \\ +\alpha(p_B-w_2)\left(\frac{2(1-\gamma-\beta)(1-\gamma-p_B)-\beta p_B+(1-\gamma)k_2+\beta(w_2-k_2)}{2(1-\gamma)(1-\gamma-\beta)}\right), & \text{if } w_1\geq k_1,\ \max\{k_2,\underline{w}_2\}\leq w_2<w_2^{(0)},\\ \pi_B^{D\dagger}(w_1,w_2)=(p_B-w_1)(1-p_B)+\alpha(p_B-w_2)(1-\frac{p_B}{1-\gamma}), & \text{if } w_1\geq k_1,\ w_2\geq\max\{w_2^{D(2)},w_2^{(0)}\};\end{cases}$$

$$\pi_B^O=\begin{cases}\pi_B^{OC}(w_2)=(p_B-w_2-t)\left(1-\frac{p_B}{1-\gamma}\right) & \\ +\alpha(p_B-w_2)\left(\frac{2(1-\gamma-\beta)(1-\gamma-p_B)-\beta p_B+(1-\gamma)k_2+\beta(w_2-k_2)}{2(1-\gamma)(1-\gamma-\beta)}\right), & \text{if } \max\{w_2^{O(1)},\underline{w}_2\}\leq w_2<w_2^{(0)},\\ \pi_B^{O\dagger}(w_2)=(p_B-w_2-t)(1-\frac{p_B}{1-\gamma})+\alpha(p_B-w_2)(1-\frac{p_B}{1-\gamma}), & \text{if } w_2\geq\max\{w_2^{O(2)},w_2^{(0)}\};\end{cases}$$

$$\pi_B^N(w_1,w_2)=0.$$

Next, we derive the optimal wholesale prices under each sourcing strategy. As $\pi_B(w_1,w_2)$ decreases in w_1, then the optimal wholesale price of the home supplier that the brand-name firm is willing to offer is equal to the production cost, that is, $w_1^H=k_1$ under Strategy H, and $w_1^D=k_1$ under Strategy D.

With Strategy D, we have the following observations.

(1) Under Strategy D without counterfeiting, as $\pi_B^D(w_1,w_2)$ decreases in w_2, then the optimal wholesale price of the overseas supplier that the brand-name firm is willing to offer is the lower bound of the feasible regions, i.e., $w_2^{D\dagger *}=\max\{w_2^{D(2)},w_2^{(0)}\}$.

(2) Under Strategy D with counterfeiting, by taking the first-order derivative of the profit function $\pi_B^{DC}(w_1,w_2)$ with respect to w_2, we obtain

$$\frac{\partial(\pi_B^{DC}(w_2))}{\partial(w_2)}=-\alpha\left(\frac{2(1-\gamma-\beta)(1-\gamma-p_B)-\beta p_B+(1-\gamma)k_2+\beta(w_2-k_2)}{2(1-\gamma)(1-\gamma-\beta)}\right)+\alpha(p_B-w_2)\left(\frac{\beta}{2(1-\gamma)(1-\gamma-\beta)}\right).$$

Then, from $\frac{\partial(\pi_B^{DC}(w_2))}{\partial(w_2)}=0$, we obtain the critical point,

$$\hat{w}_2^{DC}=k_2-\frac{2x(1-\gamma-\beta-p_B)+\beta k_2+(1-\gamma)k_2}{2\beta}=k_2-\frac{2(1-\gamma-p_B)(1-\gamma-\beta)-\beta p_B+(1-\gamma)k_2}{2\beta}+\frac{\beta(p_B-k_2)}{2\beta}.$$

If $\hat{w}_2^{DC}<w_2^{(0)}$, then, the optimal wholesale price is $w_2^{DC*}=\max\{k_2,\underline{w}_2,\hat{w}_2^{DC}\}$. As $\hat{w}_2^{DC}>\underline{w}_2$, then, $w_2^{DC*}=\max\{k_2,\hat{w}_2^{DC}\}$. We need to compare the profits of Strategy D with and without counterfeiting.

If $\hat{w}_2^{DC}\geq w_2^{(0)}$, then the optimal wholesale price is $w_2^{DC*}=w_2^{(0)}$. But this profit is dominated by the Strategy D without counterfeiting.

With Strategy O, we have the following observations.

(1) Under Strategy O without counterfeiting, as $\pi_B^O(w_2)$ decreases in w_2, then the optimal wholesale price of the overseas supplier that the brand-name firm is willing to offer is the lower bound of the feasible regions, i.e., $w_2^{O\dagger *}=\max\{w_2^{O(2)},w_2^{(0)}\}$.

(2) Under Strategy O with counterfeiting, by taking the first order derivative of the profit function $\pi_B^{OC}(w_2)$ with respect to w_2, we obtain

$$\frac{\partial(\pi_B^{OC}(w_2))}{\partial(w_2)}=-\left(1-\frac{p_B}{1-\gamma}\right)-\alpha\left(\frac{2(1-\gamma-\beta)(1-\gamma-p_B)-\beta p_B+(1-\gamma)k_2+\beta(w_2-k_2)}{2(1-\gamma)(1-\gamma-\beta)}\right)+\alpha(p_B-w_2)\left(\frac{\beta}{2(1-\gamma)(1-\gamma-\beta)}\right).$$

Then, from $\frac{\partial(\pi_B^{OC}(w_2))}{\partial(w_2)} = 0$, we obtain the critical point,

$\hat{w}_2^{OC} = k_2 - \frac{2(1-\gamma)(1-\gamma-\beta-p_B)+\beta k_2+(1-\gamma)k_2}{2\beta} - \frac{(1-\gamma-p_B)(1-\gamma-\beta)}{\alpha\beta} = k_2 - \frac{2(1+\frac{1}{\alpha})(1-\gamma-p_B)(1-\gamma-\beta)-\beta p_B+(1-\gamma)k_2}{2\beta} + \frac{\beta(p_B-k_2)}{2\beta}.$

If $\hat{w}_2^{OC} < w_2^{(0)}$, then the optimal wholesale price is $w_2^{OC*} = \max\{w_2^{O(1)}, \underline{w}_2, \hat{w}_2^{OC}\}$. We need to compare the profits under Strategy O with and without counterfeiting.

If $\hat{w}_2^{OC} \geq w_2^{(0)}$, then the optimal wholesale price is $w_2^{OC*} = w_2^{(0)}$. But this profit is dominated by the Strategy O without counterfeiting.

Recall that

$w_2^{D(2)} = k_2 + \frac{M}{\alpha(1-\frac{p_B}{1-\gamma})}$;

$w_2^{O(2)} = k_2 + \frac{K}{(1+\alpha)(1-\frac{p_B}{1-\gamma})}$;

$w_2^{O(1)} = k_2 - \frac{2(1+\frac{1}{\alpha})(1-\gamma-p_B)(x-\beta)-\beta p_B+(1-\gamma)k_2}{\beta} + \sqrt{\frac{4(1-\gamma)(1-\gamma-\beta)(K-M')}{\alpha\beta} + \left(\frac{2(1+\frac{1}{\alpha})(1-\gamma-p_B)(1-\gamma-\beta)-\beta p_B+(1-\gamma)k_2}{\beta}\right)^2}$;

$w_2^{(0)} = \max\{w_2^{(0)\prime}, \underline{w}_2\}$;

where $\underline{w}_2 = k_2 - \frac{2(x-p_B)(x-\beta)-\beta p_B+xk_2}{\beta}$, $w_2^{(0)\prime} = k_2 + \frac{\beta p_B-(1-\gamma)k_2}{\beta} - \sqrt{\frac{4(1-\gamma)(1-\gamma-\beta)e}{\alpha\beta}}$; and

$$\hat{w}_2^{DC} = k_2 - \frac{2(1-\gamma-p_B)(1-\gamma-\beta)-\beta p_B+(1-\gamma)k_2}{2\beta} + \frac{\beta(p_B-k_2)}{2\beta} > \underline{w}_2;$$
$$\hat{w}_2^{OC} = k_2 - \frac{2(1+\frac{1}{\alpha})(1-\gamma-p_B)(1-\gamma-\beta)-\beta p_B+(1-\gamma)k_2}{2\beta} + \frac{\beta(p_B-k_2)}{2\beta}.$$

We observe that $w_2^{O(1)}$, $\underline{w}_2$, $\hat{w}_2^{DC}$ and $\hat{w}_2^{OC}$ are independent of e; $w_2^{D(2)}$, $w_2^{O(2)}$ and $w_2^{(0)\prime}$ are dependent of e. Furthermore, we know that w_2^{DC*} and w_2^{OC*} are independent of e. Thus, we obtain the optimal profit functions for each sourcing strategy:

$$\pi_B^H = (p_B - k_1)(1-p_B) + \alpha(p_B - w_1 - t)\left(1 - \frac{(2-\beta)p_B - k_2}{2(1-\beta)}\right);$$

$$\pi_B^D = \begin{cases} \pi_B^{DC}(w_2^{DC*}) = (p_B - k_1)(1-p_B) \\ +\alpha(p_B - w_2^{DC*})\left(\frac{2(1-\gamma-\beta)(1-\gamma-p_B)-\beta p_B+(1-\gamma)k_2+\beta\left(w_2^{DC*}-k_2\right)}{2(1-\gamma)(1-\gamma-\beta)}\right), & \text{if } \max\{k_2, \hat{w}_2^{DC}\} \leq w_2^{(0)}, \\ \pi_B^{D\dagger}\left(w_2^{D\dagger*}\right) = (p_B - k_1)(1-p_B) + \alpha\left(p_B - w_2^{D\dagger*}\right)(1 - \frac{p_B}{1-\gamma}); \end{cases}$$

$$\pi_B^O = \begin{cases} \pi_B^{OC}\left(w_2^{OC*}\right) = \left(p_B - w_2^{OC*} - t\right)\left(1 - \frac{p_B}{1-\gamma}\right) \\ +\alpha(p_B - w_2^{OC*})\left(\frac{2(1-\gamma-\beta)(1-\gamma-p_B)-\beta p_B+(1-\gamma)k_2+\beta\left(w_2^{OC*}-k_2\right)}{2(1-\gamma)(1-\gamma-\beta)}\right), & \text{if } \max\{w_2^{O(1)}, \underline{w}_2, \hat{w}_2^{OC}\} \leq w_2^{(0)}, \\ \pi_B^{O\dagger}\left(w_2^{O\dagger*}\right) = \left(p_B - w_2^{O\dagger*} - t\right)(1 - \frac{p_B}{1-\gamma}) + \alpha\left(p_B - w_2^{O\dagger*}\right)(1 - \frac{p_B}{1-\gamma}); \end{cases}$$

where $w_2^{DC*} = \max\{k_2, \hat{w}_2^{DC}\}$, $w_2^{D\dagger*} = \max\{w_2^{D(2)}, w_2^{(0)\prime}, \underline{w}_2\}$, $w_2^{OC*} = \max\{w_2^{O(1)}, \underline{w}_2, \hat{w}_2^{OC}\}$, and $w_2^{O\dagger*} = \max\{w_2^{O(2)}, w_2^{(0)\prime}, \underline{w}_2\}$.

We next compare strategies D and O, respectively. We define $\Pi_{B2}^D(w_2^{DC*})$, $\Pi_{B2}^O(w_2^{OC*})$ are the brand-name firm's profit from the overseas market under Strategy D, Strategy O, respectively; that is, $\Pi_{B2}^D(w_2^{DC*}) = \alpha(p_B - w_2^{DC*})\left(\frac{2(1-\gamma-\beta)(1-\gamma-p_B)-\beta p_B+(1-\gamma)k_2+\beta\left(w_2^{DC*}-k_2\right)}{2(1-\gamma)(1-\gamma-\beta)}\right)$; $\Pi_{B2}^O(w_2^{OC*}) = \alpha(p_B - w_2^{OC*})\left(\frac{2(1-\gamma-\beta)(1-\gamma-p_B)-\beta p_B+(1-\gamma)k_2+\beta\left(w_2^{OC*}-k_2\right)}{2(1-\gamma)(1-\gamma-\beta)}\right)$.

Under Strategy D:

$$\pi_B^D = \begin{cases} \pi_B^{DC}\left(w_2^{DC*}\right) = \left(p_B - k_1\right)\left(1 - p_B\right) \\ +\alpha\left(p_B - w_2^{DC*}\right)\left(\frac{2(1-\gamma-\beta)(1-\gamma-p_B)-\beta p_B+(1-\gamma)k_2+\beta\left(w_2^{DC*}-k_2\right)}{2(1-\gamma)(1-\gamma-\beta)}\right), & \text{if } \max\{k_2, \hat{w}_2^{DC}\} \le w_2^{(0)\prime}, \\ \pi_B^{D\dagger}\left(w_2^{D\dagger*}\right) = \left(p_B - k_1\right)\left(1 - p_B\right) + \alpha\left(p_B - w_2^{D\dagger*}\right)\left(1 - \frac{p_B}{1-\gamma}\right). \end{cases}$$

Then,

$$\begin{aligned} &\pi_B^{D\dagger}\left(w_2^{D\dagger*}\right) \ge \pi_B^{DC}\left(w_2^{DC*}\right), \\ \Rightarrow\ &\alpha\left(p_B - w_2^{D\dagger*}\right)\left(1 - \tfrac{p_B}{1-\gamma}\right) \ge \Pi_{B2}^{D}\left(w_2^{DC*}\right), \\ \Rightarrow\ &w_2^{D\dagger*}(e) \le p_B - \frac{\Pi_{B2}^{D}\left(w_2^{DC*}\right)}{\alpha\left(1-\frac{p_B}{1-\gamma}\right)}. \end{aligned}$$

Under Strategy O:

$$\pi_B^O = \begin{cases} \pi_B^{OC}\left(w_2^{OC*}\right) = \left(p_B - w_2^{OC*} - t\right)\left(1 - \frac{p_B}{1-\gamma}\right) \\ +\alpha\left(p_B - w_2^{OC*}\right)\left(\frac{2(1-\gamma-\beta)(1-\gamma-p_B)-\beta p_B+(1-\gamma)k_2+\beta\left(w_2^{OC*}-k_2\right)}{2(1-\gamma)(1-\gamma-\beta)}\right), & \text{if } \max\{w_2^{O(1)}, \underline{w}_2, \hat{w}_2^{OC}\} \le w_2^{(0)\prime}, \\ \pi_B^{O\dagger}\left(w_2^{O\dagger*}\right) = \left(p_B - w_2^{O\dagger*} - t\right)\left(1 - \frac{p_B}{1-\gamma}\right) + \alpha\left(p_B - w_2^{O\dagger*}\right)\left(1 - \frac{p_B}{1-\gamma}\right). \end{cases}$$

$$\begin{aligned} &\pi_B^{O\dagger}\left(w_2^{O\dagger*}\right) \ge \pi_B^{OC}\left(w_2^{OC*}\right), \\ \Rightarrow\ &(1+\alpha)\left(p_B - w_2^{O\dagger*}\right)\left(1 - \tfrac{p_B}{1-\gamma}\right) - t\left(1 - \tfrac{p_B}{1-\gamma}\right) \ge \left(p_B - w_2^{OC*} - t\right)\left(1 - \tfrac{p_B}{1-\gamma}\right) + \Pi_{B2}^{O}\left(w_2^{OC*}\right), \\ \Rightarrow\ &(1+\alpha)\left(p_B - w_2^{O\dagger*}\right)\left(1 - \tfrac{p_B}{1-\gamma}\right) \ge \left(p_B - w_2^{OC*}\right)\left(1 - \tfrac{p_B}{1-\gamma}\right) + \Pi_{B2}^{O}\left(w_2^{OC*}\right), \\ \Rightarrow\ &w_2^{O\dagger*}(e) \le p_B - \frac{\left(p_B - w_2^{OC*}\right)\left(1-\frac{p_B}{1-\gamma}\right) + \Pi_{B2}^{O}\left(w_2^{OC*}\right)}{(1+\alpha)\left(1-\frac{p_B}{1-\gamma}\right)}. \end{aligned}$$

Thus, we summarize our notations for comparison as below:

$$\begin{aligned} &w_2^{D(2)} = k_2 + \frac{M}{\alpha\left(1-\frac{p_B}{1-\gamma}\right)}; \\ &w_2^{O(2)} = k_2 + \frac{K}{(1+\alpha)\left(1-\frac{p_B}{1-\gamma}\right)}; \\ &w_2^{O(1)} = k_2 - \frac{2\left(1+\frac{1}{\alpha}\right)(1-\gamma-p_B)(1-\gamma-\beta)-\beta p_B+(1-\gamma)k_2}{\beta} + \sqrt{\frac{4(1-\gamma)(1-\gamma-\beta)(K-M')}{\alpha\beta} + \left(\frac{2\left(1+\frac{1}{\alpha}\right)(1-\gamma-p_B)(1-\gamma-\beta)-\beta p_B+(1-\gamma)k_2}{\beta}\right)^2}; \\ &\hat{w}_2^{DC} = k_2 - \frac{2(1-\gamma-p_B)(1-\gamma-\beta)-\beta p_B+(1-\gamma)k_2}{2\beta} + \frac{\beta(p_B-k_2)}{2\beta}, \\ &\hat{w}_2^{OC} = k_2 - \frac{2\left(1+\frac{1}{\alpha}\right)(1-\gamma-p_B)(1-\gamma-\beta)-\beta p_B+(1-\gamma)k_2}{2\beta} + \frac{\beta(p_B-k_2)}{2\beta}; \\ &\Pi_{B2}^{D}(w_2^{DC*}) = \alpha\left(p_B - w_2^{DC*}\right)\left(\frac{2(1-\gamma-\beta)(1-\gamma-p_B)-\beta p_B+(1-\gamma)k_2+\beta\left(w_2^{DC*}-k_2\right)}{2(1-\gamma)(1-\gamma-\beta)}\right), \\ &\Pi_{B2}^{O}(w_2^{OC*}) = \alpha\left(p_B - w_2^{OC*}\right)\left(\frac{2(1-\gamma-\beta)(1-\gamma-p_B)-\beta p_B+(1-\gamma)k_2+\beta\left(w_2^{OC*}-k_2\right)}{2(1-\gamma)(1-\gamma-\beta)}\right). \end{aligned} \tag{16}$$

Then, we have the following optimal wholesale price w_2 for Strategy D and Strategy O, respectively.

(a) Under Strategy D, $w_2^D = w_2^{DC*}$ and $s^* = 1$, if $\max\{k_2, \hat{w}_2^{DC}\} \le w_2^{(0)\prime}$ and $w_2^{D*} \le p_B - \frac{\Pi_{B2}^{D}\left(w_2^{DC*}\right)}{\alpha\left(1-\frac{p_B}{1-\gamma}\right)}$; otherwise, $w_2^D = w_2^{D\dagger*}$ and $s^* = 0$;

(b) Under Strategy O, $w_2^O = w_2^{OC*}$ and $s^* = 1$, if $\max\{w_2^{O(1)}, \underline{w}_2, \hat{w}_2^{OC}\} \le w_2^{(0)\prime}$ and $w_2^{O\dagger*} \le p_B - \frac{\left(p_B - w_2^{OC*}\right)\left(1-\frac{p_B}{1-\gamma}\right) + \Pi_{B2}^{O}\left(w_2^{OC*}\right)}{(1+\alpha)\left(1-\frac{p_B}{1-\gamma}\right)}$; otherwise, $w_2^O = w_2^{O\dagger*}$ and $s^* = 0$;

where $w_2^{DC*} = \max\{k_2, \hat{w}_2^{DC}\}$, $w_2^{D\dagger*} = \max\{w_2^{D(2)}, w_2^{(0)\prime}, \underline{w}_2\}$; $w_2^{OC*} = \max\{w_2^{O(1)}, \underline{w}_2, \hat{w}_2^{OC}\}$, $w_2^{O\dagger*} = \max\{w_2^{O(2)}, w_2^{(0)\prime}, \underline{w}_2\}$. ■

B.10 Proofs For Extension 3: Endogenous Brand-Name Product and Counterfeit Prices

B.10.1 Proof of Lemma A1.

Note that when the demand of the brand-name product is $m_{B2} = 0$, it is not dual sourcing or single sourcing from the overseas supplier, because there is no market share for the brand-name firm in the overseas market. Thus, in order to focus on the cases of strategies D or O with $m_{B2} > 0$ and to examine the conditions to effectively prevent counterfeiting, in this extension, we assume that the brand-name firm has a positive market share in the overseas market, and it is possible for the overseas supplier to sell counterfeits under optimal retail prices.

It is convenient for us to define below notations:

$$\begin{aligned} \hat{p}_B^D &= \frac{2(1-\gamma)(1-\gamma-\beta)(1+w_1)+\alpha(2(1-\gamma-\beta)(1-\gamma)+(1-\gamma-\beta)k_2)+2\alpha(1-\gamma)w_2}{4(1-\gamma)(1-\gamma-\beta)+\alpha(4(1-\gamma-\beta)+2\beta)}, \\ \hat{p}_B^O &= \frac{2(1-\gamma-\beta)(1-\gamma+t)+\alpha(2(1-\gamma-\beta)(1-\gamma)+(1-\gamma-\beta)k_2)+(2(1-\gamma-\beta)+2\alpha(1-\gamma))w_2}{4(1-\gamma-\beta)+\alpha(4(1-\gamma-\beta)+2\beta)}. \end{aligned} \tag{17}$$

We assume the penalty from law enforcement e is not very high such that $w_2 < h_0(e)$, where $h_0(e) = \frac{(2(1-\gamma)+k_2)(1-\gamma-\beta)\beta-(\sqrt{\frac{4\beta(1-\gamma)(1-\gamma-\beta)e}{\alpha}}+(1-\gamma-\beta)k_2)(2(1-\gamma)-\beta)}{2\beta(1-\gamma-\beta)}$. This condition guarantees that it is possible for the overseas supplier to sell counterfeits.

Below, the proof includes two parts for strategies D and O, respectively.

Part 1: With Strategy D, we derive the counterfeiting prevention condition, and compare the counterfeiting prevention condition between this extension and the base model.

Under Strategy D, we assume $w_2 > (\underline{h}_D(w_1))^+$, where $\underline{h}_D(w_1) = \frac{(2(1-\gamma)(1+w_1)(2(1-\gamma)-\beta)-(4(1-\gamma)(1-\gamma-\beta)+\alpha(2(1-\gamma)-\beta))(2(1-\gamma)+k_2))(1-\gamma-\beta)}{\beta(4(1-\gamma)(1-\gamma-\beta)+2\alpha\gamma(2(1-\gamma)-\beta))}$. Under this condition, $\hat{p}_B^D < \frac{2(1-\gamma)(1-\gamma-\beta)+(1-\gamma)k_2+\beta(w_2-k_2)}{2(1-\gamma)-\beta}$ holds, where $\hat{p}_B^D$ is defined in Equation (17). It implies that the brand-name firm has a positive market share in the overseas market with Strategy D.

Step 1: Given p_B, we derive the overseas supplier's profit $s = 0$ and $s = 1$, respectively.

(1) If the overseas supplier does not sell the counterfeit in the market, i.e., $s = 0$, we know:

$$\begin{aligned} \pi_B^D(p_B, s=0) &= (p_B - w_1)(1 - p_B) + \alpha (p_B - w_2)(1 - \tfrac{p_B}{1-\gamma}), \\ \pi_2^D(p_B, s=0) &= \alpha (w_2 - k_2)(1 - \tfrac{p_B}{1-\gamma}). \end{aligned}$$

(2) If the overseas supplier sell the counterfeit in the market, i.e., $s = 1$, then, the brand-name firm firstly decides on the retail price p_B for the brand-name product, then the overseas supplier decides on the retail price p_2 for the counterfeit. Their profits are as follows.

$$\begin{aligned} \pi_B^D(p_B, p_2, s=1) &= (p_B - w_1)(1 - p_B) + \alpha (p_B - w_2)\left(1 - \tfrac{p_B - p_2}{1-\gamma-\beta}\right), \\ \pi_2^D(p_B, p_2, s=1) &= \alpha (w_2 - k_2)\left(1 - \tfrac{p_B - p_2}{1-\gamma-\beta}\right) + \alpha (p_2 - k_2)\left(\tfrac{p_B - p_2}{1-\gamma-\beta} - \tfrac{p_2}{\beta}\right)^+ - e. \end{aligned}$$

If both the brand-name firm and the overseas supplier get positive overseas market share, i.e., $m_{B2} = \alpha\left(1 - \frac{p_B - p_2}{\gamma-\beta}\right) > 0$, and $m_2 = \alpha\left(\frac{p_B - p_2}{1-\gamma-\beta} - \frac{p_2}{\beta}\right) > 0$, then, $p_B - (1-\gamma-\beta) < p_2 < \frac{\beta p_B}{1-\gamma}$. The profit of the overseas supplier is

$$\pi_2^D(p_B, p_2) = \alpha (w_2 - k_2)\left(1 - \frac{p_B - p_2}{1-\gamma-\beta}\right) + \alpha (p_2 - k_2)\left(\frac{p_B - p_2}{1-\gamma-\beta} - \frac{p_2}{\beta}\right) - e.$$

By taking the first order derivative of $\pi_2^D(p_B, p_2)$ with respect to p_2, we have,

$$\frac{\partial(\pi_2^D(p_B,p_2))}{\partial(p_2)} = \alpha\left(\frac{p_B+k_2-2p_2+(w_2-k_2)}{1-\gamma-\beta} - \frac{2p_2-k_2}{\beta}\right) = \alpha\left(\frac{p_B-2p_2+w_2}{1-\gamma-\beta} - \frac{2p_2-k_2}{\beta}\right).$$

From $\frac{\partial\left(\pi_2^D(p_B,p_2)\right)}{\partial(p_2)} = 0$, we obtain the critical point $\hat{p}_2^D = \frac{\beta p_B+(1-\gamma)k_2+\beta(w_2-k_2)}{2(1-\gamma)}$. Next, we need to check whether $\hat{p}_2^D$ is in the feasible region $p_B-(1-\gamma-\beta) < p_2 < \frac{\beta p_B}{1-\gamma}$. From $p_B-(1-\gamma-\beta) < \hat{p}_2^D < \frac{\beta p_B}{1-\gamma}$, we obtain, $\frac{(1-\gamma)k_2+\beta(w_2-k_2)}{\beta} < p_B < \frac{2(1-\gamma)(1-\gamma-\beta)+(1-\gamma)k_2+\beta(w_2-k_2)}{2(1-\gamma)-\beta}$. Recall that we assume the brand-name firm has a positive market share in the overseas market, i.e., $m_{B2}>0$. Thus, with Strategy D, if the overseas supplier sells the counterfeit, i.e., $s=1$, the optimal retail price p_2 for the counterfeit is

$$p_2^{D*} = \begin{cases} \frac{\beta p_B}{1-\gamma}, & \text{if } p_B \le \frac{(1-\gamma)k_2+\beta(w_2-k_2)}{\beta}, \text{[note that } m_2=0] \\ \hat{p}_2^D, & \text{if } \frac{(1-\gamma)k_2+\beta(w_2-k_2)}{\beta} < p_B < \frac{2(1-\gamma)(1-\gamma-\beta)+(1-\gamma)k_2+\beta(w_2-k_2)}{2(1-\gamma)-\beta}, \text{[note that } m_2>0] \end{cases}$$

and the overseas supplier's profit is

$$\pi_2^D(p_B, s=1) = \begin{cases} \pi_2^{DC1} = \alpha(w_2-k_2)\left(1-\frac{p_B-p_2^{D*}}{1-\gamma-\beta}\right) - e, & \text{if } p_B \le \frac{(1-\gamma)k_2+\beta(w_2-k_2)}{\beta}, \\ \hat{\pi}_2^{DC} = \alpha(w_2-k_2)\left(1-\frac{p_B-p_2^{D*}}{1-\gamma-\beta}\right) & \\ +\alpha(p_2^*-k_2)\left(\frac{p_B-p_2^{D*}}{1-\gamma-\beta} - \frac{p_2}{\beta}\right) - e, & \text{if } \frac{(1-\gamma)k_2+\beta(w_2-k_2)}{\beta} < p_B < \frac{2(1-\gamma)(1-\gamma-\beta)+(1-\gamma)k_2+\beta(w_2-k_2)}{2(1-\gamma)-\beta}, \end{cases}$$

and the brand-name firm's profit is

$$\begin{aligned} &\pi_B^D(p_B, s=1) \\ &= \begin{cases} \pi_B^{DC1} = (p_B-w_1)(1-p_B) + \alpha(p_B-w_2)\left(1-\frac{p_B-p_2^*}{1-\gamma-\beta}\right), & \text{if } p_B < \frac{(1-\gamma)k_2+\beta(w_2-k_2)}{\beta}, \\ \hat{\pi}_B^{DC} = (p_B-w_1)(1-p_B) & \\ +\alpha(p_B-w_2)\left(\frac{2(1-\gamma-\beta)(\gamma-p_B)-\beta p_B+(1-\gamma)k_2+\beta(w_2-k_2)}{2\gamma(1-\gamma-\beta)}\right), & \text{if } \frac{(1-\gamma)k_2+\beta(w_2-k_2)}{\beta} < p_B < \frac{2(1-\gamma)(1-\gamma-\beta)+(1-\gamma)k_2+\beta(w_2-k_2)}{2(1-\gamma)-\beta}. \end{cases} \end{aligned}$$

Step 2: The overseas supplier decides on whether to sell the counterfeit, $s^*(p_B)$.

For the overseas supplier, if $\pi_2^D(p_B, s=1) > \pi_2^D(p_B, s=0)$, she decides to sell the counterfeit. Otherwise, she does not sell the counterfeit. Recall that when $s=0$, the overseas supplier's profit is

$$\pi_2^D(p_B, s=0) = \alpha(w_2-k_2)\left(1-\frac{p_B}{1-\gamma}\right).$$

Note that given p_B, for the overseas supplier, there are below two scenarios.

(1) If $p_B < \frac{(1-\gamma)k_2+\beta(w_2-k_2)}{\beta}$, then, the overseas supplier's profit of counterfeiting is $\pi_2^D(p_B, s=1) = \pi_B^{DC1}$, which implies $p_2^{D*} = \frac{\beta p_B}{1-\gamma}$. Then, we know: the optimal decision is $s^*=0$, because $\pi_2^D(p_B, s=0) > \pi_2^{DC1}$ always holds.

(2) If $\frac{(1-\gamma)k_2+\beta(w_2-k_2)}{\beta} < p_B < \frac{2(1-\gamma)(1-\gamma-\beta)+(1-\gamma)k_2+\beta(w_2-k_2)}{2(1-\gamma)-\beta}$, then, the overseas supplier's profit of counterfeiting is $\pi_2^D(p_B, s=1) = \hat{\pi}_2^{DC}$, which implies $p_2^{D*} = \hat{p}_2^D(p_B) = \frac{\beta p_B+(1-\gamma)k_2+\beta(w_2-k_2)}{2(1-\gamma)}$. Then, the optimal decision is $s=0$ if $\pi_2^D(p_B, s=0) > \hat{\pi}_2^{DC}$, which means

$$\begin{aligned} &\alpha(w_2-k_2)\left(1-\frac{p_B}{1-\gamma}\right) > \alpha(w_2-k_2)\left(1-\frac{p_B-\hat{p}_2^D}{1-\gamma-\beta}\right) + \left(\alpha(\hat{p}_2^D-k_2)\left(\frac{p_B-\hat{p}_2^D}{1-\gamma-\beta} - \frac{\hat{p}_2^D}{\beta}\right) - e\right), \\ &\Rightarrow \alpha(w_2-k_2)\left(1-\frac{p_B}{1-\gamma}\right) > \alpha(w_2-k_2)\left(\frac{2(1-\gamma-\beta)(1-\gamma)-(2(1-\gamma)-\beta)p_B+(1-\gamma)k_2+\beta(w_2-k_2)}{2(1-\gamma)(1-\gamma-\beta)}\right) \\ &\quad + \left(\alpha\left(\frac{\beta p_B-(1-\gamma)k_2+\beta(w_2-k_2)}{2(1-\gamma)}\right)\frac{\beta p_B-(1-\gamma)k_2-\beta(w_2-k_2)}{2\beta(1-\gamma-\beta)} - e\right), \\ &\Rightarrow x_{low} < p_B < x_{high}, \end{aligned}$$

where $x_{low} = \frac{-\sqrt{\frac{4\beta(1-\gamma)(1-\gamma-\beta)e}{\alpha}}+((1-\gamma)k_2+\beta(w_2-k_2))}{\beta}$, $x_{high} = \frac{\sqrt{\frac{4\beta(1-\gamma)(1-\gamma-\beta)e}{\alpha}}+((1-\gamma)k_2+\beta(w_2-k_2))}{\beta}$.

Note that $x_{low} < \frac{\gamma k_2+\beta(w_2-k_2)}{\beta} < x_{high}$. Recall that we assume $w_2 < h_0(e)$, where $h_0(e) = \frac{(2(1-\gamma)+k_2)(1-\gamma-\beta)\beta-(\sqrt{\frac{4\beta(1-\gamma)(1-\gamma-\beta)e}{\alpha}}+(1-\gamma-\beta)k_2)(2(1-\gamma)-\beta)}{2\beta(1-\gamma-\beta)}$, it implies that $x_{high} < \frac{2(1-\gamma)(1-\gamma-\beta)+(1-\gamma)k_2+\beta(w_2-k_2)}{2(1-\gamma)-\beta}$. Then, if $\frac{(1-\gamma)k_2+\beta(w_2-k_2)}{\beta} < p_B < x_{high}$, the optimal decision is $s^*=0$; if $x_{high} < p_B < \frac{2\gamma(1-\gamma-\beta)+(1-\gamma)k_2+\beta(w_2-k_2)}{2(1-\gamma)-\beta}$, the optimal decision is $s^*=1$, and $\pi_B^D(p_B, s=1) = \hat{\pi}_B^{DC}$.

Thus, combining these two scenarios, the overseas supplier's optimal decision of counterfeiting is

$$s^*(p_B) = \begin{cases} 0, & \text{if } p_B \le x_{high}, \text{ [note that } m_2 = 0] \\ 1, & \text{if } x_{high} < p_B < \frac{2(1-\gamma)(1-\gamma-\beta)+(1-\gamma)k_2+\beta(w_2-k_2)}{2(1-\gamma)-\beta}. \text{ [note that } m_2 > 0] \end{cases}$$

Subsequently, the brand-name firm's profit is

$$\begin{aligned} &\pi_B^D(p_B) \\ &= \begin{cases} \pi_B^D(p_B, s=0) = (p_B-w_1)(1-p_B) + \alpha(p_B-w_2)(1-\frac{p_B}{1-\gamma}), & \text{if } p_B \le x_{high}, \\ \pi_B^D(p_B, s=1) = (p_B-w_1)(1-p_B) & \\ +\alpha(p_B-w_2)\left(\frac{2(1-\gamma-\beta)(1-\gamma-p_B)-\beta p_B+(1-\gamma)k_2+\beta(w_2-k_2)}{2\gamma(1-\gamma-\beta)}\right), & \text{if } x_{high} < p_B < \frac{2(1-\gamma)(1-\gamma-\beta)+(1-\gamma)k_2+\beta(w_2-k_2)}{2(1-\gamma)-\beta}. \end{cases} \end{aligned}$$

Step 3: The brand-name firm decides on the optimal retail price p_B^{D*}. We discuss possible cases as follow.

(1) If $p_B \le x_{high}$, which means $s=0$, the brand-name firm's profit is

$$\pi_B^D(p_B, s=0) = (p_B-w_1)(1-p_B) + \alpha(p_B-w_2)(1-\frac{p_B}{1-\gamma}).$$

In this case, only the brand-name firm decides on the optimal price p_B.

$$\begin{aligned} \frac{\partial(\pi_B^D(p_B))}{\partial(p_B)} &= ((1-p_B)-(p_B-w_1)) + \alpha((1-\frac{p_B}{1-\gamma}) - \frac{p_B-w_2}{1-\gamma}) \\ &= (1-2p_B+w_1) + \alpha(\frac{1-\gamma-2p_B+w_2}{1-\gamma}) \\ &= \frac{(1+w_1)(1-\gamma)-2p_B(1-\gamma)+\alpha(1-\gamma+w_2)+\alpha(-2p_B)}{1-\gamma}. \end{aligned}$$

From the first order condition, i.e., $\frac{\partial(\pi_B^D(p_B))}{\partial(p_B)} = 0$, the critical point of the optimal retail price is

$$p_B^{D0} = \frac{(1+w_1)(1-\gamma)+\alpha(1-\gamma+w_2)}{2(\alpha+1-\gamma)}.$$

We check whether this critical point is in the feasible region. From $p_B^{D0} \le x_{high}$, we have

$$\begin{aligned} &\frac{(1+w_1)(1-\gamma)+\alpha(1-\gamma+w_2)}{2(\alpha+1-\gamma)} \le \frac{\sqrt{\frac{4\beta(1-\gamma)(1-\gamma-\beta)e}{\alpha}}+((1-\gamma)k_2+\beta(w_2-k_2))}{\beta}, \\ &\Rightarrow w_2 \ge h_{D1}(w_1,e), \text{ where } h_{D1}(w_1,e) = \frac{((1+w_1)(1-\gamma)+\alpha(1-\gamma))\beta-2(\alpha+1-\gamma)(\sqrt{\frac{4\beta(1-\gamma)(1-\gamma-\beta)e}{\alpha}}+(1-\gamma-\beta)k_2)}{\beta(2(1-\gamma)+\alpha)}. \end{aligned}$$

Thus, with $s=0$, the brand-name firm's optimal retail price is

$$p_B^{D*}(s=0) = \begin{cases} p_B^{D0}, & \text{if } w_2 \ge h_{D1}(w_1,e), \\ x_{high}, & \text{if } w_2 < h_{D1}(w_1,e). \end{cases}$$

(2) If $x_{high} < p_B < \frac{2(1-\gamma)(1-\gamma-\beta)+(1-\gamma)k_2+\beta(w_2-k_2)}{2(1-\gamma)-\beta}$, which means $s=1$, the brand-name firm's profit is

$$\pi_B^D(p_B, s=1) = \hat{\pi}_B^{DC} = (p_B-w_1)(1-p_B) + \alpha(p_B-w_2)\left(\frac{2(1-\gamma-\beta)(1-\gamma-p_B)-\beta p_B+(1-\gamma)k_2+\beta(w_2-k_2)}{2\gamma(1-\gamma-\beta)}\right).$$

By taking the derivative of the first order condition, the critical point of the optimal retail price is

$$\hat{p}_B^D = \frac{2(1-\gamma)(1-\gamma-\beta)(1+w_1)+\alpha(2(1-\gamma-\beta)(1-\gamma)+(1-\gamma-\beta)k_2)+2\alpha(1-\gamma)w_2}{4(1-\gamma)(1-\gamma-\beta)+\alpha(4(1-\gamma-\beta)+2\beta)}.$$

We check whether this critical point $\hat{p}_B^D$ is in the feasible region of $[x_{high}, \frac{2(1-\gamma)(1-\gamma-\beta)+(1-\gamma)k_2+\beta(w_2-k_2)}{2(1-\gamma)-\beta}]$. Recall that $w_2 > (\underline{h}_D(w_1))^+$, which implies $\hat{p}_B^D < \frac{2(1-\gamma)(1-\gamma-\beta)+(1-\gamma)k_2+\beta(w_2-k_2)}{2(1-\gamma)-\beta}$ holds.

From $x_{high} < \hat{p}_B^D$, we have:

$$\frac{\sqrt{\frac{4\beta(1-\gamma)(1-\gamma-\beta)e}{\alpha}}+((1-\gamma)k_2+\beta(w_2-k_2))}{\beta} < \frac{2(1-\gamma)(1-\gamma-\beta)(1+w_1)+\alpha(2(1-\gamma-\beta)(1-\gamma)+(1-\gamma-\beta)k_2)+2\alpha(1-\gamma)w_2}{4(1-\gamma)(1-\gamma-\beta)+\alpha(4(1-\gamma-\beta)+2\beta)},$$

$\Rightarrow w_2 < h_{D2}(w_1,e)$,

where $h_{D2}(w_1,e) = \frac{(2(1-\gamma)(1-\gamma-\beta)(1+w_1)+\alpha(2(1-\gamma)+k_2)(1-\gamma-\beta))\beta-\left(\sqrt{\frac{4\beta(1-\gamma)(1-\gamma-\beta)e}{\alpha}}+(1-\gamma-\beta)k_2\right)(4(1-\gamma)(1-\gamma-\beta)+\alpha(4(1-\gamma-\beta)+2\beta))}{2\beta(2(1-\gamma)+\alpha)(1-\gamma-\beta)}$.

Thus, with $s=1$, the brand-name firm's optimal retail price is

$$p_B^{D*}(s=1) = \begin{cases} x_{high}, & \text{if } w_2 \geq h_{D2}(w_1,e), \\ \hat{p}_B^D, & \text{if } \underline{h}_D(w_1) < w_2 < h_{D2}(w_1,e). \end{cases}$$

Based on the above discussions, the brand-name firm chooses p_B^* to maximize her profit by making a comparison between $\pi_B^D(s=0)$ and $\hat{\pi}_B^{DC}(s=1)$ in overleaping region.

Note that $h_{D2}(w_1,e) < h_{D1}(w_1,e)$. Then, the optimal retail price of the brand-name firm is

$$p_B^{D*} = \begin{cases} p_B^{D0}, & \text{if } w_2 \geq h_{D1}(w_1,e), \text{[note that } m_2 = 0] \\ x_{high}, & \text{if } h_{D2}(w_1,e) \leq w_2 < h_{D1}(w_1,e), \text{[note that } m_2 = 0] \\ \hat{p}_B^D, & \text{if } (\underline{h}_D(w_1))^+ < w_2 < h_{D2}(w_1,e), \text{[note that } m_2 > 0] \end{cases}$$

where $x_{high} = \frac{\sqrt{\frac{4\beta(1-\gamma)(1-\gamma-\beta)e}{\alpha}}+((1-\gamma)k_2+\beta(w_2-k_2))}{\beta}$, $p_B^{D0} = \frac{(1+w_1)(1-\gamma)+\alpha(1-\gamma+w_2)}{2(\alpha+1-\gamma)}$, $\hat{p}_B^D = \frac{2(1-\gamma)(1-\gamma-\beta)(1+w_1)+\alpha(2(1-\gamma-\beta)\gamma+(1-\gamma-\beta)k_2)+2\alpha(1-\gamma)w_2}{4(1-\gamma)(1-\gamma-\beta)+\alpha(4(1-\gamma-\beta)+2\beta)}$.

Thus, the condition to prevent counterfeiting is $w_2 \geq w_2^{D,endog}$, where $w_2^{D,endog} = h_{D2}(w_1,e)$. That is to say, under Strategy D, $s^* = 0$ if $\hat{p}_B^D \leq \frac{\sqrt{\frac{4\beta(1-\gamma)(1-\gamma-\beta)e}{\alpha}}+((1-\gamma)k_2+\beta(w_2-k_2))}{\beta}$, where $\hat{p}_B^D$ is defined in Equation (17).

Part 2: With Strategy O, we derive the counterfeiting prevention condition, and compare the counterfeiting prevention condition between this extension and the base model.

Under Strategy O, we assume $w_2 > (\underline{h}_O)^+$, where $\underline{h}_O = \frac{(2(1-\gamma+\alpha)(2(1-\gamma)-\beta)-(4(1-\gamma-\beta)+\alpha(2(1-\gamma)-\beta))(2(1-\gamma)+k_2))(1-\gamma-\beta)}{\beta(2(1-\gamma-\beta)(2\gamma+\beta)+2\alpha\gamma(2(1-\gamma)-\beta))}$. Under this condition, $\hat{p}_B^O < \frac{2(1-\gamma)(1-\gamma-\beta)+(1-\gamma)k_2+\beta(w_2-k_2)}{2(1-\gamma)-\beta}$ holds, where $\hat{p}_B^O$ is defined in Equation (17). It implies that the brand-name firm has a positive market share in the overseas market with Strategy O.

Step 1: Given p_B, we derive the overseas supplier's profit $s=0$ and $s=1$, respectively.

(1) If the overseas supplier does not sell the counterfeit in the market, i.e., $s=0$, we know:

$$\pi_B^O(p_B, s=0) = (p_B - w_2 - t)\,(1 - \frac{p_B}{1-\gamma}) + \alpha\,(p_B - w_2)\,(1 - \frac{p_B}{1-\gamma}),$$
$$\pi_2^O(p_B, s=0) = (w_2 - k_2)\,(1 - \frac{p_B}{1-\gamma}) + \alpha\,(w_2 - k_2)\,(1 - \frac{p_B}{1-\gamma}).$$

(2) If the overseas supplier sells the counterfeit in the market, i.e., $s=1$, then, the brand-name firm firstly decides on the retail price p_B for the brand-name product, then the overseas supplier decides on the retail price p_2 for the counterfeit. Their profits are as follows.

$$\pi_B^O(p_B,p_2,s=1)=(p_B-w_2-t)\left(1-\tfrac{p_B}{1-\gamma}\right)+\alpha(p_B-w_2)\left(1-\tfrac{p_B-p_2}{1-\gamma-\beta}\right),$$
$$\pi_2^O(p_B,p_2,s=1)=(w_2-k_2)\left(1-\tfrac{p_B}{1-\gamma}\right)+\alpha(w_2-k_2)\left(1-\tfrac{p_B-p_2}{1-\gamma-\beta}\right)+\alpha(p_2-k_2)\left(\tfrac{p_B-p_2}{1-\gamma-\beta}-\tfrac{p_2}{\beta}\right)^+-e.$$

Similar with the discussion in Strategy D, we derive the optimal retail price p_2 for the overseas supplier under Strategy O by backward deduction. Thus, with Strategy O, if the overseas supplier sells the counterfeit, i.e., $s=1$, we have $\hat{p}_2^O=\frac{\beta p_B+(1-\gamma)k_2+\beta(w_2-k_2)}{2(1-\gamma)}$, and the optimal retail price is

$$p_2^{O*}=\begin{cases}\frac{\beta p_B}{1-\gamma}, & \text{if } p_B<\frac{(1-\gamma)k_2+\beta(w_2-k_2)}{\beta}, \text{[note that } m_2=0]\\ \hat{p}_2^O, & \text{if } \frac{(1-\gamma)k_2+\beta(w_2-k_2)}{\beta}<p_B<\frac{2(1-\gamma)(1-\gamma-\beta)+(1-\gamma)k_2+\beta(w_2-k_2)}{2(1-\gamma)-\beta}, \text{[note that } m_2>0]\end{cases}$$

and the overseas supplier's profit is

$$\begin{aligned}&\pi_2^O(p_B,s=1)\\&=\begin{cases}\pi_2^{OC1}=(w_2-k_2)\left(1-\frac{p_B}{1-\gamma}\right)+\alpha(w_2-k_2)\left(1-\frac{p_B-p_2^{O*}}{1-\gamma-\beta}\right)-e, & \text{if } p_B<\frac{(1-\gamma)k_2+\beta(w_2-k_2)}{\beta},\\ \hat{\pi}_2^{OC}=(w_2-k_2)\left(1-\frac{p_B}{1-\gamma}\right) & \\ +\alpha(w_2-k_2)\left(1-\frac{p_B-p_2^{O*}}{1-\gamma-\beta}\right)+\left(\alpha(p_2^*-k_2)\left(\frac{p_B-p_2^{O*}}{1-\gamma-\beta}-\frac{p_2^{O*}}{\beta}\right)-e\right), & \text{if } \frac{(1-\gamma)k_2+\beta(w_2-k_2)}{\beta}<p_B<\frac{2(1-\gamma)(1-\gamma-\beta)+(1-\gamma)k_2+\beta(w_2-k_2)}{2(1-\gamma)-\beta},\end{cases}\end{aligned}$$

and the brand-name firm's profit is

$$\begin{aligned}&\pi_B^O(p_B,s=1)\\&=\begin{cases}\pi_B^{OC1}=(p_B-w_2-t)(1-p_B)+\alpha(p_B-w_2)\left(1-\frac{p_B-p_2^{O*}}{1-\gamma-\beta}\right), & \text{if } p_B<\frac{(1-\gamma)k_2+\beta(w_2-k_2)}{\beta},\\ \hat{\pi}_B^{OC}=(p_B-w_2-t)(1-p_B) & \\ +\alpha(p_B-w_2)\left(\frac{2(1-\gamma-\beta)(1-\gamma-p_B)-\beta p_B+(1-\gamma)k_2+\beta(w_2-k_2)}{2(1-\gamma)(1-\gamma-\beta)}\right), & \text{if } \frac{(1-\gamma)k_2+\beta(w_2-k_2)}{\beta}<p_B<\frac{2(1-\gamma)(1-\gamma-\beta)+(1-\gamma)k_2+\beta(w_2-k_2)}{2(1-\gamma)-\beta}.\end{cases}\end{aligned}$$

Step 2: The overseas supplier decides on whether to sell the counterfeit, $s^*(p_B)$.

For the overseas supplier, if $\pi_2^O(p_B,s=1)>\pi_2^O(p_B,s=0)$, she decides to sell the counterfeit. Otherwise, she does not sell the counterfeit. Recall that when $s=0$, the overseas supplier's profit is

$$\pi_2^O(p_B,s=0)=(w_2-k_2)(1-\frac{p_B}{1-\gamma})+\alpha(w_2-k_2)(1-\frac{p_B}{1-\gamma}).$$

Similarly, we obtain,

$$s^*(p_B)=\begin{cases}0, & \text{if } p_B\le x_{high}, \text{[note that } m_2=0]\\ 1, & \text{if } x_{high}<p_B<\frac{2(1-\gamma)(1-\gamma-\beta)+(1-\gamma)k_2+\beta(w_2-k_2)}{2(1-\gamma)-\beta}. \text{[note that } m_2>0]\end{cases}$$

Step 3: The brand-name firm decides on the optimal retail price p_B^{O*} to maximize her profit. We discuss possible cases as follows.

(1) If $p_B\le x_{high}$, which means $s=0$, the brand-name firm's profit is

$$\pi_B^O(p_B,s=0)=(p_B-w_2-t)(1-\frac{p_B}{1-\gamma})+\alpha(p_B-w_2)(1-\frac{p_B}{1-\gamma}),$$

In this case, only the brand-name firm decides on the optimal price p_B.

$$\begin{aligned}\frac{\partial(\pi_B^O(p_B))}{\partial(p_B)} &= ((1-\frac{p_B}{1-\gamma})-\frac{p_B-w_2-t}{1-\gamma})+\alpha((1-\frac{p_B}{1-\gamma})-\frac{p_B-w_2}{1-\gamma})\\ &= \frac{(\gamma-2p_B+w_2+t)}{1-\gamma}+\alpha(\frac{1-\gamma-2p_B+w_2}{1-\gamma})\\ &= \frac{(1+\alpha)(1-\gamma+w_2)+t+(1+\alpha)(-2p_B)}{1-\gamma}.\end{aligned}$$

From the first order condition, i.e., $\frac{\partial(\pi_B^O(p_B))}{\partial(p_B)}=0$, we have,

$$p_B^{O0}=\frac{(1+\alpha)(1-\gamma+w_2)+t}{2(1+\alpha)}.$$

We check whether this critical point p_B^{O0} is in the feasible region. From $p_B^{O0}\le x_{high}$, we have

$$\begin{aligned}&\frac{(1+\alpha)(1-\gamma+w_2)+t}{2(1+\alpha)}\le\frac{\sqrt{\frac{4\beta(1-\gamma)(1-\gamma-\beta)e}{\alpha}}+((1-\gamma)k_2+\beta(w_2-k_2))}{\beta},\\ &\Rightarrow w_2\ge h_{O1}(e),\ \text{where } h_{O1}(e)=\frac{((1+\alpha)(1-\gamma)+t)\beta-2(1+\alpha)(\sqrt{\frac{4\beta(1-\gamma)(1-\gamma-\beta)e}{\alpha}}+(1-\gamma-\beta)k_2)}{\beta(1+\alpha)}.\end{aligned}$$

Thus, with $s=0$, the brand-name firm's optimal retail price is as follows:

$$p_B^{O*}(s=0)=\begin{cases}p_B^{O0}, & \text{if } w_2\ge h_{O1}(e),\\ x_{high}, & \text{if } w_2<h_{O1}(e).\end{cases}$$

(2) If $x_{high}<p_B<\frac{2(1-\gamma)(1-\gamma-\beta)+(1-\gamma)k_2+\beta(w_2-k_2)}{2(1-\gamma)-\beta}$, which means $s=1$, the brand-name firm's profit is

$$\pi_B^O(p_B,s=1)=\hat{\pi}_B^{OC}=(p_B-w_2-t)\left(1-\frac{p_B}{1-\gamma}\right)+\alpha(p_B-w_2)\left(\frac{2(1-\gamma-\beta)(1-\gamma-p_B)-\beta p_B+(1-\gamma)k_2+\beta(w_2-k_2)}{2(1-\gamma)(1-\gamma-\beta)}\right).$$

By taking the first order derivative of $\pi_B^O(p_B)$ with respect to p_B, we have,

$$\begin{aligned}\frac{\partial\left(\pi_B^O(p_B)\right)}{\partial(p_B)} &= \left(1-\frac{2p_B-w_2-t}{1-\gamma}\right)+\alpha\left(\left(\frac{2(1-\gamma-\beta)(1-\gamma-p_B)-\beta p_B+(1-\gamma)k_2+\beta(w_2-k_2)}{2(1-\gamma)(1-\gamma-\beta)}\right)+(p_B-w_2)\frac{-2(1-\gamma-\beta)-\beta}{2(1-\gamma)(1-\gamma-\beta)}\right)\\ &= \frac{1-\gamma-2p_B+w_2+t}{1-\gamma}+\alpha\left(\frac{2(1-\gamma-\beta)(1-\gamma-2p_B+w_2)-2\beta p_B+\beta w_2+(1-\gamma)k_2+\beta(w_2-k_2)}{2(1-\gamma)(1-\gamma-\beta)}\right).\end{aligned}$$

From the first order condition, i.e., $\frac{\partial\left(\pi_B^O(p_B)\right)}{\partial(p_B)}=0$, we have,

$$\hat{p}_B^O=\frac{2(1-\gamma-\beta)(1-\gamma+t)+\alpha(2(1-\gamma-\beta)(1-\gamma)+(1-\gamma-\beta)k_2)+(2(1-\gamma-\beta)+2\alpha(1-\gamma))w_2}{4(1-\gamma-\beta)+\alpha(4(1-\gamma-\beta)+2\beta)}.$$

We check whether this critical point $\hat{p}_B^O$ is in the feasible region of $[x_{high},\frac{2(1-\gamma)(1-\gamma-\beta)+(1-\gamma)k_2+\beta(w_2-k_2)}{2(1-\gamma)-\beta}]$. Recall that with Strategy O, $w_2>(\underline{h}_O)^+$, which implies that $\hat{p}_B^O<\frac{2(1-\gamma)(1-\gamma-\beta)+(1-\gamma)k_2+\beta(w_2-k_2)}{2(1-\gamma)-\beta}$.

From $x_{high}<\hat{p}_B^O$,

$$\begin{aligned}&\frac{\sqrt{\frac{4\beta(1-\gamma)(1-\gamma-\beta)e}{\alpha}}+((1-\gamma)k_2+\beta(w_2-k_2))}{\beta}<\frac{2(1-\gamma-\beta)(1-\gamma+t)+\alpha(2(1-\gamma-\beta)(1-\gamma)+(1-\gamma-\beta)k_2)+(2(1-\gamma-\beta)+2\alpha(1-\gamma))w_2}{4(1-\gamma-\beta)+\alpha(4(1-\gamma-\beta)+2\beta)},\\ &\Rightarrow w_2<h_{O2}(e),\\ &\quad\text{where } h_{O2}(e)=\frac{(2(1-\gamma-\beta)(1-\gamma+t)+\alpha(2(1-\gamma-\beta)(1-\gamma)+(1-\gamma-\beta)k_2))\beta-\left(\sqrt{\frac{4\beta(1-\gamma)(1-\gamma-\beta)e}{\alpha}}+(1-\gamma-\beta)k_2\right)(4(1-\gamma-\beta)+\alpha(4(1-\gamma-\beta)+2\beta))}{2\beta(1+\alpha)(1-\gamma-\beta)}.\end{aligned}$$

Thus, with $s=1$, the brand-name firm's optimal retail price is

$$p_B^{O*}(s=1)=\begin{cases}x_{high}, & \text{if } w_2\ge h_{O2}(e),\\ \hat{p}_B^O, & \text{if } \underline{h}_O<w_2<h_{O2}(e).\end{cases}$$

Based on the above discussions, the brand-name firm chooses p_B^* to maximize her profit by making a comparison between $\pi_B^O(s=0)$ and $\pi_B^O(s=1)$ in overleaping region.

Note that $h_{O2}(e) < h_{O1}(e)$. Then, the optimal retail price of the brand-name firm is

$$p_B^{O*} = \begin{cases} p_B^{O0}, & \text{if } w_2 \geq h_{O1}(e), \text{[note that } m_2 = 0] \\ x_{high}, & \text{if } h_{O2}(e) \leq w_2 < h_{O1}(e), \text{[note that } m_2 = 0] \\ \hat{p}_B^O, & \text{if } (\underline{h}_O)^+ < w_2 < h_{O2}(e), \text{[note that } m_2 > 0] \end{cases}$$

where $x_{high} = \frac{\sqrt{\frac{4\beta(1-\gamma)(1-\gamma-\beta)e}{\alpha}}+((1-\gamma)k_2+\beta(w_2-k_2))}{\beta}$, $p_B^{O0} = \frac{(1+\alpha)(1-\gamma+w_2)+t}{2(1+\alpha)}$, $\hat{p}_B^O = \frac{2(1-\gamma-\beta)(1-\gamma)+\alpha(2(1-\gamma-\beta)(1-\gamma+t)+(1-\gamma-\beta)k_2)+(2(1-\gamma-\beta)+2\alpha(1-\gamma))w_2}{4(1-\gamma-\beta)+\alpha(4(1-\gamma-\beta)+2\beta)}$.

Thus, the condition to prevent counterfeiting is $w_2 \geq w_2^{O,endog}$, where $w_2^{O,endog} = h_{O2}(e)$. That is to say, under Strategy O, $s^* = 0$ if $\hat{p}_B^O \leq \frac{\sqrt{\frac{4\beta(1-\gamma)(1-\gamma-\beta)e}{\alpha}}+((1-\gamma)k_2+\beta(w_2-k_2))}{\beta}$, where $\hat{p}_B^O$ is defined in Equation (17).

Thus, we have the results. ■

B.10.2 Proof of Proposition EC.1.

Part 1: With Strategy D, the overseas supplier is prevented from counterfeiting if $w_2^{D,endog} = h_{D2}(w_1, e)$. Then, we compare the threshold with the counterfeiting prevention condition under our base case. Recall that under our base case, the counterfeiting is prevented if $w_2 \geq w_2^{(0)}$, where $w_2^{(0)} = k_2 + \frac{\alpha(p_2-k_2)(\frac{p_B-p_2}{1-\gamma-\beta}-\frac{p_2}{\beta})-e}{\alpha\left(\frac{p_B-p_2}{1-\gamma-\beta}-\frac{p_B}{1-\gamma}\right)}$.

By making a comparison between the thresholds, that is, $w_2^{D,endog}$ and $w_2^{(0)}$, we obtain,

$$w_2^{D,endog} < w_2^{(0)},$$

$$\Rightarrow \frac{(2(1-\gamma)(1-\gamma-\beta)(1+w_1)+\alpha(2(1-\gamma)+k_2)(1-\gamma-\beta))\beta-\left(\sqrt{\frac{4\beta(1-\gamma)(1-\gamma-\beta)e}{\alpha}}+(1-\gamma-\beta)k_2\right)(4(1-\gamma)(1-\gamma-\beta)+\alpha(4(1-\gamma-\beta)+2\beta))}{2\beta(2(1-\gamma)+\alpha)(1-\gamma-\beta)} < k_2 + \frac{\alpha(p_2-k_2)(\frac{p_B-p_2}{1-\gamma-\beta}-\frac{p_2}{\beta})-e}{\alpha\left(\frac{p_B-p_2}{1-\gamma-\beta}-\frac{p_B}{1-\gamma}\right)},$$

$$\Rightarrow \frac{e}{\alpha\left(\frac{p_B-p_2}{1-\gamma-\beta}-\frac{p_B}{1-\gamma}\right)} - \frac{\left(\sqrt{\frac{4\beta(1-\gamma)(1-\gamma-\beta)e}{\alpha}}\right)(4(1-\gamma)(1-\gamma-\beta)+\alpha(4(1-\gamma-\beta)+2\beta))}{2\beta(2(1-\gamma)+\alpha)(1-\gamma-\beta)} + \frac{(2(1-\gamma)(1+w_1)+\alpha(2(1-\gamma)+k_2))\beta-k_2(4(1-\gamma)(1-\gamma-\beta)+\alpha(4(1-\gamma-\beta)+2\beta))}{2\beta(2(1-\gamma)+\alpha)(1-\gamma-\beta)}$$
$$< k_2 + \frac{\alpha(p_2-k_2)(\frac{p_B-p_2}{1-\gamma-\beta}-\frac{p_2}{\beta})}{\alpha\left(\frac{p_B-p_2}{1-\gamma-\beta}-\frac{p_B}{1-\gamma}\right)}.$$

We define $e_1^{D,endog}$ and $e_2^{D,endog}$ as two solutions of e satisfying $w_2^{D,endog} = w_2^{(0)}$, where $e_1^{D,endog} \leq e_2^{D,endog}$. Note that if $\frac{(2(1-\gamma)(1+w_1)+\alpha(2(1-\gamma)+k_2))\beta-k_2(4(1-\gamma)(1-\gamma-\beta)+\alpha(4(1-\gamma-\beta)+2\beta))}{2\beta(2(1-\gamma)+\alpha)(1-\gamma-\beta)} - k_2 - \frac{\alpha(p_2-k_2)(\frac{p_B-p_2}{1-\gamma-\beta}-\frac{p_2}{\beta})}{\alpha\left(\frac{p_B-p_2}{1-\gamma-\beta}-\frac{p_B}{1-\gamma}\right)} < 0$, then, the two solutions for $w_2^{D,endog} = w_2^{(0)}$ must exist and satisfy $e_1^{D,endog} < 0$ and $e_2^{D,endog} > 0$.

Thus, from $w_2^{D,endog} < w_2^{(0)}$, we have, $(e_1^{D,endog})^+ < e < (e_2^{D,endog})^+$.

Part 2: With Strategy O, the overseas supplier is prevented from counterfeiting if $w_2^{O,endog} = h_{O2}(e)$. Then, we compare the threshold with the counterfeiting prevention condition under our base case. Recall that under our base case, the counterfeiting is prevented if $w_2 \geq w_2^{(0)}$, where $w_2^{(0)} = k_2 + \frac{\alpha(p_2-k_2)(\frac{p_B-p_2}{1-\gamma-\beta}-\frac{p_2}{\beta})-e}{\alpha\left(\frac{p_B-p_2}{1-\gamma-\beta}-\frac{p_B}{1-\gamma}\right)}$).

By making a comparison between the thresholds, that is, $w_2^{O,endog}$ and $w_2^{(0)}$, we obtain,

$$w_2^{O,endog} < w_2^{(0)},$$

$$\Rightarrow \frac{(2(1-\gamma-\beta)(1-\gamma+t)+\alpha(2(1-\gamma-\beta)(1-\gamma)+(1-\gamma-\beta)k_2))\beta-\left(\sqrt{\frac{4\beta(1-\gamma)(1-\gamma-\beta)e}{\alpha}}+(1-\gamma-\beta)k_2\right)(4(1-\gamma-\beta)+\alpha(4(1-\gamma-\beta)+2\beta))}{2\beta(1+\alpha)(1-\gamma-\beta)} < k_2 + \frac{\alpha(p_2-k_2)(\frac{p_B-p_2}{1-\gamma-\beta}-\frac{p_2}{\beta})-e}{\alpha\left(\frac{p_B-p_2}{1-\gamma-\beta}-\frac{p_B}{1-\gamma}\right)},$$

$$\Rightarrow \frac{e}{\alpha\left(\frac{p_B-p_2}{1-\gamma-\beta}-\frac{p_B}{1-\gamma}\right)} - \frac{\left(\sqrt{\frac{4\beta(1-\gamma)(1-\gamma-\beta)e}{\alpha}}\right)(4(1-\gamma-\beta)+\alpha(4(1-\gamma-\beta)+2\beta))}{2\beta(1+\alpha)(1-\gamma-\beta)} + \frac{(2(1-\gamma+t)+\alpha(2(1-\gamma)+k_2))\beta-k_2(4(1-\gamma-\beta)+\alpha(4(1-\gamma-\beta)+2\beta))}{2\beta(1+\alpha)}$$
$$< k_2 + \frac{\alpha(p_2-k_2)(\frac{p_B-p_2}{1-\gamma-\beta}-\frac{p_2}{\beta})}{\alpha\left(\frac{p_B-p_2}{1-\gamma-\beta}-\frac{p_B}{1-\gamma}\right)}.$$

We define $e_1^{O,endog}$ and $e_2^{O,endog}$ as two real-value solutions of e satisfying $w_2^{O,endog} = w_2^{(0)}$, where $e_1^{O,endog} \leq e_2^{O,endog}$. Note that if $\frac{(2(1-\gamma+t)+\alpha(2(1-\gamma)+k_2))\beta - k_2(4(1-\gamma-\beta)+\alpha(4(1-\gamma-\beta)+2\beta))}{2\beta(1+\alpha)} - k_2 - \frac{\alpha(p_2-k_2)(\frac{p_B-p_2}{1-\gamma-\beta} - \frac{p_2}{\beta})}{\alpha\left(\frac{p_B-p_2}{1-\gamma-\beta} - \frac{p_B}{1-\gamma}\right)} < 0$, then, the two solutions for $w_2^{O,endog} = w_2^{(0)}$ must exist and satisfy $e_1^{O,endog} < 0$ and $e_2^{O,endog} > 0$.

Thus, from $w_2^{O,endog} < w_2^{(0)}$, we have, $(e_1^{O,endog})^+ < e < (e_2^{O,endog})^+$.

To summarize, based on the discussions under strategies D and O, we have the following sufficient conditions:

(i) Under Strategy D, if $(e_1^{D,endog})^+ < e < (e_2^{D,endog})^+$, then, $w_2^{D,endog} < w_2^{(0)}$;

(ii) under Strategy O, if $(e_1^{O,endog})^+ < e < (e_2^{O,endog})^+$, then, $w_2^{O,endog} < w_2^{(0)}$;

where

$$\begin{aligned} & w_2^{D,endog} = \frac{(2(1-\gamma)(1-\gamma-\beta)(1+w_1)+\alpha(2(1-\gamma)+k_2)(1-\gamma-\beta))\beta - \left(\sqrt{\frac{4\beta(1-\gamma)(1-\gamma-\beta)e}{\alpha}} + (1-\gamma-\beta)k_2\right)(4(1-\gamma)(1-\gamma-\beta)+\alpha(4(1-\gamma-\beta)+2\beta))}{2\beta(2(1-\gamma)+\alpha)(1-\gamma-\beta)}, \\ & e_1^{D,endog} \text{ and } e_2^{D,endog} \text{ are the solutions of } e \text{ satisfying } w_2^{D,endog} = w_2^{(0)}, \text{ and } e_1^{D,endog} \leq e_2^{D,endog}; \\ & w_2^{O,endog} = \frac{(2(1-\gamma-\beta)(1-\gamma+t)+\alpha(2(1-\gamma-\beta)(1-\gamma)+(1-\gamma-\beta)k_2))\beta - \left(\sqrt{\frac{4\beta(1-\gamma)(1-\gamma-\beta)e}{\alpha}} + (1-\gamma-\beta)k_2\right)(4(1-\gamma-\beta)+\alpha(4(1-\gamma-\beta)+2\beta))}{2\beta(1+\alpha)(1-\gamma-\beta)}, \\ & e_1^{O,endog} \text{ and } e_2^{O,endog} \text{ are the solutions of } e \text{ satisfying } w_2^{O,endog} = w_2^{(0)}, \text{ and } e_1^{O,endog} \leq e_2^{O,endog}. \end{aligned} \tag{18}$$

■